Effect of a squeezed vacuum input on optical bistability

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We discuss the optical bistable behavior of a system of N two-level atoms pumped by a coherent input field and coupled to a squeezed vacuum field by treating the optical bistability of such a system as an input-output problem. We consider the equation of motion for atomic dipole moments, coupled to the cavity field and the squeezed vacuum, and the boundary condition connecting the input and output fields with the atomic lowering operator. A simple analytical expression for the influence of the squeezed vacuum input on the bistable behavior of the output field is derived by this method. The results indicate a strong influence of the squeezed vacuum input on optical bistability. The squeezed (stretched) vacuum input field tends to increase (decrease) the range of optical bistability.

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I. INTRODUCTION

Squeezed states are nonclassical states of light. Their significance is discussed extensively in the existing literature along with their generation, detection, and physical properties [1–5]. As indicated by Gardiner [6], the squeezed vacuum leads to two dipole decay constants: a small T_2 for the dipole component in the squeezed quadrature and a correspondingly larger T_2 for the dipole component in the stretched quadrature [7]. Because the squeezed vacuum also affects the decay rate of the atomic population, the output field from a cavity that contains a system of such two-level atoms will be affected by the squeezed vacuum.

Optical bistability with squeezed vacuum input has its own distinctive features. The tunneling time, for example, with squeezed vacuum input can be as long as about 2 s instead of 0.1 s in ordinary vacuum [8]. The resonance fluorescence spectrum under the influence of the squeezed vacuum is given by Carmichael, Lane, and Walls [9]. Intrinsic optical bistability under squeezed vacuum input is discussed by Singh *et al.* [10]. There is also bistable behavior in the system of an optical parametric oscillator coupled with two-level atoms where the squeezed field is not injected but produced by the system itself [11].

Galatola *et al.* discuss the possibility of inducing switching of an optical bistable system by varying the phase of the squeezed vacuum input [12]. They give the input-output relationship and focus on the variation of the output as a function of the relative phase between the squeezed vacuum and the coherent input.

In this paper we discuss the optical bistable behavior of the output field from the system of two-level atoms that is pumped simultaneously by a coherent input field and a squeezed vacuum field (see Fig. 1). Our system is similar to the one discussed by Galatola *et al.* [12]. In our traveling-wave cavity system the coherent driving field and the squeezed vacuum field are injected via different ports, rendering the scheme more feasible experimentally than that of Ref. [12]. We consider the equation of motion for the atomic variables and obtain the input-output relationship by boundary conditions. In so doing we can neglect the coupling between the coherent pumping field and the squeezed vacuum because it does not contribute to the equation of motion for the atomic variables. We stress the importance of the input-output relationship for such a system and, with its help discuss the effect of the squeezed vacuum on the range of optical bistability. We also discuss its effect on the bistable states. Our results indicate that the atomic system exhibits phase-sensitive properties in its optical bistable behavior with squeezed vacuum input. Relative to the coherent input field, as shown in Fig. 2, the squeezed quadrature of the squeezed vacuum input field corresponds to amplitude squeezing and the stretching quadrature to phase squeezing. The output behavior depends strongly on the squeezing or the stretching quadrature fed in. The reason is that the existence of the squeezed vacuum enlarges the decay constant of atomic population and the transverse relaxation rate for the dipole component in the squeezing quadrature but reduces the transverse relaxation rate for

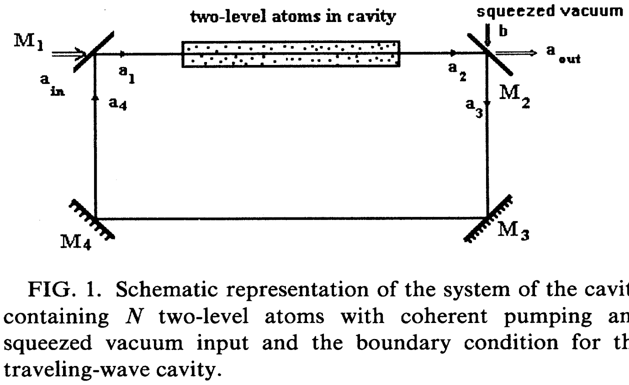

FIG. 1. Schematic representation of the system of the cavity containing N two-level atoms with coherent pumping and squeezed vacuum input and the boundary condition for the traveling-wave cavity.

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the dipole component in the stretching quadrature. Therefore, it needs higher pump field to saturate the system of two-level atoms when the dipole component S_x is in the squeezing quadrature and the system will lose optical bistability when the dipole component S_x is in stretching quadrature. This effect provides a method to detect the squeezing of the field as well as a possible broad use of optical bistability in optical information processing.

The paper is organized as follows. In Sec. II the model of a system of N two-level atoms pumped by a coherent input field and coupled to a squeezed vacuum field is discussed and the Hamiltonian and the master equation are given. In Sec. III the appropriate boundary condition for the case of traveling-wave cavity is presented. In Sec. IV we discuss the atomic dipole moments for such a sytem in steady state. In Sec. V the output field is derived by using the appropriate boundary condition and the results are given. In Sec. VI we carry out a linear stability analysis of the input-output relationship. In Sec. VII the optical bistable behavior of fluorescent light is also discussed. In Sec. VIII we briefly discuss and summarize the main results of the paper.

II. MODEL

Consider the system of N two-level atoms coupled to a laser field and a squeezed vacuum input. The Hamiltonian is

