Quantum nondemolition measurements using a crossed Kerr effect between atomic and light fields

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We analyze the coupling between the fluctuations of an atomic beam incident on an evanescent wave mirror and the fluctuations of the laser beam producing the evanescent potential barrier. We show that this coupling is equivalent to a crossed Kerr effect between an atomic field and a light field, and that the consistency of the theory requires that both fields be subject to symmetrical commutation relations and quantum fluctuations. We show in particular that the phase shift undergone by the light field during the atomic reflection process permits a quantum nondemolition measurement of the incident atomic intensity.

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INTRODUCTION

Early experiments [1] have demonstrated that atoms can be reflected off an evanescent wave created by total internal reflection of a laser beam in a dielectric prism. The progress of laser trapping and cooling of atoms has led to recent experiments where multiple bounces of atoms released from a magneto-optical trap have been observed [2]. One difficulty of such experiments is the detection of the atoms after reflection. To date, fluorescence techniques consisting of collecting the photons spontaneously emitted by the atoms after absorption from a probe laser have been used. Such detection methods are destructive: recoils during absorption-fluorescence cycles induce such a large momentum spread that most atoms do not fall onto the laser spot at the following bounce. Recently, nondestructive techniques have been suggested for circumventing this inconvenience, which are based on refraction index variations in the presence of atoms [3,4]. A conceptually simple method consists in monitoring the phase of the very laser producing the evanescent wave [3]: because the phase shift associated with total internal reflection depends on the refractive index of the boundary media [5], one expects the laser phase to exhibit variations correlated with the presence of atoms in the evanescent wave.

This detection technique raises several intriguing questions. Can it be considered a quantum nondemolition measurement? What is the atomic observable involved in the measurement? How do the laser phase-shift fluctuations correlate with those of the atomic flux? One expects intuitively that the sensitivity of the measurement will eventually be limited by the quantum fluctuations of the laser phase (shot noise). However, it is well known that such a limit is not absolute and can be overcome, provided one allows simultaneously larger fluctuations of the laser intensity. Because an infinitely precise measurement of the laser phase shift would provide "which-path" information in an atomic interferometer using evanescent wave mirrors, one expects laser intensity fluctuations to translate into phase fluctuations for the atoms. This puts forth some additional appealing questions. How do laser intensity fluctuations correlate with atomic phase fluctuations? What are the consequences of this correlation on atomic interferometers using evanescent wave mirrors? What is the quantum limit in the sensitivity of such devices? In particular, is there an atomic equivalent to the phase intensity commutation relation well known in quantum optics?

The aim of this paper is to present a detailed theoretical description of this measurement technique and give precise answers to the above questions. It is organized as follows. In Sec. I, we derive the laser phase shift induced by the presence of atoms in the evanescent wave. This phase shift is expressed as a function of the incident atomic numerical flux $I_a^{\mathrm{in}}(t)$. Symmetrically, the atomic phase shift is calculated as a function of the incident laser photon flux $I_p^{\mathrm{in}}(t)$. Section II is devoted to the analysis of the measurement process of an atomic intensity using an interferometric detection of the laser phase shift. We quantitatively evaluate the sensitivity of this measurement in two cases: one of a stationary atomic beam and another of a single atom bouncing at the evanescent wave mirror. The influence of the atomic intensity measurement process on the atomic phase shift is also considered. We show that the contrast of an atomic interferometer using an evanescent wave mirror decreases exponentially with the signal-to-noise ratio associated with the one-atom bounce detection. In Sec. III, we present an elementary description of the evanescent wave mirror in terms of input-output relations for the fluctuations of the laser and atomic phases and intensities. These relations are formally identical to those of the crossed Kerr coupling between two laser fields [6], with the important difference that an atomic beam is here substituted for one laser beam. We deduce from these relations that the measurement of the laser phase constitutes a quantum nondemolition measurement of the incident atomic flux

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$I_a^{\rm in}(t)$. Furthermore, we show that the consistency of the theory *demands* the consideration of the atomic beam as a *quantum field* subject to the same commutation relation and quantum fluctuations as the laser field.

I. CALCULATION OF THE LASER AND ATOMIC PHASE SHIFTS

Because in this paper we are only interested in *fundamental* limits of quantum measurements, we will restrict ourselves to the regime of coherent atom optics (the limit of small saturation of the atomic transition) where spontaneous emission is negligible. As shown in the Appendix, spontaneous emission is *not* a fundamental limitation: its influence can be reduced arbitrarily by increasing the frequency detuning between the laser and the atomic resonance while simultaneously increasing the laser intensity.

A. Atom-induced laser phase shift

This section is devoted to the derivation of the phase shift experienced by the laser due to total internal reflection in the presence of atoms in the evanescent wave.

1. *Presentation of the model*

We consider the simple case of a quasimonoenergetic ensemble of two-level atoms normally incident on the surface ($z=0$) of an evanescent wave mirror [1,7]. The evanescent field experienced by the atoms results from total internal reflection of a laser beam at the interface between the vacuum and a dielectric prism. The incident laser wave has amplitude $\mathcal{E}^{\rm in}$ and is perpendicularly polarized with respect to the incident plane (Fig. 1). We also assume that the detuning $\Delta=\omega_L-\omega_A$ between the laser (ω_L) and the atomic (ω_A) frequencies is positive (atomic reflection occurring on the blue side of the resonance) and chosen so that the atoms can be considered to follow adiabatically the optical potential associated with the light-shifted ground-state level. In this regime, the atoms dynamics can be accounted for by means of the Hamiltonian [1]

$$H=\frac{p^2}{2M}+\frac{\hbar\Omega_{\rm ev}^2}{4\Delta}\exp(-2\kappa z)\ , \tag{1}$$

which contains the atomic kinetic energy (first term) and the reactive part of the atom-field coupling (second term). In Eq. (1), p and $z\geq 0$ are the momentum and the position of the atomic center of mass, M is the atomic mass, $1/\kappa$ is the characteristic decay length of the evanescent wave, and $\Omega_{\rm ev}=-d|\mathcal{E}_{\rm ev}|/\hbar$ is the resonant Rabi frequency that characterizes the coupling between the atomic dipole d and the evanescent field whose amplitude at $z=0$, $\mathcal{E}_{\rm ev}$, is related to the incident amplitude $\mathcal{E}^{\rm in}$ by [5]

$$|\mathcal{E}_{\rm ev}|=\frac{2n\cos\theta}{\sqrt{n^2-1}}|\mathcal{E}^{\rm in}|\ , \tag{2}$$

n being the refraction index of the dielectric medium at the laser frequency (assumed to be real) and θ the incidence angle of the laser on the $z=0$ plane (see Fig. 1). In particular, Eq. (1) shows that the maximum velocity $v_{\rm max}$ that can be reflected by the optical potential is given by

$$\frac{1}{2}Mv_{\rm max}^2=\frac{\hbar\Omega_{\rm ev}^2}{4\Delta}\ . \tag{3}$$

In the limit where the relative amplitude of the laser intensity variations can be treated as a small perturbation, the classical trajectory of the quasimonoenergetic atoms having an average incident velocity $-\overline{v}^{\rm in}(0<\overline{v}^{\rm in}<\overline{v}_{\rm max})$ is given in a good approximation by [8,9]

$$z_{\rm cl}(t)=z_0+\kappa^{-1}\ln\cosh(t/\tau_{\rm refl}) \tag{4}$$

with

$$z_0=\kappa^{-1}\ln(\overline{v}_{\rm max}/\overline{v}^{\rm in})\ , \tag{5}$$

where $\overline{v}_{\rm max}$ is the value of $v_{\rm max}$ corresponding to the average laser intensity and

$$\tau_{\rm refl}=1/\kappa\overline{v}^{\rm in}\ . \tag{6}$$

z_0 denotes the position of the turning point of the trajectory (reached at $t=0$) and $\tau_{\rm refl}$ is the time scale for the reflection process, which corresponds to the time taken for crossing the thickness $1/\kappa$ of the optical potential at the incident velocity $\overline{v}^{\rm in}$.

