Photodetection and causality in quantum optics

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The theory of photodetection and quantum-optical coherence is formulated in the Heisenberg picture and in such a way that the causal propagation of fields at the velocity of light is manifest. Objections to the standard theory, based on a putative violation of causality, are shown to be unfounded, as is the notion that normal ordering is essential for the elimination of an infinite contribution from the vacuum field. In a similar vein we revisit the Fermi two-atom problem and explicitly demonstrate the causal nature of the interaction without invoking simplifying approximations that have recently been called into question.

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I. INTRODUCTION

The quantum theory of photodetection and optical coherence as originally formulated by Glauber [1] is central to all of quantum optics, and the many applications of the theory have required essentially no modifications of the original formulation. However, in the past few years it has been argued that the theory is inconsistent, in principle, with causality in the sense that signals should propagate no faster than the speed of light [2]. This notion of causality, which is the one assumed throughout this paper, is closely related to Fermi's model for the propagation of light in quantum electrodynamics [3]. This model consists of two atoms separated by a distance r, in an initial state at time $t=0$ such that atom A is excited, atom B is in its ground state, and there are no photons in the field. Fermi, under certain approximations, showed that the probability for atom B to be excited remains zero until the time $t=r/c$ required for light to propagate from A to B. In more recent years many authors have reexamined Fermi's model [4]. It has been shown, for instance, how a modification of Fermi's original approach leads not only to a properly continuous solution, but also to the appearance of all multiples of the retardation time r/c [5]. Based on the approximate nature of the solutions obtained for Fermi's model, it has recently been argued that no rigorous proof of causal propagation in quantum electrodynamics has been given and that in fact the theory may even admit the possibility of noncausal propagation [6].

In this paper we show that these objections to general photodetection theory and the specific Fermi model are closely related and more importantly that only slight variations on the theory in either case are sufficient to elicit causality. In neither case is conventional theory flawed in any fundamental way. After formulating the Hamiltonian and the relevant Heisenberg equations of motion in the following section, we consider the Fermi model in Sec. III and show that causality is an *exact* and rigorous consequence of the Heisenberg equations of motion. Although the recent objections to previous, approximate demonstrations of causality are not unreasonable, they are shown to be unfounded with regard to the fundamentally causal nature of interactions in quantum electrodynamics. In particular, we show that the earlier results for excitation probabilities [3,5] are correct up to a rotating-wave approximation and that they are *rigorously* correct with respect to causality. In Sec. IV we take up the objections to and proposed modifications of the standard theory of photodetection and show that they are related to the criticisms of causality proofs in the Fermi model. As in the case of the Fermi model, a closer examination of the conventional theory of photodetection and optical coherence shows that there are no violations of causality. Section V summarizes our conclusions.

II. HAMILTONIAN AND HEISENBERG EQUATIONS

In the Fermi two-atom problem it is convenient to focus attention on the model of two two-level atoms interacting with the electromagnetic field via electric-dipole transitions. The Hamiltonian for this system is [7]

$$H=\tfrac{1}{2}\hbar\omega_0[\sigma_{z1}+\sigma_{z2}]+H_F-d_jE_j(\mathbf{x}_1)\sigma_{x1}-d_jE_j(\mathbf{x}_2)\sigma_{x2}\ , \tag{1}$$

where we assume that the atoms at $\mathbf{x}_1$ and $\mathbf{x}_2$ are identical, having transition frequency ω_0 and electric dipole moment $\mathbf{d}$. The summation convention for repeated indices (j) is employed together with the conventional Pauli two-state operators for each atom. The first two terms give the form of the Hamiltonian for the unperturbed atoms and the field, respectively, and the last two terms account for the interaction of the atoms with the quantized electric field $\mathbf{E}(\mathbf{x})$ [8].

The Heisenberg equation of motion for the population difference operator σ_{z2} for atom 2, for instance, is found

from (1) to be

$$\dot{\sigma}_{z2} = -\frac{2}{\hbar} d_j E_j(\mathbf{x}_2,t)\sigma_{y2}(t)$$
$$= -\frac{2i}{\hbar} d_j E_j(\mathbf{x}_2,t)[\sigma_2(t)-\sigma_2^\dagger(t)]$$
$$= \frac{2i}{\hbar} d_j[\sigma_2^\dagger(t)E_j(\mathbf{x}_2,t)-E_j(\mathbf{x}_2,t)\sigma_2(t)] , \quad (2)$$

where σ_2 and $\sigma_2^\dagger$ are the lowering and raising operators for the two-state atom 2 ($[\sigma_2,\sigma_2^\dagger]=-\sigma_{z2}$). In the third line of Eq. (2) we have made use of the commutativity of equal-time atom and field operators. Similarly

$$\dot{\sigma}_2(t) = -i\omega_0\sigma_2(t) - \frac{i}{\hbar} d_j E_j(\mathbf{x}_2,t)\sigma_{z2}(t) . \quad (3)$$

The use of the formal solution of Eq. (3) in Eq. (2) gives

$$\langle \dot{\sigma}_{z2}(t)\rangle = -\frac{4}{\hbar^2} d_j d_k \,\mathrm{Re}\int_0^t dt' \langle E_j(\mathbf{x}_2,t)E_k(\mathbf{x}_2,t') \times \sigma_{z2}(t')\rangle e^{i\omega_0(t'-t)} . \quad (4)$$

Here the expectation value is over an initial state $|\psi_i\rangle$ with atom 2 in the lower state, so that $\sigma_2(0)|\psi_i\rangle = \langle\psi_i|\sigma_2^\dagger(0)=0$, and this has been assumed in writing Eq. (4).

III. CAUSALITY IN FERMI'S TWO-ATOM PROBLEM

The solution of the Heisenberg equation of motion for the electric field operator $\mathbf{E}(\mathbf{x},t)$ in Eq. (4) will give us an equation for the expectation value $\langle\sigma_{z2}(t)\rangle$ of the population difference of atom 2 and therefore the excitation probability

$$P_2(t) \equiv \langle\sigma_2^\dagger(t)\sigma_2(t)\rangle = \tfrac{1}{2}[1+\langle\sigma_{z2}(t)\rangle] , \quad (5)$$

in terms of the field produced by atom 1. The solution for the Heisenberg operator $\mathbf{E}(\mathbf{x},t)$ is straightforward and proceeds exactly along classical lines [7]:

$$\mathbf{E}(\mathbf{x}_2,t) = \mathbf{E}_0(\mathbf{x}_2,t)+\mathbf{E}_{\mathrm{RR}}(\mathbf{x}_2,t)+\mathbf{E}_1(\mathbf{x}_2,t) . \quad (6)$$

$\mathbf{E}_0(\mathbf{x}_2,t)$ is the vacuum field at $\mathbf{x}_2$, i.e., the source-free field corresponding to the homogeneous solution of the Maxwell equations for the Heisenberg operators. $\mathbf{E}_{\mathrm{RR}}(\mathbf{x}_2,t)$ is the radiation reaction field of atom 2 on itself and $\mathbf{E}_1(\mathbf{x}_2,t)$ is the Heisenberg operator for the field from atom 1 at the position $\mathbf{x}_2$ of atom 2 at time t. The field operator $\mathbf{E}_1(\mathbf{x}_2,t)$ has the same form as the classical field from a point electric dipole. For simplicity, but with no loss of generality for our purposes, we consider only the far-field part of $\mathbf{E}_1(\mathbf{x}_2,t)$ and write [7]

$$\mathbf{E}_1(\mathbf{x}_2,t) = -\frac{d}{c^2 r}[\hat{\mathbf{d}}-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}]\ddot{\sigma}_{x1}\left[t-\frac{r}{c}\right]\theta\left[t-\frac{r}{c}\right] . \quad (7)$$

Here $\hat{\mathbf{d}}$ is the unit vector pointing in the direction of the transition dipole matrix element and $\hat{\mathbf{r}}$ is the unit vector in the direction of $\mathbf{x}_2-\mathbf{x}_1$, i.e., $\mathbf{x}_2-\mathbf{x}_1=\hat{\mathbf{r}}r$. θ is the unit step function.

Equations (6) and (7) follow directly from the Hamiltonian (1) and the Heisenberg equations of motion for the field operators and are formally the same as their classical counterparts [7]. Thus the form (6) is essentially just a statement of the superposition principle for the electric field. Similarly the expression (7) for the field at $\mathbf{x}_2$ due to atom 1 is formally the same regardless of whether an atom 2 is actually present at $\mathbf{x}_2$. In the limit $t\to 0$, when the atom-field interaction is presumed for the sake of calculation to be "turned on," the field operators act only on the field part of the initial (direct-product) atom-field state. At $t>0$ these operators act in the full atom-field Hilbert space and thus take on a more general character in that the second and third terms on the right-hand side of (6) involve atom operators, or rather what were atom operators at $t=0$, because for $t>0$ they too act on states in the full atom-field Hilbert space. To obtain expectation values over some initial state in the Heisenberg picture, one typically employs short-time expansions, conservation laws, or some other means to relate operators at $t>0$ to operators at $t=0$ whose effects on the initial state are unambiguous.

Formally, Eqs. (4)–(7) exhibit the main point of this section: the effect of atom 1 on atom 2 is retarded by the propagation time r/c. Therefore atom 2 cannot become excited, due to the influence of atom 1, until at least a time r/c after atom 1 is excited. It is important to note that this result is *exact*. No approximations have been made in the $\theta(t-r/c)$-dependent term accounting for the effect of atom 1 on the excitation probability of atom 2 in Eq. (7).

