Photonic superguiding state in nonlinear polar crystals

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The present paper establishes the photonic superguiding theory in polar crystals with a high nonlinearity. In quantum theory it is shown that photons can sense an attractive effective interaction by exchange of virtual optical phonons. Such an interaction leads to the superguiding state. In the standing-wave superguiding state, the photons with opposite wave vectors and spins are bound into pairs. In the traveling-wave superguiding state, a propagating photon pair consists of a combination of two photons with opposite transverse wave vectors and spins. We study the particle properties of the photonic superguiding state, the most important property being that the system of photon pairs evolves without scattering attenuations. Quantum fluctuations of the standing-wave superguiding state exceed the vacuum fluctuations, while the traveling-wave superguiding state has the squeezing property. We also investigate the wave properties of the photonic superguiding state. It is found that the polar crystals with a high nonlinearity are self-defocusing media. In the standing-wave superguiding state, the system of photon pairs exists in the form of quantized vortices. In the traveling-wave superguiding state, the system of photon pairs exists in the form of quantized temporal solitons.

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I. INTRODUCTION

This paper reports detailed results of our investigation of the photonic superguiding state in polar crystals with a high nonlinearity. In a previous Brief Report [1] we showed that the system of photon pairs in the superguiding state evolves without scattering losses. However, we did not distinguish between the standing- and traveling-wave superguiding states and we also did not discuss the wave properties of the superguiding state. A comprehensive treatment of these problems is therefore necessary.

The classical Maxwell equations acquire a giant success in linear media and result in the discovery of numerous linear optical phenomena. An example is the phonon polariton in linear polar crystals, which is a particle comprised of a photon and a transverse-optical (TO) phonon. All classical optical phenomena in nonlinear media are also governed by the Maxwell equations. For example, the Maxwell equations in nonlinear media have classical soliton solutions. Basically, nonlinear optical effects originate from the nonlinear interaction of light with matter when the quantum many-body problems of light are particularly important. In order to enhance our knowledge about nonlinear optical effects greatly, we must develop quantum nonlinear optics by the quantum-field theory.

Recently the above consideration has provoked much interest in quantum effects of optical soliton propagation in nonlinear media. The quantum optical effects usually appear in one or more of the attributes of being sub-Poissonian, antibunched, or squeezed. If light exhibits a certain quantum optical effect, it has no positive nonsingular Glauber-Sudarshan P representation and is called nonclassical light. Nonclassical light is considered to be in nonclassical photon states. In the following, optical solitons first refer to temporal solitons in nonlinear optical fibers. Carter and co-workers first predicted the squeezing effect of bright solitons in nonlinear optical fibers [2–4]. This effect has now been observed in the first direct experimental test of the quantum soliton theory [5]. The bright soliton exists in the anomalous dispersion region of self-focusing media while the dark soliton is in their normal dispersion region. Using Gutkin's intertwining operator technique, Yurke and Potasek have predicted that photon antibunching statistics should result near the node of the dark soliton [6]. The photonic sub-Poissonian state often tends to occur together with the antibunched state, but the two effects are distinct. As for the two-dimensional spatial soliton in self-focusing media, Chiao, Deutsch, and Garrison have proposed a two-photon bound state which prevents the spreading of a beam of light due to diffraction [7]. Furthermore, these authors have discussed the differences between standing- and traveling-wave quantum solitons [8]. Lai and Haus have treated the quantum field theory of solitons in some detail [9].

The author has proposed the idea of the photonic superguiding state in nonlinear polar crystals [1]. Since the superguiding state is an intrinsic quantum-mechanical phenomenon, it is a new nonclassical photon state. Coherent light entering a nonlinear polar crystal can be converted into a new nonclassical light, which we refer to as superlight. In this paper we will show that photons in a polar crystal with a high nonlinearity can sense an attractive effective interaction by exchange of virtual optical phonons. The coherent state is unstable with respect to such an interaction and the superguiding state corresponding to superlight is formed through the association of photons in pairs. In the standing-wave superguiding state the two paired photons have opposite wave vectors and spins. In the traveling-wave superguiding state a propagating photon pair is the combination of the two photons with opposite transverse wave vectors and spins. There are the following contrasts between electrons and photons: (i) Electrons are charged fermions, while photons are neutral bosons; (ii) the electronic superconducting state is an equilibrium state, but the photonic superguiding state is a nonequilibrium one; and (iii) the excitation spectrum for the electronic superconducting state contains a gap, but that for the photonic superguiding state has no gap. However, there are the following similarities between the superconducting and superguiding states: (i) Supercurrent flowing in a superconductor experiences no resistance and superlight propagating in a waveguide has no scattering losses; (ii) the superconducting state excludes magnetic fields and the traveling-wave superguiding state expels vorticity fields; and (iii) the magnetic flux in a type-II superconductor is quantized and the circulation in a standing-wave superguiding state is quantized. A waveguide that can be superguiding can be called a superwaveguide. Photons in the superwaveguide can propagate as temporal solitons without dispersion. The quantum solitons in the traveling-wave superguiding state show the squeezing effect. In the present optical-communication systems, which use coherent beams of laser light, the ultimate performance is limited by the scattering losses, the dispersion effect, and the quantum noise. If the coherent input light propagates in the superwaveguide as superlight, at the same time we can obtain an ultralow energy loss, a high transmission rate, and a large signal-to-noise ratio.

Recently some new concepts of photons analogous to those of electrons have been proposed. Berry has recognized in quantum mechanics a topological phase factor arising from the adiabatic transport of a system around a closed circuit [10]. The Aharonov-Bohm effect is a manifestation of Berry's topological phase for the electron. Chiao and co-workers have given the first experimental verification of Berry's topological phase for the photon [11,12]. The photon localization in media with high-dielectric-constant scatterers has attracted extensive investigation [13,14]. The weak localization of photons has been observed in the coherent backscattering experiments [15,16] and the strong localization of photons also seems to have occurred [17,18]. A related problem is the localization of polaritons in disordered crystals and its role in optical-absorption experiments [19,20]. Yablonovitch and Gmitter [21,22] and Satpathy, Zhang, and Salehpour have proposed the idea of photon bands in a three-dimensionally-periodic dielectric structure [23]. Subsequently, John has suggested that any small deviation of the dielectric lattice from perfect periodicity will induce strongly localized electromagnetic (em) modes in the photonic band gap [24]. In structures possessing photonic band gaps, the suppression of molecular interactions and the occurrence of bound states of photons to atoms also have been predicted [25,26].

The remainder of this paper is organized as follows. Section II describes our physical model and derives the effective Hamiltonian of the photon system. The standing- and traveling-wave superguiding states are studied in Secs. III and IV, respectively. The wave properties of the superguiding state are investigated in Sec. V. Finally, Sec. VI summarizes our results and discusses possible applications of the photonic superguiding state to optical-communication systems.

II. PHOTONIC EFFECTIVE HAMILTONIAN

Let a laser light field enter a nonlinear polar crystal. The incident light field induces a macroscopic em field inside the crystal. We consider a coupled system consisting of the em field and the crystal. By nonlinearity we mean that the crystal is first-order Raman active. For convenience the crystal is taken to be of the cubic symmetry. The ion lattice within a volume V has N primitive cells. Let $\vec{r}_n$ be a suitable reference point inside the nth cell. The instantaneous displacement of the lth ion in cell n from its equilibrium position is given by the vector $\vec{s}_{nl}$. The incident light field is a linearly polarized coherent light field of a single mode. The incident frequency ω_0 is assumed to be well below the electronic transition frequencies, so that the photon-electron interaction and the related photon absorption by electrons can be neglected. This approximation allows us to make a phenomenological treatment to the interaction between the em field and the crystal.

In the ion lattice the polar vibration of an ion carries an electric dipole moment. The interaction Hamiltonian from the polar vibrations is linear in the macroscopic electric field $\vec{E}$ and can be written as [27]

$$H_{I1} = -\sum_{n,l} e_{nl}\vec{s}_{nl} \cdot \vec{E}(\vec{r}_n), \tag{1}$$

where e_{nl} is the effective charge of the nlth ion. Under the influence of the macroscopic em field, the charge center of the electron shell of an ion shifts relative to that of the nucleus and hence an electric dipole moment is induced in the ion. For the isotropic medium, the interaction Hamiltonian involving the ionic deformations is quadratic in $\vec{E}$ and has the form [28]

$$H_{I2} = -\frac{1}{2}\sum_{n,l} \int dt' \tilde{\alpha}_{nl}(t,t')\vec{E}(\vec{r}_n,t') \cdot \vec{E}(\vec{r}_n,t), \tag{2}$$

where $\tilde{\alpha}_{nl}(t,t')$ is the effective polarizability of the nlth ion. In Eqs. (1) and (2) the local electric field at an ion location is replaced by the macroscopic electric field. The Hamiltonian of the em field reads

$$H_L = \int d\vec{r} \left(\frac{\epsilon_0}{2}\vec{E}^2 + \frac{1}{2\mu_0}\vec{B}^2\right), \tag{3}$$

where ϵ_0 and μ_0 are the permittivity and permeability of vacuum. We choose for the vector potential $\vec{A}$ the Coulomb gauge $\vec{\nabla}\cdot\vec{A} = 0$ and suppose the scalar potential to be zero, so that the electric and magnetic fields are given by

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}\ , \quad \vec{B} = \vec{\nabla} \times \vec{A}\,. \tag{4}$$

In the harmonic approximation, the Hamiltonian for lattice vibrations has the form

$$H_C = \frac{1}{2}\sum_{nli} m_l \dot{s}_{nli}^2 + \frac{1}{2}\sum_{\substack{nli\\ n'l'i'}} \Phi_{nli}^{n'l'i'} s_{nli} s_{n'l'i'} \ , \qquad (5)$$

where the index i distinguishes the three rectangular components and the overdot indicates derivative with respect to time. m_l is the mass of the lth basis atom and $\Phi_{nli}^{n'l'i'}$ are called atomic force constants. The Hamiltonian of the coupled system reads

$$H = H_L + H_C + H_{I1} + H_{I2} \ . \qquad (6)$$

The above gives a classical description of the interaction of the em field with the vibrating crystal. We now discuss the quantum theory of such processes. In doing so, the coupled system is subjected to certain boundary conditions. Since the macroscopic crystal contains a large number of primitive cells, the properties of the macroscopic crystal do not depend on the choice of boundary condition. However, the properties of the macroscopic em field inside the crystal closely depend on the choice of boundary condition. Under any boundary condition, the em field is far away from thermal equilibrium and the crystal exchanges thermal energy with its surroundings. The crystal deviates slightly from equilibrium when the incident intensity of light is below the threshold of stimulated Brillouin scattering, whereas the crystal is far away from equilibrium when the incident intensity exceeds this threshold. Since plane-wave modes constitute a complete orthonormal set, they can be used for the expansion of the em field in any arbitrary geometry. In terms of the creation and annihilation operators $a_{\vec{k}\sigma}^{\dagger}$ and $a_{\vec{k}\sigma}$ of circularly polarized photons with spin $\sigma = \pm 1$, the vector potential of the em field is expanded as

$$\vec{A}(\vec{r},t) = \sum_{\vec{k}\sigma}\left(\frac{\hbar}{2V\epsilon_0\omega_{\vec{k}}}\right)^{\frac{1}{2}} \times \left[\vec{\chi}_\sigma(\vec{k}) a_{\vec{k}\sigma} e^{i\vec{k}\cdot\vec{r}} + \vec{\chi}_\sigma^{*}(\vec{k}) a_{\vec{k}\sigma}^{\dagger} e^{-i\vec{k}\cdot\vec{r}}\right] , \qquad (7)$$

where $\vec{\chi}_\sigma(\vec{k})$ and $\omega_{\vec{k}} = c|\vec{k}|$ are the polarization vector and frequency of photons, respectively. By inserting Eqs. (4) and (7) into Eq. (3), the Hamiltonian of the em field is converted into the Hamiltonian of a noninteracting system of photons

$$H_L = \sum_{\vec{k}\sigma} \hbar\omega_{\vec{k}} a_{\vec{k}\sigma}^{\dagger} a_{\vec{k}\sigma} \ , \qquad (8)$$

where the zero-point energy is removed. The acoustic vibration of an ion represents a pressure fluctuation at the ion site. If we regard the pressure at an ion site as an independent state variable of the crystal, the ion vibration refers to the optical vibration. When the phonons of the jth optical branch are created or annihilated by the operators $b_{\vec{q}j}^{\dagger}$ and $b_{\vec{q}j}$, the ion displacement can be expanded as

$$\vec{s}_{nl}(t) = \sum_{\vec{q}j}\left[\frac{\hbar}{2Nm_l\omega_j(\vec{q})}\right]^{\frac{1}{2}} \vec{e}_l(\vec{q}j)(b_{\vec{q}j} + b_{-\vec{q}j}^{\dagger}) e^{i\vec{q}\cdot\vec{r}_n} \ , \qquad (9)$$

where $\vec{e}_l(\vec{q}j)$ and $\omega_j(\vec{q})$ are the eigenvector and frequency of the jth optical branch, respectively. By putting Eq. (9) into Eq. (5), the Hamiltonian for lattice vibrations is converted into the Hamiltonian of a noninteracting system of optical phonons

$$H_C = \sum_{\vec{q}j} \hbar\omega_j(\vec{q}) b_{\vec{q}j}^{\dagger} b_{\vec{q}j} \ , \qquad (10)$$

where the zero-point energy is discarded also. The operators of photons and optical phonons obey the Bose commutation relations.