2. *Derivation of the laser phase shift*

The laser phase shift induced by the presence of atoms in the evanescent wave can be derived from reflection spectroscopy theory [10]. Its calculation proceeds in three steps: derivation of the laser-induced atomic polarization, calculation of the radiation emitted by this polarization in the direction of the reflected laser wave, and,

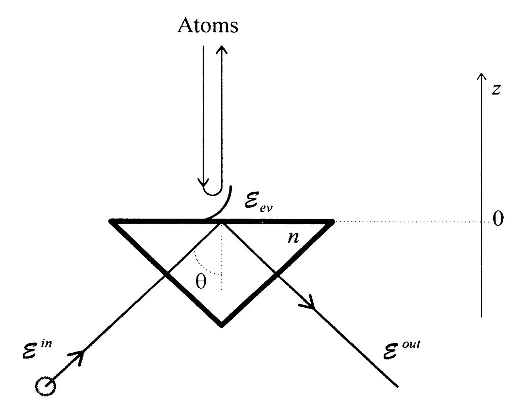

FIG. 1. Experimental configuration. A quasimonoenergetic ensemble of atoms comes at normal incidence onto the surface ($z=0$) of an evanescent wave mirror. The evanescent wave, with an amplitude $\mathcal{E}_{\rm ev}$ in the plane $z=0$, results from total internal reflection of a TE-polarized laser wave of incident amplitude $\mathcal{E}^{\rm in}$ at the interface between the vacuum ($z>0$) and a dielectric prism ($z<0$) of real refractive index n.

finally, derivation of the subsequent modification of the reflection coefficient at the interface, which gives access to the laser phase shift. Following Ref. [10], it is straightforward to show that the atom-induced laser phase shift takes the form

$$\Delta\varphi_p(t) = -\frac{\xi}{2}\left[\frac{\Gamma}{\Delta}\right]\left[\frac{\lambda^2}{S}\right]\int \rho_a(z,t)\exp(-2\kappa z)dz \tag{7}$$

with

$$\xi = \frac{3}{\pi}\frac{n\cos\theta}{n^2-1}, \tag{8}$$

where $\rho_a(z,t)$ is the instantaneous [11] atomic density distribution at time t, λ is the vacuum laser wavelength, and S is the surface characterizing the transverse dimension of the atomic wave packets, as well as the dimension of the laser spot on the vacuum-dielectric interface. Note that when deriving Eq. (7) one has assumed that both transverse profiles were square [12] and time independent.

Equation (7) shows that in order to derive the laser phase shift $\Delta\varphi_p(t)$, one needs to evaluate only the density distribution of the atoms in the immediate vicinity of the $z=0$ plane (typical spatial scale $1/\kappa$) because of the cutoff introduced in the integral by the exponential term. For values of z ranging from zero to a few $1/\kappa$ and by treating the atoms dynamics semiclassically, one has

$$\rho_a(z,t) = \int \delta(z - z_{\rm cl}(t-\tau-\tilde{T}))I_a^{\rm in}(\tau)d\tau, \tag{9}$$

where δ is the Dirac delta function, $z_{\rm cl}$ is the classical trajectory given by Eq. (4), $I_a^{\rm in}(\tau)$ is the incident numerical atomic flux (or atomic intensity) at time τ and reference position $\tilde{Z} \gg 1/\kappa$, and $\tilde{T} \gg \tau_{\rm refl}$ is the time taken by the atoms to travel from $z=\tilde{Z}$ to the turning point of their trajectory (see Fig. 2)

$$z_{\rm cl}(\tau) = \tilde{Z}, \quad z_{\rm cl}(\tau+\tilde{T}) = z_0. \tag{10}$$

Substituting Eqs. (4) and (9) into Eq. (7), one readily obtains

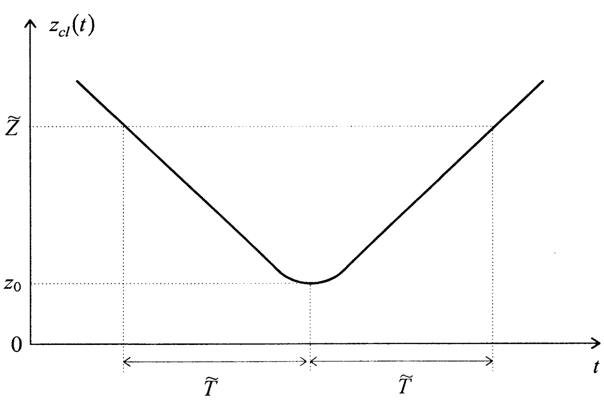

FIG. 2. Classical atomic trajectory $z_{\rm cl}(t)$ for an atom incident on the evanescent wave mirror, located at position $z=\tilde{Z}$ at time t. After a time interval $\tilde{T}$, the atom reaches the turning point of its trajectory $z=z_0$. It then comes back to position $z=\tilde{Z}$ at time $t+2\tilde{T}$.

$$\Delta\varphi_p(t) = -\frac{\xi}{2}\left[\frac{\Gamma}{\Delta}\right]\left[\frac{\lambda^2}{S}\right]\left[\frac{\bar{v}^{\rm in}}{\bar{v}_{\max}}\right]^2\int d\tau \frac{I_a^{\rm in}(\tau)}{\cosh^2\dfrac{t-\tau-\tilde{T}}{\tau_{\rm refl}}}. \tag{11}$$

In the following, we will be interested only in the variations $\delta\varphi_p$ of the laser phase shift resulting from the deviations $\delta I_a^{\rm in}$ of the atomic intensity from its average value $\bar{I}_a^{\rm in}$. Moreover, it will prove convenient to work in the frequency rather than in the time domain. Using the Fourier transform

$$f[\omega] = \int f(t)\exp(i\omega t)dt \tag{12}$$

and considering nonzero frequencies, Eq. (11) thus yields [13]

$$\delta\varphi_p[\omega] = K[\omega]\delta I_a^{\rm in}[\omega] \tag{13}$$

with

$$K[\omega] = K_0\beta[\omega], \tag{14}$$

where

$$K_0 = -\xi\left[\frac{\Gamma}{\Delta}\right]\left[\frac{\lambda^2}{S}\right]\left[\frac{\bar{v}^{\rm in}}{\bar{v}_{\max}}\right]^2\tau_{\rm refl} \tag{15}$$

and

$$\beta[\omega] = \frac{\pi\omega\tau_{\rm refl}/2}{\sinh(\pi\omega\tau_{\rm refl}/2)}. \tag{16}$$

