Gap 2π pulse with an inhomogeneously broadened line and an oscillating solitary wave

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The gap 2π pulse or gap solitary wave of self-induced transparency at an inhomogeneously broadened line of a resonant Bragg periodical structure has been found theoretically as an exact solution of the coupled-mode Maxwell-Bloch equations. In the case of a slight violation of the Bragg condition at exact two-level resonance, the analytical solution for the gap 2π pulse with phase modulation has been obtained. Furthermore, the existence of an oscillating pulse that periodically changes in amplitude and in the sign of its velocity has been numerically shown.

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I. INTRODUCTION

Interest in nonlinear optics of periodical structure has considerably grown in the last years. Most of the theoretical research in this field concerns the properties of gap solitary waves [1–11]. Due to the nonlinearity of the interaction between field and medium these localized light pulses are propagated at a Bragg frequency within the linear forbidden gap band of the periodical structure. They appear both in the resonance grating like the vector, two-wave soliton of self-induced transparency (gap 2π pulse) [1] and in the periodically modulated medium with Kerr nonlinearity [2–7,10,11]. The nonlinearity causes the localization of two strong, coupled Bragg modes within one pulse; while in linear systems the diffraction leads to pulse dissipation. The coupled-mode Maxwell-Bloch equations describe the problem of coherent interaction between an intensive field and an infinite periodical structure formed by the set of thin layers of two-level atoms. At exact resonance (δ-function spectrum line) these equations are replaced by a completely integrable sine-Gordon equation for a one-dimensional [1] and a two-dimensional [9] medium. The exact, solitonlike solution of coupled-mode equations for the field in nonlinear refractive periodic waveguide has been obtained by Aceves and Wabnitz [5]. Note that, in general, the coupled-mode equations for resonance and Kerr nonlinearity are not completely integrable, and their exact solutions are gap solitonlike waves or "gap solitaires" [11]. The process by which the gap solitary waves are formed in finite medium by an incident field was studied by numerical integration [6]. For the resonance lattice the delayed pulse reflection has been found [1,8]. The first experiments have displayed the optical bistability at the Bragg frequency [12,13], predicted earlier theoretically [14], and the nonlinear Bragg reflection from the resonant polymer, periodic structure [15]. These warrant optimism for the success of future experiments to find the gap solitaires.

In the present paper, we study the propagation of field in resonant Bragg grating with an inhomogeneously broadened spectrum line which is the more realistic physical problem. The exact gap 2π pulse solution has been obtained for two-wave, Maxwell-Bloch equations of self-induced transparency (SIT) at arbitrary line shape. The gap solitaire also exists under a slight violation of the Bragg condition in a homogeneously deformed structure at exact two-level resonance. In that case there is a small region of incident field amplitude within which the pulse evolution leads to an oscillating state with a periodically changing of (1) the field amplitude, (2) the inverse population of atoms, and (3) the sign of pulse velocity.

II. THE COUPLED-MODE MAXWELL-BLOCH EQUATIONS AND A GAP 2π PULSE SOLUTION

The one-dimensional resonant Bragg grating (RBG) is assumed to consist of periodically positioned thin layers containing two-level atoms. The incident quasimonochromatic field with wavelength λ is normal to resonance planes of the grating. The structure period d exactly satisfies the Bragg condition $d=m\lambda/2$, where m is an integer. To give a concrete expression to the model, the period of structure d is taken here to be equal to λ and the resonant layer thickness must be smaller than λ. In that case there are only two diametrically opposite points on the Ewald sphere, so it is possible to limit the number of Bragg modes using a two-wave approach. The RBG can be realized using a periodic structure of quantum wells with two-dimensional excitons in a semiconductor [16], a copolymer multilayer structure [15], or a periodic erbium-doped fiber waveguide [17].

Provided the exact two-level resonance the coherent interaction of quasimonochromatic field

$$\mathscr{E}(x,t)=\tfrac{1}{2}[E^{+}(x,t)\exp(ikx-i\omega t)+E^{-}(x,t)\exp(-ikx-i\omega t)]+\mathrm{c.c.}$$

with the resonant matter described by semiclassical, coupled-mode Maxwell-Bloch equations. These describe