We have made the common but, of course, artificial assumption that the atom-field interaction is switched on at $t=0$; this is implicit in Eq. (4) in the fact that the lower limit of integration is $t=0$ [9]. In a more realistic formulation of the problem the interaction is switched on adiabatically from $t=-\infty$ and we suppose that atom 1 is excited at, say, $t=0$, long after the interaction is fully "on." In this case the two (ground-state) atoms can interact via the van der Waals interaction for $t>0$, with both atoms remaining in their ground states. That is, the atoms will generally be "coupled" while they are both in their ground states. After $t=0$, when *one* of the atoms is presumed to be excited, the information about its excitation cannot be transmitted to the second atom until $=r/c$, as is clear from Eq. (7), which is independent of any artificial turn-on of the atom-field interaction. Therefore atom 2 cannot become excited until at least a time r/c after the first atom is excited.

Shirokov [4] and Hegerfeldt [6] have observed that an approximation made in Fermi's original calculation [3] and in various subsequent papers [4,5] was crucial to the proof of causality. Hegerfeldt [6] states that "there seems to be agreement that Fermi's local result is not correct, but that this nonlocality cannot be used for superluminal signal transmission since measurements on A and B as well as on photons are involved." However, we have just shown that the excitation of the initially unexcited atom does in fact involve the finite signal velocity c. No approximations are required [10]. Before commenting fur-

ther on specific objections to previous "causal results," we now show how the latter follow from certain *approximations.* In particular, we show how the results of Milonni and Knight [5], and consequently those of Fermi [3], follow under the very mild approximation that the transition frequency is large compared with the Rabi frequency corresponding to the electric field of one atom acting on the other.

A. Relation to previous results

The electric-field operator can be written

$$E_j(\mathbf{x},t)=E_j^{(+)}(\mathbf{x},t)+E_j^{(-)}(\mathbf{x},t)\ , \tag{8}$$

where by $E_j^{(+)}(\mathbf{x},t)$ we mean here the part of $E_j(\mathbf{x},t)$ associated with the (Heisenberg-picture) photon annihilation operators $a_{\mathbf{k}\lambda}(t)$ and likewise $E_j^{(-)}(\mathbf{x},t)=E_j^{(+)}(\mathbf{x},t)^\dagger$ the part associated with the photon creation operators $a_{\mathbf{k}\lambda}^\dagger(t)$. Since $E_j^{(\pm)}(\mathbf{x},t)$ separately commute with equal-time atomic operators, we can write Eq. (3) equivalently as

$$\dot\sigma_2(t)=-i\omega_0\sigma_2(t)-\frac{i}{\hbar}d_j[E_j^{(-)}(\mathbf{x}_2,t)\sigma_{z2}(t)+\sigma_{z2}(t)E_j^{(+)}(\mathbf{x}_2,t)] \tag{9}$$

and Eq. (2) equivalently as

$$\dot\sigma_{z2}(t)=\frac{2i}{\hbar}d_j[\sigma_2^\dagger E_j^{(+)}(\mathbf{x}_2,t)+E_j^{(-)}(\mathbf{x}_2,t)\sigma_2^\dagger(t)]+\text{H.c.} \tag{10}$$

Then, assuming as in Eq. (4) an initial state with atom 2 in its lower state, we have

$$\begin{aligned}\langle\dot\sigma_{z2}(t)\rangle=-\frac{4}{\hbar^2}d_jd_k\,\mathrm{Re}\Big[&\int_0^t dt'\langle\sigma_{z2}(t')E_k^{(+)}(\mathbf{x}_2,t')E_j^{(+)}(\mathbf{x}_2,t)\rangle e^{-i\omega_0(t'-t)}\\&+\int_0^t dt'\langle E_k^{(-)}(\mathbf{x}_2,t')\sigma_{z2}(t')E_j^{(+)}(\mathbf{x}_2,t)\rangle e^{-i\omega_0(t'-t)}\\&+\int_0^t dt'\langle E_j^{(-)}(\mathbf{x}_2,t)\sigma_{z2}(t')E_k^{(+)}(\mathbf{x}_2,t')\rangle e^{-i\omega_0(t'-t)}\\&+\int_0^t dt'\langle E_j^{(-)}(\mathbf{x}_2,t)E_k^{(-)}(\mathbf{x}_2,t')\sigma_{z2}(t')\rangle e^{-i\omega_0(t'-t)}\Big]\ .\end{aligned} \tag{11}$$

This expression is exact and equivalent to Eq. (4). To lowest order in the Rabi frequency $d_jE_j/\hbar$ we replace $\sigma_{z2}(t')$ in the integrand of Eq. (11) by its unperturbed $t=0$ value $\sigma_{z2}(0)$. Then

$$\begin{aligned}\langle\dot\sigma_{z2}(t)\rangle\cong\frac{-4}{\hbar^2}d_jd_k\,\mathrm{Re}\int_0^t dt'e^{-i\omega_0(t'-t)}[&\langle\sigma_{z2}(0)E_k^{(+)}(\mathbf{x}_2,t')E_j^{(+)}(\mathbf{x}_2,t)\rangle+\langle E_k^{(-)}(\mathbf{x}_2,t')\sigma_{z2}(0)E_j^{(+)}(\mathbf{x}_2,t)\rangle\\&+\langle E_j^{(-)}(\mathbf{x}_2,t)\sigma_{z2}(0)E_k^{(+)}(\mathbf{x}_2,t')\rangle+\langle E_j^{(-)}(\mathbf{x}_2,t)E_k^{(-)}(\mathbf{x}_2,t')\sigma_{z2}(0)\rangle]\ .\end{aligned} \tag{12}$$

In the absence of any source, $E_j^{(+)}(\mathbf{x}_2,t)$ and $E_j^{(-)}(\mathbf{x}_2,t)$ are, respectively, the positive- and negative-frequency parts of the field at $\mathbf{x}_2$, whereas in the presence of sources they are only *approximately* the positive- and negative-frequency parts. (This is discussed further in Sec. IV.) In this approximation the first, third, and fourth field correlation functions in the integrand of Eq. (12) give rapidly oscillating and ignorable contributions compared with the second correlation function. This is in fact the *rotating-wave approximation* (RWA), which can be made at the outset, in the Hamiltonian, by dropping terms $E_j^{(+)}\sigma$ and $\sigma^\dagger E_j^{(-)}$. In the RWA

$$\langle\dot\sigma_{z2}(t)\rangle\cong\frac{-4}{\hbar^2}d_jd_k\,\mathrm{Re}\int_0^t dt'e^{-i\omega_0(t'-t)}\langle E_k^{(-)}(\mathbf{x}_2,t')\sigma_{z2}(0)E_j^{(+)}(\mathbf{x}_2,t)\rangle\ . \tag{13}$$

From Eq. (6),

$$E_j^{(+)}(\mathbf{x}_2,t)=E_{0,j}^{(+)}(\mathbf{x}_2,t)+E_{\mathrm{RR},j}^{(+)}(\mathbf{x}_2,t)+E_{1,j}^{(+)}(\mathbf{x}_2,t) \tag{14}$$

and, for an initial atom-field state $|\psi_i\rangle$ with no photons in the field, $E_{0,j}^{(+)}(\mathbf{x}_2,t)|\psi_i\rangle=\langle\psi_i|E_{0,j}^{(-)}(\mathbf{x}_2,t)=0$. Thus

$$\begin{aligned}\langle\dot\sigma_{z2}\rangle&\cong\frac{-4}{\hbar^2}d_jd_k\,\mathrm{Re}\int_0^t dt'\langle[E_{\mathrm{RR},k}^{(-)}(\mathbf{x}_2,t')+E_{1,k}^{(-)}(\mathbf{x}_2,t')]\sigma_{z2}(0)[E_{\mathrm{RR},j}^{(+)}(\mathbf{x}_2,t)+E_{1,j}^{(+)}(\mathbf{x}_2,t)]\rangle e^{-i\omega_0(t'-t)}\\&=\frac{-4}{\hbar^2}d_jd_k\,\mathrm{Re}\int_0^t dt'[\langle E_{\mathrm{RR},k}^{(-)}(\mathbf{x}_2,t')\sigma_{z2}(0)E_{\mathrm{RR},j}^{(+)}(\mathbf{x}_2,t)\rangle+\langle E_{\mathrm{RR},k}^{(-)}(\mathbf{x}_2,t')\sigma_{z2}(0)E_{1,j}^{(+)}\mathbf{x}_2,t)\rangle\\&\quad+\langle E_{1,k}^{(-)}(\mathbf{x}_2,t')\sigma_{z2}(0)E_{\mathrm{RR},j}^{(+)}(\mathbf{x}_2,t)\rangle+\langle E_{1,k}^{(-)}(\mathbf{x}_2,t')\sigma_{z2}(0)E_{1,j}^{(+)}(\mathbf{x}_2,t)\rangle]e^{-i\omega_0(t'-t)}\ .\end{aligned} \tag{15}$$

Using $\sigma_x=\sigma+\sigma^\dagger$, we write the field (7) from atom 1 as

$$\mathbf{E}_1(\mathbf{x}_2,t)=-\frac{d}{c^2r}[\hat{\mathbf{d}}-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}]\theta\left[t-\frac{r}{c}\right]\times\left[\ddot{\sigma}_1\left[t-\frac{r}{c}\right]+\ddot{\sigma}_1^\dagger\left[t-\frac{r}{c}\right]\right]. \quad (16)$$

Then, in the RWA [11],

$$\mathbf{E}_1^{(+)}(\mathbf{x}_2,t)\cong-\frac{d}{c^2r}[\hat{\mathbf{d}}-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}]\theta\left[t-\frac{r}{c}\right]\ddot{\sigma}_1\left[t-\frac{r}{c}\right]$$
$$\cong\frac{d\omega_0^2}{c^2r}[\hat{\mathbf{d}}-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}]\theta\left[t-\frac{r}{c}\right]\sigma_1\left[t-\frac{r}{c}\right] \quad (17)$$

and

$$d_jE_{1,j}^{(+)}(\mathbf{x}_2,t)=\frac{d^2\omega_0^2}{c^2r}[1-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})^2]\theta\left[t-\frac{r}{c}\right]\sigma_1\left[t-\frac{r}{c}\right]. \quad (18)$$

The RWA is the most important approximation made here: it is only within the RWA that a *rigorous* proof of causality is impossible and the approximation of extending a frequency integration to $-\infty$ is required to obtain results consistent with properly causal results in Fermi's original formulation of the problem. This is discussed in Sec. III B.

