Macroscopic coherence via quantum feedback

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It is shown that quantum-nondemolition-mediated feedback is able to preserve the interference fringes of the superposition of macroscopically distinguishable quantum states.

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I. INTRODUCTION

Interest in the foundations of quantum mechanics received a great revival during the last decade [1]. One possible explanation of such a revival is the development of experimental technologies. Nowadays, experiments on single quantum objects are feasible [2] and the possibility of checking the consequences of the Copenhagen interpretation seems realizable. One of the fundamental problems is to observe quantum-mechanical features in macroscopic objects. Quantum theory, indeed, may adequately describe macroscopic objects by means of a linear superposition of states with macroscopically distinguishable properties. The signature of the superposition of quantum states is the existence of interference fringes. Even though the theory we present could be applied to a wider class of physical systems, to be more concise we will focus our attention on the possibility of observing a macroscopic superposition of quantum states in quantum optics. Thus our macroscopic states will be coherent states, and we will show that, under suitable conditions, one should be able to observe the interference fringes of the quantum superposition of two coherent states, even though the number of photons involved has to be relatively small. Macroscopic objects are never isolated because they interact with their environment. It is the coupling to the environment that causes the destruction of the quantum coherence on a very fast time scale [3]. Some years ago, Yurke and Stoler [4] showed that, after a characteristic interaction time, a Kerr medium may produce a superposition of coherent states when it is illuminated by a coherent beam. However, as soon as one tries to detect the interference fringes with some experimental apparatus, they disappear exponentially fast with the number of photons in the beam and in a time of the order of the inverse of the damping constant of the apparatus. Kennedy and Walls [5] presented a general model for dissipation with squeezed quantum fluctuations and showed that, with such a bath as a model for the experimental apparatus, not only is one able to preserve the macroscopic superposition of quantum states, thus confirming previous heuristic results [6], but one can also prepare states with low quantum noise in a quadrature phase. Despite these intriguing results, however, the fundamental question one should be able to answer is: How can a squeezed bath be realized? Our previous work [7] was devoted to showing that one possible answer to the above question is to use suitable feedback to mimic the squeezed bath, and that the result we obtain could be experimentally realized. We will show in the present paper that the same model could be applied to observe the interference fringes of the macroscopic superposition when we consider a different limit with respect to Ref. [7]. The feedback model of Ref. [7] is not the only way to simulate a squeezed bath. For example, the output of a degenerate parametric oscillator (DPO) below threshold in a cavity with a large linewidth can approximate a squeezed white input as well [8]. Anyway, we think that the feedback scheme proposed here is preferable with respect to the DPO scheme because it can be applied at any frequency and it is not limited to a single frequency (one half of the pump frequency) as in the DPO case. Moreover, the DPO output can mimic a squeezed white bath only for cavities whose linewidths are much smaller than that of the DPO cavity. This is another limitation that does not affect our feedback model of a squeezed bath. The outline of the paper is as follows: In Sec. II, we introduce the model and we apply the quantum theory of feedback recently proposed by Wiseman and Milburn [9,10] to it. In Sec. III, we study the time evolution of an optical Schrödinger cat state and we discuss the conditions under which the dissipation-induced disappearance of the interference fringes can be significantly slowed down. Section IV is for concluding remarks.

II. THE FEEDBACK MODEL

We shall describe quantum feedback by adopting the formalism recently developed by Wiseman and Milburn [9,10]. The basic ingredients of this theory essentially are the stochastic evolution of the density matrix of the system conditioned to the performed measurement, the homodyne current of the measurement process, and the way in which the current is fed back to the system. Let us consider the model first proposed in Ref. [11] for a quantum nondemolition (QND) measurement of a field quadrature of a cavity mode. We suppose that the cavity supports two different modes, described by the Bose destruction operators a and b, and that they interact by means of the interaction Hamiltonian $H_I=\hbar\chi X_\xi Y_\varphi$,

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where $X_\xi=(ae^{i\xi}+a^\dagger e^{-i\xi})/2$ and $Y_\varphi=(be^{i\varphi}+b^\dagger e^{-i\varphi})/2$ represent the generic quadratures of the two modes, i.e., X_ξ is the quadrature to be QND measured while Y_φ represents the quadrature of the meter which is part of the measurement apparatus, and χ is the coupling constant. This coupling can be realized for example for placing a nonlinear crystal within the cavity. In the interaction picture the total density matrix D evolves according to

$$\dot{D}=\mathcal{L}D-\frac{i}{\hbar}[H_I,D]+\frac{\gamma_b}{2}(2bDb^\dagger-b^\dagger bD-Db^\dagger b)\ . \quad (1)$$

