$O(\alpha^2)$ corrections to the orthopositronium decay rate

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(Received 30 November 1994)

Relativistic $O(\alpha^2)$ corrections to the orthopositronium decay rate are calculated on the basis of a local quasipotential equation. We take into account the necessary contributions resulting from the amplitude of three-photon decay, the normalization condition of the wave function, and the second-order perturbation theory.

PACS number(s): 36.10.Dr

INTRODUCTION

The process of orthopositronium annihilation, which has been investigated in quantum electrodynamics already for a long time [1], is the object of much concentrated theoretical and experimental attention. Such interest in the orthopositronium decay rate is dictated by the permanent difference between theory and experiment in the determination of the decay rate. Thus, for instance, the last experimental measurements of the orthopositronium decay rate have given the following results:

$$\Gamma^{\text{expt}}(o\text{-Ps})=\begin{cases}7.0514(14)\ \mu\text{s}^{-1}\ [2]\\ 7.0482(16)\ \mu\text{s}^{-1}\ [3]\end{cases}. \tag{1}$$

The different theoretical calculations of $\Gamma^{\text{th}}(o\text{-Ps})$ can be written in the form [4–7]

$$\Gamma^{\text{th}}(o\text{-Ps})=\Gamma_0\left[1+\tilde{A}\frac{\alpha}{\pi}+\frac{1}{3}\alpha^2\ln\alpha+\tilde{B}\left[\frac{\alpha}{\pi}\right]^2+\cdots\right]$$
$$=7.038\,31(5)\ \mu\text{s}^{-1}, \tag{2}$$
$$\Gamma_0=\frac{m\alpha^6 2(\pi^2-9)}{9\pi},$$

where the coefficient $\tilde{A}$ was calculated, $\tilde{A}_{[4]}=-10.266\pm0.011$, $\tilde{A}_{[5]}=-10.282\pm0.003$, but the coefficient $\tilde{B}$, determining the α^2 corrections, has been unknown up until the present. The discrepancy between expressions (1) and (2) is equal to 6 and 9 standard deviations. Explanation of this apparent difference between the measured and calculated values for the orthopositronium decay rate may be connected with coefficient $\tilde{B}$. A calculation of different contributions to $\tilde{B}$ was carried out recently in [8–12]. An exact determination of order $O(\alpha^8)$ contributions to the ${}^3S_1$-positronium decay rate requires employment of the consistent theory of two-particle bound states.

In this work we performed the calculation of relativistic α^2 corrections to $\Gamma(o\text{-Ps})$ on the basis of the Logunov-Tavkhelidze quasipotential method [13], in which the two-particle quasipotential equation for wave function $\psi_M(\vec{p})$ in the center-of-mass reference frame may be transformed to a local form [14],

$$\left[\frac{b^2}{2\mu_R}-\frac{\vec{p}^{\,2}}{2\mu_R}\right]\psi_M(\vec{p})=\int V(\vec{p},\vec{q},M)\psi_M(\vec{q})\frac{d\vec{q}}{(2\pi)^3}, \tag{3}$$

where $\mu_R=M/4$ is the relativistic reduced mass, the square of relative momentum of particles may be expressed through the bound state mass $M=2m+B$ (B is the binding energy) in the following manner:

$$b^2(M)=\tfrac{1}{4}M^2-m^2. \tag{4}$$

In the case of positronium the main contribution to the interaction operator $V(\vec{p},\vec{q},M)$ is defined by the modified Coulomb potential [15]

$$V^c(\vec{p},\vec{q},M)=-\frac{4\pi\alpha}{(\vec{p}-\vec{q})^2}\left[1+\frac{4b^2}{M^2}\right]. \tag{5}$$

Then the positronium ground state in momentum space is described in accordance with (3), (5) by Pauli-type wave functions [15],

$$\psi^c(\vec{q})=\frac{8\sqrt{\pi}W^{5/2}}{(\vec{p}^{\,2}+W^2)^2}\left[1-\frac{3}{8}\alpha^2\right]w_1w_2,$$
$$W=\frac{\alpha M}{4}\left[1+\frac{4b^2}{M^2}\right], \tag{6}$$

where $w_{1,2}$ are two-component Pauli spinors.