Because the wave vector of light is much smaller than the dimensions of the Brillouin zone, conservation of momentum requires that the phonons involved in optical processes be near the center of the Brillouin zone. The symmetries of the small-wave-vector lattice vibrations can then be described by irreducible representations of the crystal point group. The optical vibrations of polar crystals fall into two distinct categories, i.e., polar modes and nonpolar modes. Polar modes carry electric dipole moments and are infrared active, whereas nonpolar modes carry no electric dipole moments and are infrared inactive. In the cubic crystal, each group-theoretical threefold polar mode splits into a TO doublet and a longitudinal-optical singlet, while each nonpolar mode is threefold degenerate at zero wave vector. The charge e_{nl} in Eq. (1) is a function of the nlth ion displacement due to the polar vibration. The displacement $\vec{s}_{nl}$ here is given by Eq. (9) with nonpolar modes removed. Near the ionic equilibrium site, the charge e_{nl} can be expanded in increasing powers of the displacement as $e_{nl} = e_l(0) + \vec{\nabla}e_l \cdot \vec{s}_{nl} + \cdots$, where $e_l(0)$ and $\vec{\nabla}e_l$ are the charge and differential charge vector of the lth basis ion at the equilibrium site, respectively. In the interaction Hamiltonian H_{I1}, the linear term in $\vec{s}_{nl}$ describes the phonon-polariton effect and the higher-order terms in $\vec{s}_{nl}$ interpret the absorption of a photon with the formation of two or more TO phonons. We hypothesize that the incident frequency is well above the TO phonon frequencies of the crystal, so that the photon-phonon interaction H_{I1} linear in $\vec{E}$ and the related multiphonon absorption of a photon can be omitted.

For simplicity, the studied crystal is also taken to be centrosymmetric. Polar modes in centrosymmetric crystals have odd parity and are Raman inactive [29]. In the cubic system, the common polar crystals that are both centrosymmetric and Raman active have the fluorite structure. At this point, the studied crystal is determined as a certain crystal of the fluorite structure, such as CaF_2. In the fluorite-structure crystal, a primitive cell contains two anions and one cation giving a single nonpolar mode, which is Raman active. For the Raman-active mode, the two anions in the primitive cell move in antiphase, while the cation remains stationary. Since the following treatment has no relation to polar modes, the optical vibrations of the crystal are limited to the Raman-active mode. Now the index l is only used to distinguish the two anions in the basis. In Eqs. (9) and (10) the Raman-mode frequency $\omega_j(\vec{q})$ at small wave vectors is replaced with the zero-wave-vector value ω_R and the branch index j is deleted. Furthermore, the incident intensity is assumed to be below the threshold of stimulated Brillouin scattering, so that the pressure fluctuations and Raman-active mode of the crystal are excited thermally. The polarizability $\tilde{\alpha}_{nl}$ is a function of the temperature, pressure, and displacement at the nlth ion site. Near the equilibrium state of the ion system, the polarizability $\tilde{\alpha}_{nl}$ can be expanded as

$$\tilde{\alpha}_{nl}(t,t') = \tilde{\alpha}_l(t-t') + \delta(t-t')\left[\delta\alpha_{nl}(t) + \vec{\nabla}\alpha_l \cdot \vec{s}_{nl}(t)\right] \quad . \tag{11}$$

Here $\tilde{\alpha}_l(t-t')$ is the polarizability of the lth basis ion in the equilibrium state and its Fourier transformation $\alpha_l(\omega)$ reflects the material dispersion due to the ionic deformations. $\delta\alpha_{nl}$ is the fluctuation in the polarizability of the nlth ion due to thermal variations in the temperature and pressure. $\vec{\nabla}\alpha_l$ is the differential polarizability vector of the lth basis ion in the equilibrium state and represents the nonlinearity of the crystal. Thereby the interaction Hamiltonian H_{I2} contains three terms. The first term involving $\tilde{\alpha}_l(t-t')$ can be incorporated in the em field energy by introducing a $\vec{k}$-dependent linear dielectric function $\bar{\epsilon}(\vec{k})$. Now, in Eqs. (7) and (8), ϵ_0 is replaced with $\epsilon_0\bar{\epsilon}(\vec{k})$ and correspondingly $\omega_{\vec{k}} = c|\vec{k}|/\sqrt{\bar{\epsilon}(\vec{k})}$. We define $\alpha_n = \sum_l \alpha_{nl}$ and let $\delta\alpha_{\vec{q}}$ denote the Fourier transformation of $\delta\alpha_n$. Consequently, the second term involving $\delta\alpha_{nl}$ reduces to

$$H_f = \sum_{\vec{k}\sigma,\vec{q}} N_{\vec{k}\sigma}(\vec{q})\delta\alpha_{\vec{q}} a^\dagger_{\vec{k}+\vec{q},\sigma} a_{\vec{k}\sigma} \quad ,$$

$$N_{\vec{k}\sigma}(\vec{q}) = -\frac{\hbar N\left(\omega_{\vec{k}+\vec{q}}\omega_{\vec{k}}\right)^{\frac{1}{2}}}{2V\epsilon_0\bar{\epsilon}(\vec{k})}\vec{\chi}^*_\sigma(\vec{k}+\vec{q})\cdot\vec{\chi}_\sigma(\vec{k}). \tag{12}$$

The last term in H_{I2} is quantized as follows:

$$H_v = \sum_{\vec{k}\sigma,\vec{q}} M_{\vec{k}\sigma}(\vec{q}) a^\dagger_{\vec{k}+\vec{q},\sigma} a_{\vec{k}\sigma}(b_{\vec{q}} + b^\dagger_{-\vec{q}}),$$

$$M_{\vec{k}\sigma}(\vec{q}) = -\sum_l \frac{\hbar^{\frac{3}{2}}}{V\epsilon_0\bar{\epsilon}(\vec{k})}\left(\frac{N\omega_{\vec{k}+\vec{q}}\omega_{\vec{k}}}{8m_l\omega_R}\right)^{\frac{1}{2}} \times\left[\vec{\nabla}\alpha_l\cdot\vec{e}_l(\vec{q})\right]\vec{\chi}^*_\sigma(\vec{k}+\vec{q})\cdot\vec{\chi}_\sigma(\vec{k}). \tag{13}$$

Here we have assumed that the scattered photon has the same spin as the incident photon. This assumption will be discarded at an appropriate point. One easily finds that $\vec{\chi}^*_{\sigma'}(\vec{k}')\cdot\vec{\chi}_\sigma(\vec{k}) = (1+\sigma'\sigma\cos\theta)/2$, where θ is the angle between $\vec{k}'$ and $\vec{k}$. The Hamiltonian of the coupled system becomes

$$H = H_L + H_C + H_f + H_v \quad . \tag{14}$$

In the classical solid considered, temperature fluctuations at constant pressure travel diffusively and give rise to elastic Rayleigh scattering, but pressure fluctuations at constant entropy propagate ballistically as transverse and longitudinal sound waves and lead to spontaneous Brillouin scattering. Therefore the Hamiltonian H_f accounts for Rayleigh scattering and spontaneous Brillouin scattering. The effect of H_f on the photon system can be described by the Hamiltonian H'_f, which is defined through the relation [30]

$$\exp(H'_f/k_BT) = \langle\exp(H_f/k_BT)\rangle, \tag{15}$$

where $\langle\rangle$ denotes the ensemble average for the ion system. H'_f can be obtained by the cumulant expansion of probability theory. To the second cumulant, $H'_f = \langle H_f^2\rangle/2k_BT$. Using Einstein's fluctuation theory, we find that

$$H'_f = \frac{\langle(\delta\alpha_n)^2\rangle}{2Nk_BT}\sum_{\substack{\vec{k},\vec{k}',\vec{q}\\ \sigma,\sigma'}} N_{\vec{k}\sigma}(-\vec{q})N_{\vec{k}'\sigma'}(\vec{q}) \times a^\dagger_{\vec{k}-\vec{q},\sigma}a^\dagger_{\vec{k}'+\vec{q},\sigma'}a_{\vec{k}'\sigma'}a_{\vec{k}\sigma} \quad , \tag{16}$$

$$\frac{\langle(\delta\alpha_n)^2\rangle}{Nk_BT} = \frac{T}{VC_P}\left(\frac{\partial\alpha_n}{\partial T}\right)^2_P + \frac{1}{V\beta_S}\left(\frac{\partial\alpha_n}{\partial P}\right)^2_S \quad ,$$

where T, P, and S are the temperature, pressure, and entropy of the crystal. C_P is the constant-pressure specific heat per unit volume of the crystal and β_S is its adiabatic compressibility. Here H'_f represents a repulsive photon-photon interaction. The mechanism of repulsion is as follows. Rayleigh and spontaneous Brillouin scatterings of photons are due to inhomogeneities of the crystal on the microscopic scale. The inhomogeneous crystal possesses the energy of temperature and pressure waves. It is known that the crystal is homogeneous on the average so that photons are correlated. The conservation of energy requires that the energy of the system of correlated photons increase when the energy of temperature and pressure waves vanishes. The repulsiveness of photons is the manifestation of the increase of the system's energy. The Hamiltonian H_v represents spontaneous Raman scattering by an optical phonon. To determine the effect of H_v on the photon system, it is desirable to make the unitary transformation $H_T = e^{-iS}He^{iS}$ in which S is Hermitian and H_f is removed from H [31]. The exponential functions in the expression can be expanded. The unitary transformation can eliminate H_v from H in first order provided S is given by $i[H_L + H_C, S] = -H_v$. To second order in S, the transformed Hamiltonian is

$$H_T = H_L + H_C + \frac{1}{2}i[H_v, S], \tag{17}$$

where H_T contains an additional term. Averaging the additional term with the equilibrium phonon density matrix

$$\rho = \exp(-H_C/k_BT)/[\mathrm{Tr}\,\exp(-H_C/k_BT)], \qquad (18)$$

we find the Hamiltonian

$$H'_v = \frac{1}{2}i\langle[H_v,S]\rangle = \sum_{\substack{\vec{k},\vec{k}',\vec{q}\\ \sigma,\sigma'}} \frac{M_{\vec{k}\sigma}(-\vec{q})M_{\vec{k}'\sigma'}(\vec{q})\omega_R}{\hbar[(\omega_{\vec{k}}-\omega_{\vec{k}-\vec{q}})^2-\omega_R^2]} \times a^\dagger_{\vec{k}-\vec{q},\sigma}a^\dagger_{\vec{k}'+\vec{q},\sigma'}a_{\vec{k}'\sigma'}a_{\vec{k}\sigma} \ . \qquad (19)$$

The interaction matrix elements given by Eq. (19) can be either attractive or repulsive. If the states $\vec{k}$ and $\vec{k}-\vec{q}$ are separated by an energy smaller than $\hbar\omega_R$, an attraction is present. The system will have to adjust itself to the presence of this attraction. A physical model for this attraction is as follows. Photons are correlated when the coupling between photons and thermal optical phonons is neglected. Since the energy of the system of thermal optical phonons is constant at a certain temperature, the energy of the system of correlated photons must decrease. The effective photon-photon attraction results from this decrease in the system's energy. In this case, the conservation of energy requires there to be sufficient virtual optical phonons in the crystal to account for the energy deficit.

In deriving the Hamiltonians (16) and (19), we also obtain the change in the one-photon energy due to the interaction of photons with the crystal. This change reflects the material dispersion due to the ionic fluctuations. For this the linear dielectric function $\bar{\epsilon}(\vec{k})$ is renormalized as the linear dielectric function $\epsilon(\vec{k})$. $\epsilon(\vec{k})$ is real and spherically symmetric. $\epsilon(|\vec{k}|)$ is an ascending function of $|\vec{k}|$ and $\epsilon(\infty)=\epsilon_h$, where ϵ_h is a high-frequency dielectric constant. Then the photon frequency in the Hamiltonian (8) is rewritten as $\omega_{\vec{k}} = c|\vec{k}|/\sqrt{\epsilon(\vec{k})}$. The effective Hamiltonian of the photon system is $H_p = H_L + H'_f + H'_v$. For future study we make a remark. Since the incident light field is linearly polarized, the numbers of clockwise and counterclockwise circularly polarized photons are equal for each wave vector in the isotropic medium.