$\mathcal{L}$ describes the unperturbed motion of the mode of interest a. It is assumed that the damping rate γ_b for the mode b is sufficiently high that it can be adiabatically eliminated and, of course, it represents the meter variable by means of which we get the quantum nondemolition measurement of the quadrature X_ξ of the mode a. The evolution equation for the total density matrix conditioned by the homodyne measurement of the quadrature $Y_\delta=(be^{-i\delta}+b^\dagger e^{i\delta})/2$ is [10]

$$\begin{aligned}\dot{D}_c=&\mathcal{L}D_c-i\chi[X_\xi Y_\varphi,D_c]\\&+\frac{\gamma_b}{2}(2bD_cb^\dagger-b^\dagger bD_c-D_cb^\dagger b)\\&+\sqrt{\gamma_b}\eta\xi(t)(be^{-i\delta}D_c+D_cb^\dagger e^{i\delta}-2\langle Y_\delta\rangle_cD_c)\ ,\end{aligned} \quad (2)$$

where $0\le\eta\le1$ is the efficiency of the homodyne measurement while $\xi(t)$ is a Gaussian white noise introduced by the measurement process. By tracing over the meter variables we obtain the equation for the conditioned density matrix for the mode a,

$$\begin{aligned}\dot{\rho}_c=&\mathcal{L}\rho_c-\frac{\chi^2}{2\gamma_b}[X_\xi,[X_\xi,\rho_c]]\\&+\sqrt{g}\,\xi(t)(ie^{i(\varphi+\delta)}\rho_cX_\xi-ie^{-i(\varphi+\delta)}X_\xi\rho_c\\&+2\sin(\varphi+\delta)\langle X_\xi\rangle_c\rho_c)\end{aligned} \quad (3)$$

with $g=\eta\chi^2/\gamma_b$. Equation (3) represents the first fundamental ingredient of the theory of Wiseman and Milburn [9,10], while the second is given by the expression of the homodyne current, which can be written as [10] $I_c(t)=-2\eta\chi\sin(\varphi+\delta)\langle X_\xi(t)\rangle_c+\eta\chi\xi(t)/\sqrt{g}$. We now feed the current $I_c(t)$ back to the system by means of a superoperator $\mathcal{K}$ which we will show later on and represents the third fundamental ingredient of the theory of Wiseman and Milburn. Thus, the conditioned density matrix only due to the feedback can be written as [9] $[\dot{\rho}_c]_{fb}=I_c(t)\mathcal{K}\rho_c/\eta\chi$. In any feedback process there is, of course, a delay time due to the feedback loop, which makes the physics non-Markovian; however, it is possible to consider such a delay time much shorter than the characteristic time of the system, which usually is the inverse of the decay rate of the mode of interest. Under such an assumption, the feedback loop does not introduce any delay and we are allowed to consider the process Markovian as we have actually assumed. However, the feedback process is physically added to the evolution of the system of interest and its stochastic differential equation has to be introduced as a limit of a real process. Thus it should be considered in the Stratonovich sense [12], while the previous derivation of the stochastic equation for the conditioned density matrix Eq. (3) has to be considered in the Ito sense [13]. By using standard rules [14] to convert the Stratonvich equation into the Ito one, we can write the final unconditioned evolution equation [i.e., averaged over the Gaussian distribution of the white noise $\xi(t)$] for the reduced density matrix of the mode of interest [10]

$$\begin{aligned}\dot{\rho}=&\mathcal{L}\rho-\frac{\chi^2}{2\gamma_b}[X_\xi,[X_\xi,\rho]]\\&+\mathcal{K}(ie^{i(\varphi+\delta)}\rho X_\xi-ie^{-i(\varphi+\delta)}X_\xi\rho)+\frac{\mathcal{K}^2}{2g}\rho\ .\end{aligned} \quad (4)$$