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the slowly varying envelope of complex electric-field amplitudes of the forward and backward waves $E^{\pm}$, the dimensionless characteristic of complex atomic polarization P', and the inverse population of atoms n. The equations have the following form [1]:

$$\Omega_t^{\pm}(x,t)\pm\Omega_x^{\pm}=P'(x,t) ,$$
$$P_t'(x,t)=n(x,t)[\Omega^+(x,t)+\Omega^-(x,t)] ,$$
$$n_t(x,t)=-\mathrm{Re}\{P'^*(x,t)[\Omega^+(x,t)+\Omega^-(x,t)]\} .$$

Taking into account the inhomogeneously broadened line form $g(\Delta\omega)$ with a sufficiently small line width $\Delta\omega_0$ to be localized within the Bragg gap band, these equations are replaced by the following:

$$\Omega_t^+(x,t)+\Omega_x^+(x,t)=\int_{-\infty}^{\infty}P(x,t,\Delta\omega)g(\Delta\omega)d\Delta\omega ,$$
$$\Omega_t^-(x,t)-\Omega_x^-(x,t)=\int_{-\infty}^{\infty}P(x,t,\Delta\omega)g(\Delta\omega)d\Delta\omega ,$$
$$P_t(x,t,\Delta\omega)+i\tau_c\Delta\omega P(x,t,\Delta\omega) =n(x,t,\Delta\omega)[\Omega^+(x,t)+\Omega^-(x,t)] , \quad (1)$$
$$n_t(x,t,\Delta\omega)=-\mathrm{Re}\{P^*(x,t,\Delta\omega)[\Omega^+(x,t)+\Omega^-(x,t)]\} ,$$

where $\Omega^{\pm}=2\tau_c(\mu/\hbar)E^{\pm}$, $P=P'\exp(-i\Delta\omega t)$, cooperative time is given by $\tau_c^2=8\pi T_1/3c\rho\lambda^2$, T_1 is the atomic exited level lifetime, ρ is the density of the resonant atoms, μ is the matrix element of the projection of the transition dipole moment, c is the light velocity, $t=t'/\tau_c$ and $x=x'/c\tau_c$ are dimensionless time and space coordinates, and the difference of atoms resonance and field frequencies is $\Delta\omega=\omega-\omega_0$. The parameter of coherent interaction τ_c characterizes the mean photon lifetime in the medium preceding resonant absorption. It fixes the number of structure periods N by the condition $N>c\tau_c/d$.

By means of the transformation

$$\Omega=\Omega^++\Omega^- ,\qquad \tilde{\Omega}=\Omega^+-\Omega^- , \quad (2)$$

the Eqs. (1) are reduced to the form

$$\Omega_{tt}-\Omega_{xx}=2\int_{-\infty}^{\infty}P_t(x,t,\Delta\omega)g(\Delta\omega)d\Delta\omega , \quad (3a)$$
$$P_t+i\tau_c\Delta\omega P=n\Omega , \quad (3b)$$
$$n_t=-\mathrm{Re}(P^*\Omega) , \quad (3c)$$

and

$$\tilde{\Omega}_{tt}-\tilde{\Omega}_{xx}=-2\int_{-\infty}^{\infty}P_x(x,t,\Delta\omega)g(\Delta\omega)d\Delta\omega . \quad (4)$$

It is easy to see that Eqs. (3)–(4) are separated for the functions Ω and $\tilde{\Omega}$ and hence can be solved independently.

The left-hand side of Eq. (3a) distinguishes Eqs. (3) from the completely integrable Maxwell-Bloch equations in the SIT problem of a homogeneous medium [18]. However, Eqs. (3) for the function $g(\Delta\omega)=\delta(\omega-\omega_0)$ is known to have a sech-form soliton solution [1]. Generalizing this, one seeks the one-soliton solution of Eqs. (3) as follows:

$$\Omega(x,t)=\Omega_0\exp[i(\alpha_1x-\alpha_2t+\psi)]\mathrm{sech}\varphi , \quad (5)$$

where $\varphi=(x-vt)/v\tau$, ψ is an initial phase, v and τ are the pulse velocity and duration. Then the Bloch Eqs. (3b) and (3c) can be integrated directly

$$P(x,t,\Delta\omega)=[\beta(\Delta\omega)\mathrm{sech}\varphi\tanh\varphi-i\sigma(\Delta\omega)\mathrm{sech}\varphi] \times\exp[i(\alpha_1x-\alpha_2t+\psi)] , \quad (6)$$
$$n(x,t,\Delta\omega)=-1+2\xi(\Delta\omega)\mathrm{sech}^2\varphi ,$$

where

$$\xi(\Delta\omega)=[1+(\alpha_2-\tau_c\Delta\omega)^2\tau^2]^{-1} ,$$
$$\beta(\Delta\omega)=-\Omega_0\tau\xi(\Delta\omega) ,$$
$$\sigma(\Delta\omega)=\Omega_0\tau^2(\alpha_2-\tau_c\Delta\omega)\xi(\Delta\omega) , \quad (7)$$
$$\Omega_0^2=4\tau^{-2} . \quad (8)$$