Equation (13) shows that the relationship between atomic intensity and laser phase-shift variations takes a very simple form in the frequency domain. Each Fourier component of the incident atomic intensity spectrum leads to a proportional Fourier component in the laser phase-shift spectrum. As a consequence, the knowledge of the transfer function $K[\omega]$ describing this proportionality relation allows one to characterize the atomic intensity fluctuations from the measurement of the laser phase-shift spectrum. Note that the transfer function $K[\omega]$ exhibits a cutoff at the frequency $1/\tau_{\rm refl}$ associated with the reflection time of the atoms, which thus determines the frequency bandwidth of the measurement.

B. Laser-induced atomic phase shift

In this section we analyze the influence of the laser intensity variations on the phase shift undergone by the atoms during reflection on the evanescent wave mirror. We consider the case where the incident laser beam creating the evanescent wave is derived from an appropriately frequency and intensity stabilized source, so that its relative intensity variations are very small. In that case, the calculation of the atomic phase shift at reflection can be achieved using perturbation theory. In the framework of a semiclassical treatment of the atomic center-of-mass motion, this is accomplished by expanding the Hamiltonian (1) up to first order in the small parameter $\delta|\mathcal{E}^{\rm in}(t)|^2/|\bar{\mathcal{E}}^{\rm in}|^2$. This expansion allows one to separate the effect of the average laser intensity, described by the Hamiltonian

$$\overline{H}=\frac{p^2}{2M}+\frac{1}{2}M\overline{v}_{\max}^2\exp(-2\kappa z) \tag{17}$$

from those related to the intensity variations, accounted for by the perturbation Hamiltonian

$$\delta H=\frac{\delta|\mathcal{E}^{\rm in}(t)|^2}{|\overline{\mathcal{E}}^{\rm in}|^2}\left[\frac{1}{2}M\overline{v}_{\max}^2\right]\exp(-2\kappa z)\ . \tag{18}$$

Expression (18) arises from the proportionality between $v_{\max}^2$ and the intensity of the evanescent wave [Eq. (3)] and from the fact that the relative intensity variations of the evanescent field are equal to those of the incident laser beam [Eq. (2)].

The principle of the semiclassical perturbative calculation of the atomic phase shift follows from Ref. [9]. Besides the constant phase shift associated with the average optical potential [8], the intensity variations of the laser lead to an additional phase shift $\delta\varphi_a(t)$, which is obtained by integration of the perturbation potential (18) along the unperturbed atomic trajectory (4) corresponding to atoms reaching position $z=\tilde{Z}$ at time t *after* reflection on the evanescent wave mirror (see Fig. 2):

$$\delta\varphi_a(t)=-\frac{1}{\hbar}\int\frac{\delta|\mathcal{E}^{\rm in}(\tau)|^2}{|\overline{\mathcal{E}}^{\rm in}|^2}\left[\frac{1}{2}M\overline{v}_{\max}^2\right] \times\exp[-2\kappa z_{\rm cl}(\tau+\tilde{T}-t)]d\tau\ . \tag{19}$$

In order to treat the atomic and the laser phase shifts in as symmetric a fashion as possible, it is convenient to express the laser intensity as a numerical photon flux $I_p^{\rm in}$. These quantities are connected via the flux of the Poynting vector through the laser transverse section according to

$$\tfrac{1}{2}\epsilon_0 nc|\mathcal{E}^{\rm in}(t)|^2S\cos\theta=\hbar\omega I_p^{\rm in}(t)\ , \tag{20}$$

where ϵ_0 is the vacuum dielectric permittivity and c is the speed of light. Using Eqs. (2)–(4) and (20) and the relation $\Gamma=d^2\omega^3/3\pi\epsilon_0\hbar c^3$, Eq. (19) yields

$$\delta\varphi_a(t)=-\frac{\xi}{2}\left[\frac{\Gamma}{\Delta}\right]\left[\frac{\lambda^2}{S}\right]\left[\frac{\overline{v}^{\rm in}}{\overline{v}_{\max}}\right]^2 \times\int d\tau\frac{\delta I_p^{\rm in}(\tau)}{\cosh^2\frac{t-\tau-\tilde{T}}{\tau_{\rm refl}}}\ , \tag{21}$$

whose Fourier transform takes the simple form [13]

$$\delta\varphi_a[\omega]=K[\omega]\delta I_p^{\rm in}[\omega]\ , \tag{22}$$

where $K[\omega]$ is given by Eq. (14). The comparison between Eqs. (13) and (22) reveals several interesting features of the atom-evanescent wave interaction. First, the phase shift undergone by the laser beam due to the atoms is accompanied by a reciprocal phase shift of the atoms induced by light. Second, the variations of the atomic (light) intensity lead to variations of the laser (atomic) phase shift. Third, both effects are described by the *same* transfer function $K[\omega]$ when the light and atomic intensities are measured as numerical fluxes.

II. MEASUREMENT OF AN ATOMIC INTENSITY USING LIGHT INTERFEROMETRY

As shown in the preceding section, the variations of the phase shift undergone by the laser beam at reflection on the evanescent wave mirror directly reflect those of the incident atomic intensity [Eq. (13)]. It is therefore possible to measure the atomic intensity by monitoring the laser phase shift. In this section we study the sensitivity of such a measurement in the case where the laser phase shift is detected using light interferometry and we evaluate its influence on the atomic phase shift at reflection. Because of the symmetry between Eqs. (13) and (22), the measurement of a light intensity by atomic interferometry would proceed in a perfectly analogous way.

A. Sensitivity of the atomic intensity measurement

In this section we analyze the sensitivity of the measurement of an atomic intensity using a light interferometric detection of the laser phase shift. More precisely, we consider a Mach-Zehnder interferometer with 50%-50% beam splitters and an incident intensity I_{pi}. The measured quantity is the difference J_{pi} between the two output intensities of the interferometer (see Fig. 3). This quantity takes the general form

