

Measurement of the Bell operator and quantum teleportation

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We propose a scheme for measuring an optical version of the Bell operator, using a generalization of the Hong-Ou-Mandel interferometer. Discrete-mode calculations show this to be sufficient to allow teleportation of the state of polarization of a photon with a conditional efficiency approaching 100%. The feasibility of the scheme is investigated through full broadband calculations.

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Ever since the violation of Bell inequalities was discovered [1–3], we have understood that the transfer of information [4] in quantum systems can be quite unintuitive. One striking example is teleportation [5], the recently developed scheme for the “disembodied” transport of an unknown quantum state.

It is disembodied in the sense that the state is transported through multiple channels connecting the sender of the state and its receiver, but any channel taken separately contains no extractable information about the state being transported. By contrast, under purely unitary evolution, such as one would expect in evolution from the Schrödinger equation alone, at least one of the channels must contain some information. In the case studied here, two channels are used: one quantum and one classical; the classical signal is derived from a non-unitary collapse due to a measurement.

The proof, in principle, in Ref. [5] for quantum mechanics to allow such disembodied transport was based on discrete-mode calculations, which are most appropriate for well separated modes, such as optical modes in a high Q cavity. Such discrete-mode calculations can be “mapped” into what is performed in the laboratory in many different ways, but with each such implementation comes inherent subtleties that may make a particular mapping invalid (i.e., the sought after effect is not seen). Here we describe a proposal for performing this disembodied transport of the state of polarization of a photon (a two-state system). We perform broadband calculations to determine the likely difficulties with implementing our proposal. We find our implementation to be robust with respect to low detector efficiency, phase jiggle, and unavoidable differences between the multiple optical components needed. Our numbers suggest that successful teleportation events could be generated at the rate of about one per second, though very narrow bandwidth down-convertors are essential.

The difficult part for any implementation of teleportation is the generation of the classical signal, which requires that a measurement be made on a pair of particles, e.g., photons. This measurement must effectively collapse the incoming particles’ wave packets into one of four strongly superposed (entangled) states and produce a unique signal for each. Our scheme fails to achieve this ideal, but does work for two of the four required cases — yielding at most a 50% absolute efficiency. Here we only present detailed calculations for generating one of the four signals — yielding at most a 25% absolute efficiency. However, this signal is uniquely charac-

terized by a four-way coincidence, so even in the presence of low detector efficiency, a conditional efficiency approaching 100% should be achievable.

Our scheme for generating the classical signals generalizes the Hong-Ou-Mandel interferometer [6] by allowing for states with arbitrary polarizations. We show that the description of this device in terms of measurement of eigenstates of Bell operators [7] yields an intuitive understanding of its operation. Our implementation requires interference between independent sources as recently suggested by Zukowski *et al.* [8]. We modify their work to allow for differences between the independent light sources and we derive their continuous mode wave-packet collapse language rigorously from Glauber photodetection theory [9].

We start by reviewing the discrete-mode description of teleportation in terms of photon polarization and discuss our detection scheme based on the generalized Hong-Ou-Mandel interferometer. We then show how wave-packet collapse may be used, to a limited extent, at the amplitude level to describe photodetection of light. Finally, we calculate the amplitude for successfully teleporting the state of polarization of a single-photon state and discuss typical numbers for our scheme’s performance.

Quantum states can be more highly correlated than would be allowed by any local classical theory — this being one consequence of the violation of the Bell inequalities. The most highly correlated quantum states are eigenstates of some Bell operator [7]. A suitable version of the Bell operator, given in Ref. [7], has a complete set of eigenstates

$$|\Psi^{(\pm)}\rangle_{ab} = 1/\sqrt{2}(|\uparrow\rangle_a|\leftrightarrow\rangle_b \pm |\leftrightarrow\rangle_a|\uparrow\rangle_b),$$

$$|\Phi^{(\pm)}\rangle_{ab} = 1/\sqrt{2}(|\uparrow\rangle_a|\uparrow\rangle_b \pm |\leftrightarrow\rangle_a|\leftrightarrow\rangle_b) \quad (1)$$

for two-photon states in modes a and b , where $|\uparrow\rangle$ and $|\leftrightarrow\rangle$ are single-photon states with vertical and horizontal polarization. As pointed out in Ref. [5] a measuring device that “collapses” the wave packet of a two-particle state into this Ψ - Φ correlated basis can be used to teleport a particle.