Substitution of expressions (5)–(8) into Eq. (3a) yields the ratios for parameters v, τ, and $\alpha_{1,2}$

$$\alpha_1=\alpha_2/v , \quad (9)$$
$$\alpha_2(v^{-2}-1)=\tau_c\tau^2\int_{-\infty}^{\infty}\Delta\omega\xi(\Delta\omega)g(\Delta\omega)d\Delta\omega , \quad (10)$$
$$v^{-2}=1+2\tau^2\int_{-\infty}^{\infty}\xi(\Delta\omega)g(\Delta\omega)d\Delta\omega . \quad (11)$$

The condition of compatibility of Eqs. (10) and (11) gives the connection of values τ and α_2 [which are independent in (7)]

$$\int_{-\infty}^{\infty}\frac{(2\alpha_2-\tau_c\Delta\omega)g(\Delta\omega)}{1+\tau^2(\alpha_2-\tau_c\Delta\omega)^2}d\Delta\omega=0 . \quad (12)$$

Returning back to the wave amplitudes Ω^+ and Ω^- (2) using the solutions (5), (8), (9), (11), and (12) of Eqs. (3) and also noting from Eqs. (3a) and (4) that $\Omega_x=-\tilde{\Omega}_t$, we derive the following solutions:

$$\Omega^{\pm}=\tau^{-1}(1\pm v^{-1})\exp[i\alpha_2(x-vt)/v+i\psi] \times\mathrm{sech}[(x-vt)/v\tau] . \quad (13)$$

Thus, we have found the exact solutions (13), (11), and (12) of Eqs. (1) which describes the gap 2π pulse of self-induced transparency in the resonant two-level Bragg grating for any arbitrary form of an inhomogeneously broadened line. The gap 2π pulse consisting of two strong coupled Bragg modes (13) propagates at the frequencies within the stop gap band of the structure at the velocity v (11) depending on the spectral line $g(\Delta\omega)$, with the pulse width τ and the phase parameter α_2. This parameter can be calculated from the connection condition (12). In particular, if the function $g(\Delta\omega)$ is taken symmetrically with respect to the frequency ω_0, then the value $\alpha_2=0$ and the field envelopes (13) become the same form as in the case of the exact resonant gap 2π pulse [1], though the pulse velocities differ. For instance, for a Gaussian line form

$$g(\Delta\omega)=(\sqrt{\pi}\Delta\omega_0)^{-1}\exp[-(\Delta\omega/\Delta\omega_0)^2]$$

and a narrow homogeneous field line $(\tau_c\tau)^{-1}\ll\Delta\omega_0$ the integral (11) is calculated by approximation. It can be found that the velocity

$$v \approx [1+2\sqrt{\pi}\tau^2(\tau_c\tau\Delta\omega_0)^{-1}]^{-1/2}$$

is faster than $v=(1+2\tau^2)^{-1/2}$ for $g(\Delta\omega)=\delta(\omega-\omega_0)$ due to the small parameter $(\tau_c\tau\Delta\omega_0)^{-1}$.

III. GAP 2π PULSE AND OSCILLATING SOLITARY WAVE IN A PERTURBED RBG

Actually, the considered model of the RBG implies the existence of two different resonances that are realized simultaneously: (1) the frequency two-level resonance and (2) the space phase Bragg resonance. In the above result the spectral line is assumed narrow enough to ignore the deviation from Bragg resonance. Here we are studying the field dynamics taking into account a small violation of the Bragg condition. It will be shown that a phase-modulated, exact gap solitaire solution exists, and there is a thin region of incident pulse amplitude where the oscillating quasistable pulse is found by numerical calculation.