III. STANDING-WAVE SUPERGUIDING STATE

We first consider the configuration that the studied crystal has rectangular faces and is enclosed by perfectly reflecting mirrors, as shown in Fig. 1. In fact, we form a passive optical resonator. There is a small window in each mirror at $x=0$, $y=0$, and $z=0$. The external light field enters the resonator through the three windows within a time interval. The incident directions are along the x, y, and z axes, and the three windows are closed at the end of the time interval. The macroscopic em field induced by the incident light field is in a standing-wave configuration. In the resonator there is the blackbody radiation field besides the induced em field. According to Stefan's law, the energy density of blackbody radiation at room temperature is very much smaller than a persistent energy density of laser radiation. Correspondingly, we can neglect the effects of blackbody radiation. The em field must satisfy the boundary condition at each face,

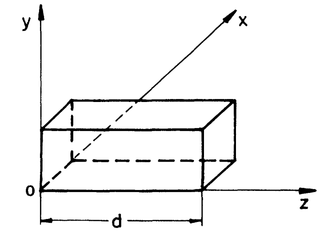

FIG. 1. The crystal has rectangular faces and is enclosed by perfectly reflecting mirrors. This forms a passive optical resonator.

$$\vec{n}\times\vec{E} = \vec{0}\ ,\quad \vec{n}\cdot\vec{B} = 0\ , \qquad (20)$$

where $\vec{n}$ is the unit vector normal to the particular face. The boundary condition (20) confines the plane-wave modes appearing in the expansion (7) to the normal modes of the resonator, so that the photons of wave vectors $\vec{k}$ and $-\vec{k}$ always occur simultaneously and have an equal number.

In Sec. II, we have seen that the interaction between photons and optical phonons can lead to an attractive interaction between the photons themselves. By high nonlinearity we mean that the repulsion in Eq. (16) can be overcome by the attraction in Eq. (19). The attractive effective interaction leads to bound photon pairs. The physical background for pairing is simple: A photon can emit or absorb a virtual optical phonon. The emission of virtual optical phonons by photons means that the photon is clothed with a polarization cloud of the lattice vibration. If a second photon is near this polarization cloud, it experiences a force of attraction. In the standing-wave configuration a photon pair is stable only if the two photons have opposite wave vectors and spins. Thus one set $\vec{k}'=-\vec{k}$ and $\sigma'=-\sigma$ in Eqs. (16) and (19). Next we let $\vec{k}$ stand for $(\vec{k},\sigma)$ and $-\vec{k}$ for $(-\vec{k},-\sigma)$. Now the pair Hamiltonian of the photon system is

$$H_p = \frac{1}{2}\sum_{\vec{k}}\hbar\omega_{\vec{k}}(a^\dagger_{\vec{k}}a_{\vec{k}} + a^\dagger_{-\vec{k}}a_{-\vec{k}}) + \sum_{\vec{k},\vec{k}'}V_{\vec{k}\vec{k}'}a^\dagger_{\vec{k}'}a^\dagger_{-\vec{k}'}a_{-\vec{k}}a_{\vec{k}}\ , \qquad (21)$$

$$V_{\vec{k}\vec{k}'} = \frac{|M_{\vec{k}}(\vec{k}'-\vec{k})|^2\omega_R}{\hbar[(\omega_{\vec{k}}-\omega_{\vec{k}'})^2-\omega_R^2]} + \frac{\langle(\delta\alpha_n)^2\rangle}{2Nk_BT}|N_{\vec{k}}(\vec{k}'-\vec{k})|^2. \qquad (22)$$

Since the spin of the scattered photon can be different from that of the incident photon, here $\vec{k}'$ stands for $(\vec{k}',\sigma')$ and $-\vec{k}'$ for $(-\vec{k}',-\sigma')$. Therefore the sum over wave vectors includes the summation over the two spins. The pair potential $V_{\vec{k}\vec{k}'}$ is real and has the symmetric properties $V_{\vec{k}\vec{k}'} = V_{-\vec{k},-\vec{k}'}$ and $V_{\vec{k}\vec{k}'} = V_{\vec{k}'\vec{k}}$. The symmetric properties are used in our treatment. It will be supposed that $V_{\vec{k}\vec{k}'} = -V_0$ if $|\omega_{\vec{k}}-\omega_0|$ and $|\omega_{\vec{k}'}-\omega_0| < \omega_R$; otherwise $V_{\vec{k}\vec{k}'} = 0$.

The diagonalization of the pair Hamiltonian (21) can be performed by the Bogoliubov transformation

$$U = \exp\left[\frac{1}{2}\sum_{\vec{k}} \varphi_{\vec{k}}(a^\dagger_{\vec{k}}a^\dagger_{-\vec{k}} - a_{\vec{k}}a_{-\vec{k}})\right], \tag{23}$$

$$a_{\vec{k}} = U^\dagger \alpha_{\vec{k}} U = \alpha_{\vec{k}}\cosh\varphi_{\vec{k}} + \alpha^\dagger_{-\vec{k}}\sinh\varphi_{\vec{k}} ,$$
$$a^\dagger_{\vec{k}} = U^\dagger \alpha^\dagger_{\vec{k}} U = \alpha^\dagger_{\vec{k}}\cosh\varphi_{\vec{k}} + \alpha_{-\vec{k}}\sinh\varphi_{\vec{k}} , \tag{24}$$

where the parameter $\varphi_{\vec{k}}$ is assumed to be real and spherically symmetric. $\alpha^\dagger_{\vec{k}}$ and $\alpha_{\vec{k}}$ are the creation and annihilation operators of quasiparticles in the photon system; they also obey the Bose commutation relations. The state vector of photon pairs in the photon system may be constructed as $|G\rangle = U|0\rangle$, where $|0\rangle$ is the vacuum state, such that $\alpha_{\vec{k}}|G\rangle = 0$. Because the incident light field is fully coherent, quasiparticles are in a coherent state of many modes. Concomitantly, we introduce the displacement operator

$$D(\eta) = \exp\left[\sum_{\vec{k}}(\eta_{\vec{k}}\alpha^\dagger_{\vec{k}} - \eta^*_{\vec{k}}\alpha_{\vec{k}})\right], \tag{25}$$

where the parameter $\eta_{\vec{k}}$ is complex and spherically symmetric. The state vector of the photon system may be written as $|\eta\rangle = D(\eta)|G\rangle$, such that $\alpha_{\vec{k}}|\eta\rangle = \eta_{\vec{k}}|\eta\rangle$. $|\eta_{\vec{k}}|^2$ characterizes the mean quasiparticle number of a mode field and can be described by a distribution function $f_{\vec{k}}(u) = |\eta_{\vec{k}}|^2$, where the argument u is the average energy density of the em field. $\eta_{\vec{k}}$ has a definite phase factor depending on its suffix $\vec{k}$. We shall assume that there is no correlation between these phase factors when $\vec{k} \neq \vec{k}'$. Strictly speaking, the phase of $\eta_{\vec{k}}$ is random to some degree, because of thermal scattering. At present one can neglect the thermal phase noise in the system of quasiparticles. We substitute Eq. (24) into Eq. (21) and let the second-order nondiagonal terms vanish. Within the framework of a mean-field theory [31], the pair Hamiltonian of the photon system becomes

$$H_p = E_p(u) + \frac{1}{2}\sum_{\vec{k}} E_{\vec{k}}(u)(\alpha^\dagger_{\vec{k}}\alpha_{\vec{k}} + \alpha^\dagger_{-\vec{k}}\alpha_{-\vec{k}}) + (\text{fourth-order nondiagonal terms}). \tag{26}$$

Here $E_{\vec{k}}(u)$ is the excitation energy of a quasiparticle defined by the equations

$$E_{\vec{k}}(u) = \left[(\hbar\omega_{\vec{k}})^2 - \Delta^2_{\vec{k}}(u)\right]^{\frac{1}{2}}, \tag{27}$$

$$\Delta_{\vec{k}}(u) = -\sum_{\vec{k}'} \frac{V_{\vec{k}\vec{k}'}\Delta_{\vec{k}'}(u)}{E_{\vec{k}'}(u)}\left[1 + 2f_{\vec{k}'}(u)\right], \tag{28}$$

where the excitation spectrum $E_{\vec{k}}(u)$ has no gap and $\Delta_{\vec{k}}(u)$ is the order parameter for pairing. The parameter $\varphi_{\vec{k}}$ is given by the relation $\tanh(2\varphi_{\vec{k}}) = \Delta_{\vec{k}}(u)/\hbar\omega_{\vec{k}}$. Equation (28) has the superguiding-state solution with the form

$$\Delta_{\vec{k}}(u) = \Delta(u)\Theta(\omega_R - |\omega_{\vec{k}} - \omega_0|),$$
$$0 < \Delta(u) < \hbar(\omega_0 - \omega_R). \tag{29}$$

The state vector $|\eta\rangle$ corresponding to Eq. (29) describes the superguiding state of the photon system. The energy of the system of photon pairs is obtained as

$$E_p(u) = \frac{1}{2}\sum_{\vec{k}}\hbar\omega_{\vec{k}}\left[\frac{\hbar\omega_{\vec{k}}}{E_{\vec{k}}(u)} - 1\right] + \frac{1}{4}\sum_{\vec{k},\vec{k}'} V_{\vec{k}\vec{k}'}\frac{\Delta_{\vec{k}'}(u)\Delta_{\vec{k}}(u)}{E_{\vec{k}'}(u)E_{\vec{k}}(u)}. \tag{30}$$

In the superguiding state the paired photons are distributed over only a small range of frequencies ($2\omega_R$) near the incident frequency and in the superguiding ground state $|G\rangle$ all the photons are paired with opposite wave vectors and spins. Equation (28) also has the normal-state solution, namely, $\Delta_{\vec{k}}(u) = 0$ for all $\vec{k}$. In this case the state vector $|\eta\rangle = D(\eta)|0\rangle$ describes the normal state of the photon system and is a coherent state of many modes. In the normal state, the quasiparticles become photons and the photons are unpaired and satisfy the energy-frequency relation $E_{\vec{k}} = \hbar\omega_{\vec{k}}$.

We rewrite the Hamiltonian of the em field in Eq. (8) as

$$H_L = \frac{1}{2}\sum_{\vec{k}}\hbar\omega_{\vec{k}}(a^\dagger_{\vec{k}}a_{\vec{k}} + a^\dagger_{-\vec{k}}a_{-\vec{k}}). \tag{31}$$

The transformation (24) is put into Eq. (31). The energy E_L of the em field is the expectation value of H_L with respect to the state vector $|\eta\rangle$ of the photon system, namely,

$$E_L = \langle\eta|H_L|\eta\rangle = \frac{1}{2}\sum_{\vec{k}}\hbar\omega_{\vec{k}}\left[\frac{\hbar\omega_{\vec{k}}}{E_{\vec{k}}(u)} - 1\right] + \sum_{\vec{k}}\frac{(\hbar\omega_{\vec{k}})^2}{E_{\vec{k}}(u)}f_{\vec{k}}(u), \tag{32}$$

where the second-order nondiagonal terms disappear under the random-phase approximation. The first and second terms represent the contributions from paired photons and quasiparticles, respectively. It is assumed that in the superguiding and normal states the em field has the same energy E_L. The energy $E(u)$ of the photon system is the expectation value of the pair Hamiltonian (26) in the state $|\eta\rangle$, namely,

$$E(u) = \langle\eta|H_p|\eta\rangle = E_p(u) + \sum_{\vec{k}} E_{\vec{k}}(u)f_{\vec{k}}(u), \tag{33}$$

where the fourth-order nondiagonal terms are discarded. In the superguiding state, Eq. (33) gives the energy $E_s(u)$ of the photon system, and in the normal state, Eq. (33)

gives the energy $E_n(u)$ of the photon system. Obviously, $E_n(u)$ is always equal to the em energy E_L and can be expressed into the superguiding-state form in Eq. (32). The difference in the energy between the normal and superguiding states is therefore

$$\begin{aligned}\delta E(u) &= E_n(u) - E_s(u)\\ &= -\frac{1}{4}\sum_{\vec{k},\vec{k}'} V_{\vec{k}\vec{k}'}\frac{\Delta_{\vec{k}'}(u)\Delta_{\vec{k}}(u)}{E_{\vec{k}'}(u)E_{\vec{k}}(u)}\\ &\quad +\sum_{\vec{k}}\left[\frac{(\hbar\omega_{\vec{k}})^2}{E_{\vec{k}}(u)} - E_{\vec{k}}(u)\right] f_{\vec{k}}(u), \end{aligned} \qquad (34)$$

where Eq. (30) is used. $\delta E(u)$ is positive and represents the condensation energy of the superguiding state. The condensation energy of the superguiding ground state is $\delta E(u_g) = \Delta^2(u_g)/4V_0$, where u_g is the em energy density corresponding to the superguiding ground state. In the normal state, the em energy stored in the resonator is carried completely by the photon system. In the superguiding state, part of the em energy, i.e., $E_s(u)$, is carried by the photon system and the other part, i.e., $\delta E(u)$, is borne by the ion system. At this point, we show that the photon system in the superguiding state has a lower energy than in the normal state and therefore the superguiding state is preferred. Furthermore, the superguiding ground state is most stable because of its maximum condensation energy.