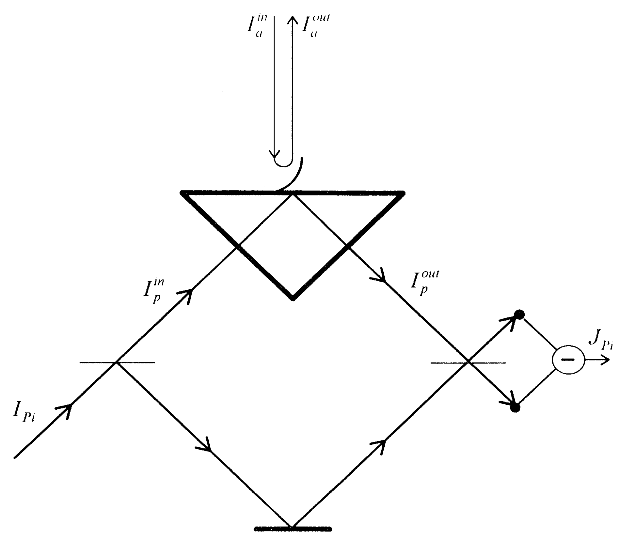

FIG. 3. Light interferometric measurement of an atomic intensity. A laser beam of intensity I_{pi} is sent into a Mach-Zehnder interferometer with 50%-50% beam splitters. It is split into two beams, one of which is used for realizing an evanescent wave mirror. This beam (intensity $I_p^{\rm in}$) experiences a phase shift at reflection on the mirror, due to its interaction with the atoms. This phase shift, which gives access to the incident atomic intensity $I_a^{\rm in}$, is monitored by measuring the difference J_{pi} between the two output intensities of the interferometer.

$$J_{pi}(t)=J_{\rm sig}(t)+J_n(t)\ , \tag{23}$$

where $J_{\rm sig}$ is a function of the phase difference $\Delta\varphi_{pi}$ between the two arms of the interferometer, which includes the phase shift due to the interaction with atoms and the difference of propagation length between the two arms of the light interferometer. Assuming for simplicity that this difference is adjusted for balancing the interferometer, the signal $J_{\rm sig}$ is proportional to the atom-induced laser phase shift $\Delta\varphi_p(t)$,

$$J_{\rm sig}(t)=I_{pi}(t)\sin[\Delta\varphi_{pi}(t)]\approx\bar{I}_{pi}\Delta\varphi_p(t)\ , \tag{24}$$

where $\bar{I}_{pi}$ denotes the average value of I_{pi}. In Eq. (23), J_n is an additional noise term associated with the random distribution of the incoming photons between the two outputs. Because we are interested in the fundamental limits of the sensitivities, we restrict ourselves to the situation where this noise term only results from quantum fluctuations. In this case, J_n is a white noise (shot noise), with a noise power equal to the main intensity of the incoming beam [14,15]

$$S_{J_n}[\omega]=\bar{I}_{pi}\ . \tag{25}$$

The definition of the sensitivity of the atomic intensity measurement depends on the time characteristics of the signal $J_{\rm sig}$. In the following, we evaluate the signal-to-noise ratio in the two cases where the atomic signal arises from a stationary atomic beam or a single pointlike atom.

1. *Measurement of the intensity of a stationary atomic beam*

In the situation of a stationary atomic beam incident on the evanescent wave mirror, we deduce from Eqs. (13), (14), and (24) that the power spectrum of the signal $J_{\rm sig}$ reflects the spectrum of the incoming atomic intensity

$$S_{J_{\rm sig}}[\omega]=\bar{I}_{pi}^2K_0^2\beta^2[\omega]S_{I_a}^{\rm in}[\omega]\ . \tag{26}$$

The sensitivity of the atomic intensity measurement is then characterized by a frequency-dependent signal-to-noise ratio $R[\omega]$,

$$R[\omega]=\frac{S_{J_{\rm sig}}[\omega]}{S_{J_n}[\omega]}=\bar{I}_{pi}K_0^2\beta^2[\omega]S_{I_a}^{\rm in}[\omega]\ . \tag{27}$$

The fluctuations of the atomic intensity are detectable provided $R[\omega]>1$. In the particular case where the intensity fluctuations of the atomic beam correspond to the shot-noise level and for detection frequencies smaller than the cutoff frequency $1/\tau_{\rm refl}$, this condition reads [16]

$$\bar{I}_{pi}\bar{I}_a^{\rm in}K_0^2>1\ . \tag{28}$$

2. *Detection of a one-atom bounce*

We now study the possibility of detecting a single atom bouncing at the evanescent wave mirror. More precisely, we consider a pointlike atom reaching the turning point of its trajectory at time $t=t_0$, hence corresponding to the atomic intensity $I_a^{\rm in}=\delta(t-t_0+\tilde{T})$ (see Sec. I A 2). In this case, the expected signal $J_{\rm sig}$ deduced from Eqs. (13) and (24) takes the form

$$J_{\rm sig}(t)=\bar{I}_{pi}K_0\beta(t-t_0)\ . \tag{29}$$

In order to extract the maximum information from the measurement, it is necessary to use a matched filter [15]. It corresponds to a linear-response function proportional to the expected signal reversed in time and leads to the filtered signal J^f,

$$J^f(t)=\int d\tau\,\beta(-\tau)J_{pi}(t-\tau)\ . \tag{30}$$

This filter is characterized by a bandwidth B, which is directly related to the reflection time of the atom on the evanescent wave mirror

$$B=\int\frac{d\omega}{2\pi}\beta^2[\omega]=\frac{1}{3\tau_{\rm refl}}\ . \tag{31}$$

Because the signal $J_{\rm sig}$ associated with the bouncing of a single atom is nonstationary, the sensitivity of the measurement is characterized by a time-dependent signal-to-noise ratio, which takes its optimum value $R_{\rm opt}$ at time $t=t_0$, where the filtered signal $J_{\rm sig}^f$ is maximum. Assuming a noise spectrum as given by Eq. (25), a straightforward calculation yields

$$R_{\rm opt}=K_0^2B\bar{I}_{pi}\ . \tag{32}$$

Consequently, the condition for detecting a single atom reads

$$K_0^2B\bar{I}_{pi}>1\ . \tag{33}$$

It is interesting to note that this relation is identical to condition (28) where the average atomic intensity $\bar{I}_a^{\rm in}$ is substituted for the bandwidth B. This corresponds to an intensity of the order of one atom per reflection time $\tau_{\rm refl}$.

B. Influence of the laser phase shift measurement on the contrast of atomic interference fringes

The possibility of detecting the bounce of a single atom by light interferometric measurement of the laser phase shift provides a way of getting which-path information in an atomic interferometer using an evanescent wave mirror. One can therefore infer that the measurement process necessarily affects the atomic phase shift in such a way that the contrast of atomic interference fringes decreases while the signal-to-noise ratio (32) increases. It is the aim of this section to present a quantitative description of this phenomenon.