It is important to emphasize that we have obtained (16) from the Hamiltonian (1) and that the RWA is made not in the Hamiltonian or the Heisenberg equations but rather in the step from (16) to (17). That is, by the "RWA" we mean here the identification of the annihilation (positive-frequency) part of the field operator with the lowering operator for the source atom as in (17); counterrotating terms are dropped. Had we made the RWA in the original Hamiltonian by dropping those terms in the interaction that lead to these counterrotating terms, we would obtain again (17) *but without the θ function.* This is consistent with the conclusions of Compagno, Passante, and Persico [4], for instance, who find that the θ function is not obtained when the RWA is made in the Hamiltonian. We comment further on this point in Sec. V.

Similarly [7]

$$d_jE_{\mathrm{RR},j}^{(+)}(\mathbf{x}_2,t)\cong\frac{2id^2\omega_0^3}{3c^3}\sigma_2(t)=i\hbar\beta\sigma_2(t), \quad (19)$$

where $\beta\equiv2d^2\omega_0^3/3\hbar c^3$ is half the Einstein A coefficient for spontaneous emission. We have ignored a term in $E_{\mathrm{RR},j}^{(+)}$ that gives rise to a single-atom frequency shift ("Lamb shift"), since it has no effect on the transition rate of interest here [7]. Within these approximations the expression (15) reduces to

$$\langle\dot{\sigma}_{z2}(t)\rangle\cong-4\beta^2\,\mathrm{Re}\int_0^t dt'e^{-i\omega_0(t'-t)}\left[\langle\sigma_2^\dagger(t')\sigma_{z2}(0)\sigma_2(t)\rangle\right.$$
$$-\frac{3i}{2k_0r}[1-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})^2]\left\langle\sigma_2^\dagger(t')\sigma_{z2}(0)\sigma_1\left[t-\frac{r}{c}\right]\right\rangle\theta\left[t-\frac{r}{c}\right]$$
$$+\frac{3i}{2k_0r}[1-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})^2]\left\langle\sigma_1^\dagger\left[t'-\frac{r}{c}\right]\sigma_{z2}(0)\sigma_2(t)\right\rangle\theta\left[t'-\frac{r}{c}\right]$$
$$+\left[\frac{3}{2k_0r}\right]^2[1-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})^2]^2\left\langle\sigma_1^\dagger\left[t'-\frac{r}{c}\right]\sigma_{z2}(0)\sigma_1\left[t-\frac{r}{c}\right]\right\rangle$$
$$\left.\times\theta\left[t'-\frac{r}{c}\right]\theta\left[t-\frac{r}{c}\right]\right], \quad (20)$$

where $k_0=\omega_0/c$.

We now make the further approximation, consistent with that made in deriving (12), of replacing $\sigma_1(t)$ and $\sigma_2(t)$ in (20) by $\sigma_1(0)\exp(-i\omega_0t)$ and $\sigma_2(0)\exp(-i\omega_0t)$, respectively,

$$\langle\dot{\sigma}_{z2}(t)\rangle\cong-4\beta^2\left[\frac{3}{2k_0r}\right]^2[1-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})^2]^2\int_0^t dt'\langle\sigma_1^\dagger(0)\sigma_{z2}(0)\sigma_1(0)\rangle\theta\left[t'-\frac{r}{c}\right]\theta\left[t-\frac{r}{c}\right]$$
$$=4\beta^2\left[\frac{3}{2k_0r}\right]^2[1-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})^2]^2\left[t-\frac{r}{c}\right]\theta\left[t-\frac{r}{c}\right], \quad (21)$$

where we have used $[\sigma_1(0),\sigma_{z2}(0)]=0$, $\sigma_2(0)|\psi_i\rangle=0$, $\sigma_{z2}(0)|\psi_i\rangle=-|\psi_i\rangle$, and $\sigma_1^\dagger(0)\sigma_1(0)|\psi_i\rangle=|\psi_i\rangle$, i.e., the initial state with atom 1 excited and atom 2 in its ground state. The approximation made here and in (12) is tantamount to the restriction to times $t\ll\beta^{-1}$, before spontaneous emission is significant. The probability (5) that atom 2 is excited at times t such that $0<t\ll\beta^{-1}$ is then given by

$$\dot{P}_2(t)\cong2\beta^2\left[\frac{3}{2k_0r}\right]^2\left[t-\frac{r}{c}\right]\theta\left[t-\frac{r}{c}\right] \quad (22)$$

for the case of $\Delta m = \pm 1$ transitions, where $\hat{\mathbf{d}}\cdot\hat{\mathbf{r}}=0$ [5]. The solution satisfying the initial condition $P_2(0)=0$ is

$$P_2(t) \cong \beta^2 \left[\frac{3}{2k_0 r}\right]^2 \left[t-\frac{r}{c}\right]^2 \theta\left[t-\frac{r}{c}\right] . \tag{23}$$

Under the approximation $\beta t \ll 1$ made here, this is exactly the first term of Milonni and Knight [5] for the excitation probability of the initially unexcited atom 2.

Thus we have shown that causality is exact in Fermi's two-atom model, but that a rotating-wave approximation is required to obtain explicit and simple expressions for probability amplitudes. These approximate expressions are identical to the amplitudes obtained by Milonni and Knight [5] for $0<t<2r/c$ and, when the two atoms are not identical, the results of Fermi [3,5].

B. Relation to previous approximations

The quantized field $\mathbf{E}(\mathbf{x},t)=\mathbf{E}^{(+)}(\mathbf{x},t)+\mathbf{E}^{(-)}(\mathbf{x},t)$, where [12]

$$\mathbf{E}^{(+)}(\mathbf{x},t)=i\sum_{\mathbf{k},\lambda}\left[\frac{2\pi\hbar\omega_k}{V}\right]^{1/2} a_{\mathbf{k}\lambda}(t)e^{i\mathbf{k}\cdot\mathbf{x}}\mathbf{e}_{\mathbf{k}\lambda} \tag{24}$$

and $\mathbf{E}^{(-)}(\mathbf{x},t)=\mathbf{E}^{(+)}(\mathbf{x},t)^\dagger$. In the rotating-wave approximation the Hamiltonian (1) is replaced by

$$H_{\mathrm{RWA}}=\tfrac{1}{2}\hbar\omega_0[\sigma_{z1}+\sigma_{z2}]+H_F - i\sum_{n=1}^{2}\sum_{\mathbf{k},\lambda}\left[\frac{2\pi\hbar\omega_k}{V}\right]^{1/2} \mathbf{d}\cdot\mathbf{e}_{\mathbf{k}\lambda} \times[\sigma_n^\dagger a_{\mathbf{k}\lambda}e^{i\mathbf{k}\cdot\mathbf{x}_n}-a_{\mathbf{k}\lambda}^\dagger\sigma_n e^{-i\mathbf{k}\cdot\mathbf{x}_n}] , \tag{25}$$

corresponding to the neglect of "energy-nonconserving" processes in which an atom makes a downward transition and a photon is annihilated or an atom makes an upward transition and a photon is created. In this approximation the Heisenberg operator $\mathbf{E}^{(+)}(\mathbf{x}_m,t), m=1,2$, is given by

$$\mathbf{E}^{(+)}(\mathbf{x}_m,t)=\mathbf{E}_0^{(+)}(\mathbf{x}_m,t) +i\sum_{\mathbf{k},\lambda}\left[\frac{2\pi\omega_k}{V}\right](\mathbf{d}\cdot\mathbf{e}_{\mathbf{k}\lambda}) \times\sum_{n=1}^{2}e^{i\mathbf{k}\cdot(\mathbf{x}_m-\mathbf{x}_n)} \times\int_0^t dt'\sigma_n(t')e^{i\omega_k(t'-t)} . \tag{26}$$

Replacing the mode summation by a continuous integration over modes $[\sum_{\mathbf{k}\lambda}\to(V/8\pi^3)\int d^3k]$ and restricting ourselves to the far field by simplicity, we obtain straightforwardly

$$\mathbf{E}_1^{(+)}(\mathbf{x}_2,t)=\frac{i}{\pi c^2 r}[\mathbf{d}-(\mathbf{d}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}] \times\int_0^t dt'\sigma_1(t')\int_0^\infty d\omega\,\omega^2\sin\frac{\omega r}{c}e^{i\omega(t'-t)} . \tag{27}$$

This field is *not retarded.* (The full field $\mathbf{E}=\mathbf{E}^{(+)}+\mathbf{E}^{(-)}$ is, of course, retarded.) However, since $\sigma_1(t')$ can be assumed to vary as $e^{-i\omega_0 t}$ in a rotating-wave approximation and only photon frequencies near ω_0 play a role in transition probabilities, it is reasonable to approximate (27) by extending the integration over ω to $-\infty$:

$$\mathbf{E}_1^{(+)}(\mathbf{x}_2,t)\cong\frac{i}{\pi c^2 r}[\mathbf{d}-(\mathbf{d}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}]\int_0^t dt'\sigma_1(t')\frac{1}{2i}\int_{-\infty}^{\infty}d\omega\,\omega^2[e^{i\omega(t'-t+r/c)}-e^{i\omega(t'-t-r/c)}] =-\frac{1}{c^2 r}[\mathbf{d}-(\mathbf{d}\cdot\hat{\mathbf{r}})\hat{\mathbf{r}}]\theta\left[t-\frac{r}{c}\right]\ddot{\sigma}_1\left[t-\frac{r}{c}\right] . \tag{28}$$

In other words, *within the RWA* an additional approximation—the extension of an integration over ω to include all negative as well as all positive frequencies—is required in order to obtain causal probability amplitudes, as noted recently by Hegerfeldt [6].