I. AMPLITUDE OF THREE-PHOTON ORTHOPOSITRONIUM DECAY

Relativistic corrections of order α^2 to the Ore-Pawell formula for Γ_0 appear, if we take into consideration the dependence of wave function and the interaction operator from the particle vector momentum of relative motion $\vec{p}$. The decay amplitude of orthopositronium can be represented as the product of an e^+e^- annihilation amplitude $M(q,P)$ times a Bethe-Salpeter wave function $\psi(q,P)$,

$$F=\int \frac{d^4q}{(2\pi)^4}\mathrm{Tr}[\psi(q,P)M(q,P)]$$
$$=(ie)^3\int \frac{d^4q_1}{(2\pi)^4}\mathrm{Tr}\left[\psi(q_1,q_2)\gamma_\lambda\frac{\hat{k}_3-\hat{q}_2+m}{(k_3-q_2)^2-m^2+i0}\right.$$
$$\left.\times\gamma_\nu\frac{\hat{q}_1-\hat{k}_1+m}{(q_1-k_1)^2-m^2+i0}\gamma_\mu\right]$$
$$\times A_\mu(k_1)A_\nu(k_2)A_\lambda(k_3)\ , \tag{7}$$

where $q_1=P/2+q=(M/2+q^0,\vec{q})$, $q_2=P/2-q=(M/2-q^0,-\mathbf{q})$ are the four momenta of electron and positron, $P=q_1+q_2$ is a total momentum, and $q=1/2(q_1-q_2)$. The decay diagram o-Ps$\rightarrow 3\gamma$, corresponding to amplitude (7), is shown on Fig. 1. It is necessary to supplement it with five diagrams with crossed photon lines k_1,k_2,k_3 for the description of total decay amplitude.

The wave function $\psi(q,P)$ obeys the two-particle Bethe-Salpeter equation, which has no physically interesting exact solutions. So for the calculation of the o-Ps decay rate it is necessary to transform F in such a way that this amplitude should contain a wave function that has a good physical interpretation and is determined by a simpler bound state equation. For changing Eq. (7) let us introduce vertex function $\Gamma(q_1,q_2)$,

$$\psi(q_1,q_2)=\frac{\hat{q}_1+m}{q_1^2-m^2+i0}\Gamma(q_1,q_2)\frac{-\hat{q}_2+m}{q_2^2-m^2+i0} \tag{8}$$

and decompose the particle propagators on positive and negative energy states. So the electron propagator can be written in the following manner:

$$\frac{\hat{q}_1+m}{q_1^2-m^2+i0}=\frac{m}{\epsilon(\vec{q})}\left[\frac{u^{(\alpha)}(\vec{q})\bar{u}^{(\alpha)}(\vec{q})}{q_1^0-\epsilon(\vec{q})+i0}+\frac{v^{(\alpha)}(-\vec{q})\bar{v}^{(\alpha)}(-\vec{q})}{q_1^0+\epsilon(\vec{q})-i0}\right]. \tag{9}$$

Considering that the particle interaction is described by local quasipotential equation (3) and a projecting vertex function on positive energy states by means of Dirac bispinors [16]

$$\Gamma^{(+)\alpha\beta}(q_1,q_2)=\frac{\bar{u}^{(\alpha)}(\vec{q})}{\sqrt{2\epsilon(\vec{q})}}\Gamma(q_1,q_2)\frac{v^{(\beta)}(\vec{q})}{\sqrt{2\epsilon(\vec{q})}}$$
$$=(M-2\sqrt{\vec{q}^{\,2}+m^2})\psi_M^{\alpha\beta}(\vec{q})\ , \tag{10}$$

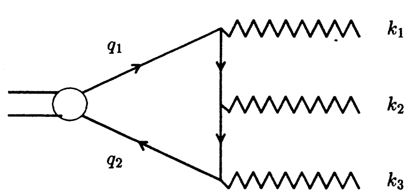

FIG. 1. Diagram for the decay amplitude of orthopositronium F.

we have obtained the following expression for amplitude F:

$$F=(ie)^3\int\frac{d^4q_1}{(2\pi)^4}\frac{2m^2}{\epsilon(\vec{q})}$$
$$\times\frac{(M-2\epsilon)\psi_M^{\alpha\beta}(\vec{q})}{(q_1^0-\epsilon(\vec{q})+i0)(-q_2^0+\epsilon(\vec{q})-i0)}\bar{v}^{(\beta)}(-\vec{q})$$
$$\times\gamma_\lambda\frac{(\hat{k}_3-\hat{q}_2+m)}{(k_3-q_2)^2-m^2+i0}\gamma_\nu\frac{(\hat{q}_1-\hat{k}_1+m)}{(q_1-k_1)^2-m^2+i0}$$
$$\times\gamma_\mu u^{(\alpha)}(\vec{q})A_\mu(k_1)A_\nu(k_2)A_\lambda(k_3)\ . \tag{11}$$