Consider a homogeneously deformed RBG where a small violation of the Bragg condition for the grating period is given by

$$\begin{aligned} &d=(1+\varepsilon)\lambda \;, \\ &\Delta\omega_0/\omega_0 \ll \varepsilon \ll 1 \;. \end{aligned} \tag{14}$$

To simplify the calculation, the field is assumed to be at an exact two-level resonance and $g(\Delta\omega)=\delta(\omega-\omega_0)$. In this case the main discrete coupled-mode Maxwell-Bloch equations [1]

$$\begin{aligned} &\Omega_t^+(x,t)+\Omega_x^+(x,t)=\Sigma_i \exp(-ikx_i)P(x_i,t)\tilde{\delta}(x-x_i) \;, \\ &\Omega_t^-(x,t)-\Omega_x^-(x,t)=\Sigma_i \exp(ikx_i)P(x_i,t)\tilde{\delta}(x-x_i) \;, \\ &P_t(x_i,t)=n(x_i,t)[\Omega^+(x_i,t)\exp(ikx_i) \\ &\qquad\qquad +\Omega^-(x_i,t)\exp(-ikx_i)] \;, \\ &n_t(x_i,t)=-\mathrm{Re}\{P^*(x_i,t)[\Omega^+(x_i,t)\exp(ikx_i) \\ &\qquad\qquad +\Omega^-(x_i,t)\exp(-ikx_i)]\} \end{aligned} \tag{15}$$

(x_i are the coordinates of the resonant layers, the function $\tilde{\delta}(x-x_i)=1$ if $x\in(x_i\pm\lambda'/2)$ and 0 at any points, $k=2\pi/\lambda'$ and $\lambda'=\lambda/c\tau_c$) after the averaging over the region $\Delta v \gg d^3$ lead to the following equations:

$$\begin{aligned} &\Omega_t^+(x,t)+\Omega_x^+(x,t)=P(x,t)\exp(-i\gamma x) \;, \\ &\Omega_t^-(x,t)-\Omega_x^-(x,t)=P(x,t)\exp(i\gamma x) \;, \\ &P_t(x,t)=n(x,t)[\Omega^+(x,t)\exp(i\gamma x) \\ &\qquad\qquad +\Omega^-(x,t)\exp(-i\gamma x)] \;, \\ &n_t(x,t)=-\mathrm{Re}\{P^*(x,t)[\Omega^+(x,t)\exp(i\gamma x) \\ &\qquad\qquad +\Omega^-(x,t)\exp(-i\gamma x)]\} \;, \end{aligned} \tag{16}$$

where $\gamma=k\varepsilon \ll 1$ and the slow variation of the function $\exp(i\gamma x)$ for the space of the period d is used. The transformation

$$\begin{aligned} &\Omega^+=\tfrac{1}{2}(\Omega'+\tilde{\Omega}')\exp[i\gamma(t-x)] \;, \\ &\Omega^-=\tfrac{1}{2}(\Omega'-\tilde{\Omega}')\exp[i\gamma(t+x)] \;, \\ &P=P'\exp(i\gamma t) \end{aligned} \tag{17}$$

passes Eqs. (16) to the following:

$$\begin{aligned} &\Omega'_{tt}-\Omega'_{xx}=2P'_t \;, \\ &P'_t+i\gamma P'=n\Omega' \;, \\ &n_t=-\mathrm{Re}(P'^*\Omega') \;. \end{aligned} \tag{18}$$

Comparing Eqs. (17) with Eqs. (3) and (4), one notes that their form coincides if the function $g(\Delta\omega)$ in Eqs. (3) and (4) is taken formally to be $\delta(\Delta\omega-\gamma/\tau_c)$. Thus, the solutions (5), (8), (11), and (12) obtained above can be used directly. For $\alpha_2=\gamma/2$ (12) and velocity (11)

$$v^{-2}=1+2\tau^2/(1+\tau^2\gamma^2/4) \tag{19}$$

the solutions of Eqs. (18) are

$$\begin{aligned} &\Omega'=2\tau^{-1}\exp[i\gamma(x-vt)/2v+i\psi]\mathrm{sech}[(x-vt)/v\tau] \;, \\ &\tilde{\Omega}'=v^{-1}\Omega' \;. \end{aligned} \tag{20}$$