In the Schrödinger or interaction pictures the use of the RWA implies a restriction on the "essential states" and the approximation of including all negative frequencies is made in an integral involving familiar energy denominators [5]. Without some essential-state truncation of the Hilbert space, the solution of the two-atom problem in the Schrödinger (or interaction) picture is somewhat unwieldy for the purpose of proving causality. The Hamiltonian that enforces such a truncation involves the *unretarded* fields $\mathbf{E}^{(\pm)}$ in an essential way and consequently an approximation that effectively avoids a spurious nonretarded contribution, namely, the extension of frequency integrals to $-\infty$, must be made in order to elicit causality. By working in the Heisenberg picture with the full field $\mathbf{E}$ and without any RWA, on the other hand, one can easily show that the interaction is perfectly causal, as we have done. The one drawback of the Heisenberg picture is that it does not allow excitation probabilities to be calculated quite as directly as in the Schrödinger picture.

As noted, the approximation made in the Schrödinger picture in order to exhibit causality evolves an extension of a frequency integral with energy denominators to all negative energies (or frequencies). To establish the connection with the form of the approximation made in the Schrödinger picture, we approximate $\sigma_1(t')$ in (27) by $\sigma_1(t)\exp[-i\omega_0(t'-t)]$, the "Markovian approximation," which is part of the Weisskopf-Wigner approximation in the theory of spontaneous emission [7]:

$$d_j E^{(+)}_{1,j}(\mathbf{x}_2,t)\cong\frac{id^2}{\pi c^2 r}[1-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})^2]\sigma_1(t)\times\int_0^\infty d\omega\,\omega^2\sin\frac{\omega r}{c}\int_0^t dt' e^{i(\omega-\omega_0)(t'-t)}\,. \tag{29}$$

For $\omega_0 t \gg 1$ the integral over time can be replaced by its well-known approximation involving the δ function and Cauchy principal part [13]

$$\int_0^t dt' e^{i(\omega\pm\omega_0)(t'-t)}\rightarrow\pi\delta(\omega\pm\omega_0)-i\mathrm{P}\left[\frac{1}{\omega\pm\omega_0}\right] \tag{30}$$

or, equivalently,

$$\frac{1}{\omega\pm\omega_0-i\epsilon}=\pi\delta(\omega\pm\omega_0)-i\mathrm{P}\left[\frac{1}{\omega\pm\omega_0}\right]\quad(\epsilon\rightarrow 0^+)\,. \tag{31}$$

Thus

$$d_j E^{(+)}_{1,j}(\mathbf{x}_2,t)\cong\frac{id^2}{\pi c^2 r}[1-(\hat{\mathbf{d}}\cdot\hat{\mathbf{r}})^2]\sigma_1(t)\times\int_0^\infty d\omega\,\omega^2\frac{\sin(\omega r/c)}{\omega-\omega_0-i\epsilon}\,. \tag{32}$$

When the integral over ω is now extended to $-\infty$, one obtains causal, outgoing waves; this is precisely the approximation made by Milonni and Knight [5] and essentially the same approximation made by Fermi [3].

Regarding Hegerfeldt's theorem that the initially unexcited atom B "starts to move out of the ground state immediately and is thus influenced by atom A instantaneously," [6] we note that the theorem as proved applies *regardless of whether atom A is present* and therefore, in our opinion, should not be used as an argument against causality in the two-atom interaction. Such "immediate influences" are associated with the fact that the assumed initial state is not an eigenstate of the interacting atom-field system: a true eigenstate of the system involves an *admixture* of "bare" states such as the state with atom A excited, atom B unexcited, and the field in its unperturbed ground (vacuum) field, the state with A unexcited, B excited, and the field in its ground state, etc. Such admixtures involving excited, unperturbed atomic (and field) states occur even in the case of a *single* atom coupled to the field and are associated with phenomena, such as the Lamb shift, involving virtual transitions. In the two-atom case, however, there are no *interatomic* immediate influences before the time r/c after the system is presumed to be prepared in an eigenstate of the unperturbed atom-field system. This is the content of the operator equation (7), which makes no reference to any specific states, bare or dressed [14].

To illustrate the nature of the immediate influences when a system is supposed to be in an unperturbed state at $t=0$, consider the problem of a *single* two-level atom interacting with a single field mode. (The single-mode assumption here is made only for simplicity and is certainly not essential for the present discussion.) The Hamiltonian for this system— *without making the RWA* —is

$$H=\tfrac{1}{2}\hbar\omega_0\sigma_z+\hbar\omega a^\dagger a-iC(a-a^\dagger)(\sigma+\sigma^\dagger) \tag{33}$$

and the Heisenberg equations of motion for σ, σ_z, and a are

$$\dot{\sigma}=-i\omega_0\sigma+\frac{C}{\hbar}(a-a^\dagger)\sigma_z\,, \tag{34}$$

$$\dot{\sigma}_z=\frac{2C}{\hbar}(\sigma a-a^\dagger\sigma)+\mathrm{H.c.}, \tag{35}$$

and

$$\dot{a}=-i\omega a+\frac{C}{\hbar}(\sigma+\sigma^\dagger)\,. \tag{36}$$

It is convenient to define the slowly varying operator

$$S(t)=\sigma(t)e^{i\omega_0 t}\,, \tag{37}$$

in terms of which we obtain the following formal equation for the expectation value of the population difference operator σ_z:

$$\langle\dot{\sigma}_z(t)\rangle=\frac{2C^2}{\hbar^2}\int_0^t dt'[\langle S(t)S(t')\rangle e^{i(\omega-\omega_0)t'}e^{-i(\omega+\omega_0)t}+\langle S(t)S^\dagger(t')\rangle e^{i(\omega+\omega_0)(t'-t)}-\langle S^\dagger(t')S(t)\rangle e^{-i(\omega-\omega_0)(t'-t)}-\langle S(t')S(t)\rangle e^{-i(\omega+\omega_0)t'}e^{i(\omega-\omega_0)t}]+\mathrm{c.c.} \tag{38}$$

We have assumed an initial state such that the field is in its vacuum state, but no approximations have been made.

The terms involving $\omega+\omega_0$ do not arise when one makes the RWA. Even without the RWA such terms do not contribute to "real" transitions over times long compared with a few periods of oscillation. For short times, however, we have

$$\langle\dot{\sigma}_z(t)\rangle\cong\frac{2C^2}{\hbar^2}\langle S(0)S^\dagger(0)\rangle\int_0^t dt' e^{i(\omega+\omega_0)(t'-t)}+\mathrm{c.c.}=\frac{4C^2}{\hbar^2}\frac{\sin(\omega+\omega_0)t}{(\omega+\omega_0)}\,, \tag{39}$$

where we have used the operator identity $S^2(0)=0$ and also the expectation values $\langle S(0)S^\dagger(0)\rangle=1$ and $\langle S^\dagger(0)S(0)\rangle=0$ appropriate to the case of the atom initially in its lower state. Thus

$$\dot{P}(t)=\frac{1}{2}\langle\dot{\sigma}_z(t)\rangle=\frac{2C^2}{\hbar^2}\frac{\sin(\omega+\omega_0)t}{\omega+\omega_0} \tag{40}$$

and the probability of the atom being excited over short times is

$$P(t)=\frac{2C^2}{\hbar^2}\left[\frac{1}{\omega+\omega_0}\right]^2[1-\cos(\omega+\omega_0)t]\,. \tag{41}$$

For $\omega=\omega_0$ and $\omega_0 t \ll 1$, for instance,

$$P(t)\cong\frac{C^2}{\hbar^2\omega_0^2}\omega_0^2 t^2=\frac{C^2}{\hbar^2}t^2 , \tag{42}$$

i.e., there is a nonvanishing probability for $t \ll \omega_0^{-1}$ that the atom, initially in its lower state with no photons in the field, is excited. This is consistent with the energy-time uncertainty relation: for short enough times "energy-nonconserving" transitions are possible. Over times long in the sense of the energy-time uncertainty relation, of course, only energy-conserving processes contribute to real transition rates and the non-RWA terms manifest themselves only through virtual transitions contributing to energy-level shifts [15].

These results are entirely consistent with Hegerfeldt's theorem based on continuity requirements. The point we wish to make is that immediate influences of the type suggested by Hegerfeldt are present even in the absence of a second atom and that they pose no difficulty for the proof of causality in the Fermi problem, where we have shown without any approximations that there is no excitation of atom 2 due specifically to the initially excited atom 1.

IV. CAUSALITY IN PHOTODETECTION

The quantum theory of photodetection [1] leads to normally ordered field correlation functions such as

$$G_{ij}^{(1)}(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2)=\langle E_i^{(-)}(\mathbf{x}_1,t_1)E_j^{(+)}(\mathbf{x}_2,t_2)\rangle , \tag{43}$$

$$\begin{aligned}G_{ijkl}^{(2)}&(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2;\mathbf{x}_3,t_3;\mathbf{x}_4,t_4)\\&=\langle E_i^{(-)}(\mathbf{x}_1,t_1)E_j^{(-)}(\mathbf{x}_2,t_2)\\&\quad\times E_k^{(+)}(\mathbf{x}_3,t_3)E_l^{(+)}(\mathbf{x}_4,t_4)\rangle .\end{aligned} \tag{44}$$

As noted, the annihilation and creation parts of the field correspond precisely to the positive- and negative-frequency parts only in the absence of sources.