In the normal state, the behavior of photons is governed by the Hamiltonian (14) of the coupled system. The normal photons suffer spontaneous Raman scattering by optical phonons as well as Rayleigh and spontaneous Brillouin scatterings. In the superguiding state, photon pairs and single quasiparticles are present together. Since Rayleigh and spontaneous Brillouin scatterings are overcome by attractive photon-phonon-photon interaction, the resultant photon pairs incorporate the optical phonons and hence do not suffer any scatterings. Furthermore, the system of photon pairs is a condensate because a macroscopically large number of photon pairs occupy a single-quantum state of zero wave vector and zero spin. However, the fourth-order nondiagonal terms in the Hamiltonian (26) represent the interaction between quasiparticles. As this interaction originates from the scattering mechanisms, individual quasiparticles experience scatterings like the normal photons. Because of continual scattering in the resonator, quasiparticles are distributed continuously in the whole wave-vector space. The wave vector $\vec{k}$ is specified by the frequency ω and the travel direction Ω of quasiparticles, where Ω corresponds to a polar angle θ and an azimuthal angle ϕ. The frequency spectrum of quasiparticles can be described by a Gaussian distribution with frequency width $2\omega_b$ centered at the incident frequency ω_0. ω_b is an increasing function of temperature, but always far smaller than ω_0. We write the distribution function of quasiparticles in the separation form of variables

$$f_{\vec{k}}(u) = N(u)g(\Omega)\exp\left[-\left(\frac{\omega-\omega_0}{\omega_b}\right)^2\right]. \qquad (35)$$

Here $g(\Omega)$ represents the direction distribution of quasiparticles and must satisfy the spherical symmetry

$$g(\Omega) = 1,\ 0 \le \theta \le \pi,\ \ 0 \le \phi \le 2\pi. \qquad (36)$$

The average energy density of the em field is $u = E_L/V$, where E_L is given by Eq. (32). Therefore, Eq. (32) determines the functional relation of $N(u)$ to u. For inelastic scatterings of quasiparticles, the crystal both absorbs and emits energy, but the absorbing ability is higher than the emitting one. The energy of the system of quasiparticles will decay out as time goes on. In describing the energy loss, we use, rather than u, time t as the argument of the function N. If the laser radiation into the resonator is stopped at the instant $t = 0$, we can write

$$N(t) = N_0 e^{-t/t_c}, \qquad (37)$$

where t_c is the quasiparticle lifetime. N_0 is determined by the em energy density u_0 at the initial instant through Eq. (32).

It remains for us to calculate the order parameter of the superguiding state. In the calculation, we neglect the frequency dispersion in the dielectric response of the crystal and consider the case of $\omega_0 \gg \omega_R$. From Eq. (32), one can acquire the expression of $N(u)$

$$N(u) = \frac{V\omega_0^2}{\hbar^2 G_0}\frac{u - \hbar\omega_0\hbar\omega_R G_0 V^{-1}\left[\frac{\hbar\omega_0}{E(\omega_0,u)} - 1\right]}{Z(u) + \sqrt{\pi}\omega_0\omega_b\left(\frac{3}{2}\omega_b^2+\omega_0^2\right)},$$

$$Z(u) = \int_{\omega_0-\omega_R}^{\omega_0+\omega_R} d\omega\left[\frac{\hbar\omega}{E(\omega,u)} - 1\right]\omega^3 \times \exp\left[-\left(\frac{\omega-\omega_0}{\omega_b}\right)^2\right], \qquad (38)$$

where $E(\omega,u) = \left[(\hbar\omega)^2 - \Delta^2(u)\right]^{\frac{1}{2}}$ and $G_0 = V\omega_0^2\epsilon_h^{\frac{3}{2}}/\pi^2\hbar c^3$ is the density of photonic states at the incident energy $\hbar\omega_0$. In the normal state where $\Delta(u) = 0$, $N(u)$ increases proportionally with the em energy density u. In the superguiding state, Eq. (28) can be converted into an integral form

$$1 = \frac{\hbar\omega_0\lambda}{E(\omega_0,u)} + \frac{\hbar\lambda N(u)}{\omega_R\omega_0}\int_{\omega_0-\omega_R}^{\omega_0+\omega_R} d\omega\frac{\omega^2}{E(\omega,u)} \times \exp\left[-\left(\frac{\omega-\omega_0}{\omega_b}\right)^2\right], \qquad (39)$$

where $\lambda = 2V_0G_0\omega_R/\omega_0$ denotes the nonlinear photon-phonon coupling strength. The order parameter $\Delta(u)$ of the superguiding state is determined through the combination of Eq. (38) with Eq. (39). It is found that for $\lambda_0 = (2\omega_R/\omega_0)^{\frac{1}{2}}$, the superguiding state can occur only if $\lambda_0 < \lambda < 1$. There are no quasiparticles in the superguiding ground state and so one demands $N(u_g) = 0$. Equation (39) gives the ground-state order parameter $\Delta(u_g) = \hbar\omega_0(1-\lambda^2)^{\frac{1}{2}}$ and thereby Eq. (38) yields the ground-state energy density $u_g = (\hbar\omega_0)^2(1-\lambda)/2VV_0$. When $N(u) > 0$, the corresponding u must be larger than u_g for nonzero $\Delta(u)$. u_g is the lower cutoff energy density of the superguiding state. The order parameter $\Delta(u)$ determined by Eqs. (38) and (39) is a decreasing function of u, which vanishes at the critical energy density u_c. A simple expression for the critical energy density is acquired,

$$u_c = u_g \left[\frac{3}{2}\left(\frac{\omega_b}{\omega_0}\right)^2 + 1\right]\left[\mathrm{erf}\left(\frac{\omega_R}{\omega_b}\right)\right]^{-1}, \quad (40)$$

where erf(x) is the error function. As a result, another condition for superguiding is that $u_g \le u < u_c$. For the order parameter we also derive the limiting behavior

$$\Delta^2(u) = (\hbar\omega_0)^2 \frac{2(1-\lambda)}{\lambda}\left(1 - \frac{u}{u_c}\right), \quad u_c - u \ll u_c. \quad (41)$$

This result is characteristic of a mean-field theory.

Inserting Eqs. (35) and (37) into Eq. (32), we find the time evolution of the em energy density,

$$u(t) = u_s(u) + u_n(u)e^{-t/t_c}, \quad (42)$$

where the first and second terms represent the contributions from paired photons and quasiparticles, respectively. It is trivial to write the expressions for $u_s(u)$ and $u_n(u)$ from Eq. (32). Under the condition $\lambda_0 < \lambda < 1$, whether the standing-wave superguiding state will occur depends on the em energy density u_0 at the initial instant. If $u_0 < u_g$, the photon system is always in the normal state. When $u_g \le u_0 < u_c$, the photon system enters the superguiding state. For $u_0 > u_c$ the normal state occurs first. Then the energy density $u(t)$ of the em field exponentially attenuates with time t like Eq. (37). Once $u(t)$ is reduced below u_c, the superguiding state appears. The photon system undergoes a nonequilibrium phase transition from the normal to the superguiding state. The energy of the system of quasiparticles is lost after a sufficiently long time $t \gg t_c$. Finally, the superguiding state becomes the superguiding ground state. The superguiding ground state is an equilibrium state in which the physical variables are independent of time and there are no macroscopic currents. If $u_s(\vec{r})$ represents the ground-state energy density of the em field at position $\vec{r}$, the em field exerts an isotropic pressure $P = \frac{1}{3}u_s(\vec{r})$ on the surfaces of the resonator.

The macroscopic em field in the resonator is no longer a transverse em one and the quantized em field is given by Eqs. (4) and (7). The expectation value of the electric or magnetic field in the superguiding state corresponds to the classical description of an em wave. Since the superguiding state $|\eta\rangle$ is not an eigenstate of the electric field $\vec{E}$, the single measurements reveal unavoidable fluctuations of the measured field strength about the expectation value. We need to examine mean-square fluctuations of the electric field in the superguiding state. For the photon operator $a_{\vec{k}}$, one may introduce two Hermitian operators $a_{1\vec{k}}$ and $a_{2\vec{k}}$ by $a_{\vec{k}} = a_{1\vec{k}} + ia_{2\vec{k}}$, where $\vec{k}$ denotes $\vec{k}\sigma$. $a_{1\vec{k}}$ and $a_{2\vec{k}}$ may be identified as the two quadrature components of a mode field in the plane-wave expansion of the electric field. The variances of the quadrature operators in the superguiding state are

$$\langle\eta|(\Delta a_{1\vec{k}})^2|\eta\rangle = \langle\eta|(\Delta a_{2\vec{k}})^2|\eta\rangle = \frac{1}{4}\frac{\hbar\omega_{\vec{k}}}{E_{\vec{k}}(u)}. \quad (43)$$

The vacuum state is a minimum uncertainty state and has the equal variance $\frac{1}{4}$ in each quadrature phase. At the critical energy density u_c, the superguiding state becomes a coherent state whose fluctuations are equal to the vacuum fluctuations. The fluctuations of the superguiding state become increasingly larger than the vacuum fluctuations with decreasing of the em energy density. At the ground-state energy density u_g, Eq. (43) gives a maximum variance of about $\frac{1}{4\lambda}$ at the incident frequency. The reason for this is as follows. To have a higher degree of order at lower u, the photon system makes adjustments on a microscopic scale. These adjustments appear in the form of excess fluctuations. However, the em field becomes a coherent light field when it leaves the resonator into the vacuum. One can use various methods of nonlinear optics to generate a squeezed state from the coherent output state, which has fewer fluctuations in one quadrature phase than the coherent state at the expense of increased fluctuations in the other quadrature phase.

We may remove the four lateral mirrors of the enclosed resonator and retain only the two perfectly reflecting mirrors in the xy plane. Therefore we form an open optical resonator. If the open resonator contained an ordinary dielectric medium, the em energy in the resonator would suffer diffraction losses. In fact, our open resonator contains a nonlinear polar crystal. In Sec. V we will show that the em field in such an open resonator exists in the form of optical vortex solitons and has no diffraction losses. The photonic superguiding theory in the enclosed resonator applies basically to the open resonator. In the following we just point out the two differences. Since scattered quasiparticles can escape out of the open resonator, the quasiparticle lifetime t_c in the open resonator is much shorter than that in the enclosed resonator. If $u_s(\vec{r})$ represents the ground-state energy density of the em field at position $\vec{r}$, the em field exerts a pressure $P = u_s(\vec{r})$ on the end faces of the open resonator.

Now we estimate the orders of magnitude of the relevant physical quantities. The pair potential V_0 is found from Eqs. (13) and (22) as $V_0 \approx (\alpha_1\hbar\omega_0/\epsilon_0\epsilon_h\omega_R)^2/2V\Omega m$, where we have ignored the repulsive term from the fluctuations in Eq. (22). Ω is the volume of primitive cells and m and α_1 represent the mass and the differential polarizability of one basis anion, respectively. Thus an approximate expression for the nonlinear photon-phonon coupling strength is obtained as

$$\lambda \approx \frac{\hbar\omega_0^3\alpha_1^2}{\pi^2\epsilon_0^2\sqrt{\epsilon_h}c^3\Omega m\omega_R}. \quad (44)$$

We take CaF_2 crystals, for example [32]. CaF_2 crystals have a wide band gap $E_g = 12.2$ eV. In CaF_2, the frequencies of the TO and the Raman-active mode are

$\hbar\omega_T = 33.10$ meV and $\hbar\omega_R = 40.92$ meV, respectively. If we use $\omega_0 = 20\omega_R$, the prerequisite for superguiding that $\hbar\omega_T \ll \hbar\omega_0 \ll E_g$ is satisfied. The cell volume is given by $\Omega = a^3/4$ with the lattice constant a. The parameters concerned are given as follows: $a = 5.4629$ Å, $m = 18.9984$ u, and $\epsilon_h = 2.047$. To the author's knowledge, the differential polarizability values of anions in the fluorite-structure crystals are unavailable at present. Therefore, we regard α_1 as an adjustable parameter and set $\alpha_1 = 2.510 \times 10^{-24}$ F m. These values specify a certain crystal of the fluorite structure. The calculation yields $\lambda_0 = 0.316$ and $\lambda \approx 0.536$. One can find that $\lambda_0 < \lambda < 1$. This shows that the superguiding state can indeed occur in this crystal. For the ground-state energy density we also get an approximate expression

$$u_g \approx (1-\lambda)\Omega m(\epsilon_0\epsilon_h\omega_R/\alpha_1)^2. \tag{45}$$

Putting the above parametric values into Eq. (45) yields $u_g \approx 0.120$ J/m^3. The energy density of blackbody radiation u_B in the resonator is determined by Stefan's law $u_B = 4\sigma T^4/c$, where the coefficient σ is called the Stefan-Boltzmann constant. At room temperature $T = 300$ K, $u_B = 6.138\times 10^{-6}$ J/m^3, and so $u_B \ll u_g$. The omission of blackbody radiation is therefore reasonable. Although the critical energy density u_c is an increasing function of temperature, u_c has an order of magnitude of 1 J/m^3, at the most. In the standing-wave configuration, the threshold energy density of stimulated Brillouin scattering of light has an order of magnitude $u_t \approx 10^3$ J/m^3. Thus we see $u_c \ll u_t$. The superguiding state is meaningful only in this case.