Suppose we have an arbitrary (but unknown) state $|\phi\rangle_1 = \alpha|\uparrow\rangle_1 + \beta|\leftrightarrow\rangle_1$ in mode 1. Teleportation is based on the following decomposition:

$$\begin{aligned}
 |\phi\rangle_1|\Psi^{(-)}\rangle_{23} = & -\frac{1}{2}[|\Psi^{(+)}\rangle_{12}\hat{R}^{(1)}|\phi\rangle_3 + |\Psi^{(-)}\rangle_{12}\hat{R}^{(2)}|\phi\rangle_3 \\
 & + |\Phi^{(+)}\rangle_{12}\hat{R}^{(3)}|\phi\rangle_3 + |\Phi^{(-)}\rangle_{12}\hat{R}^{(4)}|\phi\rangle_3], \quad (2)
 \end{aligned}$$

with the operators $\hat{R}^{(j)}$ ($j=1, \dots, 4$) given by

$$\hat{R}^{(1)} = \sigma_z, \quad \hat{R}^{(2)} = \mathbb{1}, \quad \hat{R}^{(3)} = i\sigma_y, \quad \hat{R}^{(4)} = -\sigma_x, \quad (3)$$

in terms of the usual Pauli spin matrices and the 2×2 identity matrix $\mathbb{1}$, using the representation $|\phi\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$. Measurement of $|\phi\rangle_1|\Psi^{(-)}\rangle_{23}$ in the Ψ - Φ basis of modes 1 and 2 will transfer the original unknown state $|\phi\rangle_1$ to mode 3, becoming $|\phi\rangle_3$ up to one of the operations $\hat{R}^{(j)}$. The teleportation process is completed when the classical information about which of the four correlated basis states was detected (two bits of information) is relayed to the site of particle 3 so the appropriate operation $\hat{R}^{(j)T}$ can transform the state of this mode to $|\phi\rangle_3$.

This transport of the photon's state has been achieved by two channels: one quantum (mode 3) and one classical (the two bits generated from detecting modes 1 and 2). Each channel by itself carries *no* information about the original state $|\phi\rangle_1$: the quantum channel by itself is in a completely mixed state, hence carrying no information; and the classical channel is worthless by itself because each of the results of measurement occurs with equal probability independent of $|\phi\rangle_1$.

It is becoming more accepted [10] to view collapse of the wave packet due to measurement in terms of a pure unitary evolution of the measurement apparatus and system, but coupled to a large "environment," which is traced out. In this view we would be forced to admit that the "classical" message is carrying the information about the state of the system through its subtle correlations to the environment. Nonetheless, since these correlations are apparently inaccessible, the classical message appears to be an information-free channel. Surprisingly, however, this quantum teleportation scheme is a situation where the effect of these correlations is writ large in the reappearance of the unknown state in mode 3.

The experiment we propose consists of a scheme to measure the Bell operator optically and hence collapsing states to the Ψ - Φ basis. The detection scheme is shown in Fig. 1; it consists of combining the incoming single-photon modes \hat{b}_1 and \hat{b}_2 at a 50-50 beam splitter to form the outgoing modes \hat{c}_1 and \hat{c}_2 . These outgoing modes are detected by a pair of polarization sensitive photodetectors (constructed by placing polarizers in front of ordinary detectors). For simplicity, we shall assume that the beam splitter imparts equal phase shifts to each polarization so that, after absorbing other extraneous phases, we have

$$\hat{c}_1^j = 1/\sqrt{2}(\hat{b}_1^j + \hat{b}_2^j), \quad \hat{c}_2^j = 1/\sqrt{2}(\hat{b}_1^j - \hat{b}_2^j), \quad (4)$$

with polarization $j = \uparrow, \leftrightarrow$ and inverse relations given by interchanging fields \hat{b} and \hat{c} .