The integral function in (11) has three poles in the lower half plane of the complex variable q_1^0,

$$q_1^0=\begin{cases}\sqrt{\vec{q}^{\,2}+m^2},\\ \omega_1-\sqrt{\omega_1^2+m^2},\\ M-\omega_3-\sqrt{\omega_3^2+m^2}.\end{cases} \tag{12}$$

Taking in mind that the electron and positron are in the $^3S_1$ bound state, let us introduce a relativistic projection operator on this state, in which the exact dependence from vector momentum $\vec{q}$ must be considered [17],

$$\hat{\pi}=u(\vec{q})\bar{v}(-\vec{q})=\frac{1}{2\sqrt{2}}\frac{(\hat{q}_1+m)(1+\gamma^0)\hat{\epsilon}(-\hat{q}_2+m)}{2m(\epsilon(\vec{q})+m)}\ , \tag{13}$$

where the polarization vector of orthopositronium ϵ_μ must satisfy the condition

$$\sum_{\text{pol}}\epsilon_\mu\epsilon_\nu^*=-g_{\mu\nu}+\frac{P_\mu P_\nu}{M^2}\ . \tag{14}$$

After insertion of the $\hat{\pi}$ operator we have considered the contribution of the first pole (12) in amplitude (11), which takes the form, under these conditions,

$$F=(ie)^3\int\frac{d\vec{q}}{(2\pi)^3}\frac{2m^2}{\epsilon(\vec{q})}\psi_M(\vec{q})A_\mu(k_1)A_\nu(k_2)A_\lambda(k_3)$$
$$\times\mathrm{Tr}\left[\hat{\pi}\gamma_\lambda\frac{(\hat{k}_3-\hat{q}_2+m)}{(k_3-q_2)^2-m^2+i0}\right.$$
$$\left.\times\gamma_\nu\frac{(\hat{q}_1-k_1+m)}{(q_1-k_1)^2-m^2+i0}\gamma_\mu\right]\ , \tag{15}$$

where the substitutions $q_1^0=\epsilon(\vec{q}),q_2^0=M-\epsilon(\vec{q})$ are required in the annihilation amplitude, $\psi_M(\vec{q})$ is a quasipotential wave function of orthopositronium in momentum representation.

Further calculations of a squared modulus of amplitude $|F|^2$, performed by means of the system of analytical calculations FORM [18], have the following main features. Doing the decomposition of denominators of spinor particles propagators on $|\vec{q}|/m$ we take into account the terms proportional to $|\vec{q}|/m, |\vec{q}|^2/m^2$. If the numerator of F contained the vector momentum $\vec{q}$ in second degree then we used the substitution $\vec{q}=0$ in the F denominator and carried out the angle integration: $\int(\vec{q}\vec{k}_1)(\vec{q}\vec{k}_2)d\Omega_{\vec{q}} = 4\pi/3\vec{q}^2(\vec{k}_1\vec{k}_2)$. For parts of the F numerator proportional to $\vec{q}$ in the first degree it is necessary to decompose the denominator of (15) on $|\vec{q}|/m$ and after that to do the angle integration. The typical integral on angle variables in (15) has the following form:

$$I=\int\frac{d\Omega_{\vec{x}}}{(1-\vec{n}_1\vec{x})(1+\vec{n}_3\vec{x})},\quad \vec{n}_1=\frac{\vec{k}_1}{\omega_1},\quad \vec{n}_3=\frac{\vec{k}_3}{\omega_3},$$

$$\vec{x}=\frac{\vec{q}}{\epsilon(\vec{q})}. \tag{16}$$

It may be calculated exactly by means of Feynman parametrization. Thereafter we make the decomposition on $|\vec{q}|/m$ with necessary accuracy,

$$I=\int_0^1 dy\int\frac{d\Omega_{\vec{x}}}{[1+\vec{x}(y\vec{n}_3+\bar{y}\vec{n}_1)]^2}$$
$$=-\frac{2\pi\epsilon^2(\vec{q})}{q\sqrt{A(m^2+\vec{q}^2A)}}\ln\frac{[1+(\vec{q}^2/m^2)A]^{1/2}-x\sqrt{A}}{[1+(\vec{q}^2/m^2)A]^{1/2}+x\sqrt{A}}$$
$$\approx 4\pi\left[1+\frac{\vec{q}^2}{m^2}\left[1-\frac{2}{3}A\right]\right],$$
$$A=1+\frac{m(m-\omega_1-\omega_3)}{\omega_1\omega_3}. \tag{17}$$