IV. TRAVELING-WAVE SUPERGUIDING STATE

Having studied the standing-wave superguiding state, we are able to discuss the traveling-wave superguiding state in what follows. As shown in Fig. 2, the traveling-wave configuration is determined by a cylindrical dielectric waveguide whose core is occupied by the first crystal and whose cladding corresponds to the second crystal. The physical quantities in the first and second crystals are marked with subscripts 1 and 2. The two high-frequency dielectric constants must satisfy the inequality $\epsilon_{h1} > \epsilon_{h2}$. The laser light field is normally incident on the end face $z = 0$ of the core at the instant $t = 0$. In Sec. II we assumed the incident light field to be a monochromatic field. Now the incident light field is required to be a quasimonochromatic field of a central frequency ω_0. The incident light field excites a macroscopic em field in the waveguide, which propagates with a group velocity v_g. At time t we investigate a length of waveguide in the interval z to $z+\Delta z$, where $z+\Delta z = v_g t$. Δz is so small macroscopically that the em field has the same intensity in this interval. The vector potentials $\vec{A}_1$ and $\vec{A}_2$ of the em field can be expanded into the form of Eq. (7). The inequality $\epsilon_{h1} > \epsilon_{h2}$ limits the plane-wave modes appearing in the expansion (7) to the guided modes. Each wave vector of the guided modes is separated into $\vec{k} = \vec{K} + \vec{Q}$,

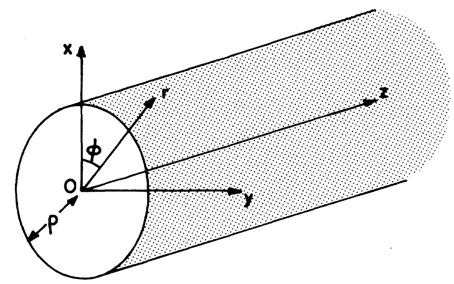

FIG. 2. Section of a cylindrical dielectric waveguide. The z axis coincides with the waveguide axis of symmetry and ρ is the the core radius.

where $\vec{K}$ and $\vec{Q}$ are the components parallel to and transverse to the z axis. The axial wave vector $\vec{K}$ is real everywhere. $\vec{Q}$ is real in the core, but imaginary in the cladding. For a fixed K, the values of allowed Q in the core are in the range $0 \leq Q < K\tan\theta_c$, where θ_c is the complement of the critical angle of total internal reflection defined by $\sqrt{\epsilon_{h1}}\cos\theta_c = \sqrt{\epsilon_{h2}}$. While propagating in the core, a guided wave undergoes a series of total internal reflections at the core-cladding interface. Consequently, the energy carried by the guided modes is confined to the vicinity of the core and diffraction effects are eliminated. The axial wave number K is a function of the incident frequency ω. The function $K(\omega)$ must be specified by the Maxwell equations together with the boundary condition at the core-cladding interface,

$$\vec{n}\times(\vec{E}_2-\vec{E}_1)=\vec{0},\quad \vec{n}\times(\vec{B}_2-\vec{B}_1)=\vec{0}\,, \tag{46}$$

where $\vec{n}$ is a unit vector normal to the interface [33]. Next we introduce the waveguide frequency $F = \omega_0\rho(\epsilon_{h1} - \epsilon_{h2})^{\frac{1}{2}}/c$, where ρ is the core radius, and the profile height parameter $\Delta = (\epsilon_{h1}-\epsilon_{h2})/2\epsilon_{h1}$. If $0 < F < 2.4048$, the waveguide supports only the fundamental mode. Under the weak-guidance approximation $\Delta \ll 1$, the propagation constant $K_0 = K(\omega_0)$ of the fundamental mode is the largest value of K_0 determined by the eigenvalue equation

$$U\frac{J_1(U)}{J_0(U)} = W\frac{K_1(W)}{K_0(W)}, \tag{47}$$

where $U = \rho\left(\epsilon_{h1}\omega_0^2/c^2 - K_0^2\right)^{\frac{1}{2}}$ and $F^2 = U^2 + W^2$. $J_0(x)$ and $J_1(x)$ are the Bessel functions of the first kind, while $K_0(x)$ and $K_1(x)$ are the modified Bessel functions of the second kind. By the definition $v_g^{-1} = (\partial K/\partial\omega)_{\omega=\omega_0}$, the group velocity v_g of the fundamental mode is obtained as

$$v_g = \frac{c}{\sqrt{\epsilon_{h1}}}\left\{1+\Delta\frac{U^2}{F^2}\left[1-2\frac{K_0^2(W)}{K_1^2(W)}\right]\right\}. \tag{48}$$

The fundamental mode photons of wave vectors $\vec{K}+\vec{Q}$ and $\vec{K}-\vec{Q}$ always occur simultaneously and have an equal number. By exchange of virtual optical phonons, the fundamental mode photons can experience an attractive effective interaction. Such an interaction leads to the traveling-wave superguiding state, in which a propagating photon pair is the combination of $(\vec{K}+\vec{Q},\sigma)$ with $(\vec{K}-\vec{Q},-\sigma)$ and thus has the composite momentum $2\hbar\vec{K}$. The propagating photon pair is created by the operator $a^{\dagger}_{\vec{K}+\vec{Q},\sigma}a^{\dagger}_{\vec{K}-\vec{Q},-\sigma}$. In general, the creation operator of a propagating photon pair is $a^{\dagger}_{\vec{K}+\vec{k},\sigma}a^{\dagger}_{\vec{K}-\vec{k},-\sigma}$, where $\vec{k}$ is an arbitrary wave vector. If $\vec{k}$ has a nonzero axial component, however, the pairing probability of these two photons is almost zero. The two photons with different axial wave vectors propagate independently. Where $a^{\dagger}_{\vec{K}+\vec{k},\sigma}a^{\dagger}_{\vec{K}-\vec{k},-\sigma}$ is concerned, we therefore let $\vec{k}=\vec{Q}$. In Eqs. (16) and (19) we set $\vec{k}=\vec{K}+\vec{Q}$, $\vec{k}'=\vec{K}-\vec{Q}$, and $\sigma'=-\sigma$. Next $\vec{K}+\vec{Q}$ is used for $(\vec{K}+\vec{Q},\sigma)$ and $\vec{K}-\vec{Q}$ for $(\vec{K}-\vec{Q},-\sigma)$. The pair Hamiltonian of the photon system in the core region reads, consequently,

$$H_p=\frac{1}{2}\sum_{\vec{K},\vec{Q}}\hbar\omega_{\vec{K}+\vec{Q}}\left(a^{\dagger}_{\vec{K}+\vec{Q}}a_{\vec{K}+\vec{Q}}+a^{\dagger}_{\vec{K}-\vec{Q}}a_{\vec{K}-\vec{Q}}\right)+\sum_{\substack{\vec{K},\vec{Q}\\ \vec{Q}'}}V_{\vec{K}+\vec{Q},\vec{K}+\vec{Q}'}a^{\dagger}_{\vec{K}+\vec{Q}'}a^{\dagger}_{\vec{K}-\vec{Q}'}a_{\vec{K}-\vec{Q}}a_{\vec{K}+\vec{Q}}\ , \quad (49)$$

$$V_{\vec{K}+\vec{Q},\vec{K}+\vec{Q}'}=\frac{|M_{\vec{K}+\vec{Q}}(\vec{Q}'-\vec{Q})|^2\omega_R}{\hbar[(\omega_{\vec{K}+\vec{Q}}-\omega_{\vec{K}+\vec{Q}'})^2-\omega_R^2]}+\frac{\langle(\delta\alpha_n)^2\rangle}{2Nk_BT}|N_{\vec{K}+\vec{Q}}(\vec{Q}'-\vec{Q})|^2. \quad (50)$$

Here the pair potential is real and satisfies the symmetric properties $V_{\vec{K}+\vec{Q},\vec{K}+\vec{Q}'}=V_{\vec{K}-\vec{Q},\vec{K}-\vec{Q}'}$ and $V_{\vec{K}+\vec{Q},\vec{K}+\vec{Q}'}=V_{\vec{K}+\vec{Q}',\vec{K}+\vec{Q}}$. It will be supposed that $V_{\vec{K}+\vec{Q},\vec{K}+\vec{Q}'}=-V_0$ if $\left|\omega_{\vec{K}+\vec{Q}}-\omega_0\right|$ and $\left|\omega_{\vec{K}+\vec{Q}'}-\omega_0\right|<\omega_R$; otherwise $V_{\vec{K}+\vec{Q},\vec{K}+\vec{Q}'}=0$. We also need the Hamiltonian of the em field in the core region, which from Eq. (8) takes the form

$$H_L=\frac{1}{2}\sum_{\vec{K},\vec{Q}}\hbar\omega_{\vec{K}+\vec{Q}}\left(a^{\dagger}_{\vec{K}+\vec{Q}}a_{\vec{K}+\vec{Q}}+a^{\dagger}_{\vec{K}-\vec{Q}}a_{\vec{K}-\vec{Q}}\right). \quad (51)$$

The double sums over $\vec{K}$ and $\vec{Q}$ in Eqs. (49) and (51) represent a sum over wave vectors $\vec{k}=\vec{K}+\vec{Q}$ in three dimensions. The direction Ω of $\vec{k}=\vec{K}+\vec{Q}$ is specified by a polar angle θ and an azimuthal angle ϕ. The double sums are calculated in the following way. For a fixed $\vec{K}$ we first find the sum over $\vec{Q}$ in the region $0\le Q<K\tan\theta_c$ and $0\le\phi\le 2\pi$. Then we find the sum over the axial wave vectors $\vec{K}(\omega)$ in a small interval near $\vec{K}_0$. The sum over $\vec{K}(\omega)$ reflects the fact that the incident light field is a quasimonochromatic field of a central frequency ω_0. Since $\vec{Q}'$ is a two-dimensional wave vector, the sum over $\vec{Q}'$ in Eq. (49) is inconsistent with the three-dimensional photon system. The reason for this is that in deriving Eq. (49) we replace a small three-dimensional wave vector $\vec{k}'$ with $\vec{Q}'$. Therefore $\vec{Q}'$ is replaced self-consistently by $\vec{k}'$ in the end.