Suppose a coincidence is found by the detectors in Fig. 1 that might be described by annihilating both a \hat{c}_1 and a \hat{c}_2 photon

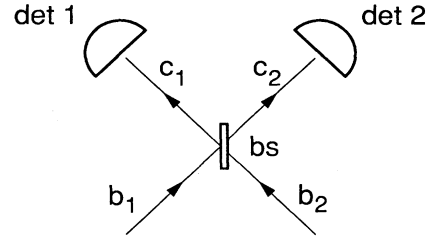


FIG. 1. Detection scheme for collapsing an incoming two-photon state into $|\Psi^{(-)}\rangle_{12}$ of Eq. (1); it generalizes the action of the Hong-Ou-Mandel interferometer [6]. Modes \hat{b}_1 and \hat{b}_2 , each in a single-photon state, are combined at a 50-50 beam splitter (bs). A coincidence between the photodetectors [within the narrow coincidence of Eq. (15)] is the signal for a successful collapse.

$$\hat{c}_1^j \hat{c}_2^k = \frac{1}{2}(\hat{b}_1^j \hat{b}_1^k - \hat{b}_1^j \hat{b}_2^k + \hat{b}_2^j \hat{b}_1^k - \hat{b}_2^j \hat{b}_2^k). \quad (5)$$

Let us restrict the incident state to a two-photon state with one photon in each mode \hat{b}_1 and \hat{b}_2 ; then this coincidence is effectively annihilating these photons to the vacuum state

$$\langle 0 | \hat{c}_1^j \hat{c}_2^k = \begin{cases} \pm (1/\sqrt{2}) {}_{12}\langle \Psi^{(-)} |, & j \neq k \\ 0, & j = k \end{cases} \quad (6)$$

where we have dropped the $\hat{b}_1^j \hat{b}_1^k$ and $\hat{b}_2^j \hat{b}_2^k$ terms by our choice of the incident state (this annihilation to vacuum by photodetectors will be made rigorous when we discuss Glauber photodetector theory). Note that for $j \neq k$ we have collapsed the state to one of the Ψ - Φ basis states needed for teleportation. In general, such coincidences between detectors 1 and 2 can only occur for purely antisymmetric incoming states; when these polarizations are unequal this is possible for the incoming state $|\Psi^{(-)}\rangle_{12}$, but when these polarizations are equal no such antisymmetric state is possible — this impossibility is what leads to a failure of coincidence and explains from a new point of view the operation of the Hong-Ou-Mandel interferometer [6].

If, instead, both photons are found in detector 1, we find

$$\langle 0 | \hat{c}_1^j \hat{c}_1^k = \begin{cases} (1/\sqrt{2}) {}_{12}\langle \Psi^{(+)} |, & j \neq k \\ (1/\sqrt{2})({}_{12}\langle \Phi^{(+)} | + {}_{12}\langle \Phi^{(-)} |), & j = k = \uparrow \\ (1/\sqrt{2})({}_{12}\langle \Phi^{(+)} | - {}_{12}\langle \Phi^{(-)} |), & j = k = \leftrightarrow \end{cases} \quad (7)$$

(with a similar result, up to an overall sign, if both photons are found in detector 2). Thus, this detection scheme successfully distinguishes two of the four needed states; the other two are indistinguishable, here, because of a degeneracy. This device measures the Bell operator faithfully for optical states — the degeneracy we see is also found there [7].