We consider an atomic interferometer consisting of an atomic beam of intensity I_{ai} sent into a Mach-Zehnder interferometer using 50%-50% atomic beam splitters and an evanescent wave mirror (the other mirror is assumed to be perfect such as, for example, a magnetic mirror). In addition, the phase shift of the laser creating the evanescent wave is measured using light interferometry (see Fig. 4). Similarly to the case of Sec. II A, the output signal of the atomic interferometer J_{ai}, equal to the difference between the two output atomic intensities, is of the form

$$J_{ai}=I_{ai}\sin\Delta\varphi_{ai}\ , \tag{34}$$

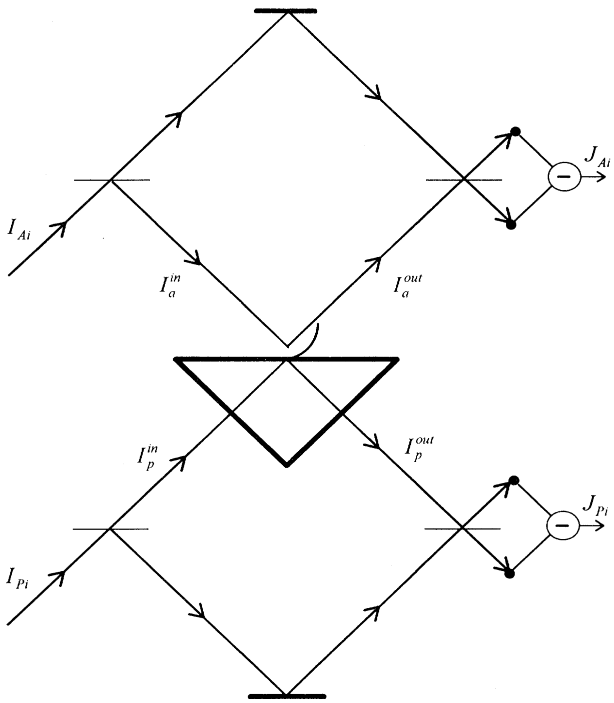

FIG. 4. Influence of the atomic intensity measurement on the contrast of an atomic interferometer. An atomic beam of intensity I_{ai} is sent into a Mach-Zehnder atomic interferometer with 50%-50% beam splitters. It is split into two beams, one of which is reflected on an evanescent wave mirror. This beam (intensity I_a^{in}) experiences a phase shift at reflection on the mirror due to its interaction with the fluctuating intensity of the laser beam creating the evanescent wave. This phase shift may result in a loss of contrast of the atomic interference fringes detected by measuring the difference J_{ai} between the two output intensities of the atomic interferometer. The other parameters are the same as in Fig. 3.

where $\Delta\varphi_{ai}$ is the sum of the laser-induced atomic phase shift given by Eq. (22) and of a constant phase factor $\Delta\Phi$ associated with the difference between the propagation length in the two arms of the interferometer, which can be adjusted for detecting the atomic interference fringes.

Using the fact that the variations of the input laser and atomic intensities are not correlated, the time average $\bar{J}_{ai}$ of the output signal J_{ai} can be readily shown to read

$$\bar{J}_{ai} = C\bar{I}_{ai}\sin\Delta\Phi \ , \tag{35}$$

where

$$C = \overline{\cos[\delta\varphi_a(t)]} \tag{36}$$

is the contrast of the atomic interference fringes. Considering that $\delta\varphi_a$ is a Gaussian variable, one further obtains

$$C = \exp(-\tfrac{1}{2}\overline{\delta\varphi_a^2}) \ . \tag{37}$$

The contrast C is thus directly related to the variance $\overline{\delta\varphi_a^2}$ of the instantaneous atomic phase shift induced by the intensity noise of the laser creating the evanescent wave. The intensity of this laser is half the intensity of the light sent into the light interferometer and its intensity fluctuations are at the shot-noise level provided those of the laser incident on the interferometer are also. Using Eq. (22), it is then straightforward to show that

$$\overline{\delta\varphi_a^2} = \tfrac{1}{2}K_0^2 B\bar{I}_{pi} \ . \tag{38}$$

The comparison between Eqs. (32), (37), and (38) shows that the loss of contrast of the atomic interference fringes is directly related to the optimum signal-to-noise ratio associated with the detection of a one-atom bounce

$$C = \exp(-R_{\text{opt}}/4) \ . \tag{39}$$

It is interesting to note that whatever the characteristics of the atomic intensity incident on the atomic interferometer, the interference fringes contrast is related to the ability of detecting a *single* atom bouncing on the evanescent wave mirror or, equivalently, to the possibility of obtaining which-path information in the atomic interferometer. Equation (39) indeed shows that when such information is available [that is, when condition (33) is fulfilled], the contrast of the atomic interferometer is reduced. Note, however, that a good contrast for the atomic interference fringes is not exclusive of the measurement of the atomic intensity fluctuations, provided these fluctuations are sufficiently large [compare Eqs. (28) and (33)].

At a somewhat deeper level, Eq. (39) is related to the fact that in quantum optics, the phase (used for measuring the atomic intensity) and the intensity (resulting in the atomic phase shift) of the electric laser field are noncommuting quantum operators that are subject to a Heisenberg uncertainty relation [17]. Hence the less the laser phase fluctuations, the larger the laser intensity fluctuations. Consequently, the better the sensitivity in the atomic intensity measurement, the larger the atomic phase-shift perturbation and hence the reduction of the contrast of the atomic interference fringes. In order to get a better understanding of this quantum property, we will now present a more precise, quantum description of the atom-evanescent wave coupled system.

III. QUANTUM FIELD DESCRIPTION OF THE ATOM-LASER COUPLING

We have so far described light interferometric measurements of atomic intensities and their influence on atomic interferometers by considering the interferometers as a whole. The aim of this section is to provide a more general way of describing these systems by considering their elementary building blocks. This approach will allow us to shed light on previously discussed properties such as the reciprocity of the atom-light coupling and the measurement-perturbation relations. It will also provide a way of describing rigorously the quantum nondemolition properties of the atomic intensity measurement process. We will first present an elementary phenomenological description of the evanescent wave mirror in terms of input-output relations for the quantum fluctuations of the laser and atomic phases and intensities. We will then show how these relations can be derived rigorously in the framework of quantum field theory.

A. Input-output relations for the laser and atomic phase and intensity operators

In the previous sections, we have considered phase shifts, which are the quantities of interest in interferometric measurements. However, a more powerful description of the atom-evanescent wave interaction can be obtained by considering laser and atomic *phases* rather than *phase shifts*. *More precisely,* we characterize the evanescent wave mirror by input-output relations for the fluctuations of laser and atomic phases and intensities. Furthermore, we show that the consistency of the theory demands the consideration of these quantities as quantum operators subject to commutation relations.

1. Crossed optoatomic Kerr effect

In Sec. I A we derived the laser phase shift resulting from the presence of atoms in the evanescent wave. This phase shift has to be interpreted as a difference between the phase of the laser *after* total internal reflection φ_p^{out} and the phase of the laser *before* the reflection process φ_p^{in},

$$\varphi_p^{\text{out}}=\varphi_p^{\text{in}}+\Delta\varphi_p \ . \tag{40}$$

Symmetrically, it is natural to interpret the atomic phase shift calculated in Sec. I B as a difference between output and input atomic phases

$$\varphi_a^{\text{out}}=\varphi_a^{\text{in}}+\Delta\varphi_a \ . \tag{41}$$

We will give a more precise definition of those atomic phases in the following.