We now consider the diagonalization of the pair Hamiltonian (49) following the Bogoliubov transformation

$$a_{\vec{K}+\vec{Q}}=\alpha_{\vec{K}+\vec{Q}}\cosh\varphi_{\vec{K}+\vec{Q}}+\alpha^{\dagger}_{\vec{K}-\vec{Q}}\sinh\varphi_{\vec{K}+\vec{Q}}\ ,$$
$$a^{\dagger}_{\vec{K}+\vec{Q}}=\alpha^{\dagger}_{\vec{K}+\vec{Q}}\cosh\varphi_{\vec{K}+\vec{Q}}+\alpha_{\vec{K}-\vec{Q}}\sinh\varphi_{\vec{K}+\vec{Q}}\ , \quad (52)$$

where the parameter $\varphi_{\vec{K}+\vec{Q}}$ is real and has the symmetry $\varphi_{\vec{K}+\vec{Q}}=\varphi_{\vec{K}-\vec{Q}}$. The state vector $|\eta\rangle$ of the photon system should satisfy $\alpha_{\vec{K}+\vec{Q}}|\eta\rangle=\eta_{\vec{K}+\vec{Q}}|\eta\rangle$, where the parameter $\eta_{\vec{K}+\vec{Q}}$ is complex and meets $\eta_{\vec{K}+\vec{Q}}=\eta_{\vec{K}-\vec{Q}}$. Therefore $|\eta\rangle$ may be constructed as

$$|\eta\rangle=\exp\left[\sum_{\vec{K},\vec{Q}}\left(\eta_{\vec{K}+\vec{Q}}\alpha^{\dagger}_{\vec{K}+\vec{Q}}-\eta^{*}_{\vec{K}+\vec{Q}}\alpha_{\vec{K}+\vec{Q}}\right)\right]\times\exp\left[\frac{1}{2}\sum_{\vec{K},\vec{Q}}\varphi_{\vec{K}+\vec{Q}}\left(a^{\dagger}_{\vec{K}+\vec{Q}}a^{\dagger}_{\vec{K}-\vec{Q}}-a_{\vec{K}+\vec{Q}}a_{\vec{K}-\vec{Q}}\right)\right]|0\rangle. \quad (53)$$

The mean quasiparticle number of the $(\vec{K}+\vec{Q})$-wave field in the state $|\eta\rangle$ is $\left|\eta_{\vec{K}+\vec{Q}}\right|^2=f_{\vec{K}+\vec{Q}}(u)$, where $f_{\vec{K}+\vec{Q}}(u)$ is still given by Eq. (35). Now, in Eq. (35), $g(\Omega)=1$ for $0\le\theta<\theta_c$ and the Gaussian frequency distribution is due to the quasimonochromatic incident field itself as well as the intrinsic scatterings. Putting Eq. (52) into Eq. (49), we obtain the diagonalized pair Hamiltonian

$$H_p=E_p(u)+\frac{1}{2}\sum_{\vec{K},\vec{Q}}E_{\vec{K}+\vec{Q}}(u)\times\left(\alpha^{\dagger}_{\vec{K}+\vec{Q}}\alpha_{\vec{K}+\vec{Q}}+\alpha^{\dagger}_{\vec{K}-\vec{Q}}\alpha_{\vec{K}-\vec{Q}}\right). \quad (54)$$

Here $E_p(u)$ denotes the energy of the system of propagating photon pairs. The excitation energy $E_{\vec{K}+\vec{Q}}(u)$ of a quasiparticle is defined by the equations

$$E_{\vec{K}+\vec{Q}}(u)=\left[\left(\hbar\omega_{\vec{K}+\vec{Q}}\right)^2-\Delta^2_{\vec{K}+\vec{Q}}(u)\right]^{\frac{1}{2}}, \quad (55)$$

$$\Delta_{\vec{K}+\vec{Q}}(u)=-\sum_{\vec{k}'}\frac{V_{\vec{K}+\vec{Q},\vec{K}+\vec{k}'}\Delta_{\vec{K}+\vec{k}'}(u)}{E_{\vec{K}+\vec{k}'}(u)}\times\left[1+2f_{\vec{K}+\vec{k}'}(u)\right]. \quad (56)$$

In Eq. (56) the two-dimensional wave vector $\vec{Q}'$ has been replaced self-consistently by the three-dimensional wave vector $\vec{k}'$. In calculating Eq. (56), we let $\vec{K}+\vec{k}'=\vec{K}'+\vec{Q}'$ and convert the sum over $\vec{k}'$ into the double sums over $\vec{K}'$ and $\vec{Q}'$. The transformation (52) is also inserted into Eq. (51). Then the expectation value of the Hamiltonian (51) in the state $|\eta\rangle$ gives the energy of the em field in the core region

$$E_L = \frac{1}{2}\sum_{\vec{K},\vec{Q}} \hbar\omega_{\vec{K}+\vec{Q}} \left[\frac{\hbar\omega_{\vec{K}+\vec{Q}}}{E_{\vec{K}+\vec{Q}}(u)} - 1\right] + \sum_{\vec{K},\vec{Q}} \frac{\left(\hbar\omega_{\vec{K}+\vec{Q}}\right)^2}{E_{\vec{K}+\vec{Q}}(u)} f_{\vec{K}+\vec{Q}}(u). \quad (57)$$

In the above we only present some useful descriptions and expressions. Some other results are similar to those in Sec. III and will be neglected for brevity.

In the traveling-wave superguiding state the order parameter $\Delta_{\vec{K}+\vec{Q}}(u)$ still has the form in Eq. (29). Adopting the method in Sec. III, we can find the order parameter $\Delta(u)$. The argument u is the em energy density defined by the relation $u = E_L/V$, where $V = \pi\rho^2\Delta z$ is the volume of the core region. The double sums over $\vec{K}$ and $\vec{Q}$ in Eqs. (56) and (57) are converted into the double integrals over frequencies ω and $\vec{Q}$. For a fixed ω, one finds the integral over $\vec{Q}$ in the region $0 \le Q < \omega\sqrt{\epsilon_{h1}}\sin\theta_c/c$ and $0 \le \phi \le 2\pi$. If we introduce the density of photonic states G_0 at the central frequency of incidence by

$$G_0 = \frac{V\omega_0^2\epsilon_{h1}^{\frac{3}{2}}}{2\pi^2\hbar c^3}(1-\cos\theta_c), \quad (58)$$

then Eqs. (57) and (56) are transformed into Eqs. (38) and (39), respectively. The evaluation of the simultaneous equations (38) and (39) comes to the order parameter $\Delta(u)$ of the traveling-wave superguiding state. In solving the equations, we also obtain the ground-state energy density u_g and the critical energy density u_c for the traveling-wave superguiding state. Since the em energy density u is not a good physical quantity for the waveguide, we introduce the light intensity I in the core region by the relation $I = uv_g$. The ground-state light intensity I_g and the critical light intensity I_c are given by

$$I_g = (\hbar\omega_0)^2(1-\lambda)v_g/2VV_0,$$
$$I_c = I_g\left[\frac{3}{2}\left(\frac{\omega_b}{\omega_0}\right)^2 + 1\right]\left[\operatorname{erf}\left(\frac{\omega_R}{\omega_b}\right)\right]^{-1}. \quad (59)$$

For the nonlinear photon-phonon coupling strength λ defined in Sec. III, the traveling-wave superguiding state still requires that $\lambda_0 < \lambda < 1$. If $I_g \le I < I_c$, then the traveling-wave superguiding state is realized.

In the traveling-wave superguiding state, a macroscopically large number of photon pairs occupy every quantum state of wave vector $2\vec{K}$ and zero spin, where $\vec{K}$ varies only in a small interval near $\vec{K}_0$. This forms a condensate. The condensate travels without scattering losses and is called superlight. However, individual quasiparticles are scattered and a scattered quasiparticle deviates from its former travel direction. A part of scattered quasiparticles escapes out of the core, while the other part is reflected from the interface back into the core due to partial and total internal reflections. Therefore, the quasiparticle number passing through the cross section of the core per unit time exponentially decreases with the transit time $t = z/v_g$. For this, Eqs. (35) and (37) are substituted into the expression (57) of the em energy in the core region near the position z. Through the relations $u = E_L/V$ and $I = uv_g$, the light intensity $I(z)$ at the propagation distance z can be expressed as

$$I(z) = I_s(I) + I_n(I)e^{-\beta z}, \quad (60)$$

where $\beta = 1/t_c v_g$ is the absorption coefficient of quasiparticles in the superwaveguide. $I_s(I)$ is the superlight intensity and the second term gives the intensity of quasiparticles. According to Eq. (57), $I_s(I)$ and $I_n(I)$ are functions of the light intensity I. Under the condition $\lambda_0 < \lambda < 1$, whether the traveling-wave superguiding state will occur depends on the incident light intensity I_0 at the end face $z = 0$. If $I_0 < I_g$, a propagating photon system is always in the normal state. When $I_g \le I_0 < I_c$, the photon system arriving at each position z enters the superguiding state. For $I_0 > I_c$, the normal state occurs first. Then the light intensity $I(z)$ falls like $I(z) = I_0 e^{-\beta z}$. Once $I(z)$ is reduced below I_c, the superguiding state appears. The intensity of quasiparticles becomes zero after a sufficiently long distance $z \gg \beta^{-1}$. Finally the superguiding state becomes the superguiding ground state. The ground state of the traveling-wave superguiding state is a steady state in which the physical variables are independent of time and the em field propagates with the persistent superlight intensity I_g in the core.

We need to examine quantum fluctuations of the light field in the traveling-wave superguiding state. For the photon operator $a_{\vec{K}+\vec{Q}}$ determining the light field, one may introduce two Hermitian operators $a_{1\vec{K}+\vec{Q}}$ and $a_{2\vec{K}+\vec{Q}}$ by $a_{\vec{K}+\vec{Q}} = a_{1\vec{K}+\vec{Q}} + ia_{2\vec{K}+\vec{Q}}$, where $\vec{K} + \vec{Q}$ denotes $(\vec{K} + \vec{Q}, \sigma)$. $a_{1\vec{K}+\vec{Q}}$ and $a_{2\vec{K}+\vec{Q}}$ represent the two quadrature components of a wave field. Note that $Q \ll K$ under the weak-guidance approximation. The variances of the quadrature operators in the traveling-wave superguiding state are thus found as follows:

$$\langle\eta|\left(\Delta a_{1\vec{K}+\vec{Q}}\right)^2|\eta\rangle = \frac{1}{4}\exp\left(2\varphi_{\vec{K}+\vec{Q}}\right),$$
$$\langle\eta|\left(\Delta a_{2\vec{K}+\vec{Q}}\right)^2|\eta\rangle = \frac{1}{4}\exp\left(-2\varphi_{\vec{K}+\vec{Q}}\right), \quad (61)$$

where $\tanh(2\varphi_{\vec{K}+\vec{Q}}) = \Delta_{\vec{K}+\vec{Q}}(I)/\hbar\omega_{\vec{K}+\vec{Q}}$. This equation tells us that the variance of one quadrature phase is less than that of the vacuum, whereas the variance of the other quadrature phase is increased. The traveling-wave superguiding state has the squeezing property. At the critical light intensity I_c, the traveling-wave superguiding state becomes a coherent state and so the squeezing effect vanishes. The squeezing effect becomes more and more strong as the traveling-wave superguiding state approaches the ground state. The squeezing mechanism of the traveling-wave superguiding state is as follows. Since photon pairs propagate without scattering losses, the photon pairs with a composite momentum $2\hbar\vec{K}$ have a well-defined particle number. Phase and particle number are conjugate observables and thus obey the Heisenberg uncertainty principle. As they are not simultaneously measurable, the phase experiences quantum diffusion during propagation. There are interference effects in the quantum phase diffusion. The quantum fluctuations are reduced in certain quadrature phase due to the destructive interference. In a term squeezing arises due to self-phase-modulation.

The nonlinear photon-phonon coupling strength in the traveling-wave configuration can be expressed as $\lambda = \lambda_s(1-\cos\theta_c)/2$, where λ_s denotes the nonlinear photon-phonon coupling strength in the standing-wave configuration and is given by Eq. (44). If we determine the first crystal in the core as the fluorite-structure crystal chosen in the numerical calculation of Sec. III, the first crystal can be superguiding in the standing-wave configuration. Since $\lambda_s \approx 0.536$, $\lambda \ll \lambda_0$ under the weak-guidance approximation. Therefore, the first crystal does not support the traveling-wave superguiding state. In order to support the traveling-wave superguiding state, the first crystal must have a much higher nonlinearity than the fluorite-structure crystal in Sec. III. In the following numerical calculation, we determine the first crystal in the core as another crystal of the fluorite structure. The differential polarizability of anions in this crystal has a value $\alpha_1 = 3.559 \times 10^{-23}$ F m. The values of the relevant parameters given in Sec. III continue in use here; for example, $\epsilon_{h1} = 2.047$. For the second crystal in the cladding, we only require it to be a dielectric with high-frequency dielectric constant $\epsilon_{h2} = 2.014$. The core radius of the waveguide has a typical value $\rho = 2.10$ μm. One acquires the waveguide frequency $F = 1.576$ and the profile height parameter $\Delta = 0.008$, so that the single-mode and weak-guidance conditions are satisfied. We solve numerically Eqs. (47) and (48) and gain the group velocity $v_g = 2.097 \times 10^8$ m s^{-1}. The calculation gives $\lambda \approx 0.433$ and therefore $\lambda_0 < \lambda < 1$. Consequently, the first crystal can be superguiding in the traveling-wave configuration. Because of $\lambda_s \gg 1$, however, the first crystal cannot be superguiding in the standing-wave configuration. Since $I_g = u_g v_g$, the ground-state light intensity can be estimated from the approximate expression (45) as $I_g \approx 15.34$ W/cm^2. Although the critical light intensity I_c is an increasing function of temperature, I_c has an order of magnitude of 10^2 W/cm^2, at the most. In the traveling-wave configuration, the threshold intensity of stimulated Brillouin scattering of light has an order of magnitude $I_t \approx 10^7$ W/cm^2. Therefore we see $I_c \ll I_t$. The traveling-wave superguiding state is meaningful only in this situation.

At the end of this section, let us inspect the propagation of the laser light field in a single crystal placed in the vacuum. In Sec. V we will show that the studied crystal is a self-defocusing medium. Recently, Swartzlander and Law have observed that in a self-defocusing medium the em field can propagate as optical vortex solitons [34]. Therefore the propagation of the laser light beam in the crystal can carry no diffraction losses. There are two prerequisite conditions for forming bound photon pairs in any traveling-wave configuration. The first one is that the traveling-wave configuration has no diffraction effect. The second one is that the traveling-wave configuration confines the em energy density near the propagation axis. In the waveguide configuration the two conditions are satisfied. In the present configuration, the second condition is rather difficult to meet when the first one is met. The reason for this is as follows. The energy density of an optical vortex diverges from the vortex axis and so the beam area becomes very wide. Since there are no bound photon pairs, the propagation of the laser light beam in a single crystal accompanies scattering losses. In conclusion, the traveling-wave superguiding state can hardly occur in a single crystal.