$$H=H_0+H' , \tag{2.1}$$

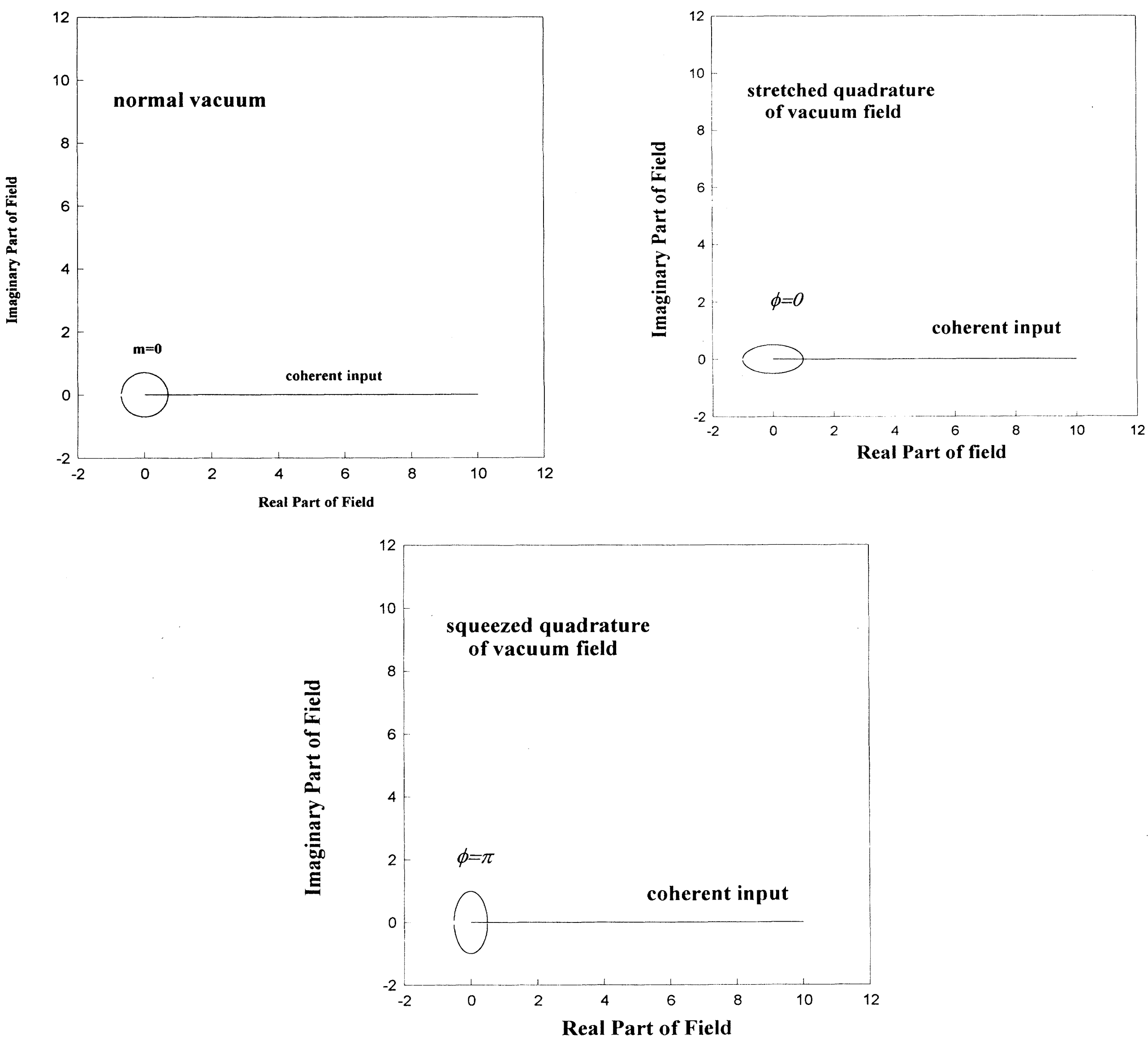

FIG. 2. Comparison of normal and squeezed vacuum fields: (a) normal vacuum field exhibiting circular variance; (b) squeezed vacuum field, corresponding to $\varphi=\pi$, with the major axis of the variance ellipse perpendicular to the direction of amplitude; (c) stretched vacuum field, corresponding to $\varphi=0$ with the minor axis of the variance ellipse along the direction of amplitude.

where

$$H_0=\frac{\hbar\delta}{2}\sum_{i=1}^{N}\sigma_i^z+\hbar\int d\omega\,\omega b^\dagger(\omega)b(\omega) \tag{2.2}$$

and

$$H'=\hbar\int d\omega\, g(\omega)b^\dagger(\omega)\sum_{i=i}^{N}\sigma_i^- e^{-i\omega_0 t}e^{-ik_z z_i}+\frac{\hbar\Omega^*}{2}\sum_{i=1}^{N}\sigma_i^- e^{-ik_z z_i}+\text{H.c.}\ , \tag{2.3}$$

where b and $b^\dagger$ are annihilation and creation operators of the squeezed vacuum field, σ_i^- and σ_i^+ are atomic lowering and raising operators for the ith atom, k_z is the magnitude of the wave vector, z_i is the position of the ith atom, and Ω is the Rabi frequency characterizing the coupling between the atom and the cavity field. Microscopically $\Omega=2G\langle a_{\text{cav}}\rangle$, where G is the coupling between the atom and the quantized cavity field. Macroscopically $\Omega=\mu E_{\text{cav}}/\hbar$, where E_{cav} is the classical cavity field and μ is the classical dipole moment. Furthermore, g is the coupling between the squeezed vacuum and atoms and δ is the detuning $\delta=\omega_a-\omega_0$ between the transition frequency ω_a of the atom and the frequency of the coherent field ω_0. The properties of the squeezed vacuum field are described by the expressions

$$\langle b^\dagger(t)b(t')\rangle=n\delta(t-t')\ , \tag{2.4}$$

$$\langle b(t)b(t')\rangle=m\delta(t-t')\ , \tag{2.5}$$

where $m=|m|e^{i\varphi}$ is a complex number and $\varphi/2$ is the phase angle of the amplitude quadrature with respect to classical pumping field. The physical meaning of n and m are given in Fig. 2. Figure 2(a) shows the circular variance of a normal vacuum input field ($m=0$). In Fig. 2(b) we show a vacuum field squeezed in the amplitude quadrature corresponding to $\varphi=\pi$ (squeezing quadrature) and the variance ellipse in the direction of amplitude is illustrated. In Fig. 2(c) we show a vacuum field squeezed in phase quadrature corresponding to $\varphi=0$ (stretching quadrature) and the variance ellipse perpendicular to the direction of the amplitude is illustrated.

We discuss the problem from the viewpoint of input-output by considering the equation of motion for atomic variables and connecting the input and output by boundary conditions. The master equation for zero detuning is

$$\frac{d\rho}{dt}=\frac{i\Omega^*}{2}(\rho S^- - S^-\rho)+\frac{i\Omega}{2}(\rho S^+ - S^+\rho)+\Lambda_a\rho+L\rho\ , \tag{2.6}$$

where

$$\Lambda_a=(\gamma_\perp-\gamma_\parallel)\sum_{i=1}^{N}\tfrac{1}{4}([\sigma_i^z,\rho\sigma_i^z]+[\sigma_i^z\rho,\sigma_i^z]) \tag{2.7}$$

describes the collisional decay of the atoms. The last term in Eq. (2.6) describes radiative decay in the presence of a squeezed vacuum. Its explicit form is given by

$$\begin{aligned}L\rho=&\frac{\gamma_\parallel(n+1)}{2}(2S^-\rho S^+ - S^+S^-\rho-\rho S^+S^-)\\&+\frac{\gamma_\parallel n}{2}(2S^+\rho S^- - S^-S^+\rho-\rho S^-S^+)\\&-\frac{\gamma_\parallel m}{2}(2S^+\rho S^+ - S^+S^+\rho-\rho S^+S^+)\\&-\frac{\gamma_\parallel m^*}{2}(2S^-\rho S^- - S^-S^-\rho-\rho S^-S^-)\end{aligned} \tag{2.8}$$

and

$$S^-=\sum_{i=1}^{N}\sigma_i^-\exp(-ik_z z_i),\quad S_z=\sum_{i=1}^{N}\sigma_z^i,\quad S^+=(S^-)^*\ . \tag{2.9}$$

Here S^-, S^+, and S_z are collective dipole operators, $\gamma_\parallel$ and $\gamma_\perp$ are longitudinal and transverse relaxation rates, respectively, $\gamma=\gamma_\parallel=2\pi g^2(\omega_0)$, and n and m characterize the squeezed vacuum fluctuations as specified by Eqs. (2.4) and (2.5).

