Higher-order squeezing in a boson-coupled two-mode system

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We consider a model for nondegenerate cavity fields interacting through an intervening boson field. The quantum correlations introduced in this manner are manifest through their higher-order correlation functions, where a type of squeezed state is identified.

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I. INTRODUCTION

Quantum mechanics dictates that there is an inherent uncertainty in the simultaneous measurement of noncommuting observables. A single, cavity electromagnetic field mode has complementary quadratures (or fields) that do not commute and therefore their uncertainties are constrained by a Heisenberg uncertainty relation. Amplifiers act to magnify the quantum fields to a macroscopic level, but at the same time they inject additional noise into the fields. Thus they cannot be used to determine the state of the microscopic fields with precision exceeding that dictated by Heisenberg's result.

Squeezed state generation of electromagnetic fields provides a means of reducing uncertainty in one electric field quadrature at the expense of a larger uncertainty in its conjugate partner [1,2]. It is one realization of nonclassical states (ideally, minimum uncertainty states) that has received wide attention. Ordinarily, in single or multimode squeezing, the fluctuations of linear combinations of the field operators are considered [1]; however, Hillery [3] introduced quadratic combinations of the field operators as a type of higher-order squeezing [4]. The higher-order combinations are examined to help elucidate the nature of the phase space occupied by the squeezed states.

We consider a two-mode model originally developed to study stimulated Raman scattering [5,6]. In a cavity environment the model has features of amplifiers [7,8] in which quantum states are rendered macroscopic and therefore classically measurable, while at the same time the fields retain some quantum mechanical correlations. The introduction of both Stokes and anti-Stokes fields indirectly coupled through a boson field, whose origin stems either from phonons or weak atomic excitation of the medium, is an interesting two-mode quantum system. It differs from several previous two-mode systems (see, e.g., [1,8,9]) because the two modes are coupled through the intermediate field that acts like a reservoir.

In this paper we provide an analysis of two modes coupled via a reservoir of bosons. The Hamiltonian and its solution within the quantum Markovian approximation is sketched in the following section; details are relegated to the Appendix. The results are examined in Sec. III; the emphasis is placed on higher-order squeezing found in the fields because squeezing of the linear combinations of the operators is not present in this model. The type of higher-order squeezing found is in the variance of the variables defined by Hillery, so-called sum or difference squeezing variables; they are used to infer that quantum correlations exist between the electromagnetic fields and the boson fields. Finally, the results are summarized and discussed in Sec IV.

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II. MODEL

We investigate the model Hamiltonian for a stimulated Raman scattering process with undepleted laser field e_L, which can be treated classically. The fields in the interaction are the Stokes field, subscript S, and anti-Stokes field, subscript A, that are coupled through a boson field with multiple modes [5,6]:

$$\mathcal{H} = \hbar\omega_S a_S^\dagger a_S + \hbar\omega_A a_A^\dagger a_A + \sum_l \hbar\omega_{Bl} a_{Bl}^\dagger a_{Bl} - \sum_l (\hbar g_l e_L a_S^\dagger a_{Bl}^\dagger + \hbar\kappa^* e_L a_A^\dagger a_{Bl} + \text{H.c.}). \quad (1)$$

This model has a bath of bosons, e.g., phonons; in other words, the excitations have energies spread over a range of frequencies. In this model the bosons are responsible for coupling the electromagnetic fields and for introducing damping as well. The Heisenberg equations of motion are given by

$$\begin{aligned}\frac{da_S}{dt} &= -i\omega_S a_S + i\sum_l g_l e_L a_{Bl}^\dagger,\\ \frac{da_A}{dt} &= -i\omega_A a_A + i\sum_l \kappa_l^* e_L a_{Bl},\\ \frac{da_{Bl}}{dt} &= -i\omega_{Bl} a_{Bl} + i g_l e_L a_S^\dagger + i\kappa_l e_L^* a_A. \quad (2)\end{aligned}$$