The sum of photon energies $\omega_1+\omega_2+\omega_3=M$. Appearing after the trace calculation in (15) and summing up on photon polarization in $|F|^2$, the products of photon momenta are determined from the momentum conservation law, $k_1k_2=M(M/2-\omega_3),\ldots$. The momentum integration may be done, using the exact ground state wave function and the residue theory. It should be particularly emphasized that the relativistic correction of order α^2 results from the calculation of residue in the second-order pole of wave function [12]. As a result the differential three-photon decay rate was derived,

$$\frac{d\Gamma}{dx_1dx_2}=\frac{2m\alpha^6}{9\pi}\left\{\left[\frac{x_3-1}{x_1x_2}\right]^2+\alpha^2\left[\frac{f(x_2x_3)}{16x_1x_2^3x_3^3}\right]+\text{two other cyclic permutations}\right\},$$
$$f(x_2,x_3)=7x_2^4x_3^2+2x_2^4x_3+2x_2^4+26x_2^3x_3^3-12x_2^3x_3^2-24x_2^3x_3-68x_2^3+7x_2^2x_3^4-12x_2^2x_3^3+24x_2^2x_3^2-120x_2^2x_3$$
$$+118x_2^2+2x_2x_3^4-24x_2x_3^3-120x_2x_3^2+204x_2x_3-84x_2+2x_3^4-68x_3^3+118x_3^2-84x_3+32, \tag{18}$$

where $x_i=2\omega_i/M$. After analytical integration of (18) on photon frequencies by means of the "REDUCE" system [19] we have obtained the following infrared finite value of a relativistic correction to the total decay rate:

$$\Gamma=\Gamma_0\left[1+\frac{3\pi^2(7\pi^2-32)}{32(\pi^2-9)}\left[\frac{\alpha}{\pi}\right]^2\right]. \tag{19}$$

The biggest contribution of an α^2 correction (19) comes from the terms proportional to $\vec{q}^2/m^2$, which appear from the decomposition of particle energies q_1^0,q_2^0 in (15) $[66(\alpha/\pi)^2]$. Formula (19) is not the final result because there are other sources of $O(\alpha^8)$ contributions to $\Gamma(o\text{-Ps})$. First of all, as it follows from (11) and (12) that in the studied decay rate the α^2 corrections appear, if we consider two other poles on q_1^0 at an accurate calculation of amplitude F. Computing the contribution of these poles to Γ we must keep in mind that now the proportional to the α^2 term $(M-2\epsilon)$ in the numerator of (11) is not cancelled by a similar factor in the denominator as it was in the case of the first pole (12). So we have transposed $M-2\epsilon(\vec{q})\approx m\alpha^2/2$ and have ignored the dependence of amplitude F (11) from the vector momentum $\vec{q}$ in both the denominator and the numerator. After numerical integration of the corresponding differential decay rate on $\omega_1,\omega_2,\omega_3$, we have obtained that the given contribution to the total decay rate is equal to

$$\Delta\Gamma_{\text{pol}}=5.4\left[\frac{\alpha}{\pi}\right]^2\Gamma_0. \tag{20}$$

Of special note is the influence of the normalization factor of the wave function and the quantity W (6) on the quantity of the α^2 correction (when obtained (19) we have not taken into account the normalization term $[1-3/8\alpha^2]$, and have used the simplified expression for W: $W=\alpha m/2$). The order $O(\alpha^8)$ contribution caused by these factors is calculated analytically and turns out to be negative,

$$\Delta\Gamma_\psi=\Gamma_0\left[\frac{M}{2m}\left[1+\frac{4b^2}{M^2}\right]\right]^3\left[1-\frac{3}{8}\alpha^2\right]^2-\Gamma_0$$
$$\approx-\frac{15\pi^2}{8}\left[\frac{\alpha}{\pi}\right]^2\Gamma_0, \tag{21}$$

resulting in some decreasing of correction, obtained in (19) and (20).