We now see that the two phases φ and δ, representing the phase of the quadrature of the b mode interacting with X_ξ, and the phase of the homodyning measured b quadrature, respectively, are not independent of each other and we can set $\phi=\varphi+\delta$. By measuring the Y_δ quadrature we actually get a QND measurement [15] of X_ξ. This will be a perfect QND measurement as long as we can consider the coupling constant χ very big and the efficiency $\eta=1$. Thus, Eq. (4) describes the evolution of the density matrix of the signal mode a once a QND measurement of its quadrature X_ξ has been performed and part of the information obtained is fed back to it by means of the action of the superoperator $\mathcal{K}$. Let us assume that the following form of the feedback superoperator holds, $\mathcal{K}\rho=-iG[X_\theta,\rho]$, where G is a constant representing the gain or "efficiency" of the feedback process. It can be obtained by feeding back the photocurrent to the cavity with a driving term in the Hamiltonian involving a different quadrature θ of the signal mode, where θ is another phase parameter that can be controlled by the experimeter. We finally obtain

$$\begin{aligned}\dot{\rho}=&\mathcal{L}\rho-\frac{\Gamma}{2}[X_\xi,[X_\xi,\rho]]-G\cos\phi[X_\theta,[X_\xi,\rho]]\\&+iG\sin\phi[X_\theta,\{X_\xi,\rho\}]-\frac{G^2}{2g}[X_\theta,[X_\theta,\rho]]\ .\end{aligned} \quad (5)$$

with $\Gamma=\chi^2/\gamma_b$ and we have introduced { } to denote the anticommutator. We now consider the standard Liouvillian for the signal mode $\mathcal{L}\rho=(\gamma_a/2)(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a)$. After writing explicitly the commutators and the anticommutator, Eq. (5) can finally be written as

$$\begin{aligned}\dot{\rho}=&\frac{\gamma}{2}(N+1)(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a)+\frac{\gamma}{2}N(2a^\dagger\rho a-aa^\dagger\rho-\rho aa^\dagger)-\frac{\gamma}{2}M(2a^\dagger\rho a^\dagger-a^\dagger a^\dagger\rho-\rho a^\dagger a^\dagger)\\&-\frac{\gamma}{2}M^*(2a\rho a-aa\rho-\rho aa)+i\frac{G}{4}\sin\phi[(aae^{i(\xi+\theta)}+a^\dagger a^\dagger e^{-i(\xi+\theta)})+2\cos(\xi-\theta)a^\dagger a,\rho]\ ,\end{aligned} \quad (6)$$

where

$$\gamma=\gamma_a+G\sin(\xi-\theta)\sin\phi\ , \tag{7a}$$

$$N=\frac{1}{\gamma}\left[\frac{\Gamma}{4}+\frac{G^2}{4g}+\frac{G}{2}\cos(\xi-\theta+\phi)\right] , \tag{7b}$$

$$M=-\frac{1}{\gamma}\left[\frac{\Gamma}{4}e^{-2i\xi}+\frac{G^2}{4g}e^{-2i\theta}+\frac{G}{2}e^{-i(\xi+\theta)}\cos\phi\right] . \tag{7c}$$

We have already shown [7] that, under suitable conditions, this equation gives almost perfect squeezing of the measured quadrature of the mode a in the steady state, thus allowing the interpretation that a squeezed bath can be physically realized in such a way. Using a QND measurement step (or any other intracavity measurement scheme) in the feedback loop is a *necessary* condition for obtaining nonclassical effects. In fact, as shown by the "no go" theorems of Ref. [10], any feedback loop based on extracavity measurements, such as direct photocounting or homodyne detection, is not able to produce nonclassical states of light if the internal dynamics does not have the capability of producing them by itself.

III. TIME EVOLUTION OF AN OPTICAL SCHRÖDINGER CAT

The dynamics of a field mode in the presence of a squeezed bath has been already discussed in Ref. [5], where it is shown that a squeezed bath is more efficient than a thermal or vacuum bath for the detection of the linear superposition of macroscopically distinguishable states. In fact, the inevitable destruction of the corresponding interference pattern due to the presence of dissipation can be significantly slowed down by the squeezing of the bath quantum fluctuations. Therefore, it is interesting to study the time evolution of these Schrödinger-cat-like states and to see if a preservation of macroscopic quantum coherence similar to that discuss in Ref. [5] can be achieved in the case of the physically realizable feedback model presented here.