Even though this detection scheme cannot be used to teleport a state every time, it will, however, allow 100% efficiency conditioned on successful collapse to either of $|\Psi^{(\pm)}\rangle_{12}$. It is sufficient to look at detection of the state $|\Psi^{(-)}\rangle_{12}$, so we will consider only coincidences between detectors 1 and 2. To check the feasibility of implementing

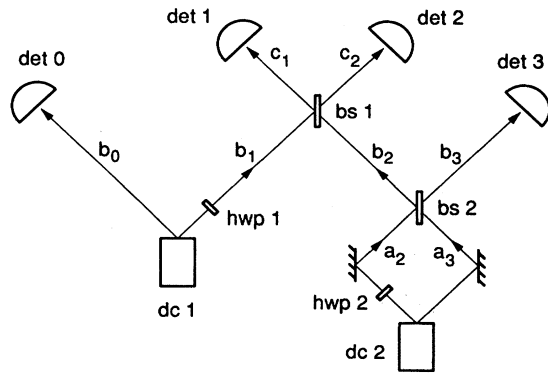


FIG. 2. A schematic for an apparatus to teleport the state of polarization of mode \hat{b}_1 to mode \hat{b}_3 . The parametric down-converters (dc) generate independent photon pairs: mode \hat{b}_1 has arbitrary polarization set by a half wave plate (hwp); and modes \hat{b}_2 and \hat{b}_3 form the conditional superposed state $|\Psi^{(-)}\rangle_{23}$ [13]. Detector 0 ensures mode \hat{b}_1 is a single-photon state. A successful teleportation event is uniquely signalled by a four-way coincidence with a narrow time constraint only on detectors 1 and 2 (Fig. 1 describes the detection scheme).

this discrete-mode scheme we shall consider broadband calculations for the implementation of the scheme shown in Fig. 2. A pair of down-converters are pumped by continuous-wave sources and coincidences well within a coherence time at detectors 1 and 2, as suggested by Zukowski *et al.* [8], are the gates for successful teleportation of mode \hat{b}_1 to mode \hat{b}_3 . To ensure the further assumption of only single-photon states being present at any one time in each mode we require a four-way coincidence among all four detectors; however, as we will see, “tight” coincidences [satisfying Eq. (15)] are only required between detectors 1 and 2 [8].

Consider the special case of broadband photodetection from modes with single-photon states. Glauber’s probability for output from a photodetector [9] (with single-photon state $|\psi\rangle$ incident upon it) is given in terms of the matrix elements of the positive part of the electric field operator $\hat{E}^{(+)}(t)$ as

$$\begin{aligned} I_D(t) &\propto \langle \psi | \hat{E}^{(-)}(t) \hat{E}^{(+)}(t) | \psi \rangle \\ &= \sum_i \langle \psi | \hat{E}^{(-)}(t) | i \rangle \langle i | \hat{E}^{(+)}(t) | \psi \rangle \\ &= |\langle 0 | \hat{E}^{(+)}(t) | \psi \rangle|^2 \propto |\langle 0 | \hat{a}(t) | \psi \rangle|^2, \end{aligned} \quad (8)$$

where we have summed over a complete set of states for which only the vacuum will contribute as an intermediate state; in the last step we have approximated the electric field operator by the Fourier transform of the broadband annihilation operator

$$\hat{a}(t) = \int (d\Delta/2\pi) e^{-i\Delta t} \hat{a}(\Delta), \quad (9)$$

which is valid as long as the bandwidth of the incident light is not too large (here and henceforth we write operators in a rotating frame about some fiducial frequency ω , which we take as the central frequency of down-converter 1). Thus, photodetection of single-photon states may be written at the amplitude level as collapse or annihilation of mode $\hat{a}(t)$ to vacuum [11].