The positive-frequency part of the field $\mathbf{E}(\mathbf{x},t)$ may be defined formally as

$$\mathbf{E}^{(+)}(\mathbf{x},t)=\lim_{\epsilon\to 0^+}\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{dt'\mathbf{E}(\mathbf{x},t-t')}{t'-i\epsilon} . \tag{45}$$

Thus if $\mathbf{E}(\mathbf{x},t)$ has a Fourier expansion involving $e^{\pm i\omega t}$, the replacement of (45) by a contour integral along the real axis and over a semicircle in the upper half plane picks out the $e^{-i\omega t}$ components, whereas closure of the contour in the lower half plane ensures that there are no negative-frequency components $e^{i\omega t}$, $\omega>0$.

In the absence of sources the quantized field is given by (24) with $a_{\mathbf{k}\lambda}(t)=a_{\mathbf{k}\lambda}(0)e^{-i\omega_k t}$:

$$\mathbf{E}_0^{(+)}(\mathbf{x},t)=i\sum_{\mathbf{k},\lambda}\left[\frac{2\pi\hbar\omega_k}{V}\right]^{1/2}a_{\mathbf{k}\lambda}(0)e^{-i\omega_k t}e^{i\mathbf{k}\cdot\mathbf{x}}\mathbf{e}_{\mathbf{k}\lambda} . \tag{46}$$

With sources, however, $a_{\mathbf{k}\lambda}(t)\neq a_{\mathbf{k}\lambda}(0)e^{-i\omega_k t}$ [cf. Eq. (36)] and Eq. (24) does not in fact correspond exactly to the positive-frequency part of the field defined by (45). That is, Eqs. (24) and (45) in general define two different fields. Neither definition gives a "causal" (retarded) field, although of course the full electric-field operator $\mathbf{E}^{(+)}+\mathbf{E}^{(-)}$ is retarded and is the same regardless of whether (24) or (45) is used to define $\mathbf{E}^{(\pm)}$. The nonretarded character of $\mathbf{E}^{(\pm)}(\mathbf{x},t)$ has raised objections about the general validity of the standard theory of photodetection based on normally ordered field correlation functions [2]. We now address these objections.

Consider first the simplest case, the measurement of the intensity of an optical field. For the detector we assume at first a two-level atom for which, under the assumption that the atom is initially in its lower state [Eq. (4)],

$$\begin{aligned}\langle\dot\sigma_z(t)\rangle=-\frac{4}{\hbar^2}d_j d_k\,\mathrm{Re}\int_0^t dt'\langle E_j(\mathbf{x},t)E_k(\mathbf{x},t')\\\times\sigma_z(t')\rangle e^{i\omega_0(t'-t)} .\end{aligned} \tag{47}$$

We are interested of course in the more practical situation in which the detector is not a two-level system but in fact has a continuum of possible final states, such that stimulated emission from an excited state is negligible compared with absorption from the ground state and the detector is taken to be unsaturable. In the context of our idealized two-level system, this means we can take $\sigma_z(t')\cong\sigma_z(0)$ in (47) and use the assumption $\sigma_z(0)|\psi_i\rangle=-|\psi_i\rangle$ that the detector atom is initially in its ground state:

$$\langle\dot\sigma_z(t)\rangle\cong\frac{4}{\hbar^2}d_j d_k\,\mathrm{Re}\int_0^t dt'\langle E_j(\mathbf{x},t)E_k(\mathbf{x},t')\rangle e^{i\omega_0(t'-t)} . \tag{48}$$

Here the field may be written

$$E_j(\mathbf{x},t)=E_{0,j}(\mathbf{x},t)+E_{\mathrm{RR},j}(\mathbf{x},t)+E_{s,j}(\mathbf{x},t) , \tag{49}$$

where $E_{s,j}(\mathbf{x},t)$ is the "external" source field due, for instance, to a thermal source or a laser. Since the "full" source field consisting of both positive- and negative-frequency parts is of course retarded [cf. Eq. (7)], we can write $E_{s,j}(\mathbf{x},t)=F_j(\mathbf{x},t)\theta(t-r/c)$ and therefore

$$E_j(\mathbf{x},t)=E_{0,j}(\mathbf{x},t)+E_{\mathrm{RR},j}(\mathbf{x},t)+F_j(\mathbf{x},t)\theta\left[t-\frac{r}{c}\right] . \tag{50}$$

We are simply indicating explicitly here the retarded nature of the field from the source at a distance r from the detector atom at $\mathbf{x}$.

We can now proceed as in the example of two atoms, writing out all the interference terms that appear when (50) is used in (48). We first write

$$E_{0,j}(\mathbf{x},t)=E_{0,j}^{(+)}(\mathbf{x},t)+E_{0,j}^{(-)}(\mathbf{x},t) , \tag{51}$$

$$E_{\mathrm{RR},j}(\mathbf{x},t)=E_{\mathrm{RR},j}^{(+)}(\mathbf{x},t)+E_{\mathrm{RR},j}^{(-)}(\mathbf{x},t) , \tag{52}$$

$$F_j(\mathbf{x},t)=F_j^{(+)}(\mathbf{x},t)+F_j^{(-)}(\mathbf{x},t) , \tag{53}$$

where the positive-frequency parts of the fields $\mathbf{E}_0$, $\mathbf{E}_{\mathrm{RR}}$, and $\mathbf{F}$ are defined formally by (45) and the negative-frequency parts by Hermitian conjugation. The positive- and negative-frequency parts, therefore, have Fourier components $e^{-i\omega t}$ and $e^{i\omega t}$, respectively, where all fre-

quencies ω are of course positive definite. From (30) it then follows that only the normally ordered combination $\langle E_j^{(-)}(\mathbf{x},t)E_k^{(+)}(\mathbf{x},t')\rangle$ will contribute to (48) for times $t \gg \omega_0^{-1}$, i.e., only this combination of positive- and negative-frequency parts of the field produces energy-conserving transitions:

$$\langle \dot{\sigma}_z(t)\rangle \cong \frac{4}{\hbar^2} d_j d_k \,\mathrm{Re}\int_0^t dt' \left[\langle E^{(-)}_{\mathrm{RR},j}(\mathbf{x},t)E^{(+)}_{\mathrm{RR},j}(\mathbf{x},t')\rangle + \theta\left(t'-\frac{r}{c}\right)\langle E^{(-)}_{\mathrm{RR},j}(\mathbf{x},t)F_j^{(+)}(\mathbf{x},t')\rangle + \theta\left(t-\frac{r}{c}\right)\langle F_j^{(-)}(\mathbf{x},t)E^{(+)}_{\mathrm{RR},k}(\mathbf{x},t')\rangle + \theta\left(t-\frac{r}{c}\right)\theta\left(t'-\frac{r}{c}\right)\langle F_j^{(-)}(\mathbf{x},t)F_j^{(+)}(\mathbf{x},t')\rangle \right] e^{i\omega_0(t'-t)}, \tag{55}$$

where we have used $E_0^{(+)}(\mathbf{x},t)|\psi_i\rangle = \langle\psi_i|E_0^{(-)}(\mathbf{x},t)=0$. From (19) we have, furthermore,

$$\begin{aligned} d_j d_k \langle E^{(-)}_{\mathrm{RR},j}(\mathbf{x},t)E^{(+)}_{\mathrm{RR},k}(\mathbf{x},t')\rangle &\cong (\hbar\beta)^2\langle \sigma^\dagger(t)\sigma(t')\rangle \\ &\cong (\hbar\beta)^2\langle \sigma^\dagger(0)\sigma(0)\rangle e^{-i\omega_0(t'-t)} = 0 \end{aligned} \tag{56}$$

and in similar fashion $\langle E^{(-)}_{\mathrm{RR},j}(\mathbf{x},t)F_j^{(+)}(\mathbf{x},t')\rangle \cong 0$, $\langle F_j^{(-)}(\mathbf{x},t)E^{(+)}_{\mathrm{RR},k}(\mathbf{x},t')\rangle \cong 0$ under the perturbation-theoretic assumption, already made in the step from (47) to (48), that the detector atom is only weakly perturbed by the external field. Thus

$$\langle \dot{\sigma}_z(t)\rangle \cong \frac{4}{\hbar^2} d_j d_k \theta\left(t-\frac{r}{c}\right) \times \mathrm{Re}\int_{r/c}^t dt' \langle F_j^{(-)}(\mathbf{x},t)F_j^{(+)}(\mathbf{x},t')\rangle \times e^{i\omega_0(t'-t)}. \tag{57}$$

There are three points we wish to stress about the simple result (57). The first is that the appearance of the step function $\theta(t-r/c)$ is *exact*, i.e., the influence of the external field on the atom is properly causal independently of the approximations made in going from the exact expression (47) to the approximation (57). This is obvious from (50), and in fact the proof of causality in the Fermi model is seen to be a special case where the "external" field is just the field from a second atom. Second, the appearance of a normally ordered field correlation function *is* an approximation—the consequence of considering energy-conserving transitions at times $t \gg \omega_0^{-1}$ long enough for the transition frequency to be resolvable in the sense of the energy-time uncertainty relation. Finally, we note that it is important, for the purpose of exhibiting causality, to include the step function $\theta(t-r/c)$ explicitly in Eq. (50) *before* making the approximation leading to the normally ordered field correlation function: without the step function the result (57) for the excitation rate in second-order perturbation theory is *not* manifestly causal, for the positive- and negative-frequency parts of the field are themselves not retarded, as already noted.

$$\langle \dot{\sigma}_z(t)\rangle \cong \frac{4}{\hbar^2} d_j d_k \,\mathrm{Re}\int_0^t dt' \langle E_j^{(-)}(\mathbf{x},t)E_k^{(+)}(\mathbf{x},t')\rangle \times e^{i\omega_0(t'-t)}. \tag{54}$$

From (51)–(53), therefore,

These points carry over to the case of a more realistic model for a photodetector, which we now consider.