III. BOUNDARY CONDITION

The boundary condition to connect the pump field and the atomic lowering operator is discussed by Gardiner and co-workers [6,14,15] in input-output problems for the case of a standing-wave cavity. Here we consider a traveling-wave cavity as an example. In Fig. 1 a traveling-wave cavity with a classical input field and squeezed vacuum is shown. The classical pumping field a_{in} is injected at mirror M_1 and the squeezed vacuum field is injected at mirror M_2. The output field a_{out} is at mirror M_2. The bistable absorber consisting of a collection of two-level atoms is placed between M_1 and M_2. We have the following relations at various locations in the cavity:

$$a_1=\tilde{t}a_{\text{in}}+\tilde{r}a_4 \tag{3.1}$$

at mirror M_1,

$$a_2=a_1+\frac{iGL}{c}S^- \tag{3.2}$$

from mirror M_1 to mirror M_2, where G is the coupling between the atom and the field and L is the cavity length, so that L/c is the interaction time,

$$a_{\text{out}}=\tilde{t}a_2+\tilde{r}b \tag{3.3}$$

for the output at M_2,

$$a_3=\tilde{r}a_2+\tilde{t}b \tag{3.4}$$

for the cavity field at mirror M_2, and

$$a_4=a_3e^{i\varphi_0} \tag{3.5}$$

from M_2 through M_3 and M_4 to M_1, where φ_0 is the phase shift from M_2 to M_1. It should be noted that φ_0 depends on the cavity detuning.

For perfect resonance and steady state, we have the boundary condition for our present case

$$a_{\rm out}=a_{\rm in}+\frac{iGL}{\tilde{t}c}S^{-}+2\tilde{r}b\ . \tag{3.6}$$

Note that when $\langle b\rangle=0$ and $\langle a_{\rm out}\rangle=\tilde{t}\langle a_{\rm cav}\rangle$ for steady state, then we reobtain the relationship between the input field and the intracavity field first given by Lugiato [16,17],

$$\langle a\rangle-\alpha_0=\frac{iG}{k}\langle S^{-}\rangle\ , \tag{3.7}$$

where G represents the coupling between the cavity field and the atom. $\alpha_0=(V/8\pi\hbar\omega_0)^{1/2}(E_i/\sqrt{T})$ is the scaled injected coherent field, E_i being its classical amplitude, and $\langle a\rangle=(V/8\pi\hbar\omega_0)^{1/2}E_{\rm cav}=(V/8\pi\hbar\omega_0)^{1/2}(E_{\rm out}/\sqrt{T})$. $T=\tilde{t}^2$ is the transmissivity of the cavity, k is the cavity damping constant, and V is the volume of the cavity.

IV. ATOMIC DIPOLE MOMENT

The source term in Eqs. (3.6) and (3.7) is the atomic dipole S^{-}. Next we will derive an expression for its steady-state value.

From the master equation (2.6) we can derive the relations

$$\frac{d\langle S_z\rangle}{dt}=-\gamma_{\parallel}(2n+1)\langle S_z\rangle-\gamma_{\parallel}N-i\Omega\langle S^{+}\rangle+i\Omega^{*}\langle S^{-}\rangle\ , \tag{4.1}$$

$$\frac{d\langle S^{-}\rangle}{dt}=-\gamma_{\perp}(2n\beta+1)\langle S^{-}\rangle-\gamma_{\parallel}m\langle S^{+}\rangle+\frac{i\Omega}{2}\langle S_z\rangle\ , \tag{4.2}$$

and

$$\frac{d\langle S^{+}\rangle}{dt}=-\gamma_{\perp}(2n\beta+1)\langle S^{+}\rangle-\gamma_{\parallel}m^{*}\langle S^{-}\rangle-\frac{i\Omega^{*}}{2}\langle S_z\rangle\ , \tag{4.3}$$

where we let $\gamma=\gamma_{\parallel}$, $\gamma_{\perp}=1/T_2$ is the atomic dipole decay constant [13], N is the number of atoms in the cavity, and $\beta=\gamma_{\parallel}/2\gamma_{\perp}$. In the steady state we obtain from (4.1)–(4.3)

$$\langle S_z\rangle+N'+ip_1\langle S^{+}\rangle-ip_1^{*}\langle S^{-}\rangle=0\ , \tag{4.4}$$

$$\langle S^{-}\rangle-\alpha\langle S^{+}\rangle-\frac{ip_2}{2}\langle S_z\rangle=0\ , \tag{4.5}$$

$$\langle S^{+}\rangle-\alpha^{*}\langle S^{-}\rangle+\frac{ip_2^{*}}{2}\langle S_z\rangle=0\ . \tag{4.6}$$

In the above expressions $\alpha=-2m\beta/(2n\beta+1)$ is a measure of the squeezing and $N'=N/(2n+1)$. Furthermore, we introduced

$$p_1=\frac{\Omega}{(2n+1)\gamma_{\parallel}} \tag{4.7}$$

and

$$p_2=\frac{\Omega}{(2n\beta+1)\gamma_{\perp}}\ . \tag{4.8}$$

Equations (4.4)–(4.6) yield

$$\langle S^{-}\rangle=\langle S^{+}\rangle^{*}=\frac{i(p_2+\alpha p_2^{*})N'}{\{2(1-|\alpha|^2)+(p_1p_2^{*}+p_1^{*}p_2)+(\alpha^{*}p_1p_2+\alpha p_1^{*}p_2^{*})\}} \tag{4.9}$$

and

$$\langle S_z\rangle=-\frac{2(1-|\alpha|^2)N'}{\{2(1-|\alpha|^2)+(p_1p_2^{*}+p_1^{*}p_2)+(\alpha^{*}p_1p_2+\alpha p_1^{*}p_2^{*})\}}\ . \tag{4.10}$$

If we consider the simple case that both α and p are real, then we can write

$$\langle S^{-}\rangle=\langle S^{+}\rangle^{*}=\frac{ip_2N'}{2(1-\alpha+p_1p_2)} \tag{4.11}$$

and

$$\langle S_z\rangle=-\frac{(1-\alpha)N'}{(1-\alpha+p_1p_2)}\ . \tag{4.12}$$

These equations give the influence of squeezed vacuum on the atomic polarization $\langle S^{-}\rangle$ and inversion $\langle S_z\rangle$.

V. OUTPUT FIELD

From the boundary condition (3.7) we can get an input-output relationship. Substituting Eq. (4.9) into Eq. (3.7) and normalizing the fields by letting $\langle a\rangle=\sqrt{N_s}x$ and $\alpha_0=\sqrt{N_s}y$, where N_s is the saturation photon number ($N_s=\gamma_{\perp}\gamma_{\parallel}/4G^2$), yield an input-output relationship to describe the effect of squeezed vacuum input on the intracavity field

$$y=x+\frac{2C\{(1+2n\beta)x-2m\beta x^{*}\}}{\{(2n+1)[(2n\beta+1)^2-4|m^2|\beta^2]+(2n\beta+1)|x|^2-2\beta(m^{*}x^2+mx^{*2})\}}\ , \tag{5.1}$$

where $C = G^2N/2k\gamma_\perp$ is the atomic cooperative parameter. This expression was first given by Galatola *et al.* [12] from a different method. It is easy to see that without squeezed vacuum input ($n = m = 0$), we recover the expression first given by Lugiato [17]:

$$y = x\left[1 + \frac{2C}{1+x^2}\right] . \tag{5.2}$$

In the simple case where both the field x and the squeezing parameter m are real we have

$$y = x\left[1 + \frac{2C}{(1+\alpha)(2n+1)(2n\beta+1)+x^2}\right] . \tag{5.3}$$

Figure 3 shows the bistability curves of the output field versus the coherent input field with squeezed vacuum input for $C=20$ and $\gamma_\parallel = 2\gamma_\perp$. We exhibit the results for two cases: $n=0.1$ in Fig. 3(a) (moderately squeezed vacuum) and $n=0.5$ in Fig. 3(b) (strongly squeezed vacuum).