Finally, passing from functions Ω' and $\tilde{\Omega}'$ (20) to wave amplitudes Ω^+ and Ω^- (17), one gets the exact solution of Eqs. (16),

$$\begin{aligned} &\Omega^+=\tau^{-1}(1+v^{-1})\exp\{i\gamma[(x-vt)/2v-(x-t)]+i\psi\} \\ &\qquad \times\mathrm{sech}[(x-vt)/v\tau] \;, \\ &\Omega^-=\tau^{-1}(1-v^{-1})\exp\{i\gamma[(x-vt)/2v+(x+t)]+i\psi\} \\ &\qquad \times\mathrm{sech}[(x-vt)/v\tau] \;, \\ &n=-1+2(1+\tau^2\gamma^2/4)^{-1}\mathrm{sech}^2[(x-vt)/v\tau] \;, \\ &P=(1+\tau^2\gamma^2/4)^{-1}\{-2\tanh[(x-vt)/v\tau] \\ &\qquad\qquad \times\mathrm{sech}[(x-vt)/v\tau] \\ &\qquad\qquad +i\gamma\tau\,\mathrm{sech}[(x-vt)/v\tau]\} \\ &\qquad \times\exp[i\gamma(x-vt)/2v+i\psi] \;. \end{aligned} \tag{21}$$

The gap 2π pulse (21) propagates in a perturbed, deformed RBG at the speed v (19) and connects two waves with different amplitudes and phase modulations as well as an inverse population pulse.

As it was noted above, the exact solution (21) is just the partial solution of Eqs. (16). To get a boundary condition under which this gap solitary wave varies and to make clear its stability, we simulated numerically the process of producing the localized pulse in a perturbed RBG under irradiation of the medium boundary by an incident field. The numerical integration of the set of Eqs. (15) uses the method of characteristics and takes into account (14) and the boundary conditions

$$\begin{aligned} &\mathrm{Re}\Omega^+(x=0,t)=\Omega_0\mathrm{sech}[(t-t_0)/\tau_0] \;, \\ &\mathrm{Im}\Omega^+(x=0,t)=0 \;, \\ &\Omega^-(x=l,t)=0 \;, \quad \Omega^\pm(x,t=0)=0 \;, \\ &n(x,t=0)=-1 \;, \quad P(x,t=0)=0 \;. \end{aligned} \tag{22}$$

The perturbation parameter was chosen to be $\varepsilon = 10^{-5}$.

The stable pulse described by the solutions (21) and (19) is excited in the grating if the incident field is strong enough, when the amplitude Ω_0 is larger than the critical one Ω_{cr} by 5%, where the value Ω_{cr} corresponds to the delayed reflection regime [1,8].

The intriguing result is obtained at the incident field amplitudes within this thin 5% band, where an oscillating pulse arises. Figures 1–3 illustrate the dynamics of forward and backward fields as well as that of medium inversion occurring during the motion of the oscillating pulse. The pulse of real incident field (22) going into the medium acquires the imaginary component due to the perturbation $i\gamma$ (18) and its form is close to solution (21) at small velocity $v > 0$, but there is an essential energy loss. The imaginary field component grows and causes additional emission of excited atoms into the backward mode. As a result, the maximum inversion decreases from $n_0 \approx 1$ to 0.8; the backward field amplitude becomes larger than the forward one $|\Omega^-| > |\Omega^+|$ and, after the pulse stops the move direction changes the sign ($v < 0$). Afterward, the pulse atomic subsystem absorbs part of the field. The maximum inversion returns to $n_0 \approx 1$ again and the speed sign is changed $v > 0$ due to the field amplitude ratio $|\Omega^+| > |\Omega^-|$. To each oscillation of pulse, there corresponds a loop on the trajectory of the Bloch vector $R = \{\mathrm{Re}P, \mathrm{Im}P, n\}$ on the Bloch sphere. The energy loss becomes small, and the described nonlinear beating in the system of strong coupled modes is quasistable. The number of pulse oscillations depends on the incident field amplitude Ω_0 and reached the value 35 in our computer experiments. As a result of the oscillating pulse evolution the pulse stops and decays.

The frequency of oscillations ω_{os} is much larger than the phase modulation γ in solution (21), and moreover ω_{os} weakly depends on γ. The perturbation only stimulates the transition from a solitonlike regime of pulse propagation to a new, oscillating, quasistable state with a fast rotating field phase. The moment of transition depends on γ. The analytical solution of Eqs. (18) for an oscillating pulse has not been found. Note that the stable, localized oscillating solution of nonlinear evolution equations has been obtained by Calogero and Degasperis [19] for a special class of solvable nonlinear equations.