It is then possible to characterize completely the atom-laser coupled system in terms of the following input-output relations for the phase and intensity fluctuations:

$$\delta\varphi_p^{\text{out}}[\omega]=\delta\varphi_p^{\text{in}}[\omega]+K[\omega]\delta I_a^{\text{in}}[\omega] \ , \tag{42a}$$

$$\delta I_p^{\text{out}}[\omega]=\delta I_p^{\text{in}}[\omega] \ , \tag{42b}$$

$$\delta\varphi_a^{\text{out}}[\omega]=\delta\varphi_a^{\text{in}}[\omega]+K[\omega]\delta I_p^{\text{in}}[\omega] \ , \tag{42c}$$

$$\delta I_a^{\text{out}}[\omega]=\delta I_a^{\text{in}}[\omega] \ . \tag{42d}$$

These relations have the same form as those describing the coupling between two laser fields interacting with a Kerr medium [6,18] (crossed Kerr effect). It is important to note, however, that the present situation exhibits two original features. First, the coupling constant $K[\omega]$ is frequency dependent, as a result of the finite bouncing time of the atoms. Second, an atomic beam has been substituted here for one laser beam. One can thus consider that the present coupling corresponds to a crossed *optoatomic Kerr effect.*

2. Definition of phase and intensity operators

The transformation relations (42) have been obtained from the results of Sec. I where the variations of laser and atomic observables were described in a classical manner. However, it is well known from the semiclassical theory in quantum optics that quantum fluctuations of phase and intensity can be represented by classical random variables [14]. One can thus consider that Eqs. (42) effectively take into account the quantum fluctuations of light. Because the laser phase and intensity operators are coupled to the atomic phase and intensity [see Eqs. (42)], the consistency of the theory requires that the atomic phase and intensity also have quantum fluctuations. The aim of this section is to make precise the definition of the corresponding operators and to derive their commutation relations. We first consider the case of light field operators. We will then use the reciprocity between the laser and atomic variables to obtain similar relations for atomic fields.

In the framework of quantum optics theory, the laser electric field is described by a time-dependent operator that is a linear superposition of photon creation and annihilation operators [19]. In the simple case of a one-dimensional scalar laser field, the complex amplitude of the electric field (normalized so that the intensity corresponds to a numerical photon flux) in the rotating frame associated with the laser frequency ω_L reads [14]

$$E(t)=\int\frac{d\omega}{2\pi}e^{-i(\omega-\omega_L)t}a_k \ , \tag{43}$$

where a_k is the annihilation operator associated with photons of frequency ω and wave vector $k=\omega/c$ (c being the speed of light). This operator satisfies the commutation relation

$$[E(t),E^\dagger(t')]=\delta(t-t') \ . \tag{44}$$

We define the two quadrature operators E_1 and E_2 of the electric field as

$$E_1(t)=E(t)+E^\dagger(t) \ , \tag{45}$$

$$E_2(t)=i[E^\dagger(t)-E(t)] \ , \tag{46}$$

which yield

$$E(t)=\frac{E_1(t)+iE_2(t)}{2} \ . \tag{47}$$

Let us first consider an electric-field quantum state corresponding to a nonzero average amplitude and small relative quantum fluctuations. The mean value of the field amplitude is a complex number having a modulus equal to the square root of the mean laser intensity $\bar{I}_p$ and a well-defined phase $\bar{\varphi}_p$,

$$\langle E\rangle=\sqrt{\bar{I}_p}e^{i\bar{\varphi}_p} \ . \tag{48}$$

A linear expansion of E in the intensity and phase fluctuations yields the deviations $\delta E(t)$ of the electric field from its average value

$$\delta E(t)=\langle E\rangle\left[\frac{\delta I_p(t)}{2\bar{I}_p}+i\delta\varphi_p(t)\right] . \tag{49}$$

The intensity and phase fluctuations of the laser field thus appear as being proportional to the fluctuations of the field quadratures. Assuming for simplicity $\bar{\varphi}_p=0$, the comparison between Eqs. (47) and (49) shows that

$$\delta I_p(t)=\sqrt{\bar{I}_p}\,\delta E_1(t)\ , \tag{50}$$

$$\delta\varphi_p(t)=\frac{1}{2\sqrt{\bar{I}_p}}\delta E_2(t)\ . \tag{51}$$

Note that these expressions hold in the general case $\bar{\varphi}_p\neq 0$ with an appropriate redefinition of E_1 and E_2. It is then straightforward using Eqs. (50), (51), and (44) to derive the commutation relation between the laser intensity and phase operators:

$$[\delta I_p(t),\delta\varphi_p(t')]=\delta(t-t')\ , \tag{52}$$

which reads in the frequency domain

$$[\delta I_p[\omega],\delta\varphi_p[\omega']]=i2\pi\delta(\omega+\omega')\ . \tag{53}$$

In the case where the quantum fluctuations of the electric field are comparable or larger than the average amplitude of the field (this is in particular the case of a zero average amplitude), it is no longer possible to perform a linear expansion of E around its average value. More precisely, the intensity operator is still well defined, but it becomes difficult to give a completely satisfactory definition of a phase operator. Hence the commutator (53) becomes inappropriate for characterizing the quantum fluctuations of the field. It must then be substituted for by the commutator between the electric field operator itself and the light intensity operator

$$[E(t),I_p(t')]=E(t)\delta(t-t') \tag{54}$$

or, equivalently,

$$[E^\dagger(t),I_p(t')]=-E^\dagger(t)\delta(t-t')\ . \tag{55}$$

This commutator generalizes Eq. (53). It is well defined *whatever the characteristics of the electric-field quantum state* and is equivalent to the commutator (53) in the situation where the linear expansion (49) is legitimate.

We now consider the case of *atomic* phase and intensity. As previously mentioned, one expects from the reciprocity between the atomic and laser variables in the atom-light coupling considered in this paper that commutation relations similar to Eqs. (53) and (54) also apply for the atomic variables. In a first step, we may assume that beside the atomic intensity operator, it is possible to define an atomic phase operator (for a general case, see Sec. II B). Equations (42) can thus be considered as input-output relations for the laser and atomic phase and intensity operators. A general property of such relations is their unitarity. This implies that the commutation relations between two given operators are the same for the output as for the input quantities. Since atomic and laser phase operators commute with each other at the input of the evanescent wave mirror, it follows from input-output relations (42) and from the laser field commutation relation (53) that *the atomic intensity and phase are also noncommuting operators obeying the same commutation relation*

$$[\delta I_a[\omega],\delta\varphi_a[\omega']]=i2\pi\delta(\omega+\omega')\ . \tag{56}$$

We will show in Sec. III B that such a commutation relation may be justified using the general framework of quantum field theory.

3. *Quantum nondemolition properties of the atomic intensity measurement*

As previously mentioned, the equations (42) characterizing the atom-light coupling at the evanescent wave mirror have exactly the same form as those corresponding to the optical crossed Kerr effect. It is well known that this effect can be used for realizing quantum nondemolition measurements of a light beam intensity [6,18]. Similarly, the optoatomic crossed Kerr effect allows one to perform quantum nondemolition measurements of an atomic intensity: First, the laser phase-shift detection is an actual *measurement* of the atomic intensity because the fluctuations of the output laser phase reflect those of the incoming atomic intensity [Eq. (42a)]. Second, this property still holds at the level of *quantum* fluctuations, when intensities and phases are treated as time-dependent *operators* written in the Heisenberg representation. Third, the *nondemolition* character of the measurement is ensured by the fact that the measured observable, that is, the atomic intensity, is unchanged by the interaction with light [Eq. (42d)].