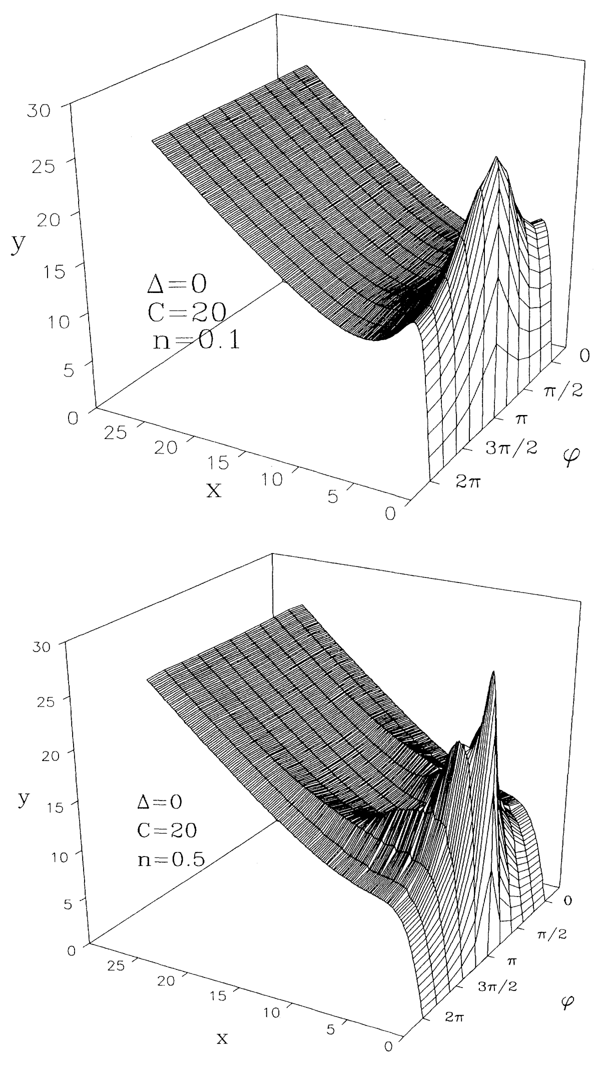

FIG. 3. Phase sensitivity of the input-output relationship with squeezed vacuum. (a) $n=0.1$; (b) $n=0.5$, $0 \le \varphi \le 2\pi$.

In Fig. 3(a) we see that the input-output relationship at each phase angle φ is changing. At $\varphi = \pi$ we have a large bi- stable region and at $\varphi = 0$ we have a small bistable region. The bistable region increases as φ increases from 0 to π. In Fig. 3(b) we find a different behavior, viz., as φ increases from 0 to π, there is an additional local maximum between 0 and π. The figure is symmetric about $\varphi = \pi$ and at $\varphi = \pi$ the bistable region reaches its absolute maximum. In Fig. 4 we consider the case of Eq. (5.3) and plot the input-output relationship. The case when $n=0$ corresponds to classical input without squeezed vacuum and in each case with squeezed vacuum there are two diferent choices for the value of m. The positive and the negative values of m correspond to the squeezed ($\varphi = \pi$) and the stretched ($\varphi = 0$) vacuum field, respectively. For a given value of n, we always consider the two cases of perfect squeezing [$m = \sqrt{n(n+1)}$, $\varphi = \pi$] and stretching [$m = \sqrt{n(n+1)}$, $\varphi = 0$]. In Fig. 4(a) we compare the input-output curves for $n = 0.1$ to the curve for $n = 0$. We see that for the squeezed case the output has a large change only near the first turning point and a minor change near the second turning point. The turning field value for the first turning point y_M is larger than that without squeezing and the range of the bistable region is increased. For the stretching case, both turning points are shifted towards smaller input field values, and the bistable region is much smaller than that without squeezing. Figure 4(b) shows similar results for the case of $n=0.5$. We see that for negative m the bistable behavior is lost, but for positive m the system is strongly bistable. Figure 4(c) shows the input-output for $n = 0$, $0.1 (m=0)$, $0.5 (m=0)$. We see that there is a tendency to decrease the optical bistability with the increase of n. Since the squeezed vacuum field enlarges the atomic inversion decay rate [6], the system tends to behave like a linear one (saturation is negligible). As we see in Figs. 4(a) and 4(b), there are different tendencies for squeezed and stretched vacuum input. The squeezed vacuum makes the system exhibit a more pronounced optical bistable behavior. This is because the atomic system is phase sensitive and, for the perfectly squeezed or stretched vacuum input, there is a phase transition only between two components (between S_x and S_z for squeezing or between S_y and S_z for stretching) of the atomic spin vector; the third component (S_y or S_x) is kept unchanged, as shown by Gardiner [6]. Then $S^- = S_x - iS_y$ varies only as a function of S_x (or S_y). So the output field is affected by the squeezed vacuum field. Without the input field the spin vector has spherical symmetry. With the coherent input field the spin vector has circular symmetry. We see that the symmetry of the spin vector is broken with the squeezed vacuum input field.

Figure 5 shows the input-output curves with and without the squeezed vacuum input field for the case of $\gamma_\parallel = \gamma_\perp$ [7]. In Fig. 5(a) we compare the input-output curves for $n=0.1$ to the curve for $n=0$. We see that with the squeezed vacuum the bistable region is slightly larger than in ordinary vacuum. On the other hand, the stretched vacuum decreases the bistable region, as shown in Fig. 4. Figure 5(b) gives the comparison of input-output curves for $n=0.5$ with that for $n=0$. We see that the optical bistability is decreased for both squeezed and stretched vacuum cases. The reason is that the collision is a random process that helps restore the original symmetry of the spin vector. It also enlarges the phase transition rate and averages the phase transitions among the three components of spin dipole moment S_x, S_y, and S_z. Thus collisions suppress the effect of the squeezed vacuum field by decreasing phase sensitivity in the spin system.

The turning points can be derived from the relation $dy/dx=0$. From Eq. (5.3) we have the results

$$x_{M,m}=\{C-(1-\alpha)(2n+1)(2n\beta+1)\pm[C^2-4C(1-\alpha)(2n+1)(2n\beta+1)]^{1/2}\}^{1/2}\,, \tag{5.4}$$

where x_M and x_m are the output fields for the first and the second turning points, respectively (so we have $x_M<x_m$). Bistable behavior can only occur under the condition that $C\geq 4(1-\alpha)(2n+1)(2n\beta+1)$. Without the squeezed vacuum field the critical value for bistability