The equations can be simplified by introducing the Markovian approximation [5,6] and using the interaction picture:

$$a_j = A_j \exp(-i\omega_j t - i\Delta t), \;\; j = S, A, \tag{3}$$

where the detuning parameter is

$$\Delta = \omega_L - (\omega_S + \omega_A)/2. \tag{4}$$

The parameters introduced from the Markovian approximation with the boson excitation frequency $\omega_B = \omega_L - \omega_S$ are

$$\begin{aligned} \gamma_S &= 2\pi|g(\omega_B)|^2\rho(\omega_B), \\ \gamma_A &= 2\pi|\kappa(\omega_B)|^2\rho(\omega_B), \\ \gamma_{SA} &= 2\pi g(\omega_B)\kappa^*(\omega_B)\rho(\omega_B) \end{aligned} \tag{5}$$

and the laser field is

$$E_L = e_L \exp(i\omega_L t). \tag{6}$$

The reduced equations of motion are written as

$$\begin{aligned} \frac{dA_S}{dt} &= \left(\frac{1}{2}\gamma_S|E_L|^2 + i\Delta\right) A_S + \frac{1}{2}\gamma_{SA}E_L^2 A_A^\dagger + E_L F_S, \\ \frac{dA_A^\dagger}{dt} &= \left(-\frac{1}{2}\gamma_A|E_L|^2 - i\Delta\right) A_A^\dagger \\ &\quad -\frac{1}{2}\gamma_{SA}^* E_L^{*2} A_S + E_L^* F_A^\dagger. \end{aligned} \tag{7}$$

The Langevin forces due to the boson field are

$$\begin{aligned} F_S &= i\sum_l g_l a_{Bl}^\dagger(0)\exp\{i[(\omega_S - \omega_A)/2 + \omega_{Bl}]t\}, \\ F_A &= i\sum_l \kappa_l^* a_{Bl}(0)\exp\{-i[(\omega_S - \omega_A)/2 + \omega_{Bl}]t\}; \end{aligned} \tag{8}$$

they satisfy the quantum fluctuation-dissipation theorem.

The solution of the equations of motion is straightforward and some details are provided in the Appendix. The calculation of the moments is done by determining the characteristic function of the operators in normal-ordered form. The usual definition of the two-mode operators is a linear combination of the creation and annihilation operators. However, we find that the model discussed here does not yield the usual squeezed state correlations between the Stokes and anti-Stokes fields. The coupling through the reservoir is also expected to degrade the coherence developed between the Stokes and the anti-Stokes fields during evolution. It is therefore surprising that the fields do display quantum coherence in the higher-order correlations between the fields. To show this we adopt of the definitions of sum squeezing and difference squeezing used by Hillery [3].

Sum and difference squeezing operators are quadratic in the fields and it represents one particular example of higher-order squeezing; experiments on higher-order squeezing and quantum correlations were examined in a series of papers by Hong and Mandel [4]. The particular variations we examine in this paper can be measured by mixing the fields together in a second-order nonlinear material by either sum-frequency generation or by difference-frequency generation [3] and measuring the variance of the new field by techniques already developed for normal squeezing. For example, when the Stokes and anti-Stokes fields are mixed in a $\chi^{(2)}$ material, the output field operator is related to the product of the two by $A_0 = \chi^{(2)}A_S A_A$. The new field A_0 is then measured by the usual homodyning methods. An extension of this transformation concept can also be used in third order, i.e., $\chi^{(3)}$ nonlinear materials, as well by applying a classical pump field E_p. By nondegenerate four-wave mixing, the output field is given by $A_0 = \chi^{(3)}E_p^* A_S A_A$.