II. SECOND-ORDER PERTURBATION THEORY

Second-order perturbation theory correction in the energy spectrum of positronium is determined in our case by the following expression [15]:

$$\Delta B = \langle \psi^c | \Delta V | \psi^c \rangle \langle \psi^c | \frac{\partial \Delta V}{\partial B} | \psi^c \rangle + \sum_{n=2}^{\infty} \frac{\langle \psi^c | \Delta V | \psi_n^c \rangle \langle \psi_n^c | \Delta V | \psi^c \rangle}{B_1^c - B_n^c} . \quad (22)$$

The part of total quasipotential ΔV, entering in (22), consists in one-photon, two-photon interactions and an annihilation term. With necessary accuracy the $\Delta V(\vec{p},\vec{q},M)$ may be expressed in the following form (see Ref. [14]):

$$\Delta V(\vec{p},\vec{q},M) = -\frac{\pi\alpha}{(\vec{p}-\vec{q})^2} \left[\frac{6(\vec{p}^{\,2}-b^2)}{m^2} - \frac{6\vec{p}(\vec{p}-\vec{q})}{m^2} + \frac{(\vec{p}-\vec{q})^2}{m^2} \right] + \frac{7\pi\alpha}{6m^2}(\vec{\sigma}_1\vec{\sigma}_2) + \frac{\alpha^2\pi^2}{m|\vec{p}-\vec{q}|} + \frac{3\pi\alpha}{2m^2} + V^{\rm an} , \quad (23)$$

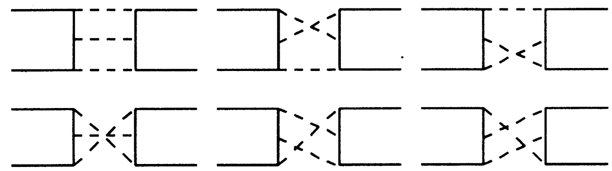

FIG. 2. Feynman diagrams of three-photon orthopositronium annihilation.

and the annihilation part $V^{\rm an}$ for orthopositronium is determined by six diagrams of three-photon annihilation (see Fig. 2). In the process of the construction of quasipotential $V^{\rm an}$ with necessary accuracy we may choose the relative particle momenta in initial and final states $\vec{p}=\vec{q}=0$. Then we obtain (see Ref. [20])

$$V^{\rm an}(o\text{-Ps}) = -\frac{8i(\pi^2-9)\alpha^3}{9m^2} . \quad (24)$$

Averaging (24) on the Coulomb wave function of the ground state gives us the main contribution to the decay rate $\Gamma = -2{\rm Im}E$ of expression (2). In order to use the formula (22) let us modify the sum over the Coulomb states by means of Schwinger's representation for Coulomb Green's function (see Ref. [21]),

$$\sum_{n=2}^{\infty} \frac{\psi_n^{c*}\psi_n^c}{B_1^c - B_n^c} = -\frac{64\pi}{\alpha W^4} \left[R(\vec{p},\vec{q}) + \frac{\pi^2 W^5 \delta(\vec{p}-\vec{q})}{4(\vec{p}^{\,2}+W^2)} + \frac{W^6}{4(\vec{p}^{\,2}+W^2)(\vec{p}-\vec{q})^2(\vec{q}^{\,2}+W^2)} \right] . \quad (25)$$

The quantity $R(\vec{p},\vec{q})$ describes the particle interaction through the exchange of two, three, and so on Coulomb photons and it has the following form:

$$R(\vec{p},\vec{q}) = \frac{W^8}{(\vec{p}^{\,2}+W^2)^2(\vec{q}^{\,2}+W^2)^2} \left[\frac{5}{2} - \frac{4W^2}{(\vec{p}^{\,2}+W^2)} - \frac{4W^2}{(q^2+W^2)} + \frac{1}{2}\ln A + \frac{2A-1}{\sqrt{4A-1}} \arctan\sqrt{4A-1} \right] ,$$

$$A = \frac{(\vec{p}^{\,2}+W^2)(\vec{q}^{\,2}+W^2)}{4W^2(\vec{p}-\vec{q})^2} . \quad (26)$$

The calculation of integrals with function $R(\vec{p},\vec{q})$ appearing in (22) may be done on the basis of the Caswell-Lepage identity (see Ref. [22]),

$$\frac{1}{\pi^4}\int \frac{d\vec{p}d\vec{q}}{W^6} R(\vec{p},\vec{q}) f(p) = -\frac{4}{\pi}\int_0^{\infty} \frac{Wp^2dp}{(p^2+W^2)^7} f(p) \left[\ln 2 - \frac{5}{2} + \frac{W}{p}\arctan\frac{p}{W} - \frac{1}{2}\ln\left[1+\frac{p^2}{W^2}\right] + \frac{4W^2}{p^2+W^2} \right] . \quad (27)$$

In the table below we present the results of such calculations of basic integrals by means of (27), which are determined by quantity $R(\vec{p},\vec{q})$ and give the contribution to the decay rate of orthopositronium.