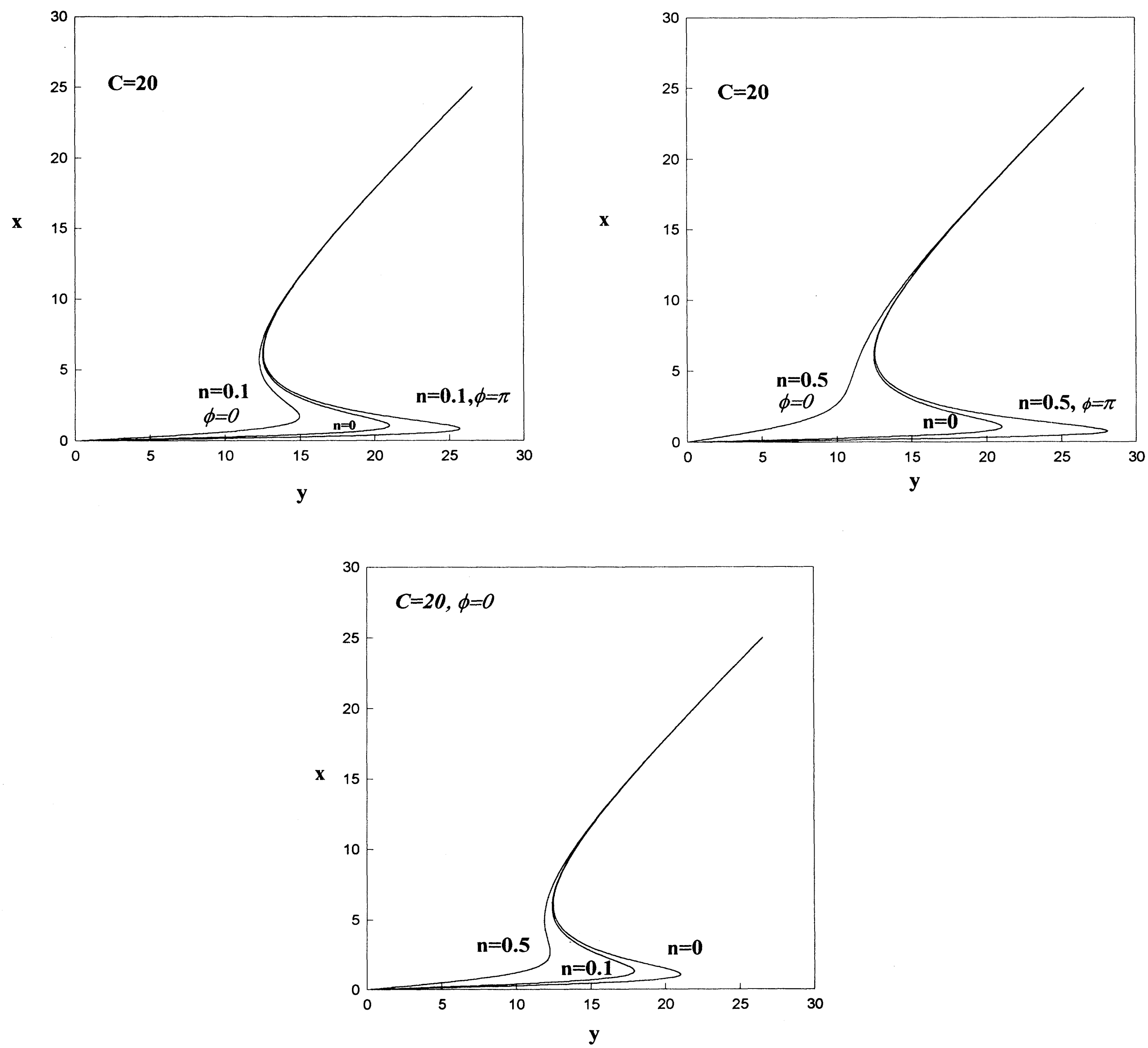

FIG. 4. Input-output relationship with $(n>0)$ and without $(n=0)$ squeezed vacuum input for $C=20$ and $\gamma_{\parallel}=2\gamma_{\perp}$. (a) $n=0,0.1$; (b) $n=0,0.5$; (c) $n=0,0.1,0.5$, $m=0$. With given n, a squeezed (stretched) vacuum increases (reduces) the range of optical bistability.

is $C=4$, but with the squeezed vacuum field ($n=0.5$, $\varphi=\pi$) the critical value is $C=3$. Thus, with the squeezed vacuum field, one can get optical bistability with fewer atoms than in the presence of normal vacuum.

Figure 6(a) shows the change of y_M and y_m versus C for $n=0$, 0.1, and 0.5 with perfect squeezing ($\varphi=\pi$), where y_M and y_m are scaled amplitudes of the input field for the first and the second turning points, respectively. For larger n, we have a larger range of optical bistability for a given value of C. Figure 6(b) shows the corresponding curves for stretching ($\varphi=0$) and we see that one needs a larger value of C to have bistability in this case.

Figure 7 shows the ratio of y_M to y_m versus C for $n=0$, 0.1, and 0.5. For given n the ratio increases (decreases) with increasing (decreasing) the phase φ from 0 to π and for the stretched vacuum input the optical bistability decreases with increasing average photon number in the vacuum field.

VI. STABILITY OF THE STEADY STATE

The stability of the output field is discussed starting from the input-output relationship by employing a standard method of mapping analysis. In Fig. 1 we consider that the expectation value of the atomic lowering opera-

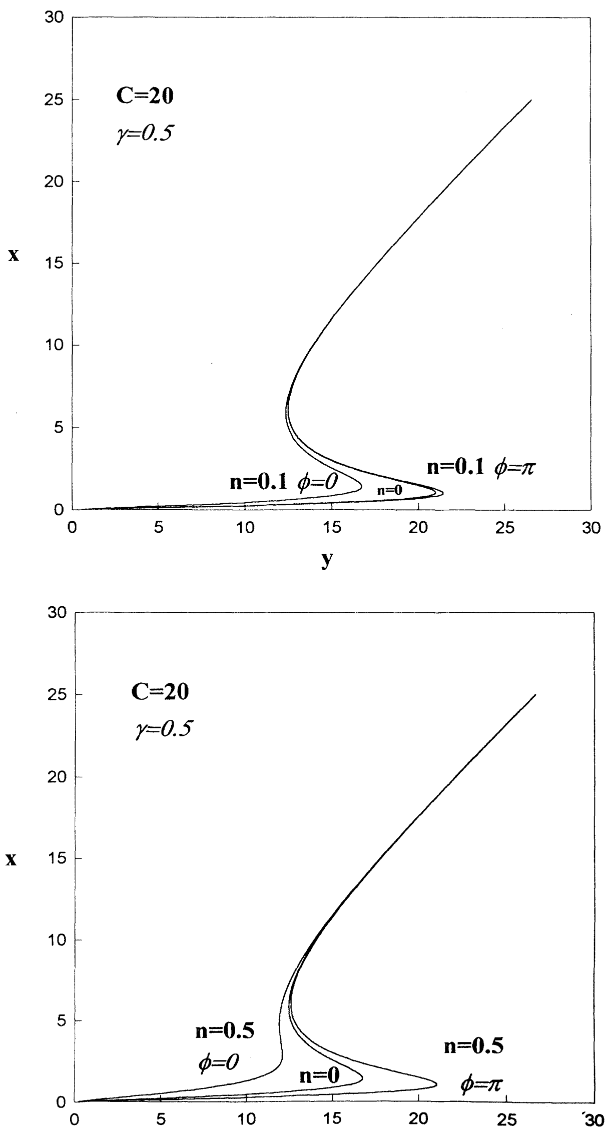

FIG. 5. Input-output relationship for $C=20$ and $\gamma_{\parallel}=\gamma_{\perp}$. (a) $n=0,0.1$; (b) $n=0,0.5$. The effect of the collision tends to symmetrize the S_x and S_y components of the spin vector and then to counter the effect of the squeezed vacuum input.