In Fig. 2 parametric down-converters (dc) 1 and 2 supply photon pairs, when pumped at low intensity, generating states well approximated by [12]

$$\begin{aligned} |\Psi_{dc1}\rangle_{01} &= \int (d\Delta/2\pi) f(\Delta) \hat{b}_0^{\dagger}(\Delta) [\alpha \hat{b}_1^{\dagger}(-\Delta) \\ &\quad + \beta \hat{b}_1^{\leftrightarrow}(-\Delta)] |0\rangle, \\ |\Psi_{dc2}\rangle_{23} &= e^{-i\delta_p} \int (d\Delta'/2\pi) g(\Delta') \hat{a}_2^{\dagger}(\delta + \Delta') \\ &\quad \times \hat{a}_3^{\dagger}(\delta - \Delta') |0\rangle, \end{aligned} \quad (10)$$

where we have allowed dc 2 to be operated at a central frequency δ above that of dc 1 and the half-wave plates 1 and 2 have rotated mode \hat{b}_1 to an arbitrary linear polarization orientation $\alpha \hat{e}_{\uparrow} + \beta \hat{e}_{\leftrightarrow}$ and mode \hat{a}_2 to be horizontally polarized. The symbol δ_p is the phase difference (including phase jiggle) between the independent pumps; since this only contributes to the overall phase it will be dropped from our expressions.

Beam splitter 2 takes $|\Psi_{dc2}\rangle_{23}$ and produces a conditional state $|\Psi^{(-)}\rangle_{23}$ [13]

$$\begin{aligned} |\Psi^{(-)}\rangle_{23} &= \frac{1}{2} \int (d\Delta'/2\pi) g(\Delta') [\hat{b}_2^{\dagger}(\delta - \Delta') \hat{b}_3^{\dagger}(\delta + \Delta') \\ &\quad - \hat{b}_2^{\leftrightarrow}(\delta + \Delta') \hat{b}_3^{\dagger}(\delta - \Delta')] |0\rangle \end{aligned} \quad (11)$$

(only this piece of this state contributes when a full four-way coincidence is observed). A four-way coincidence will have an amplitude given by

$$\begin{aligned} &\langle 0 | \hat{b}_0^{\dagger}(t_0) \hat{c}_1^{\dagger}(t_1) \hat{c}_2^{\dagger}(t_2) \hat{b}_3^{\dagger}(t_3) | \Psi_{dc1}\rangle_{01} | \Psi^{(-)}\rangle_{23} \\ &= e^{-i\delta t_3} / 4 \{ \alpha \delta_{j\uparrow} \delta_{k\uparrow} \delta_{\ell\leftrightarrow} [e^{-i\delta t_1} f(t_0 - t_2) g(t_3 - t_1) - e^{-i\delta t_2} f(t_0 - t_1) g(t_3 - t_2)] \\ &\quad - \beta \delta_{j\leftrightarrow} \delta_{k\leftrightarrow} \delta_{\ell\uparrow} [e^{-i\delta t_1} f(t_0 - t_2) g(t_1 - t_3) - e^{-i\delta t_2} f(t_0 - t_1) g(t_2 - t_3)] \\ &\quad - \delta_{j\leftrightarrow} \delta_{k\uparrow} [\alpha \delta_{\ell\uparrow} e^{-i\delta t_1} f(t_0 - t_2) g(t_1 - t_3) + \beta \delta_{\ell\leftrightarrow} e^{-i\delta t_2} f(t_0 - t_1) g(t_3 - t_2)] \\ &\quad + \delta_{j\uparrow} \delta_{k\leftrightarrow} [\alpha \delta_{\ell\uparrow} e^{-i\delta t_2} f(t_0 - t_1) g(t_2 - t_3) + \beta \delta_{\ell\leftrightarrow} e^{-i\delta t_1} f(t_0 - t_2) g(t_3 - t_1)] \}, \end{aligned} \quad (12)$$