Finally, it is interesting to note that in the case (considered throughout this paper) where the atom-light interaction is considered as purely dispersive, the evanescent wave mirror is analogous to a perfect Kerr medium. In particular, because of the absence of any added noise, it realizes an ideal quantum nondemolition situation [18] and the only remaining criterion is the sensitivity of the measurement defined as in Sec. II.

4. *Atomic intensity measurement and phase perturbation*

We now return to the relation (39) between the sensitivity to a one-atom bounce and the contrast of an atomic interferometer. We show that the description of the evanescent wave mirror in terms of input-output relations for the laser and atomic phase and intensity operators [Eqs. (42)] sheds light on its physical significance. In the framework of this description, it indeed becomes unnecessary to give a precise description of the apparatus used for detecting the atom-induced laser phase shift. It is sufficient to notice that when a one-atom bounce is detected on the output phase of light, the noise superimposed on the signal of interest is the input phase noise of the laser, characterized by its power spectrum $S^{\rm in}_{\varphi_p}$. Using an optimal filtering of the signal (see Sec. II A 2), the corresponding signal-to-noise ratio may be written

$$R_{\rm opt}=\frac{K_0^2 B}{S^{\rm in}_{\varphi_p}}\ . \tag{57}$$

Consider now the perturbation of the atomic phase in this measurement process. Fluctuations are added on the atomic phase due to the noise of the light intensity coming onto the evanescence wave mirror, characterized by its power spectrum $S^{\rm in}_{I_p}$. The variance of this added fluctuations reads

$$\overline{\delta\varphi_a^2} = K_0^2 B S_{I_p}^{\text{in}} . \tag{58}$$

The laser-induced atomic phase noise and the one-atom bounce signal-to-noise ratio are thus related through

$$\overline{\delta\varphi_a^2} = R_{\text{opt}} S_{I_p}^{\text{in}} S_{\varphi_p}^{\text{in}} . \tag{59}$$

As a consequence of the commutation relation (53), the noise spectra associated with laser phase and intensity obey a Heisenberg inequality [14]

$$S_{I_p}^{\text{in}} S_{\varphi_p}^{\text{in}} \geq \tfrac{1}{4} . \tag{60}$$

Combining Eqs. (59) and (60), it is then possible to deduce the following inequality between the variance of the laser-induced atomic phase noise and the optimal one-atom bounce signal-to-noise ratio:

$$\overline{\delta\varphi_a^2} \geq \tfrac{1}{4} R_{\text{opt}} . \tag{61}$$

We can now discuss more precisely the physical significance of relation (39) that was derived in Sec. II B. It clearly appears as the consequence of the Heisenberg inequality between the noncommuting operators associated with the laser phase and intensity. Note, however, that in relation (39), which was obtained for a specific interferometer (50%-50% beam splitters, detection of the difference between the two output intensities in a balanced configuration), the added atomic phase noise is *twice* the minimum added noise imposed by the Heisenberg inequalities. This shows that this specific configuration does not provide the optimal quantum nondemolition technique for measuring the atomic intensity. In contrast, a homodyne detection of the laser phase shift would allow one to design such an optimal technique.

B. Quantum-field-theory description of the atom-field coupling

In the present state of the art of experimental atom optics, atomic coherent states or more generally atomic states with a well-defined phase cannot yet be obtained. Therefore, the description of the atom-laser coupling at the evanescent wave mirror considered in Sec. III A is not completely satisfactory since it makes use of atomic phase and intensity operators. It would thus be worth having a more general theoretical description that would remain valid for any atomic state. We show now that such a description can be obtained in the framework of quantum field theory [20,21].

1. Atomic quantum field

For the sake of simplicity, we restrict ourselves to the simple case of nonrelativistic, spin zero atoms in a one-dimensional space. Using the formalism of second quantization, we describe such particles by a scalar bosonic field $\Psi(z,t)$, which can be represented in the form

$$\Psi(z,t) = \int \frac{dk}{2\pi} e^{-i(\hbar k^2/2M)t} e^{ikz} b_k , \tag{62}$$

where b_k is the annihilation operator corresponding to the atomic mode of wave vector k and frequency $\omega_k = \hbar k^2/2M$ (M being the atomic mass), the commutation relation between annihilation (b_k) and creation ($b_{k'}^\dagger$) operators being

$$[b_k, b_{k'}^\dagger] = 2\pi\delta(k-k') . \tag{63}$$

When describing a nearly monoenergetic atomic beam (the case of interest in this paper), the relevant atomic quantum modes are those associated with a wave vector nearly equal to $\bar{k} = M\bar{v}/\hbar$, the wave vector corresponding to the average atomic velocity $\bar{v}$. For these modes, the dispersion relation may be approximated by

$$\omega_{\bar{k}+k} \approx \frac{\hbar\bar{k}^2}{2M} + \frac{\hbar\bar{k}k}{M} = \frac{M\bar{v}^2}{2\hbar} + k\bar{v} . \tag{64}$$

This allows us to define a slowly varying complex amplitude $\tilde{\Psi}(t)$

$$\tilde{\Psi}(t) = \sqrt{\bar{v}} \int \frac{dk}{2\pi} e^{-ik\bar{v}t} b_k , \tag{65}$$

which is related to the field Ψ through

$$\Psi(z,t) \approx e^{-i(M\bar{v}^2/2\hbar)t} e^{i(M\bar{v}/\hbar)} \frac{1}{\sqrt{\bar{v}}} \tilde{\Psi} \left[t - \frac{z}{\bar{v}} \right] . \tag{66}$$

Using Eq. (63), one gets the commutation relation

$$[\Psi(z,t), \Psi^\dagger(z',t')] \approx \frac{1}{\bar{v}} \delta(t-t'-\bar{v}(z-z')) \tag{67}$$

or, equivalently,

$$[\tilde{\Psi}(t), \tilde{\Psi}^\dagger(t')] = \delta(t-t') , \tag{68}$$

which appears as the analog for the atomic quantum field of Eq. (44). In quantum field theory, the commutation relation of the fields is directly related to their propagation and Eq. (67) shows that the approximation (64) of the atomic dispersion relation is equivalent to the semiclassical treatment of atomic motion in quantum field theory. This approximation also allows one to simplify the general expression of the atomic intensity operator [22]

$$I_a(z,t) = \frac{i\hbar}{2M} \{ [\partial_z \Psi^\dagger(z,t)] \Psi(z,t) - \Psi^\dagger(z,t) [\partial_z \Psi(z,t)] \} \tag{69}$$

to the form

$$I_a(t) \approx \tilde{\Psi}^\dagger(t) \tilde{\Psi}(t) , \tag{70}$$

which, using Eq. (68), leads to the commutation relation between the atomic quantum field and the atomic intensity operator

$$[\tilde{\Psi}(t), I_a(t')] = \tilde{\Psi}(t) \delta(t-t') . \tag{71}$$

As emphasized in Sec. III A 2 [Eq. (54)] for the analogous case of the laser electric field, Eq. (71) generalizes the commutation relation (56) between the atomic phase and intensity operators for arbitrary atomic states. In particular, Eq. (71) yields Eq. (56) in the case of atomic states associated with a well-defined phase.