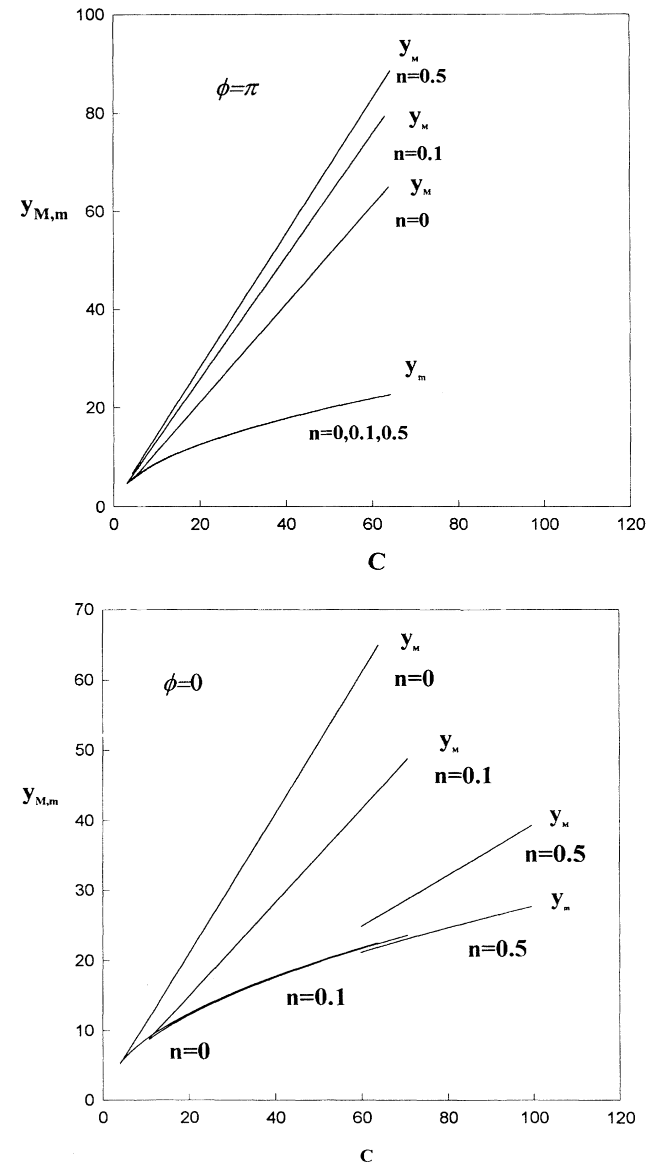

FIG. 6. Values of the first and second turning points vs C for $n=0,0.1,0.5$ and (a) $\varphi=\pi$ and (b) $\varphi=0$. The curve shows that for the strong, squeezed vacuum field (large value of n and $\varphi=\pi$) there is a large range of optical bistability for the system. For stretched vacuum input ($\varphi=0$), one needs large C to achieve optical bistability.

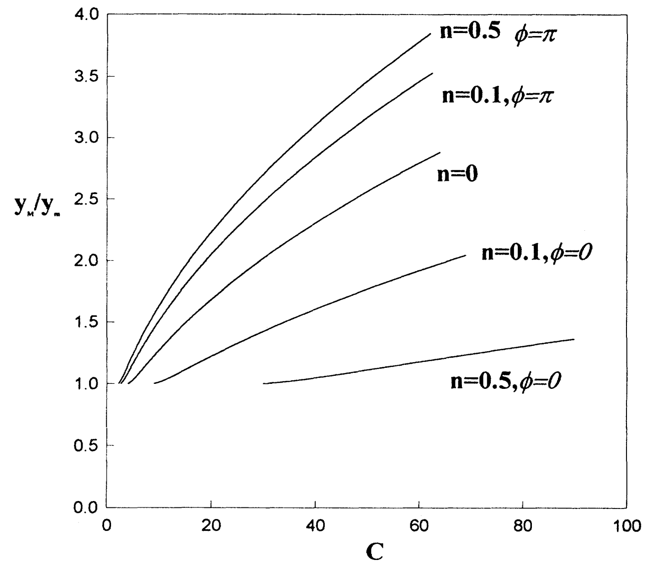

FIG. 7. Ratio of y_M to y_m versus C. (a) $n=0$; (b) $n=0.1$, $\varphi=\pi$; (c) $n=0.1$, $\varphi=0$; (d) $n=0.5$, $\varphi=\pi$; (e) $n=0.5$, $\varphi=0$. A higher ratio means a larger range of optical bistability.

tor $\langle S^- \rangle$ during the time period of nth round trip t_n is related to the expectation value of the cavity field $\langle a_2(t_{n-1}) \rangle$ during the previous time period of the $(n-1)$th round-trip t_{n-1}. From Eq. (3.6) we have

$$\langle a_{\rm out}(t_n) \rangle = \langle a_{\rm in}(t_n) \rangle + \frac{iGL}{\tilde{\tau} c} \langle S^- [\langle a_{\rm out}(t_{n-1}) \rangle] \rangle \ . \tag{6.1}$$

Now we perform a linear stability analysis by expanding the output field about its steady-state value; assuming that the deviation is real, we can represent the field as

$$\langle a_{\rm out}(t_n) \rangle = \langle a_{\rm out} \rangle_{\rm SS} + \delta(t_n) \ . \tag{6.2}$$

The substitution of Eq. (6.2) into Eq. (6.1) yields the mapping relationship for the deviations

$$\delta(t_n) = \left[1 - \frac{\partial y}{\partial x} \right] \delta(t_{n-1}) \ . \tag{6.3}$$

A stable steady state requires the following convergence condition to hold [7]:

$$\left| \left[1 - \frac{\partial y}{\partial x} \right] \right| < 1 \ , \tag{6.4}$$

which gives two separate conditions

$$\frac{\partial y}{\partial x} > 0 \tag{6.5}$$

and

$$\frac{\partial y}{\partial x} < 2 \ . \tag{6.6}$$

Equation (6.5) gives the usual result, which predicts that the negative slope region is unconditionally unstable. In addition, Eq. (6.6) predicts that parts of the positive slope regions may also become unstable. The physical mechanisms for the instabilities predicted from Eqs. (6.5) and (6.6) are different. In the negative slope region the decrease of the input field leads to an output increase; therefore the system is unstable. In Fig. 8 the stable and unstable regions of the output field with squeezed vacuum are shown for (a) $n=0,0.1$ $(\varphi=\pi,0)$ and (b) $n=0,0.5$ $(\varphi=\pi,0)$. We see that the upper branch and a small portion on the lower branch, near the first turning point, are stable and the negative slope part between the first and the second turning points and most of the lower branch are unstable.