V. WAVE PROPERTIES OF SUPERGUIDING STATES

Sections II–IV have discussed the particle properties of the superguiding state by using the Hamiltonian approach. Because of the wave-particle duality of light, one wants to know the wave properties of the superguiding state. The wave properties must be determined from the Maxwell equations. If we can write the constitutive equations of the crystal under study and find the solution for the resulting Maxwell equations with appropriate boundary conditions, then all the wave properties of the superguiding state will be predictable.

As we have seen, the pair Hamiltonians (21) and (49) lead to the standing- and traveling-wave superguiding states, respectively. The pair Hamiltonians (21) and (49) are expressed by the operators $a_{\vec{k}\sigma}$ and $a^\dagger_{\vec{k}\sigma}$ of circularly polarized photons with spin $\sigma = \pm 1$. Now we need the pair Hamiltonian in terms of the electric field $\vec{E}$ and the magnetic field $\vec{B}$. The electric and magnetic fields are related to the vector potential through Eq. (4) and the quantized vector potential in terms of $a_{\vec{k}\sigma}$ and $a^\dagger_{\vec{k}\sigma}$ is given by Eq. (7). With these transformations, in terms of $\vec{E}$ and $\vec{B}$ the pair Hamiltonians (21) and (49) can be cast into the same form

$$H_p = \int d\vec{r} \left[\frac{\epsilon_0}{2}\vec{E}^2 + \frac{1}{2\mu_0}\vec{B}^2 + \frac{1}{2}\vec{E}\cdot\vec{P}^{(1)} + \frac{3}{4}\vec{E}\cdot\vec{P}^{(3)} \right], \tag{62}$$

where $\vec{P}^{(1)}$ is the linear polarization and $\vec{P}^{(3)}$ is the third-order nonlinear polarization. At this point, it is useful to note that the linear dielectric function $\epsilon(|\vec{k}|)$ introduced in Sec. II is actually an even function of the bare photon frequency $\omega_{\vec{k}} = c|\vec{k}|/\sqrt{\epsilon_h}$, namely, $\epsilon(|\vec{k}|) = \epsilon(\omega_{\vec{k}}) = \epsilon(-\omega_{\vec{k}})$. Then the constitutive equations of $\vec{P}^{(1)}$ and $\vec{P}^{(3)}$ are deduced as

$$\vec{P}^{(1)} = \epsilon_0 \int dt' \left[\epsilon(t-t') - \delta(t-t')\right] \vec{E}(t'),$$
$$\vec{P}^{(3)} = \epsilon_0 \chi^{(3)} \vec{E}^3. \tag{63}$$

$\epsilon(t)$ reflects the noninstantaneous dependence between the linear polarization and the electric field and its

Fourier transformation $\epsilon(\omega_{\vec{k}})$ represents the material dispersion. $\chi^{(3)}$ is the third-order nonlinear susceptibility defined by the relation

$$\frac{3\hbar^2\chi^{(3)}\omega_{\vec{k}}\omega_{\vec{k}'}}{8V\epsilon_0\epsilon(\vec{k})\epsilon(\vec{k}')}\left\{\delta+2\left[\vec{\chi}^*_{\sigma'}(\vec{k}')\cdot\vec{\chi}_\sigma(\vec{k})\right]^2\right\}=V_{\vec{k}\sigma,\vec{k}'\sigma'}\ , \tag{64}$$

where $\omega_{\vec{k}}$ is the renormalized photon frequency and $\delta=0$ and 1, respectively, for the standing- and traveling-wave configurations. We have neglected the weak dependence of $\chi^{(3)}$ on wave vectors and spins. $\chi^{(3)}$ is negative in the high-nonlinear ion crystal and so the high-nonlinear ion crystal is a self-defocusing medium. The nonlinearity here is a quantum effect of the interaction of photons with the crystal and can not be inferred from any classical theory. In the above, $\vec{E}$ and $\vec{B}$ are q numbers. Here we consider their expectation values in the state vector $|\eta\rangle$ of the photon system. In the following, $\vec{E}$ and $\vec{B}$ are still used to denote the expectation values and are thus c numbers. $\vec{E}$ and $\vec{B}$ must obey the classical Maxwell equations in connection with the constitutive relations (63). The Maxwell equations can be reduced to the wave equation for $\vec{E}$

$$\vec{\nabla}^2\vec{E}-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E}=\mu_0\frac{\partial^2}{\partial t^2}\left[\vec{P}^{(1)}+\vec{P}^{(3)}\right]. \tag{65}$$

The wave equation can describe nonlinear optical effects in the crystal. The solution of the wave equation has to be carried out under certain boundary and initial conditions, which are imposed by the physical nature of the problem to be treated.

The em field in the superguiding state is formed jointly by photon pairs and quasiparticles. However, the linear superposition principle of fields fails because of the nonlinearity of the wave equation (65). In what follows we consider the case when the photon system approaches the superguiding ground state. In this case, the intensity of quasiparticles tends to zero and the em field corresponds to a condensate. Concomitantly, the electric field $\vec{E}$ evolves without scattering attenuations. In contrast, the em field in the normal state is formed completely by quasiparticles, or photons, and so the electric field $\vec{E}$ evolves with scattering attenuations.

We first investigate the wave properties of the superguiding state in an open optical resonator. With reference to Fig. 1, the open resonator consists of two plane mirrors set parallel to one another and separated by a distance d. If the dimensions of the plane mirrors are far larger than the distance d, the plane mirrors can be regarded as infinitely extended. Therefore, the open resonator become a (2+1)-dimensional resonator. Under the scalar-wave approximation, the em field in this resonator is assumed to be a transverse electric wave. Since the incident light field is a monochromatic field of frequency ω_0, the electric field excited in the (2+1)-dimensional resonator may be written as

$$\vec{E}(\vec{r},t)=\vec{e}_r\mathrm{Re}\left[\Psi(\vec{r})e^{-i\omega_0t}\right], \tag{66}$$

where $\vec{e}_r$ is a unit radial vector orthogonal to the z axis and the complex amplitude $\Psi(\vec{r})$ must vanish at $z=0$ and $z=d$. Putting Eq. (66) into Eq. (65) yields the differential equation for $\Psi(\vec{r})$,

$$\left[-\vec{\nabla}^2-\frac{\epsilon_h\omega_0^2}{c^2}-\frac{\chi^{(3)}\omega_0^2}{c^2}|\Psi|^2\right]\Psi=0. \tag{67}$$

This is the Ginzburg-Landau equation investigated thoroughly [35–38]. In cylindrical coordinates (r,ϕ,z), Eq. (67) possesses single-vortex solutions of the form

$$\Psi(\vec{r})=Ae^{im\phi}\sin\left(\frac{n\pi z}{d}\right)f_{mn}(r), \tag{68}$$

where $m=\pm1,\pm2,\pm3,...$, and n is a positive integer such that $n\pi/d<\sqrt{\epsilon_h}\omega_0/c$. $f_{mn}(r)$ is real, $f_{mn}(0)=0$, and $f_{mn}(\infty)=1$. The exact solution for $f_{mn}(r)$ has to be found numerically. A is the background amplitude. The vortices with $|m|$ and $n>1$ are energetically unfavorable and only the vortices with $|m|$ and $n=1$ can really exist. To a good approximation we have $f(r)=\tanh(r/r_0)$, where r_0 is a fit constant. Equation (68) can be interpreted as the wave function for a vortex in the condensate. The em energy density $u_s(\vec{r})$ of the vortex is determined by the amplitude of Ψ as $u_s=\epsilon_0\epsilon_h|\Psi|^2$. We will show that the phase of Ψ determines the vortex velocity. The line $r=0$ is known as the axis of the vortex and r_0 is its outer radius. The arbitrariness of the z axis means that there are a series of parallel straight vortex lines in the condensate. Since the condensate has no rigid-body rotation, the vortex lines with $m=\pm1$ always occur in pairs. An optical vortex is highly stable and can be called a soliton. The transverse Laplacian $\vec{\nabla}^2_\perp$ accounts for diffraction. The effect of diffraction is to produce a transverse flow of the em energy and thereby the field intensity tends to become low. In the self-defocusing medium the refraction index decreases with intensity and, owing to the law of refraction, the em field spreads into regions of low intensity. The formation of optical vortex solitons is due to the counterbalanced effects of diffraction and nonlinear refraction. Therefore, optical vortex solitons have the desirable property that they do not suffer from diffraction.

Then we discuss the wave properties of the superguiding state in the waveguide shown in Fig. 2. The light wave incident in the waveguide is assumed to be a quasi-monochromatic light pulse with central frequency ω_0 and duration τ. The waveguide supports only the fundamental mode with propagation constant K_0 so that the modal dispersion is absent. Under the weak-guidance approximation, the fundamental mode is a nearly transverse em wave. The electric field in the fundamental mode may be written in the form

$$\vec{E}(\vec{r},t)=\vec{\chi}\mathrm{Re}\left[R(r)\psi(z,t)e^{i(K_0z-\omega_0t)}\right], \tag{69}$$

where $\vec{\chi}$ is the unit polarization vector orthogonal to the z axis and r is the radial coordinate. $R(r)$ is the dimensionless transverse profile of the electric field and $\psi(z,t)$ is its slowly varying envelope. The electric field given by

Eq. (69) must obey the wave equation (65). $R(r)$ is the solution of the Bessel equation of order zero. Continuity of $R(r)$ and $dR(r)/dr$ throughout the waveguide gives rise to the eigenvalue equation (47) for $K_0 = K(\omega_0)$. To reflect the material dispersion, in Eq. (47) we replace ϵ_{h1} and ϵ_{h2} with the linear dielectric functions $\epsilon_1(\omega_0)$ and $\epsilon_2(\omega_0)$ of the core and the cladding, respectively. $K(\omega)$ represents the waveguide dispersion and can be written in terms of the effective linear dielectric function $\epsilon^*(\omega)$ of the waveguide as $K^2(\omega) = \epsilon^*(\omega)(\omega/c)^2$. The group velocity dispersion is due to the material dispersion and the waveguide dispersion. Within the framework of the slowly varying envelope approximation, $\psi(z,t)$ satisfies the dynamical evolution equation

$$i\left(\frac{\partial}{\partial z} + \frac{1}{v_g}\frac{\partial}{\partial t}\right)\psi - \frac{K_2}{2}\frac{\partial^2\psi}{\partial t^2} + \frac{K_0\chi^{(3)*}\Gamma}{2\epsilon^*(\omega_0)}|\psi|^2\psi = 0, \tag{70}$$

where Γ is a geometrical factor given by

$$\Gamma = \frac{\int_0^\infty 2\pi r dr |R(r)|^4}{\int_0^\infty 2\pi r dr |R(r)|^2}. \tag{71}$$

$v_g = (\partial K/\partial\omega)^{-1}_{\omega=\omega_0}$ is the group velocity of the fundamental mode and $K_2 = \left(\partial^2 K/\partial\omega^2\right)_{\omega=\omega_0}$ describes the effect of group-velocity dispersion. $\chi^{(3)*}$ is the effective third-order nonlinear susceptibility of the waveguide and describes the effect of self-defocusing. Below we change into a reference frame moving with the light pulse by the transformation of variables

$$s = \frac{1}{\tau}\left(t - \frac{z}{v_g}\right), \quad x = -\frac{|K_2|}{2\tau^2}z, \qquad y = \tau\left[\frac{K_0\,|\chi^{(3)*}|\,\Gamma}{2\epsilon^*(\omega_0)\,|K_2|}\right]^{\frac{1}{2}}\psi \ . \tag{72}$$

Equation (70) then reduces to the dimensionless form

$$i\frac{\partial y}{\partial x} + \sigma\frac{\partial^2 y}{\partial s^2} + 2|y|^2 y = 0 \ , \tag{73}$$

where $\sigma = +1$ stands for the normal dispersion $K_2 > 0$ and $\sigma = -1$ for the anomalous dispersion $K_2 < 0$. Equation (73) is the nonlinear Schrödinger equation investigated thoroughly [39–42]. In the normal dispersion region of $\sigma = +1$, the boundary condition in Eq. (73) is $|y| \longrightarrow 0$ at $s \longrightarrow \pm\infty$. In this case, Eq. (73) possesses solutions of temporal bright solitons and the one-soliton bright pulse is given by

$$y(x,s) = 2\eta\frac{\exp[-4i(\xi^2 - \eta^2)x - 2i\xi s]}{\cosh[2\eta(4\xi x + s)]}, \tag{74}$$

where 2η and -4ξ are the soliton's amplitude and velocity. In the anomalous dispersion region of $\sigma = -1$, the boundary condition to Eq. (73) becomes $|y| \longrightarrow y_0$ =const at $s \longrightarrow \pm\infty$. In this case, Eq. (73) admits solutions of temporal dark solitons and the one-soliton dark pulse has the form

$$y(x,s) = y_0\frac{(\lambda - i\nu)^2 + \exp Z}{1 + \exp Z}\exp(2iy_0^2 x), \qquad Z = 2\nu y_0(s - 2\lambda y_0 x), \quad \lambda^2 = 1 - \nu^2 \ , \tag{75}$$

where the parameter ν characterizes the soliton intensity. Both temporal solitons have the desirable property that they do not suffer from dispersion. The physical mechanism for this is simple: The self-defocusing nonlinearity in the waveguide may compensate for the group-velocity dispersion and therefore leads to propagation of solitary waves without distortion.