Interference fringes are the relevant signature of the linear superposition and they can be detected by using homodyne techniques [4] where the output current is proportional to an experimentally adjustable field quadrature. To be more precise, quantum interference can be seen in the probability distribution of the QND measured quadrature X_ξ, $P(x_\xi)=\langle x_\xi|\rho(t)|x_\xi\rangle$, where $|x_\xi\rangle$ is the eigenstate of X_ξ with eigenvalue x_ξ. This probability distribution can be obtained from the exact solution of Eq. (6), once the initial condition for the density matrix is chosen of the form $\rho(0)=\sum_{\alpha,\beta}N_{\alpha,\beta}|\alpha\rangle\langle\beta|$, (where $|\alpha\rangle$, $|\beta\rangle$ are coherent states of the mode a), by relating the characteristic function $\chi(\lambda,\lambda^*;t)$ to the generalized P function

$$P_{AB}(\sigma,\sigma^*;t)=\frac{1}{\pi^2}\int d^2\lambda\chi(\lambda,\lambda^*,t)\exp\{\lambda^*(\sigma+A(t)) -\lambda(\sigma+B(t))^*\}\ , \tag{8}$$

and by using the identity [5]

$$\rho(t)=\int d^2\sigma P_{AB}(\sigma,\sigma^*)\frac{|\sigma+A(t)\rangle\langle\sigma+B(t)|}{\langle\sigma+B(t)|\sigma+A(t)\rangle}\ , \tag{9}$$

where

$$A(t)=\frac{\alpha e^{2i\xi}+\beta^*}{e^{2i\xi}-e^{2i\theta}}e^{-(\gamma-\gamma_a/2)t}-\frac{\alpha e^{2i\theta}+\beta^*}{e^{2i\xi}-e^{2i\theta}}e^{-\gamma_a t/2}\ , \tag{10a}$$

$$B^*(t)=\frac{\alpha+\beta^* e^{-2i\xi}}{e^{-2i\xi}-e^{-2i\theta}}e^{-(\gamma-\gamma_a/2)t}-\frac{\alpha+\beta^* e^{-2i\theta}}{e^{-2i\xi}-e^{-2i\theta}}e^{-\gamma_a t/2}\ . \tag{10b}$$

After some Gaussian integrations, one obtains

$$P(x_\xi,t)=\sum_{\alpha,\beta}N_{\alpha,\beta}\frac{\langle\beta|\alpha\rangle}{\sqrt{2\pi\sigma_x^2(t)}}\times\exp\left\{-\frac{[x_\xi-\delta_{\alpha,\beta}(t)]^2}{2\sigma_x^2(t)}\right\}\ , \tag{11}$$

where

$$\sigma_x^2(t)=\tfrac{1}{2}+\nu(t)+\mathrm{Re}\{\mu(t)e^{2i\xi}\}\ , \tag{12}$$

$$\delta_{\alpha,\beta}(t)=\frac{A(t)e^{i\xi}+B^*(t)e^{-i\xi}}{\sqrt{2}}\ . \tag{13}$$

with

$$\nu(t)=\left[\frac{G^2}{4g}-\frac{G}{2}\frac{\sin\phi}{\sin(\xi-\theta)}\right]\left[\frac{1-e^{-(2\gamma-\gamma_a)t}}{(2\gamma-\gamma_a)}\right]+\frac{\Gamma}{4}\left[\frac{1-e^{-\gamma_a t}}{\gamma_a}\right]+\frac{G}{2}\cot(\xi-\theta)\sin(\xi-\theta+\phi)\times\left[\frac{1-e^{-\gamma t}}{\gamma}\right]\ , \tag{14a}$$

$$\mu(t)=\frac{ie^{i(\xi-\theta)}\sin(\xi-\theta)}{e^{2i\theta}-e^{2i\xi}}\left[\frac{G^2}{2g}-G\frac{\sin\phi}{\sin(\xi-\theta)}\right]\times\left[\frac{1-e^{-[2\gamma-\gamma_a]t}}{(2\gamma-\gamma_a)}\right]+\frac{ie^{-i(\xi-\theta)}\sin(\xi-\theta)}{e^{2i\theta}-e^{2i\xi}}\frac{\Gamma}{2}\left[\frac{1-e^{-\gamma_a t}}{\gamma_a}\right]+i\frac{G\sin(\xi-\theta+\phi)}{e^{2i\theta}-e^{2i\xi}}\left[\frac{1-e^{-\gamma t}}{\gamma}\right]\ . \tag{14b}$$

As a special case of the general result Eq. (11), we consider the initial superposition treated by Yurke and Stoler [4], produced by unitary evolution of a coherent state in a nonlinear medium $\rho(0)=[e^{-i\pi/4}|\alpha\rangle+e^{i\pi/4}|-\alpha\rangle][e^{i\pi/4}\langle\alpha|+e^{-i\pi/4}\langle-\alpha|]/2$. With this choice Eq. (11) simplifies to

$$P(x_\xi,t)=\tfrac{1}{2}\{p_\alpha^2(x_\xi,t)+p_{-\alpha}^2(x_\xi,t)+2p_\alpha(x_\xi,t)p_{-\alpha}(x_\xi,t)\times\sin[\Omega(x_\xi,t)]|\langle\alpha|-\alpha\rangle|^{\eta(t)}\}\ . \tag{15}$$