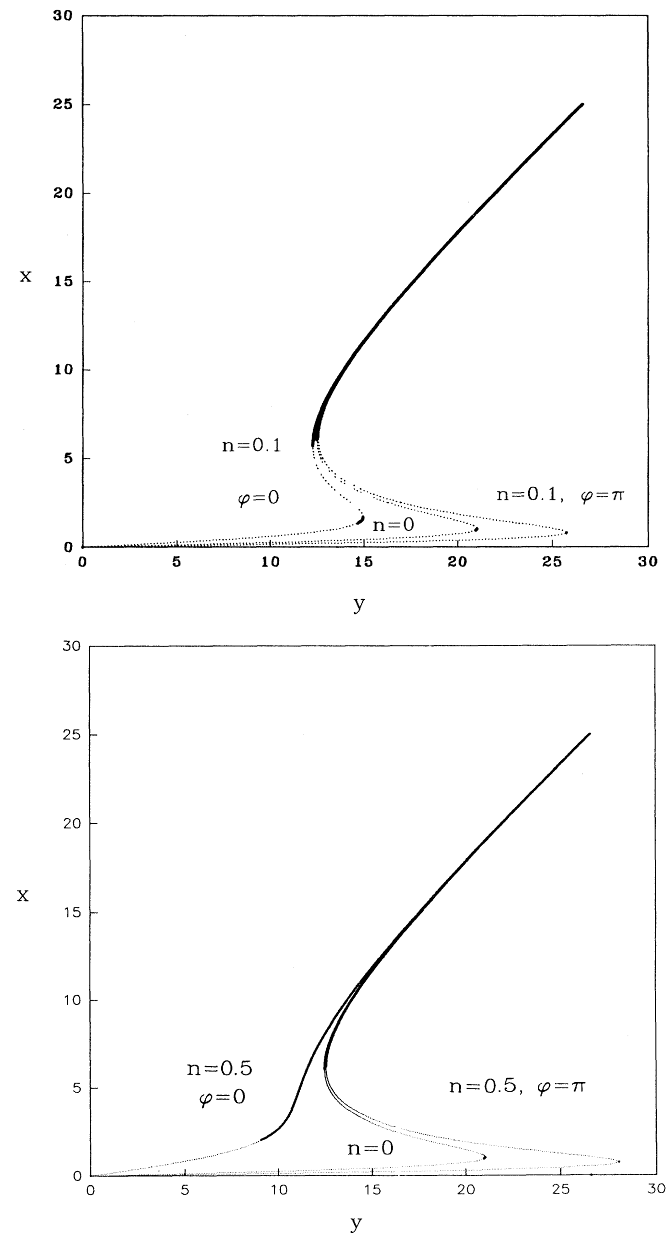

FIG. 8. Stability of the input-output relation. (a) $n=0,0.1$ $(\varphi=0,\pi)$; (b) $n=0,0.5$ $(\varphi=0,\pi$ and $m<0)$. The bold part of the curve is stable and the thin part of the curve is unstable.

When we consider the time dependence of the field in more detail following the method of Sec. III, the following relationship can be obtained from the boundary conditions:

$$\langle a_{\rm cav}(t_n)\rangle = \tilde{t}\langle a_{\rm in}(t_n)\rangle + \tilde{r}^2\langle a_{\rm cav}(t_{n-1})\rangle + \frac{iGL}{c}\langle S^-[\langle a_{\rm cav}(t_{n-1})\rangle]\rangle \ . \tag{6.7}$$

If a linear stability analysis is performed by using Eq. (6.2) we obtain

$$\delta(t_n) = \left\{\tilde{r}^2 + \tilde{t}\left[1 - \frac{\partial y}{\partial x}\right]\right\}\delta(t_{n-1}) \ , \tag{6.8}$$

which implies that

$$\left|\tilde{r}^2 + \tilde{t}\left[1 - \frac{\partial y}{\partial x}\right]\right| < 1 \ . \tag{6.9}$$

From here one can immediately get the unstable region

$$\frac{\partial y}{\partial x} > 1 - \tilde{t} > 0 \ . \tag{6.10}$$

The difference between Eqs. (6.10) and (6.5) predicts additional unstable regions on both the upper and the lower branches near the turning points. The ranges of those regions depend on the feedback of the cavity. When $\tilde{r}=0$ ($\tilde{t}=1$), one can recover Eq. (6.5). Our conclusion is that

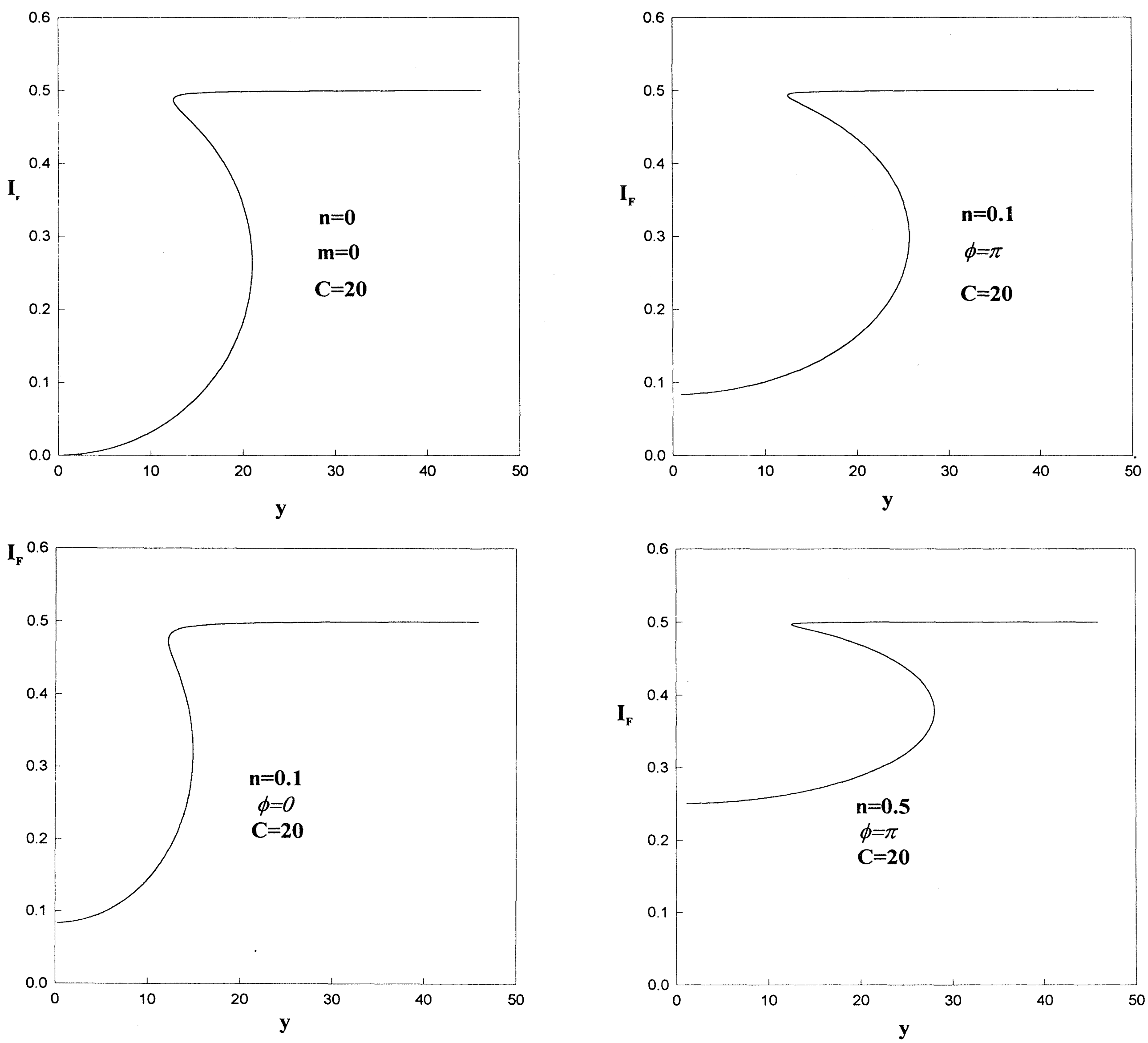

FIG. 9. Hysteresis cycle of fluorescent intensity I_F. (a) $n=0$; (b) $n=0.1$, $\varphi=\pi$; (c) $n=0.1$, $\varphi=0$; (d) $n=0.5$, $\varphi=\pi$; (e) $n=0.5$, $\varphi=0$. The influence of the squeezed vacuum on the fluorescent light is shown to be similar to that on the transmitted output field. Note that when there is no pump field ($y=0$) there is still fluorescent light.