A. Sum squeezing

For sum squeezing we define the operators

$$\begin{aligned} V_1 &= \frac{1}{2}(A_S^\dagger A_A^\dagger + A_S A_A), \\ V_2 &= \frac{i}{2}(A_S^\dagger A_A^\dagger - A_S A_A). \end{aligned} \tag{9}$$

The moments of these operators are calculated by using the characteristic function discussed in the Appendix. For instance, the first and second moments of V_1 are

$$\langle V_1\rangle = \frac{1}{2}\left(\frac{\partial^2 C_N}{\partial\beta_S\partial\beta_A} + \frac{\partial^2 C_N}{\partial(-\beta_S^*)\partial(-\beta_A^*)}\right)\Bigg|_{\{\beta_i=0\}} \tag{10}$$

and

$$\begin{aligned} \left\langle V_1^2\right\rangle = \frac{1}{4}\Bigg(& \frac{\partial^4 C_N}{(\partial\beta_S)^2(\partial\beta_A)^2} + \frac{\partial^4 C_N}{(\partial(-\beta_S^*))^2(\partial(-\beta_A^*))^2} \\ & +2\frac{\partial^4 C_N}{\partial\beta_S\partial(-\beta_S^*)\partial\beta_A\partial(-\beta_A^*)} \\ & +1 + \frac{\partial^2 C_N}{\partial\beta_S\partial(-\beta_S^*)} + \frac{\partial^2 C_N}{\partial\beta_A\partial(-\beta_A^*)}\Bigg)\Bigg|_{\{\beta_i=0\}}. \end{aligned} \tag{11}$$

The standard deviations ΔV_i are constructed from both operators; their product satisfy the Heisenberg inequality

$$\Delta V_1 \Delta V_2 \geq \frac{1}{4}\langle N_A + N_S + 1\rangle. \tag{12}$$

The operators are in a quantum state, said to be sum squeezed in the V_1 direction when the variance of V_1 satisfies the inequality

$$(\Delta V_1)^2 < \frac{1}{4}\langle N_A + N_S + 1\rangle. \tag{13}$$

To determine whether the dynamics produces a higher-order squeezed state, we define the shifted variance

$$\delta V_1^2 = (\Delta V_1)^2 - \frac{1}{4}\langle N_A + N_S + 1\rangle, \tag{14}$$

which is negative in the region of the quantum state.

B. Difference squeezing

For the definition of difference squeezing, we define

$$W_1 = \frac{1}{2}(A_S A_A^\dagger + A_S^\dagger A_A),$$
$$W_2 = \frac{i}{2}(A_S A_A^\dagger - A_S^\dagger A_A). \quad (15)$$

The state is difference squeezed in the W_1 operator when the variance of the operator satisfies the inequality $(\langle N_S\rangle > \langle N_A\rangle)$

$$(\Delta W_1)^2 < \frac{1}{4}\langle N_S - N_A\rangle . \quad (16)$$

The moments are calculated from the characteristic function, as discussed already in the preceding subsection. We also define a shifted variance of W_1 in analogy with Eq. (14)

$$\delta W_1^2 = (\Delta W_1)^2 - \frac{1}{4}\langle N_A - N_S\rangle \ , \quad (17)$$

which is negative when the state is squeezed along the W_1 direction.

III. RESULTS

The calculations of the preceding section have been examined for a variety of parameters. The model incorporates the damping of the Stokes photons and anti-Stokes photons and the magnitudes and phases of the initial fields and the coupling coefficients. Values of the variables are chosen to illustrate the phenomena.

The choice of initial state for the Stokes and anti-Stokes fields is dictated by experimental conditions. Two experimentally useful states are the coherent state and the chaotic state, but other situations, such as a Fock state or a squeezed vacuum state, could also be identified. The squeezed state is generally difficult to achieve as an initial state and, moreover, it is often the desired final state, so we do not consider it further here. We restrict our discussion to combinations of the first two cases and examine their quantum correlations.