For a system with a ground-state energy E_g and a manifold of excited state $\{E_a\}$, (57) generalizes to the following expression for the rate $\dot{P}(t)$ at which electrons make transitions out of the ground state:

$$\dot{P}(t) \cong \frac{2}{\hbar^2}\sum_a d_{ag,j} d_{ga,k} R(a)\theta\left(t-\frac{r}{c}\right) \times \mathrm{Re}\int_{r/c}^t dt' \langle F_j^{(-)}(\mathbf{x},t)F_k^{(+)}(\mathbf{x},t')\rangle \times e^{i\omega_{ag}(t'-t)}, \tag{58}$$

where $\omega_{ag} = (E_a - E_g)/\hbar$ and $R(a)$ gives the probability, which will depend on the physical characteristics of the detector, of actually counting a photoelectron of energy E_a. Following Glauber [1], we define a sensitivity function

$$s_{jk}(\omega) \equiv 2\pi\frac{1}{\hbar^2}\sum_a R(a) d_{ag,j} d_{ga,k}\delta(\omega-\omega_{ag}) \tag{59}$$

in terms of which

$$\dot{P}(t) \cong \theta\left(t-\frac{r}{c}\right)\frac{2}{2\pi}\mathrm{Re}\int_{r/c}^t dt' \langle F_j^{(-)}(\mathbf{x},t)F_k^{(+)}(\mathbf{x},t')\rangle \times \int_{-\infty}^{\infty} d\omega\, s_{jk}(\omega) e^{i\omega(t'-t)}. \tag{60}$$

Two approximations have been made in the derivation of (60): (i) the replacement of $\langle\sigma_z(t')\rangle$ by its initial value $\langle\sigma_z(0)\rangle$, amounting to conventional second-order perturbation theory for the calculation of absorption rates, and (ii) the restriction to energy-conserving transitions implicit in the assumption $t \gg \omega_0^{-1}$ made in the two-level formulation. Except for the appearance of the step function $\theta(t-r/c)$ and the time r/c appearing as the lower limit of integration in (60), our result is essentially identical to that of Glauber [1]. Glauber defines an ideal broadband detector such that $s_{jk}(\omega) \cong \mathrm{const} \equiv s_{jk}$ for all frequencies ω (or actually for all frequencies within the

bandwidth of the external field [1]), so that

$$\int_{-\infty}^{\infty} d\omega\, s_{jk}(\omega) e^{i\omega(t'-t)} = 2\pi s_{jk}\delta(t'-t) \tag{61}$$

and

$$\dot{P}(t) \cong \theta\left[t-\frac{r}{c}\right] s_{jk}\langle F_j^{(-)}(\mathbf{x},t) F_k^{(+)}(\mathbf{x},t)\rangle \ . \tag{62}$$

A. Causality

As already noted, the appearance of the step function $\theta(t-r/c)$ in (57)–(60) is exact. Moreover, this causal feature is implicit in the original formulation by Glauber [1] since he begins with the *full* electric-field operator and proceeds to normally ordered field correlation functions involving $\mathbf{E}^{(\pm)}$ under the condition of energy-conserving transitions. In other words, properly retarded effects of the external field on the detector responding to it are implicit in the Glauber theory, although as a practical matter one simply writes expressions such as

$$\dot{P}(t) \cong s_{jk}\langle F_j^{(-)}(\mathbf{x},t) F_k^{(+)}(\mathbf{x},t)\rangle \tag{63}$$

instead of (62) and similarly for higher-order correlation functions.

This contradicts the claim by Bykov and Tatarskii [2] that the Glauber theory violates "causality." Their claim is based on the presumption that the photon-counting rate, for instance, fundamentally involves (63) rather than (62) and therefore that causality is violated owing to the nonretarded character of $\mathbf{F}^{(\pm)}(\mathbf{x},t)$. As we have shown, causality in the sense meant in this context is trivially implicit in the Glauber theory, beginning as it does with the full, properly retarded electric field. Normally ordered field correlation functions appear at a later stage in the theory and follow from a long-time, energy-conservation approximation much the same as in the derivation of Fermi's golden rule.

The putative violation of causality, according to Bykov and Tatarskii (BT) [1], "requires changing the determination of the correlation functions and in particular the velocity of photocounting." They suggest the replacement of (63), for instance, by

$$\dot{P}_{\mathrm{BT}}(t) = s_{jk}\langle :E_{s,j}(\mathbf{x},t) E_{s,k}(\mathbf{x},t):\rangle \ , \tag{64}$$

where the colons as usual denote normal ordering. The full external field operator $\mathbf{E}_s(\mathbf{x},t)$ is employed in order to guarantee causality, which, as we have shown, is in fact already implicit in the conventional theory. The normal ordering of the field product in (64) is employed "to avoid an infinite contribution of vacuum fluctuations" [2]. Normal ordering is discussed in the following subsection.

The replacement of (4) by (54) involves a rotating-wave approximation, which of course is not an essential part of the standard theory. Retention of non-RWA terms leads to expressions such as

$$\dot{P}(t) = s_{jk}\langle E_{s,j}(\mathbf{x},t) E_{s,k}(\mathbf{x},t)\rangle \ , \tag{65}$$

which differs from (64) in that it includes an antinormally ordered term $\langle E_{s,j}^{(+)}(\mathbf{x},t) E_{s,k}^{(-)}(\mathbf{x},t)\rangle$. As discussed below, this term does not arise in a formulation of photodetection theory that accounts for dissipation as well as fluctuations in the response of the detector. In other words, the modification of standard photon-counting theory suggested by Bykov and Tatarskii, stemming from the erroneous notion that the standard theory violates causality, amounts only to dropping the rotating-wave approximation and does not lead to anything essentially new. Aside from this it is not clear, as a practical matter, what is gained in this context by going beyond the RWA: optical pulses so short as to necessitate non-RWA terms are also short compared with resolving times of even the fastest photoconductive detectors [16].

The fact that an *ab initio* use of the RWA in the Hamiltonian leads to an apparent violation of causality is thus seen to be the basis not only of Hegerfeldt's criticism of previous work on the Fermi model, but also the criticism of the Glauber theory of photodetection by Bykov and Tatarskii. In either context there are no violations of causality when one works from the start with a complete Hamiltonian including the possibility of virtual (non-RWA) transitions.

B. Normal ordering

We have noted that Bykov and Tatarskii [2] introduce normal ordering as in (64) to eliminate the infinite quantity $\langle E_j^{(+)}(\mathbf{x},t) E_k^{(-)}(\mathbf{x},t)\rangle$, the infinity arising from the vacuum field $E_{0,j}(\mathbf{x},t)$. We will now show that such a term appears in a more fundamental formulation of photon-counting theory *even when normal ordering is employed.* However, as we shall see, such a term is unphysical in this context and does not appear at all in the way claimed by Bykov and Tatarskii.

Our derivation of $\dot{P}(t)$ has led to normal ordering as a consequence of the RWA. If we proceed directly from the two-level result (47) to its multilevel generalization without any RWA, we obtain instead of (58)

$$\begin{aligned}\dot{P}(t) &\cong \frac{2}{\hbar^2}\sum_a d_{ag,j} d_{ga,k} R(a)\,\mathrm{Re}\int_{r/c}^{t} dt'\langle E_j(\mathbf{x},t) E_k(\mathbf{x},t')\rangle e^{i\omega_{ag}(t'-t)} \\ &= \frac{2}{2\pi}\,\mathrm{Re}\int_{r/c}^{t} dt'\langle E_j(\mathbf{x},t) E_k(\mathbf{x},t')\rangle \int_{-\infty}^{\infty} d\omega\, s_{jk}(\omega) e^{i\omega(t'-t)} \\ &\to s_{jk}\langle E_j(\mathbf{x},t) E_k(\mathbf{x},t)\rangle \ ,\end{aligned} \tag{66}$$

where in the last step we have gone to the limit of ideal broadband detection. Bykov and Tatarskii propose to eliminate the divergence by normal ordering, replacing (66) by (64).

Care must be exercised in applying the non-RWA expression (66). For one thing, the vacuum field has an infinite bandwidth and therefore the limit of idealized broadband detection is inapplicable. Just as important is the fact that in dealing with vacuum field contributions we cannot in general ignore radiation reaction, i.e., we cannot completely separate the vacuum fluctuations driving the detector from its own internal dissipation [7]. Indeed the result (66) as it stands does not distinguish among the vacuum, radiation reaction, or external field due to the sources causing the photoabsorption. We will now proceed more carefully to an expression for the absorption rate without any RWA. By accounting for radiation reaction, we will show that the infinite term that Bykov and Tatarskii propose to eliminate by normal ordering is present even with normal ordering and moreover is present regardless of what ordering is employed. It has the form given by Bykov and Tatarskii only in the limit of perfect broadband detection, which is not applicable to this term. More importantly, the term in question will be shown to be without physical significance for photodetection.