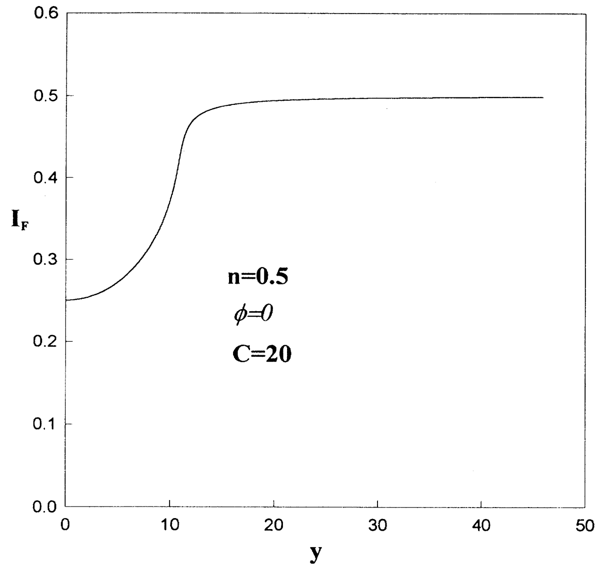

FIG. 9. (*Continued*).

the stability of the optical bistability depends on the parameters of the cavity and therefore the feedback is restricted to a certain range for stable self-consistent solutions.

VII. FLUORESCENT LIGHT

The spectrum of fluorescent light $I(\omega)$ is proportional to the Fourier transform of the time correlation function $F(t)=\sum_{i=1}^{N}\langle\sigma_i^+(t)\sigma_i^-(0)\rangle_{st}$,

$$I(\omega)\propto\frac{1}{\pi}\text{Re}\int dt\,\exp[-i(\omega-\omega_0)t]F(t)\ . \quad (7.1)$$

The integrated spectrum $I_F=\int d\omega\, I(\omega)$ is given by

$$I_F\propto\sum_{i=1}^{N}\langle\sigma_i^+\sigma_i^-\rangle_{st}=N+\langle S_z\rangle_{st}$$
$$=N\frac{(2n+1)(1-\alpha)(2n+1-\beta)+x^2}{(2n+1)(1-\alpha)(2n\beta+1)+x^2}\ . \quad (7.2)$$

In Fig. 9 we show the hysteresis cycle of the total fluorescent intensity I_F, which is proportional to the population of the upper level, for $n=0$, 0.1, and 0.5. When there is squeezed vacuum input but no pump field ($y=0$), there is still fluorescent light. The reason is that the squeezed vacuum input changes the population of the upper level. The stronger the squeezing, the stronger the fluorescent light. The effect of squeezed vacuum input on the fluorescent light is similar to that on transmitted light. We also notice that for a given value of n, negative m can make the bistability disappear.

VIII. DISCUSSION AND SUMMARY

The optical bistable behavior of a system of N two-level atoms in a resonator with squeezed vacuum input is discussed. We start from the equation of motion for the atomic dipole moment, which is coupled to the cavity field and to the squeezed vacuum field, and derive the expectation value of the atomic lowering operator S^- in steady state. Then we connect the coherent input field with the intracavity field and the atomic lowering operator by an appropriate boundary condition. A simple expression that exhibits the effect of squeezed vacuum on the optical bistability of the output field is given. Our results show that, since the squeezed vacuum input destroys the symmetry of the S_x and S_y components of Bloch vector, a squeezed vacuum input will increase the optical bistable range for a given system that is described by the cooperative parameter C and a stretched vacuum input will decrease the optical bistability of the system. Compared with the case of ordinary vacuum field ($n=m=0$), the optical bistability of the system with the squeezed vacuum input now depends on both n and m. A squeezed vacuum field (large value of n and positive m) will enhance the range of optical bistability. The value of the input field for the first turning point increases significantly and the value of the input field for the second turning point almost does not change. We also see that the stretched vacuum input reduces the range of optical bistability significantly since our results give the two extreme cases of a perfectly squeezed input $[m=\sqrt{n(n+1)},\ \varphi=\pi]$ and a perfectly stretched input field $[m=\sqrt{n(n+1)},\ \varphi=0]$. The x-quadrature stretched field ($\varphi=0$) can even make the optical bistability disappear and make the system linear. This effect provides us with the possibility to use the squeezed vacuum input to change the behavior of the two-level atom system as we need.

The stability of the input-output curve is discussed by using the standard method of linear stability analysis. Our results show that the stability of the steady state depends on the structure of the cavity and the slope of the input-output curve. For the squeezed vacuum input the region of bistability is increased and the stability of the output field is also affected via the influence of the squeezed vacuum field on the input-output relationship. When $\tilde{r}=1$ the entire upper branch of the input-output curve is stable and there is a small stable reigon near the first turning point on the lower branch. When $\tilde{r}<1$ a small portion near the second turning point on the upper branch and a small portion near the first turning point on the lower branch become unstable.

Optical bistable behavior of the fluorescent light is also discussed. The hysteresis cycle of the total fluorescent intensity vs input exhibits a behavior similar to bistable behavior of the output field. The influence of squeezed vacuum is similar for both transmitted and fluorescent light. Our result shows that when there is no coherent input field, there is still fluorescent light with squeezed vacuum input. This effect is due to the fact that the squeezed vacuum input changes the expectation value of $\langle S_z\rangle$.

The effect of imperfectly squeezed vacuum input ($n\neq0$, $m=0$) is that it reduces the range of optical bistability since the squeezed vacuum field changes the atomic transition rate $[\gamma'=(2n+1)\gamma]$ [6]. Hence the time that the atom spends in the upper level is shorter compared with the case with ordinary vacuum input field ($n=0$) and the optical bistability of the system decreases [as shown in Fig. 4(c)]. But any amount of squeezing ($|m|>0$, $\pi/2<\varphi<3\pi/2$) has the tendency to increase the range of optical bistability. The total effect of a perfectly squeezed vacuum input [$m=\sqrt{n(n+1)}$, $\varphi=\pi$] is to increase the bistable range of the system.

Our results show that the optical bistability with a stretched vacuum input strongly depends on the cooperative parameter C, which is proportional to the number of atoms. With a stretched vacuum input, a larger value of C is required for the system to have the optical bistable behavior. With squeezed vacuum, one can get optical bistability for $C<4$, i.e., with fewer atoms in the system.

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