The first two terms $p^2_{\pm\alpha}(x_\xi,t)$ describe the two Gaussian probability hills corresponding to the two coherent states $|\pm\alpha\rangle$ of the initial superposition and they are explicitly given by

$$p^2_{\pm\alpha}(x_\xi,t)=\frac{1}{\sqrt{2\pi\sigma_x^2(t)}}\exp\left\{-\frac{(x_\xi\mp g(\alpha,t))^2}{2\sigma_x^2(t)}\right\}, \tag{16}$$

where

$$g(\alpha,t)=\frac{1}{2\sqrt{2}\sin^2(\xi-\theta)}\mathrm{Re}\{[e^{i\xi}(\alpha+\alpha^*e^{-2i\theta})(1-e^{-2i(\xi-\theta)})]e^{-\gamma_a t/2}+[(\alpha e^{i\xi}+\alpha^*e^{-i\xi})(1-e^{2i(\xi-\theta)})]e^{-(\gamma-\gamma_a/2)t}\}. \tag{17}$$

The third term in Eq. (15) describes the quantum interference between the two coherent states in the presence of the measurement apparatus, where the function

$$\Omega(x_\xi,t)=\frac{x_\xi}{2\sqrt{2}\sin^2(\xi-\theta)\sigma_x^2(t)}\mathrm{Im}\{[e^{i\xi}(\alpha-\alpha^*e^{-2i\theta})(1-e^{-2i(\xi-\theta)})]e^{-\gamma_a t/2}+[(\alpha e^{i\xi}-\alpha^*e^{-i\xi})(1-e^{2i(\xi-\theta)})]e^{-(\gamma-\gamma_a/2)t}\} \tag{18}$$

gives the probability oscillations associated with the interference fringes and the factor $|\langle\alpha|-\alpha\rangle|^{\eta(t)}=\exp[-2|\alpha|^2\eta(t)]$ describes the suppression of quantum coherence due to dissipation. It is clear that this suppression is almost immediate for macroscopically distinguishable states (i.e., large $|\alpha|$), unless $\eta(t)\simeq 0$. It is, therefore, important to analyze the behavior of this decoherence function $\eta(t)$, which is equal to

$$\eta(t)=1-\frac{e^{-(2\gamma-\gamma_a)t}}{2\sigma_x^2(t)}. \tag{19}$$

To be more specific, if we want to see whether or not the proposed QND-mediated feedback is able to facilitate the detection of macroscopic quantum coherence, we have to compare $\eta(t)$ with the corresponding decoherence function of a standard vacuum bath, which is given by [5] $\eta_{\mathrm{vac}}(t)=1-\exp(-\gamma_a t)$. This function shows that in the standard case, after a time $t\simeq 1/\gamma_a$, it is $\eta_{\mathrm{vac}}(t)\simeq 1$ and, therefore, the quantum interference is quickly washed out. On the contrary, by using Eq. (19), it is possible to find a very important consequence of the feedback mechanism studied in this paper: it is possible to choose the feedback parameters so that $\eta(t)<\eta_{\mathrm{vac}}(t)$, thereby slowing down the destruction of the interference pattern due to the inevitable presence of damping. To be more precise, if we consider the "stable" case $\gamma>\gamma_a/2$ in which the system can reach the steady state showing squeezed quantum fluctuations [7], even though the condition $\eta(t)<\eta_{\mathrm{vac}}(t)$ can be satisfied, the presence of the lower bound $\eta(t)\geq\gamma_a t/(1+\gamma_a t)$, for any choice of the parameters compatible with the stability condition, only produces a small enhancement of the interference fringes visibility with respect to the standard vacuum bath. We have no need, however, to stay below threshold if we wish to observe the interference fringes of the superposition of the two coherent states at not too large time, i.e., times smaller than the critical time when we expect that the above description will break up and the system will begin to oscillate in a complicated way. If we disregard the above stability constraint, the function $\eta(t)$ can be reduced well below its vacuum value. This can be better shown in Fig. 1, where we have chosen $\gamma=-7\gamma_a$ with $|\alpha|=5$ and $\gamma_a t=0.09$. In the inset is shown the interference pattern which is completely washed out if, at the same time, one considers $\eta_{\mathrm{vac}}(t)$ instead of Eq. (19). The preservation of macroscopic quantum coherence in both the stable and the unstable cases is not as good as the one that can be obtained in principle from a pure *theoretical* squeezed bath, in the case of maximum squeezing $|M|=\sqrt{N(N+1)}$. In fact, as shown in Ref. [5], in this case the decoherence can be significantly slowed down because the initial slope of $\eta(t)$ can be arbitrarily decreased by increasing the bath parameter N, which is completely free in this abstract model. On the contrary, in our phys-