A final problem is to determine the flow pattern of the em energy in the superguiding state. As shown before, the em energy in the superguiding state is the sum of the energy of the condensate and that of the quasiparticle system. Since there are no scatterings, the condensate has a definite propagation direction at each position $\vec{r}$. Therefore, the energy flow of the condensate can be described by the Poynting vector $\vec{S}(\vec{r},t)$. In contrast, the Poynting vector cannot be defined for the quasiparticle system, for the system experiences scatterings and has many propagation directions at each position. The Poynting vector can be written as $\vec{S}(\vec{r},t) = u_s(\vec{r},t)\vec{v}_s(\vec{r},t)$, where $u_s(\vec{r},t)$ and $\vec{v}_s(\vec{r},t)$ are the energy density and propagation velocity of the condensate at position $\vec{r}$. The condensate satisfies the conservation law of energy,

$$\frac{\partial u_s}{\partial t} = -\vec{\nabla}\cdot\vec{S}. \tag{76}$$

In the preceding paragraph, we have seen that the condensate in the traveling-wave superguiding state has no vortices. This indicates that in the traveling-wave superguiding state there is no vorticity, that is, $\vec{\nabla}\times\vec{v}_s = \vec{0}$. The expulsion of the vorticity field in the traveling-wave superguiding state is an analog of the Meissner effect in the superconducting state. For the (2+1)-dimensional resonator, we assume that the photon system approaches the superguiding ground state. In this case, the Poynting vector can be expressed in terms of the electric and magnetic fields as $\vec{S} = \mu_0^{-1}\vec{E}\times\vec{B}$. Using Eq. (66) for $\vec{E}$ and the Maxwell equation for $\vec{B}$, we obtain a quantum-mechanical expression of $\vec{S}$

$$\vec{S} = \frac{1}{2i\mu_0\omega_0}\left(\Psi^*\vec{\nabla}\Psi - \Psi\vec{\nabla}\Psi^*\right). \tag{77}$$

The insertion of Eq. (68) into Eq. (77) yields the rotation velocity of the condensate round a vortex axis

$$\vec{v}_s = m\frac{c^2}{\epsilon_h\omega_0 r}\vec{e}_\phi \ , \tag{78}$$

where $\vec{e}_\phi$ is a unit vector parallel to the azimuthal direction. For any contour round the vortex axis, we can find the circulation

$$\oint \vec{v}_s \cdot d\vec{l} = m\frac{2\pi c^2}{\epsilon_h \omega_0} \ . \tag{79}$$

This last equation reveals that the circulation is quantized and the circulation quantum is $2\pi c^2/\epsilon_h\omega_0$. The quantization of the circulation in the standing-wave superguiding state is similar to the quantization of the magnetic flux in a type-II superconductor.

For the enclosed optical resonator, the nonlinear wave equation (65) is rather difficult to solve. Therefore, we do not know the wave properties of the superguiding state in the enclosed optical resonator. This is a subject for future study. In the present section we have seen that the wave properties of the superguiding state depend greatly on the third-order nonlinear susceptibility $\chi^{(3)}$. An approximate expression for $\chi^{(3)}$ is obtained from Eq. (64) as

$$\chi^{(3)} \approx -\frac{4(\alpha_1/\omega_R)^2}{3\epsilon_0 \Omega m(\delta+2)}. \tag{80}$$

In contrast to the nonlinear photon-phonon coupling strength λ, the third-order nonlinear susceptibility $\chi^{(3)}$ depends only on the characteristic parameters of the crystal. By using the values of the characteristic parameters given in Sec. III, the third-order nonlinear susceptibility has an order of magnitude $\chi^{(3)} \approx -9.545 \times 10^{-11}$ $\mathrm{m^2\,V^{-2}}$. The crystal that can show the standing-wave superguiding state has thus a high self-defocusing nonlinearity. If we use the value of α_1 given in Sec. IV, the third-order nonlinear susceptibility has an order of magnitude $\chi^{(3)} \approx -1.279 \times 10^{-8}$ $\mathrm{m^2\,V^{-2}}$. The crystal that can show the traveling-wave superguiding state has an even higher self-defocusing nonlinearity.

VI. DISCUSSION

There has been a deep interest in superfluids and superconductors. For some time it has been recognized that there is a marked similarity between the two [43]. The chief characteristic of both is their ability to sustain particle currents at a constant velocity for long periods of time without any driving force. The common physical mechanism is that below transition temperature a macroscopically large number of particles condenses in a single-quantum state. These effects for helium atoms and electrons in the condensed-matter systems have a certain generality and many important concepts are certain to be applicable to photons in nonlinear polar crystals. In this paper we have explored the idea of the photonic superguiding state in nonlinear polar crystals. The photonic superguiding effect is similar to, but not congruent with, the electronic superconducting effect. The electron system undergoes a second-order phase transition from the normal to the superconducting state, while the photon system undergoes a nonequilibrium phase transition from the normal to the superguiding state. All the differences originate from the fact that the crystal electrons are in an equilibrium state, while the laser photons are far away from thermal equilibrium. We have constructed the microscopic theory of the photonic superguiding state. The heart of the theory is that the repulsive photon-photon interaction from Rayleigh and spontaneous Brillouin scatterings is overcome by the attractive interaction via optical phonons, leading to photon pairs which incorporate the optical phonons and hence are not hampered by them. Consequently, photon pairs do not suffer such intrinsic scatterings as Rayleigh, Brillouin, and Raman scatterings. For the sake of convenience we have used the idealized single crystals for the object of study. The object of study can be the real single crystals that include imperfections. Furthermore, the object of study can be polycrystals, mixed crystals, and glassy solids, in which there are microcrystalline disorder, substitutional disorder, and bond network disorder, respectively. Imperfections and kinds of disorder cause extrinsic scatterings of light. Nevertheless, the system of photon pairs is a highly stable condensate and the condensate is not quickly destroyed by extrinsic scatterings.

The transition of the electron system from the normal to the superconducting state is connected with a change in the gauge symmetry of the system's state. In the superguiding transition of the photon system, what symmetry is broken? In order to answer the question, we introduce the density matrix of the photon system ρ. In Secs. III and IV the photon system in the normal state was assumed to be in an ideal coherent state of many modes $|\eta\rangle = D(\eta)|0\rangle$. In this case, $\rho = |\eta\rangle\langle\eta|$. In fact, there is the thermal phase noise in the normal state and so $\rho = \overline{|\eta\rangle\langle\eta|}$, where the overbar denotes the average over some randomness of the phase angles. Because of this average, the density matrix of the normal state has a phase angle rotation invariance [44]. The superguiding ground state is a pure state $|\eta\rangle = U|0\rangle$ and the density matrix of the state takes the form $|\rho\rangle = |\eta\rangle\langle\eta|$. The density matrix of the superguiding ground state apparently lacks phase angle rotation symmetry. We conclude that in the transition from the normal to the superguiding state, the phase symmetry is spontaneously broken. The symmetry change of the photonic system accompanies a variation of the system's entropy. The relation between the entropy and the density matrix is given by $S = -k_B \mathrm{Tr}(\rho \ln \rho)$. Obviously, the entropy S decreases in the transition from the normal to the superguiding state and $S = 0$ in the superguiding ground state. The fact that the entropy of the superguiding state is less than that of the normal state demonstrates that the superguiding state has a higher degree of order than the normal state.

Now we summarize the macroscopic properties of the photonic superguiding state. (i) If a configuration can superguide an em wave, then the nonlinear photon-phonon coupling strength λ must satisfy the relation $\lambda_0 < \lambda < 1$. (ii) The standing-wave superguiding state occurs if the em energy density u is in the range $u_g \le u < u_c$ and the traveling-wave superguiding state occurs if the light intensity I is in the range $I_g \le I < I_c$. (iii) The system of photon pairs evolves without scattering attenuations, so that the standing-wave superguiding state has a persistent em energy density and the traveling-wave superguiding state has a persistent superlight intensity. (iv) Quantum fluctuations of the standing-wave superguiding state exceed the vacuum fluctuations, while the traveling-wave superguiding state has the squeezing property. (v) The superguiding crystal is a self-defocusing medium. (vi) In the open resonator the em field can exist in the form of vortex solitons without diffraction and in the superwaveguide the em wave can propagate as temporal solitons without dispersion. (vii) The superguiding state in the open resonator reveals the quantization of circulation and the traveling-wave superguiding state expels vorticity fields. The observation of the superguiding state requires that the superguiding crystal is ideally perfect and that the incident frequency is well below the electronic transition frequencies, but well above the TO phonon frequencies. According to the above properties, one can devise many experiments to check the superguiding state. Here we consider a low-temperature optical transmission experiment in the superwaveguide. At low temperatures, thermally induced fluctuations in the superwaveguide are very small and so the superguiding state is easy to realize. The superguiding state is characterized by the critical light intensity I_c. The absorption coefficient of the superwaveguide drops suddenly at I_c and becomes zero at the ground-state light intensity I_g. For a sufficiently long propagation distance, the intensity of quasiparticles already decays into zero and only the persistent superlight intensity propagates in the superwaveguide. If we find this persistent superlight intensity, we can assert that the superwaveguide is indeed superguiding.

In the present optical communication systems, the transmitters all use coherent pulses of laser light and the transmission media are highly flexible waveguides, i.e., optical fibers. There are three physical effects which limit the transmission of pulses in optical fibers: scattering, dispersion, and noise. Scattering reduces the intensity of pulses in optical fibers and so places an upper limit on the propagation distance of pulses. For a long haul system larger than the limiting distance, one needs repeaters to amplify the signal. Dispersion causes the pulses to spread out and eventually overlap to such an extent that all of the information is lost. This imposes an upper limit on the transmission rate of pulses. Noise degrades the signal and impairs the system performance. The quantum noise is a limitation on the signal-to-noise ratios. The present optical fibers are almost exclusively made from polar materials with a self-focusing nonlinearity. The scattering losses such as Rayleigh scattering exist inevitably in these fibers. For example, a well-fabricated silica fiber has a loss of 0.2 dB/km at 1.53 μm, which almost corresponds to its intrinsic scattering-loss value. If we want to suppress the scattering losses of optical fibers it is necessary to develop the superwaveguides, which are made from polar materials with a self-defocusing nonlinearity. The propagation of light pulses in the superwaveguides gets rid of the dispersion effect. In fact there is no limitation on the transmitted power in the superwaveguides. If the incident light intensity exceeds the threshold of stimulated Brillouin scattering, initially the laser light field is in the coherent state. During propagation, the laser light field evolves into the traveling-wave superguiding state. The information is carried by the quadrature phase of the superguided light field with reduced quantum fluctuations. Utilization of the superwaveguides as the transmission media enables us to realize repeaterless optical communications with a high bit rate and a large signal-to-noise ratio. At the receivers of such optical communication systems, one can perform quantum nondemolition measurements without destroying the signal. The presentation of the photonic superguiding state is based on the experimental fact that infrared optical fibers have extremely low scattering losses compared with silica fiber [45]. Halides are considered to be the most appropriate candidates for the superwaveguides because their band gap is large and their multiphonon absorption is located in a longer wavelength region than that for silica glass.

In conclusion, photons in a polar crystal with a self-defocusing nonlinearity can sense an attractive effective interaction by exchange of virtual optical phonons. The coherent input state of photons is unstable with respect to such an interaction and a superguiding state is formed through the association of photons in pairs. In the standing-wave superguiding state the two paired photons have opposite wave vectors and spins. In the traveling-wave superguiding state a propagating photon pair is the combination of the two photons with opposite transverse wave vectors and spins. The photon system undergoes a nonequilibrium phase transition from the coherent to the superguiding state. The system of photon pairs evolves without scattering attenuations. The traveling-wave superguiding state has the squeezing property and supports quantum solitons without dispersion. If the photons propagating in a superwaveguide enter the superguiding state, at the same time we can obtain an ultralow energy loss, a high transmission rate, and a large signal-to-noise ratio.

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