Furthermore, it is important to note that Eq. (71) holds for fermions as well as for bosons. Following the same approach (semiclassical treatment of the atomic motion) one would obtain for a fermion field an *anticommutation* relation of the form

$$\{\tilde{\Psi}(t), \tilde{\Psi}^{\dagger}(t')\} = \delta(t-t') \ . \tag{72}$$

However, one can readily check that the relation of interest in our problem, i.e., the relation between the atomic quantum field and the atomic intensity operator, is still a *commutation* relation and is the same as for bosons. As a consequence, *the properties and relations that are considered in this paper are independent of the quantum statistical nature of the atoms.*

2. Input-output transformation relations for the laser and atomic quantum fields

We are now able to generalize the relations (42) to the case of arbitrary atomic quantum states. This is achieved by writing input-output relations for the quantum electromagnetic and atomic fields involving only *phase shifts*

$$E^{\text{out}}(t) = E^{\text{in}}(t)\exp[i\Delta\varphi_p(t)] \ , \tag{73a}$$

$$\tilde{\Psi}^{\text{out}}(t) = \tilde{\Psi}^{\text{in}}(t)\exp[i\Delta\varphi_a(t)] \ . \tag{73b}$$

More explicitly, using the expressions of the phase shifts in terms of the intensity operators, one obtains the expressions

$$E^{\text{out}}(t) = E^{\text{in}}(t)\exp\left[i\int d\tau\, K(\tau) I_a^{\text{in}}(t-\tau)\right] \ , \tag{74a}$$

$$\tilde{\Psi}^{\text{out}}(t) = \tilde{\Psi}^{\text{in}}(t)\exp\left[i\int d\tau\, K(\tau) I_p^{\text{in}}(t-\tau)\right] \ . \tag{74b}$$

These relations, associated with the commutation relations (54) and (71) between the fields and intensities, provide a complete and consistent quantum description of the optoatomic crossed Kerr effect, which can be summarized by the following properties.

(i) The commutation relations for light and atomic operators are identical.

(ii) The input-output relations are unitary. In particular, the commutation relations for the output fields are the same as for the input fields.

(iii) There is a reciprocity relation between the effect of the atoms on light and of light on atoms. More precisely, both effects are described by the *same* coupling coefficient $K[\omega]$.

In the case where the laser and atomic quantum states are such that well-defined phases exist, this description corresponds to the previously established equations (42), (53), and (56).

CONCLUSION

In this paper, we have analyzed the coupling between the fluctuations of the laser beam producing the evanescent wave and the fluctuations of an atomic beam incident on an evanescent wave mirror. We have derived input-output relations for these fluctuations, which are characteristic of a crossed Kerr effect. We have shown that the consistency of the theory demands the consideration of the atoms as a quantum field subject to the same commutation relations as the light field. The framework of quantum field theory indeed provides a unique way of understanding some remarkable properties of the evanescent wave mirror. Finally, we have emphasized that these properties are the same for bosons and fermions and must therefore be considered as general and basic properties of intensity and phase for any quantum particles.

The semiclassical formalism used for our derivations should prove interesting for describing various atom optics experiments. For example, it would allow one to analyze the situation where atomic bounces are detected via the phase shift of an additional probe running wave [4]. Also, the presence of a cavity on the laser field can be handled very simply since it merely results in a modification of the coupling coefficient $K[\omega]$, proportional to the cavity finesse [23]. Among future developments, the presence of a cavity on the atomic field, the influence of the gravity field, and atomic gyrometry seem appealing.

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APPENDIX: INFLUENCE OF SPONTANEOUS EMISSION ON THE ATOMIC INTENSITY MEASUREMENTS

Spontaneous emission has been neglected throughout this paper. We discuss here the validity of this approximation with regard to the atomic intensity measurements considered in Sec. II. The influence of spontaneous emission can be readily evaluated in the limit $\Gamma/\Delta \ll 1$. It results in a modification of Eq. (42a) of the form

$$\delta\varphi_p^{\text{out}}[\omega] = \delta\varphi_p^{\text{in}}[\omega] + K[\omega]\delta I_a^{\text{in}}[\omega] + \delta\varphi_p^{\text{sp}}[\omega] \ . \tag{A1}$$

First, one notes that the optoatomic crossed Kerr coupling [Eqs. (14) and (15)]

$$K[\omega] = K_0\beta[\omega] \ , \tag{A2}$$

$$K_0 = -\xi\left[\frac{\Gamma}{\Delta}\right]\left[\frac{\lambda^2}{S}\right]\left[\frac{\bar{v}^{\text{in}}}{\bar{v}_{\max}}\right]^2 \tau_{\text{refl}} \tag{A3}$$

is *not* modified in the limit $\Gamma/\Delta \ll 1$. Second, the laser phase fluctuations due to spontaneous emission result in a supplementary term $\delta\varphi_p^{\text{sp}}[\omega]$, which is not correlated with $\delta\varphi_p^{\text{in}}[\omega]$. These fluctuations are characterized by a noise spectrum that reaches its maximum value at zero frequency

$$S_{\varphi_p}^{\text{sp}}[0] = 2\frac{\Gamma}{\Delta}\frac{\bar{I}_a^{\text{in}}}{\bar{I}_{pi}}|K_0| \ . \tag{A4}$$

In an interferometric measurement of the laser phase, this leads to an added noise spectrum for the signal J_{sig} (see Sec. II),

$$S_J^{\rm sp}[0]=2\frac{\Gamma}{\Delta}\bar{I}_a^{\rm in}\bar{I}_{pi}|K_0| \ . \tag{A5}$$

This spectrum has to be compared on one hand with the photon noise of the incoming beam [Eq. (25)]

$$S_{J_n}[0]=\bar{I}_{pi} \tag{A6}$$

and on the other hand with the fluctuations of the atomic intensity signal [Eq. (26)]

$$S_{J_{\rm sig}}[0]=K_0^2\bar{I}_{pi}^2\bar{I}_a^{\rm in} \ . \tag{A7}$$

The condition for spontaneous emission to be negligible is

$$S_J^{\rm sp}[0]\ll S_{J_n}[0] \tag{A8}$$

or, equivalently,

$$\frac{\Gamma}{\Delta}\bar{I}_a^{\rm in}|K_0|\ll 1 \ . \tag{A9}$$

It is worth emphasizing that this condition is not contradictory with condition $S_{J_{\rm sig}}[0]>S_{J_n}[0]$ [Eq. (28)] for detectability of atomic intensity fluctuations. Both conditions can indeed be fulfilled simultaneously provided Δ/Γ and $\bar{I}_{pi}$ are sufficiently large. In particular, they may be satisfied in the present state-of-the-art experiments [3].

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