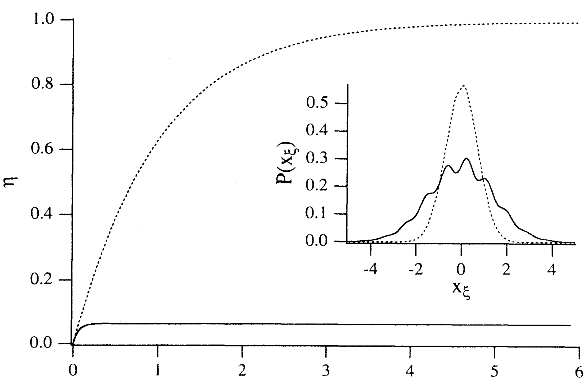

FIG. 1. The exponent η vs $\gamma_a t$ for the vacuum case (dashed line) and for the model with feedback, Eq. (19) (full line). The inset shows the probability distribution of the QND measured quadrature X_ξ at $\gamma_a t=0.09$. In the model with feedback (full line) an interference pattern still persists, while it has already disappeared in the vacuum case (dashed line). Parameter values are $\gamma=-7\gamma_a$, $|\alpha|=5$, $G^2\sin^2(\xi-\theta)/g=10^{-4}$.

ically realizable model, the bath parameters M and N are complicated functions of the feedback parameters [Eqs. (7b) and (7c)] and they cannot be varied in an arbitrary way.

IV. CONCLUSIONS

We have shown that the adoption of an appropriate feedback loop based on a QND measurement of a field quadrature simulates the presence of a squeezed bath. When the feedback scheme is used in the unstable case, it is able to significantly slow down the destruction of the interference fringes associated with macroscopic quantum coherence caused by dissipation.

In this paper, we are not concerned with the *generation* of optical Schrödinger cats. We simply assume that a linear superposition of coherent states is prepared in some way at $t=0$, and we propose to use feedback to hinder the decoherence mechanism produced by dissipation, always present in any measurement process. The relevant point of this paper is that the use of a suitable feedback scheme can appreciably increase the decoherence time of a generic optical Schrödinger cat.

At first sight, this paper seems to have a close relationship with the paper by Brune *et al.* [16], where an adaptation of a QND measurement of the number of photons stored in a high-Q cavity is proposed for the generation of Schrödinger cat states. However in our model, the QND measurement of the field quadrature X_ξ plays a quite different role. In fact, in Ref. [16], the QND measurement of the photon number is performed by detecting the dispersive phase shift produced by the field on the wave function of nonresonant atoms crossing the cavity. When a coherent state is initially present in the cavity and the atom velocities are conveniently selected, an atom detection projects the field into a linear superposition of two coherent states with different phases. Therefore, the QND measurement is used for *generating* an optical Schrödinger cat. On the contrary, the QND measurement of the field quadrature in our model is an important step in the feedback loop which, when used in the unstable case, is able to significantly increase the decoherence time of any linear superposition of coherent states and permits the observation of the interference fringes by homodyne detection. Analogous considerations can be made on the relation between our work and Refs. [17], where a back-action evading apparatus based on a parametric amplifier is proposed for the production of superposition of macroscopically distinct states. In fact, also in these papers a QND scheme is used for *generating* optical Schrödinger cats, not for slowing down the destruction of macroscopic quantum coherence due to the measurement.

For an experimental realization of the above scheme, the number of photons in the beam has to be adequately small but with modern technologies this is not a great experimental problem. What could be more difficult to realize is the QND measurement, which is essential in the above discussion; however, very recently Grangier *et al.* [18] were able to perform a QND measurement via feedback, even though the feedback mechanism in their case turned out to be completely internal to the system. This fact and the intriguing possibility of using the QND-mediated feedback seems to us appealing enough to warrant the interest of experimenters despite its intrinsic difficulty.

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