It is convenient for the present discussion to write the field operator at point $\mathbf{x}$ as

$$E_j(\mathbf{x},t)=E_{\mathrm{RR},j}(\mathbf{x},t)+E_{e,j}(\mathbf{x},t)\ , \tag{67}$$

where the external field $\mathbf{E}_e(\mathbf{x},t)$ accounts for the vacuum field as well as the fields from all sources except the atom at $\mathbf{x}$. Since equal-time matter and field operators commute, we can write [cf. Eq. (2)]

$$\dot{\sigma}_z(t)=-\frac{2i}{\hbar}d_j[E_j^{(-)}(\mathbf{x},t)\sigma(t)+\sigma(t)E_j^{(+)}(\mathbf{x},t)]+\mathrm{H.c.}\ , \tag{68}$$

where $\mathbf{E}^{(\pm)}(\mathbf{x},t)$ are defined as in (24) and we have chosen a normal ordering of these field operators. Thus

$$\begin{aligned}\langle\dot{\sigma}_z(t)\rangle&=-\frac{2i}{\hbar}d_j[\langle E_j^{(-)}(\mathbf{x},t)\sigma(t)\rangle+\langle\sigma(t)E_j^{(+)}(\mathbf{x},t)\rangle]+\mathrm{c.c.}\\&=-\frac{2i}{\hbar}d_j[\langle E_{e,j}^{(-)}(\mathbf{x},t)\sigma(t)\rangle+\langle\sigma(t)E_{e,j}^{(+)}(\mathbf{x},t)\rangle]+\mathrm{c.c.}\\&\quad-\frac{2i}{\hbar}d_j[\langle E_{\mathrm{RR},j}^{(-)}(\mathbf{x},t)\sigma(t)\rangle+\langle\sigma(t)E_{\mathrm{RR},j}^{(+)}(\mathbf{x},t)\rangle]+\mathrm{c.c.}\\&\equiv\langle\dot{\sigma}_z(t)\rangle_e+\langle\dot{\sigma}_z(t)\rangle_{\mathrm{RR}}\ ,\end{aligned} \tag{69}$$

where $\langle\dot{\sigma}_z(t)\rangle_e$ and $\langle\dot{\sigma}_z(t)\rangle_{\mathrm{RR}}$ correspond to the terms involving $E_{e,j}^{(\pm)}(\mathbf{x},t)$ and $E_{\mathrm{RR},j}^{(+)}(\mathbf{x},t)$, respectively. We now calculate these two contributions to $\langle\dot{\sigma}_z(t)\rangle$ up to second order in the matter-field coupling.

To evaluate $\langle\dot{\sigma}_z(t)\rangle_e$ we solve the equation [cf. Eq. (3)]

$$\dot{\sigma}(t)=-i\omega_0\sigma(t)-\frac{i}{\hbar}d_kE_k(\mathbf{x},t)\sigma_z(t) \tag{70}$$

to lowest order in the Rabi frequency $d_kE_k(\mathbf{x},t)/\hbar$ and use the result in the expression for $\langle\dot{\sigma}_z(t)\rangle_e$ given by (69). Thus

$$\sigma(t)\cong\sigma(0)e^{-i\omega_0t}-\frac{i}{\hbar}d_k\int_0^t dt'E_k(\mathbf{x},t')\sigma_z(0)e^{i\omega_0(t'-t)} \tag{71}$$

and consequently

$$\begin{aligned}\langle\dot{\sigma}_z(t)\rangle_e\cong&-\frac{2i}{\hbar}d_j[\langle E_{e,j}^{(-)}(\mathbf{x},t)\sigma(0)\rangle e^{-i\omega_0t}+\langle\sigma(0)E_{e,j}^{(+)}(\mathbf{x},t)\rangle e^{-i\omega_0t}]+\mathrm{c.c.}\\&-\frac{2}{\hbar^2}d_jd_k\int_0^t dt'\langle E_{e,j}^{(-)}(\mathbf{x},t)E_k(\mathbf{x},t')\sigma_z(0)\rangle e^{i\omega_0(t'-t)}+\mathrm{c.c.}\\&-\frac{2}{\hbar^2}d_jd_k\int_0^t dt'\langle E_k(\mathbf{x},t')\sigma_z(0)E_{e,j}^{(+)}(\mathbf{x},t)\rangle e^{i\omega_0(t'-t)}+\mathrm{c.c.}\end{aligned} \tag{72}$$

Since the detector atom at $t=0$ may be assumed to be uncorrelated from the external field at $t=0$ and $\langle\sigma(0)\rangle=0$ for the atom initially in its ground state,

$$\langle E_{e,j}^{(-)}(\mathbf{x},t)\sigma(0)\rangle=\langle E_{e,j}^{(-)}(\mathbf{x},t)\rangle\langle\sigma(0)\rangle=0 \tag{73}$$

and likewise $\langle\sigma(0)E_{e,j}^{(+)}(\mathbf{x},t)\rangle=0$. To remain to second order in the transition matrix elements, furthermore, we approximate $E_k(\mathbf{x},t')$ in (72) by the external field $E_{e,k}(\mathbf{x},t')$. Thus, assuming the detector atom is initially in its ground state, so that $\sigma_z(0)$ may effectively be replaced by -1,

$$\langle \dot{\sigma}_z(t)\rangle_e \cong \frac{4}{\hbar^2} d_j d_k \,\mathrm{Re}\int_0^t dt' [\langle E_{e,j}^{(-)}(\mathbf{x},t) E_{e,k}(\mathbf{x},t')\rangle + \langle E_{e,k}(\mathbf{x},t') E_{e,j}^{(+)}(\mathbf{x},t)\rangle] e^{i\omega_0(t'-t)} \tag{74}$$

or, equivalently,

$$\dot{P}_e(t) \cong \frac{2}{\hbar^2} d_j d_k \,\mathrm{Re}\int_0^t dt' \langle :E_{e,j}(\mathbf{x},t) E_{e,k}(\mathbf{x},t'):\rangle e^{i\omega_0(t'-t)}$$
$$\rightarrow \frac{2}{2\pi} \,\mathrm{Re}\int_0^t dt' \langle :E_{e,j}(\mathbf{x},t) E_{e,k}(\mathbf{x},t'):\rangle \int_{-\infty}^{\infty} d\omega\, s_{jk}(\omega) e^{i\omega(t'-t)} \tag{75}$$

when we proceed as in (57)–(60).

To evaluate the contribution $\dot{P}_{\mathrm{RR}}(t) = \frac{1}{2}\langle \dot{\sigma}_z(t)\rangle_{\mathrm{RR}}$ from radiation reaction, we begin with the exact expression [7]

$$d_j E_{\mathrm{RR},j}^{(+)}(\mathbf{x},t) = \frac{2id^2}{3\pi c^3} \int_0^{\infty} d\omega\, \omega^3 \int_0^t dt' \sigma_x(t') e^{i\omega(t'-t)} . \tag{76}$$

This result follows from the $r \rightarrow 0$ limit of the non-RWA form of (27), i.e., (27) with σ replaced by $\sigma + \sigma^\dagger = \sigma_x$. We now make the approximation, based as usual on the assumption of weak matter-field coupling, that

$$\sigma_x(t') \cong \sigma(t) e^{-i\omega_0(t'-t)} + \sigma^\dagger(t) e^{i\omega_0(t'-t)} . \tag{77}$$

Then

$$-\frac{2i}{\hbar} d_j E_{\mathrm{RR},j}^{(+)}(\mathbf{x},t) \cong \frac{4d^2}{3\pi\hbar c^3} \left[\sigma(t) \int_0^t dt' \int_0^{\infty} d\omega\, \omega^3 e^{i\omega(t'-t)} e^{-i\omega_0(t'-t)} \right.$$
$$\left. + \sigma^\dagger(t) \int_0^t dt' \int_0^{\infty} d\omega\, \omega^3 e^{i\omega(t'-t)} e^{i\omega_0(t'-t)} \right] \tag{78}$$

and therefore

$$-\frac{2i}{\hbar} d_j \langle \sigma(t) E_{\mathrm{RR},j}^{(+)}(\mathbf{x},t)\rangle \cong \frac{4d^2}{3\pi\hbar c^3} \int_0^t dt' \int_0^{\infty} d\omega\, \omega^3 e^{i\omega(t'-t)} e^{i\omega_0(t'-t)} \tag{79}$$

and

$$-\frac{2i}{\hbar} d_j \langle E_{\mathrm{RR},j}^{(-)}(\mathbf{x},t)\sigma(t)\rangle \cong 0 \tag{80}$$

when we use $\sigma^2(t) = 0$ and $\langle \sigma(t)\sigma^\dagger(t)\rangle \cong \langle \sigma(0)\sigma^\dagger(0)\rangle = 1$ under the assumption that the detector atom is initially in its ground state.

Now from (46) and the usual free-space mode continuum limit it follows that

$$\int_0^{\infty} d\omega\, \omega^3 e^{i\omega(t'-t)} = \frac{3\pi c^3}{2\hbar d^2} d_j d_k \langle E_{0,j}^{(+)}(\mathbf{x},t) E_{0,k}^{(-)}(\mathbf{x},t')\rangle = \frac{3\pi c^3}{2\hbar d^2} d_j d_k \langle E_{0,j}(\mathbf{x},t) E_{0,k}(\mathbf{x},t')\rangle . \tag{81}$$

Therefore we can write $\dot{P}_{\mathrm{RR}}(t)$ as defined by (69) in terms of the vacuum field correlation function:

$$\dot{P}_{\mathrm{RR}}(t) = \frac{1}{2}\langle \dot{\sigma}_z(t)\rangle_{\mathrm{RR}} \cong \frac{2}{\hbar^2} d_j d_k \,\mathrm{Re}\int_0^t dt' \langle E_{0,j}(\mathbf{x},t) E_{0,k}(\mathbf{x},t')\rangle e^{i\omega_0(t'-t)}$$
$$\rightarrow \frac{2}{2\pi} \,\mathrm{Re}\int_0^t dt' \langle E_{0,j}(\mathbf{x},t) E_{0,k}(\mathbf{x},t')\rangle \int_{-\infty}^{\infty} d\omega\, s_{jk}(\omega) e^{i\omega(t'-t)} . \tag{82}$$

The complete expression for the rate at which the detector is excited is then

$$\dot{P}(t) = \dot{P}_e(t) + \dot{P}_{\mathrm{RR}}(t)$$
$$= \frac{2}{2\pi} \,\mathrm{Re}\int_0^t dt' \langle :E_{e,j}(\mathbf{x},t) E_{e,k}(\mathbf{x},t'):\rangle \int_{-\infty}^{\infty} d\omega\, s_{jk}(\omega) e^{i\omega(t'-t)}$$
$$+ \frac{2}{2\pi} \,\mathrm{Re}\int_0^t dt' \langle E_{0,j}(\mathbf{x},t) E_{0,k}(\mathbf{x},t')\rangle \int_{-\infty}^{\infty} d\omega\, s_{jk}(\omega) e^{i\omega(t'-t)} \tag{83}$$

and for a broadband detector

$$\dot{P}(t)=s_{jk}\langle :E_j(\mathbf{x},t)E_k(\mathbf{x},t):\rangle + s_{jk}\langle E_{0,j}(\mathbf{x},t)E_{0,k}(\mathbf{x},t)\rangle \ . \tag{84}$$

Thus the *complete* excitation rate, including the contribution from radiation reaction, leads in the limit of an idealized broadband detector to an infinite vacuum field contribution that Bykov and Tatarskii [2] propose to eliminate by normal ordering. We see from (75) that the external field contribution does lead naturally to a normally ordered correlation function and therefore has no vacuum field contribution. However, the reaction field, which of course must also be included, gives a contribution that is equivalent, owing to the fluctuation-dissipation relation between radiation reaction and vacuum field fluctuations [7], to a vacuum field term.