where $f(t)$ and $g(t)$ are the real Fourier transforms (time domain) of the functions $f(\Delta)$ and $g(\Delta)$, respectively. The single-mode calculations showed that we want coincidences with different polarizations between detectors 2 and 3, this being the signal for detecting state $|\Psi^{(-)}\rangle$. Finally, we suppose that detector 3 is filtered by a polarizer with orientation $\alpha' \vec{e}_{\uparrow} + \beta' \vec{e}_{\leftarrow}$. The probability for a four-way coincidence is now

$$I_D(t_0, t_1, t_2, t_3) \propto \frac{1}{16} \delta_{j \leftrightarrow k} |\alpha^* \alpha' e^{i\delta t_1} f(t_0 - t_2) g(t_1 - t_3) + \beta^* \beta' e^{i\delta t_2} f(t_0 - t_1) g(t_3 - t_2)|^2 + \frac{1}{16} \delta_{j \uparrow} \delta_{k \leftarrow} |\alpha^* \alpha' e^{i\delta t_2} f(t_0 - t_1) g(t_2 - t_3) + \beta^* \beta' e^{i\delta t_1} f(t_0 - t_2) g(t_3 - t_1)|^2, \quad (13)$$

which shows that the count in detector 0 occurs roughly within an inverse bandwidth time of a count in either detector 1 or 2 (similarly for detector 3 and the other of 1 or 2). In a parametric amplifier the functions $f(t)$ and $g(t)$ may be chosen to be symmetric to ensure that either photon from the pair has an equal chance of appearing at the detector first [6]. In this way the condition for perfect “fringe” visibility of the teleported photon as we rotate the polarizer in front of detector 3 is given by

$$f^2(t_0 - t_2) g^2(t_1 - t_3) + f^2(t_0 - t_1) g^2(t_3 - t_2) = 2 \cos[\delta(t_2 - t_1)] f(t_0 - t_2) f(t_0 - t_1) g(t_1 - t_3) g(t_3 - t_2). \quad (14)$$

This will be approximately satisfied when the time difference $t_2 - t_1$ is much smaller than the other relevant time scales

$$t_2 - t_1 \ll 2\pi/\delta, \quad 2\pi/B_f, \quad 2\pi/B_g, \quad (15)$$

where B_f and B_g are the optical bandwidths of the functions $f(\Delta)$ and $g(\Delta)$, respectively.

Using the well-defined rising edge of the detectors' outputs for our timing gates we could, not unreasonably, ask for detectors with resolution times of $\tau \approx 100$ psec [14]. Condition (15) then implies a very narrow bandwidth (taking $B = B_f = B_g \ll 2\pi/\tau$) of say $B \approx 2$ GHz. Narrower bandwidths have been obtained by placing the down-converters inside cavities [15], though to ensure that each photon from a pair leaves its cavity through a distinct mode we require type-II parametric down-converters [16].

Consider now some typical numbers for this scheme. Let η be the detector efficiency and $\langle n \rangle \ll 1$ be the number of photons per mode produced by our down-converters (and hence the probability for getting a down-converted pair from each parametric down-converter in an inverse bandwidth time of $2\pi/B$). Then a coincidence (to within a fraction f of an inverse bandwidth time; note that this implies $B \approx 2\pi f/\tau$) between detectors 1 and 2 as well as detecting events in detectors 0 and 3 within roughly an inverse bandwidth time will occur with probability $f \langle n \rangle^2 \eta^4/8$. Thus, we would expect to see one successful event per time

$$16\pi/(f \langle n \rangle^2 \eta^4 B). \quad (16)$$

Taking $\eta \approx 0.25$, [14] $f \approx 0.03$, $\langle n \rangle \approx 0.01$ (to ensure that we only see single-photon states—Ref. [15] achieves $\langle n \rangle \approx 2$ which is higher than needed here), and $B \approx 2$ GHz we expect one successful teleportation event every 2 sec. It is easy to convince oneself [14] that fake coincidences due to dark counts, within a time $2\pi/B$, are insignificant.

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