There are several parameters occurring in the model and appearing in Sec. II. The dynamical parameters, i.e., those appearing in the evolution equations, have been previously defined. We note that the detuning was assumed to be small in our model and this parameter is set to zero. The initial states of the fields represent another set of important parameters, but the discussion of these is relegated to the Appendix. The boson field is considered to be in a chaotic state with an average number of excitations $\bar{n}_B$; when the Stokes and/or anti-Stokes fields are in a chaotic state, then their phases are randomized and their statistical properties are also represented by their average photon number $\bar{n}_S$ and $\bar{n}_A$, respectively. We do not discuss the Fock state and the squeezed initial state, although the results are easily derived using formulas in the Appendix. When the Stokes and anti-Stokes fields are in coherent states, in addition to the average photon number, the phase of the fields ϕ_S and ϕ_A is also needed.

The three-dimensional plot of Fig. 1 is a display of the shifted variance of the operator V_1 versus the phase $\phi = 2\phi_L + \psi_S - \psi_A$ and the interaction time t. The Stokes and anti-Stokes fields are both initially in a coherent state $n_S = n_A = 2$ and the reservoir is in the vacuum state $\bar{n}_V = 0$. The time has been scaled to the product $\gamma|E_L|^2$, where E_L is the laser field amplitude and in the results presented here we set $\gamma = \gamma_S = \gamma_A$, i.e., the damping constants are equal. The region of the surface with negative ordinate values corresponds to the case when light is V_1-sum squeezed and for large times squeezing occurs near the point $\phi = \pi/2$. The squeezing is more apparent in Fig. 2, which displays the shifted variance of V_1 for three different values of the phase. The phase value of $\phi = \pi/2$ continues to decrease as the interaction time increases.

As the average number of excitations is increased in the boson reservoir, the squeezing deteriorates. Figure 3 shows the effect of a small increase in the average; in this case $\phi = 0.4\pi$. The minimum squeezing rises until it becomes tangent to the zero value of the variance. For values of $\bar{n}_V$ above this point, no squeezing is found. Squeezing is found only for a range of interaction times when the phase is not precisely $\phi_L = \pi/4$. The range of values where squeezing can be expected is given by the contour plot in Fig. 4. The outer contour $\bar{n}_V = 0$ has the largest domain to find the squeezed state and the domain shrinks as the average number of excitations is increased.

The values of the reservoir variable $\bar{n}_V$ over which squeezing can be observed, even for a small time interval, depends upon the phase ϕ. Figure 5 is a plot of the $\bar{n}_V$ versus the phase ϕ and the summary phase of the initial coherent fields $\psi = \phi_S + \phi_A$. Near $\phi = \pi/2$, the range of values over which the squeezing occurs becomes narrower as the excitations in the bath increase. Away from this value, the amount of squeezing is severely restricted by the bath excitation and the phase ψ. Squeezing is found in the regime below the contour lines.

When both the Stokes and the anti-Stokes fields are initially in a chaotic state, the sum-squeezing variable

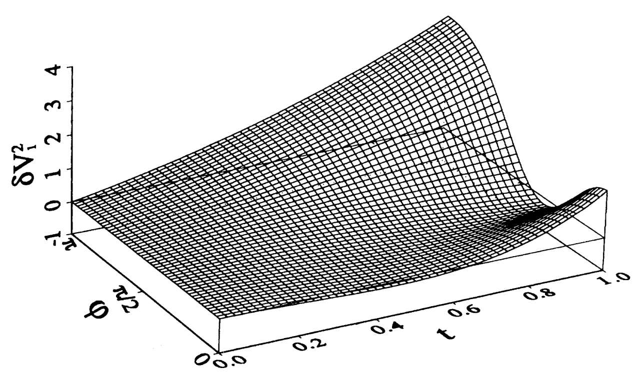

FIG. 1. Surface and contour plot of the sum squeezing shifted variance versus the phase $\phi = 2\phi_L + \psi_S - \psi_A$ and the interaction time for initially coherent Stokes and anti-Stokes fields. The zero value lines separate the regions of sum squeezing and excess fluctuations.