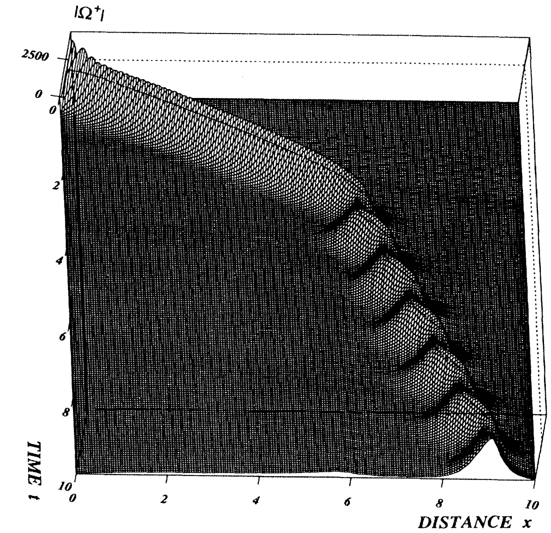

FIG. 2. Dynamics of the forward wave amplitude modulus $|\Omega^+(x,t)|$ of the oscillating pulse (arbitrary units).

IV. CONCLUSION

At the propagation of a gap 2π pulse of self-induced transparency in the RBG with an inhomogeneously

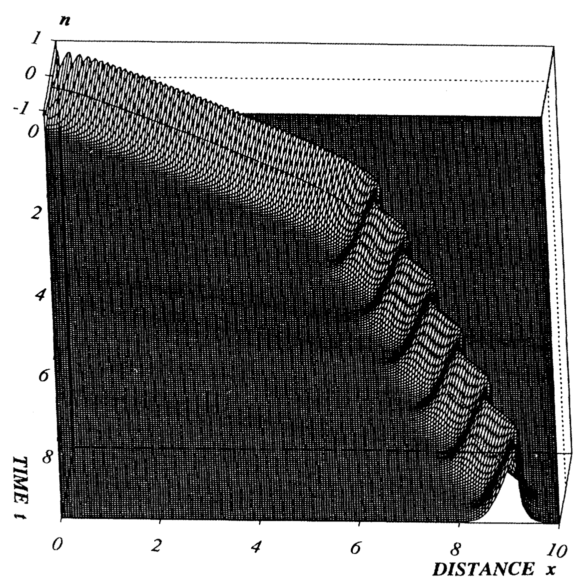

FIG. 1. Space-time dependence of medium inversion $n(x,t)$ induced by the propagating oscillating pulse.

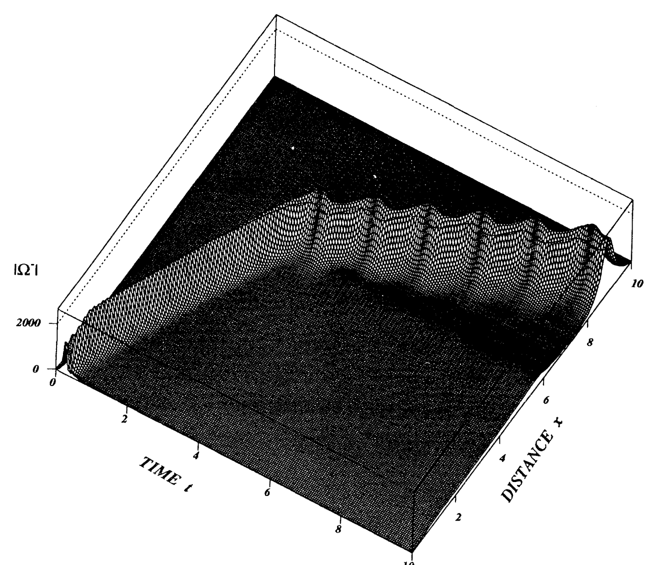

FIG. 3. Dynamics of the backward wave amplitude modulus $|\Omega^-(x,t)|$ of the oscillating pulse (arbitrary units).

broadened line, both dissipative processes—the resonant absorption and the diffractive Bragg dissipation—are suppressed due to nonlinearity. The existence of gap solitary waves in resonant and Kerr periodical media shows that the phenomenon of nonlinear suppression of Bragg diffractive scattering of a field pulse is rather general and can be realized in physical systems with different types of nonlinearity. The simple estimates show that the gap 2π pulse can be observed experimentally, for instance, in a periodical structure of GaAs quantum wells with two-dimensional excitons at low temperature [16]. When an exciton density $\rho=2\times10^{19}$ m^{-3}, $\lambda=806$ nm, and $T_1=180$ ps, the cooperative time $\tau_c=0.6$ ps and the number of structure layers N, which is necessary to form a localized pulse, is fixed by the expression $N>c\tau_c/d=225$.

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