We have obtained this result using normal ordering in order to show that normal ordering does not eliminate the infinite contribution as claimed by Bykov and Tatarskii. In fact, the result (84) is independent of the ordering of equal-time matter-field operators since these operators commute.

It must be noted, however, that the infinite contribution to (84) is only an artifice of the broadband limit. The latter rests on the approximation that the detector response is relatively flat within the bandwidth of the field. This approximation cannot be made for the vacuum field, which has *infinite* bandwidth. We now consider more carefully the effect of the vacuum field without taking the broadband limit as in (84).

Returning to (79), but without taking the broadband detector limit, and using (30),

$$\begin{aligned}-\frac{2i}{\hbar}d_j\langle\sigma(t)E^{(+)}_{\mathrm{RR},j}(\mathbf{x},t)\rangle &\cong \frac{4d^2}{3\pi\hbar c^3}\int_0^\infty d\omega\,\omega^3\left[\pi\delta(\omega+\omega_0)-i\mathrm{P}\left[\frac{1}{\omega+\omega_0}\right]\right]\\ &=-\frac{4id^2}{3\pi\hbar c^3}\mathrm{P}\int_0^\infty\frac{d\omega\,\omega^3}{\omega+\omega_0}\ .\end{aligned} \tag{85}$$

Therefore, for a ground-state atom,

$$\langle\dot{\sigma}_z(t)\rangle_{\mathrm{RR}}=-2\,\mathrm{Re}\left[\frac{2i}{\hbar}d_j\langle\sigma(t)E^{(+)}_{\mathrm{RR},j}(\mathbf{x},t)\rangle\right]=0\ . \tag{86}$$

The second term in large square brackets in (85) corresponds to the ground-state level shift [17] and does not, as (86) shows, contribute to the transition rate. Moreover, being related to the Lamb shift, it is independent of the field due to sources other than the detector itself and therefore is not physically relevant to photodetection.

The interplay of vacuum field fluctuations and radiation reaction as exemplified by Eq. (86) is well known [7]. The fact that there is no vacuum field contribution to an absorption rate, for instance, is actually a consequence of the cancellation of such a contribution by the effect of radiation reaction. There is therefore no "spontaneous absorption" associated with the vacuum field. Depending on the choice of operator orderings [7], one can ascribe radiation level shifts to the vacuum field, but, as shown by (86), for instance, such shifts are quite distinct from energy-conserving transitions that would be registered by a photodetector. It is straightforward to generalize the preceding results for photon-counting theory to higher-order field correlation functions. For the rate at which photons are counted jointly at two identical broadband detectors at $(\mathbf{x}_1,t_1)$ and $(\mathbf{x}_2,t_2)$, for instance, we obtain

$$\begin{aligned}R(\mathbf{x}_1,t_1;\mathbf{x}_2,t_2)=s_{jm}s_{kl}\theta\left[t_1-\frac{r_1}{c}\right]\theta\left[t_2-\frac{r_2}{c}\right]&\\ \times\langle E^{(-)}_j(\mathbf{x}_1,t_1)E^{(-)}_k(\mathbf{x}_2,t_2)&\\ \times E^{(+)}_l(\mathbf{x}_1,t_1)E^{(+)}_m(\mathbf{x}_1,t_1)\rangle\ ,&\end{aligned} \tag{87}$$

where r_1 and r_2 are the distances from the (point) source to $\mathbf{x}_1$ and $\mathbf{x}_2$. Once again causality is manifested explicitly in the appearance of the step functions $\theta(t_j-r_j/c)$. The RWA has been employed in writing this result and again it is the RWA that leads naturally to a normally ordered field correlation function.

V. RELATIONS TO PREVIOUS WORK AND SUMMARY

The rotating-wave approximation can lead to apparent violations of causality, i.e., retardation, in theoretical analyses involving the propagation of light. Such violations, of course, are artificial and are eliminated by working from the start with a Hamiltonian that accounts for non-RWA terms associated primarily with virtual transitions and level shifts.

The Heisenberg picture allows one to include non-RWA terms very easily, at least formally, and therefore to explicitly account for the finite propagation of light and causality. Using the Heisenberg picture, we have considered the two-atom model of light propagation treated by Fermi and others and have shown explicitly that standard QED theory contains the expected causality. We have shown how the results of Milonni and Knight, which reduce to those of Fermi under certain simplifying approximations [5], may be derived in the Heisenberg picture.

We have also shown that causality is implicit in the standard photodetection theory as originally formulated by Glauber. The non-RWA contributions obtained by Bykov and Tatarskii [2], based on an erroneous claim that the Glauber theory violates causality, are in principle already contained in the standard theory but are negligible in the vast majority of photon-counting experiments of any practical interest. The proposed modifications of the standard theory [2] do not therefore amount to anything new, although we have argued on theoretical grounds that the normal ordering in the "modified" theory is also based on erroneous presumptions and, in particular, the neglect of the reaction fields of the detector electrons.

Our results concerning the RWA are not inconsistent with those of De Haan or Compagno, Passante, and Persico [4]. The latter authors, for instance, show that if non-RWA, energy-nonconserving terms are dropped at

the outset from the Hamiltonian, then the field operators calculated from the approximate Hamiltonian are not retarded. Our use of the term RWA here is somewhat different, as noted following Eq. (18), in that it refers to the omission of counterrotating terms only after calculating the field based on the full Hamiltonian including energy-nonconserving terms. In other words, the neglect of counterrotating terms is simply made at a later point in the calculation. Regardless of the point in the calculations where counterrotating terms are dropped, such terms are fundamentally necessary to our approach, as in the work of De Haan and Compagno, Passante, and Persico for the formal demonstration of causality.

Our approach is motivated primarily by the considerations in Sec. IV, where the fully retarded, non-RWA electric-field operator is used in writing (47) and a normally ordered field product appear only *after* approximations akin to those used in the derivation of Fermi's golden rule. The causal form of this product, that is, the appearance of the θ function in (62), for instance, follows from our use of the non-RWA electric-field operator. Its normally ordered form follows from energy conservation or, in other words, a "long-time" approximation. Had we made the RWA straightaway in the Hamiltonian, the positive- and negative-frequency parts of the field, and therefore their normally ordered product, would not be causal. In other words, we have followed an approach that, in our opinion, shows most clearly how a trivial modification of standard photodetection theory exhibits its correctly causal character.

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[7] See, for instance, P. W. Milonni, *The Quantum Vacuum: An Introduction to Quantum Electrodynamics* (Academic, San Diego, 1994), and references therein. We assume, without any loss of generality, for our purposes that the transition dipole moment **d** is real.

[8] The field operator **E** here actually corresponds to the electric displacement vector, but for our purposes it will not be essential to distinguish between **E** and **D**. For a discussion of this point see E. A. Power, in *Physics and Probability,* edited by W. T. Grandy, Jr. and P. W. Milonni (Cambridge University Press, Cambridge, 1993). It should also be noted that all our results follow also from the minimal coupling form of the atom-field interaction involving the vector potential: in the Coulomb gauge the static, unretarded dipole-dipole interaction between two atoms is exactly canceled by a static coupling arising from the $\mathbf{A}\cdot\mathbf{p}$ coupling. We intend to discuss these points in more detail in a future paper dealing with dispersive dielectric media.

[9] The "sudden switching" assumption is discussed in the context of the natural line shape by P. W. Milonni, R. J. Cook, and J. R. Ackerhalt, Phys. Rev. A **40**, 3764 (1989).

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[11] $\mathbf{E}_1^{(+)}(\mathbf{x}_2,t)$ also has a (non-RWA) term involving $\sigma_1^\dagger$, but this term plays no role in the present discussion. See J. R. Ackerhalt, Phys. Rev. A **17**, 471 (1978).

[12] We follow a standard notation where V is a quantization volume, eventually allowed to become infinite, and $\mathbf{e}_{\mathbf{k}\lambda}$ is a unit polarization vector. The field is quantized in free space, but quantization in terms of any other complete set of mode functions does not affect our conclusions.

[13] See, for instance, Ref. [7] or W. Heitler, *The Quantum Theory of Radiation,* 3rd ed. (Oxford University Press, London, 1960), p. 183.

[14] In this connection the reason for Dirac's preference for the Heisenberg picture [P. A. M. Dirac, Phys. Rev. **139**, B684 (1965)] is worth recalling: ". . .if one starts with zero in the Heisenberg picture, it remains zero, while if one starts with vacuum in the Schrödinger picture, it does not remain the vacuum. In that way the Heisenberg picture avoids the worst difficulties of the Schrödinger picture."

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