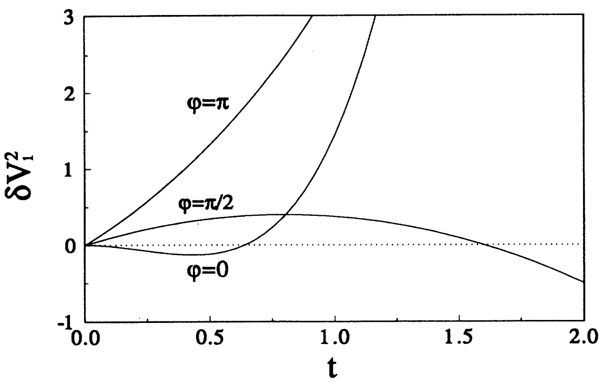

FIG. 2. Time slices of the surface plot in Fig. 1. The phase ϕ has the values 0, $\pi/2$, and π.

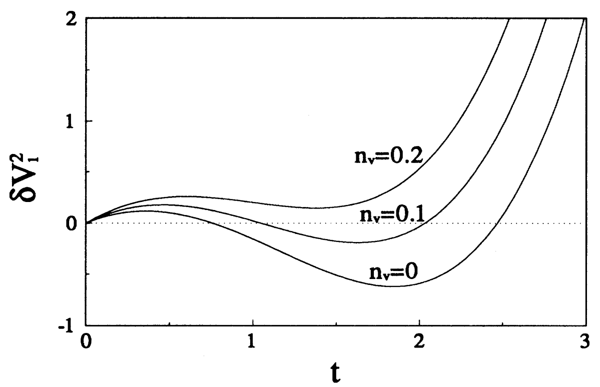

FIG. 3. Sum squeezing shifted variance versus time for three values of the average value of the boson field: $\bar{n}_v = 0, 0.1, 0.2$. The laser phase is $\phi_L = 0.2\pi$.

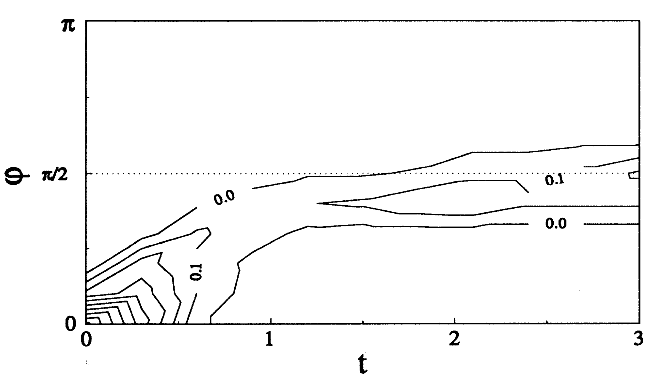

FIG. 4. Contour plot of the region of the interaction time versus phase ϕ space where squeezing is found. The numbers on the lines denote the values of $\bar{n}_V$.

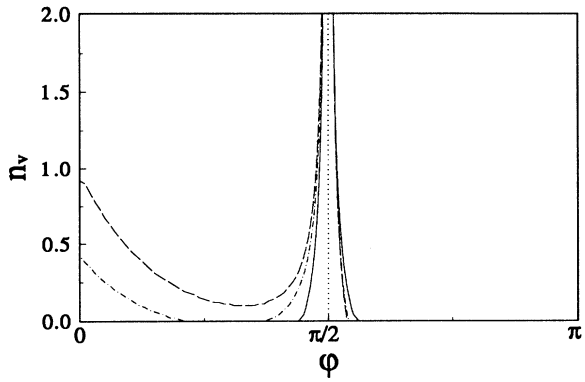

FIG. 5. Plot of the boundaries between the squeezed and the nonsqueezed states for various values of the average number of bosons versus the phase ϕ: the summary phase $\psi = 0$ is denoted by a dashed line, and $\psi = 0.1\pi$ by a dash-dotted line, and $\psi = \pi/2$ by a solid line.

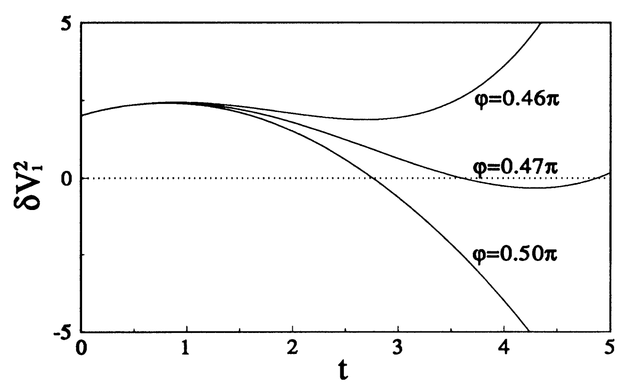

FIG. 6. Sum squeezing shifted variance versus time for initially chaotic Stokes and anti-Stokes fields.

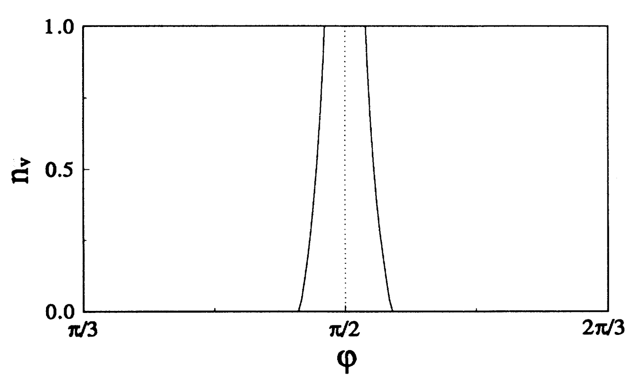

FIG. 7. Boundary between the squeezed and the nonsqueezed state regimes for initially chaotic Stokes and anti-Stokes fields.

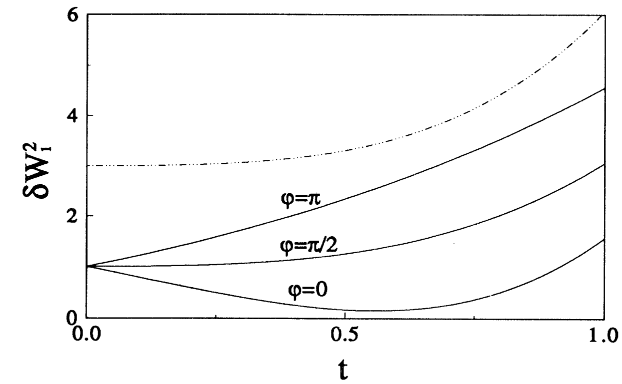

FIG. 8. Difference squeezing shifted variance for two separate cases: the initial Stokes and the anti-Stokes fields are coherent (solid lines) and the initial fields are chaotic (dash-dotted line).

V_1 still shows squeezing and the phase $\phi = \pi/2$ is very robust to the values of the initial state (Fig. 6). We note that the initial value of the shifted variance has been changed by the initial chaotic state of the variables. The range of phases over which squeezing is observed has been greatly reduced as well. Figure 7 is a plot similar to Fig. 5; the squeezed state region is found only close to the value $\phi = \pi/2$ and as the number of bath excitations increases, the range of phases over which squeezing is found decreases.

No squeezing was found for the variable W_1 with either coherent or chaotic initial states. The behavior of the shifted variance of W_1 is shown in Fig. 8 for initially coherent states and one value for initially chaotic states. In the coherent state cases, the variance decreases for a certain range of the phase ϕ, but does not become negative for any value of the phase. For chaotic initial states the variance is a monotonically increasing function of the interaction time.

IV. SUMMARY

In this paper we have examined a special model for the interaction between two modes in a cavity mediated by a boson reservoir field [5,6]. We find sum squeezing, a form of higher-order squeezing, over a range of interaction times and initial states. There are two salient features of our results: first, the intermediate field has a continuous spectrum of a reservoir, but still the two fields develop quantum mechanical correlations; second, the quantum nature of the correlations is not manifest through the usual first-order or even simple higher-order correlations among the operators but through special combinations of the field operators.

There are other models where the fields are mediated by either electronic or acoustic fields, e.g., a polariton or Brillouin scattering model [5,6,10]; these processes are analogous to the present model where the directly coupled fields are not detected in an experiment. In such cases experiments designed to measure higher-order correlations can reveal the underlying quantum correlations induced through the fields.

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APPENDIX

The linear equations of motion Eqs. (5) can be directly solved and the annihilation operators have the form

$$A_S(t) = u_S(t)a_S + v_S(t)a_A^\dagger + \sum_l w_{Sl}(t)a_{Bl}^\dagger,$$
$$A_A(t) = u_A(t)a_A + v_A(t)a_S^\dagger + \sum_l w_{Al}(t)a_{Bl}. \qquad \text{(A1)}$$

The operators on the right-hand side are with respect to the initial state. The normal characteristic function after reducing the intermediate reservoir in the dynamical equations is expressed as an average over an initial distribution of complex amplitudes $\{\xi_S, \xi_A\}$, which is the coherent state representation for the initial field operators appearing in Eq. (A1),

$$C_N(\beta_S,\beta_A,t) = \langle e^{-\{B_S(t)|\beta_S|^2 - B_A(t)|\beta_A|^2 + [D_{SA}(t)\beta_S^*\beta_A^* + \text{c.c.}] + [\beta_S\xi_S^*(t) + \beta_A\xi_A^*(t) - \text{c.c.}]\}}\rangle, \qquad \text{(A2)}$$

where we assume that $\Delta = 0$ in Eq. (7) and define

$$\xi_S(t) = u_S(t)\xi_S + v_S(t)\xi_A^*,$$
$$\xi_A(t) = u_A(t)\xi_A + v_A(t)\xi_S^*. \qquad \text{(A3)}$$

The angular brackets denote the average over the initial states of the Stokes and the anti-Stokes fields. The various cases are discussed in Sec. II. The coefficients in the above expressions are obtained from a long but straightforward solution of the Heisenberg equations of motion and the subsequent reduction of the boson modes in the normal characteristic function using disentangling theorems. The results are, setting $\Gamma = (\gamma_S - \gamma_A)|E_L|^2/2$,

$$u_S(t) = \frac{1}{\gamma_S - \gamma_A}(\gamma_S e^{\Gamma t} - \gamma_A),$$
$$u_A(t) = \frac{1}{\gamma_S - \gamma_A}(\gamma_S - \gamma_A e^{\Gamma t}),$$
$$v_S(t) = -v_A(t) = \frac{\sqrt{\gamma_S \gamma_A}}{\gamma_S - \gamma_A}(e^{\Gamma t} - 1)e^{i(2\phi_L + \psi_S - \psi_A)},$$
$$B_S(t) = \frac{1}{(\gamma_S - \gamma_A)^2}\left[\gamma_S^2(e^{2\Gamma t} - 1) + 2\gamma_S\gamma_A(1 - e^{\Gamma t})\right] + \frac{\gamma_A \bar{n}_V}{\gamma_S - \gamma_A}(e^{2\Gamma t} - 1),$$
$$B_A(t) = \frac{\gamma_S\gamma_A}{(\gamma_S - \gamma_A)^2}(e^{\Gamma t} - 1)^2 + \frac{\gamma_A \bar{n}_V}{\gamma_S - \gamma_A}(1 - e^{2\Gamma t}),$$
$$D_{SA}(t) = \frac{\sqrt{\gamma_S\gamma_A}}{\gamma_S - \gamma_A}\left(\frac{1}{\gamma_S - \gamma_A}(e^{\Gamma t} - 1)(\gamma_A - \gamma_S e^{\Gamma t}) + \bar{n}_V(e^{2\Gamma t} - 1)\right)e^{i(2\phi_L + \psi_S - \psi_A)}. \quad \text{(A4)}$$

The phases are defined by $E_L = |E_L|\exp(i\phi_L), g = |g|\exp(i\psi_S)$, and $\kappa = |\kappa|\exp(i\psi_A)$. Explicit forms for the functions $w_{Sl}(t)$ and $w_{Al}(t)$ are not required; their correlations satisfy the following relations, derived from conservation of the number of excitations in the Hamiltonian operator and commutation relations [7,8]:

$$|u_S(t)|^2 - |v_S(t)|^2 - \sum_l |w_{Sl}(t)|^2 = 1, \quad \text{(A5)}$$

$$|u_A(t)|^2 - |v_A(t)|^2 + \sum_l |w_{Al}(t)|^2 = 1, \quad \text{(A6)}$$

and

$$u_S(t)v_A(t) - v_S(t)u_A(t) - \sum_l w_{Sl}(t)w_{Al}(t) = 0. \quad \text{(A7)}$$

The case $\gamma_S = \gamma_A$ can be obtained from the above results by applying l'Hôspital's rule.

The calculation of the sum–squeezing variance is lengthy but straightforward. The normal characteristic function is applied to Eqs. (10) and (11). The result for the sum squeezing shifted variance of V_1 is

$$\delta V_1^2 = \frac{1}{4}\langle\{[(\xi_S(t)\xi_A(t)]^2 + 4D_{SA}(t)\xi_S(t)\xi_A(t) + 2(D_{SA})^2 + 2D_{SA}^*\xi_S(t)\xi_A(t) + \text{c.c.}\} + 2[|\xi_S(t)\xi_A(t)|^2 + B_S(t)|\xi_A(t)|^2 + B_A(t)|\xi_S(t)|^2 + |D_{SA}(t)|^2 + B_S(t)B_A(t)]\rangle - \frac{1}{4}\langle\xi_S(t)\xi_A(t) + D_{SA}(t) + \text{c.c.}\rangle^2 \quad \text{(A8)}$$

and for the difference squeezing variable W_1 we have

$$\delta W_1^2 = \frac{1}{4}\langle\{[\xi_S(t)\xi_A^*(t)]^2 + 2D_{SA}^*(t)\xi_S(t)\xi_A(t) + \text{c.c.}\} + 2[|\xi_S(t)\xi_A(t)|^2 + B_S(t)|\xi_A(t)|^2 + B_A(t)|\xi_S(t)|^2 + |D_{SA}(t)|^2 + B_S(t)B_A(t) + |\xi_A(t)|^2 + B_A(t)]\rangle - \frac{1}{4}\langle\xi_S(t)\xi_A(t)^* + \text{c.c.}\rangle^2. \quad \text{(A9)}$$

The averages are performed over the initial conditions. There are two cases considered in this paper: fields initially in a chaotic state or a coherent state. They are defined by the following expressions. The coherent state is an eigenstate of the annihilation operator. The displacement operator generating these states from the vacuum state is defined by

$$D(\alpha)|0\rangle = e^{(\alpha a^\dagger - \alpha^* a)}\,|0\rangle\,; \quad \text{(A10)}$$

the coefficient is $\alpha = \sqrt{n}\exp(i\phi)$. A chaotic system is characterized by averaging over a distributed set of states. The variables are Gaussian distributed and the average of the number operator is denoted by an overbar

$$\langle a^\dagger a\rangle = \bar{n}. \quad \text{(A11)}$$

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