$f(p)$	1	$\frac{W^2}{W^2+p^2}$	$\frac{W}{p}\arctan\frac{p}{W}$
$\frac{1}{\pi^4}\int \frac{d\vec{p}d\vec{q}}{W^6} R(\vec{p},\vec{q}) f(p)$	$\frac{3}{2}$	$-\frac{1}{8}$	$1-\frac{\pi^2}{12}$

The quasipotential (23) has a clear dependence from the energy of the bound state. So the first term of formula (22) also gives the contribution to the orthopositronium decay rate. It has the following form:

$$\langle \psi^c|\Delta V|\psi^c\rangle \langle \psi^c|\frac{\partial \Delta V}{\partial B}|\psi^c\rangle = \int \frac{d\vec{p}}{(2\pi)^3}\psi^c(\vec{p})V^{\mathrm{an}}(o\text{-Ps})\psi^c(\vec{q})\frac{d\vec{q}}{(2\pi)^3}\int \frac{d\vec{k}}{(2\pi)^3}\psi^c(\vec{k})\frac{\partial}{\partial B}\left[\frac{6\pi\alpha b^2}{m^2(\vec{k}-\vec{r})^2}\right]\psi^c(\vec{r})\frac{d\vec{r}}{(2\pi)^3}$$
$$= -\frac{i}{2}\Gamma_0\frac{3}{4}\alpha^2 , \tag{28}$$

Summing up all terms in expression (22), we have obtained the following value of the α^2 correction to the $\Gamma(o\text{-Ps})$ at second-order perturbation theory:

$$\Delta\Gamma = \frac{m\alpha^8(\pi^2-9)}{18\pi}\left[\frac{19}{2}-\frac{\pi^2}{3}\right] . \tag{29}$$

III. DISCUSSION OF THE RESULTS

We have considered above the contributions of the kind $O(\alpha^8)$ in the decay rate of positronium triplet state $^3S_1$. These contributions were obtained from the basic amplitude of three-photon decay (19) and (20), the renormalization of wave function, describing the positronium ground state (21), and the corrections of second-order perturbation theory (29). Summing the results (19)–(21), and (29) we get the total contribution in the following form:

$$\Gamma_{\mathrm{tot}} = \Gamma_0\left[1+41.9\left[\frac{\alpha}{\pi}\right]^2\right] , \tag{30}$$

where the factor π in the denominator of the second term was separated by an analogy with the radiative corrections of the same order on α.

As pointed out above in the last few years there were some publications (see Refs. [8–12]), devoted to the calculation of α^2 corrections to the orthopositronium decay rate by means of different approaches. Our calculational scheme in Sec. I is close to that of Ref. [12]. This is especially true with regard to the momentum space integration of expression (15), which is not reduced to the substitution $\vec{q}^{\,2}\to b^2$ [as might be expected from Eq. (3)], but to the replacement $\vec{q}^{\,2}=-3/4\alpha^2m^2$. Such a substitution takes into account the contribution of the wave function (6) pole. Our numerical result (19) for the α^2 correction due to the main three-photon orthopositronium decay amplitude agrees well with expression $\alpha^2(27\pi^2-204)/16(\pi^2-9)\approx 4,5\alpha^2$, found in Ref. [12]. The relativistic α^2 correction (21) coming from the normalization condition of the wave function must be considered together with a similar correction of second-order perturbation theory (29) because of specific dependence of quasipotential (23) from the bound state mass M. In the sum of terms (21) and (29) we have obtained the contribution, which is close in magnitude to the relativistic correction $(-0,6\alpha^2)$ determined in Ref. [12] due to the modification of the wave function. Let us also point out that the coefficient $\tilde{B}=41.9$, derived in this article, is in good agreement with the result $\tilde{B}=46\pm 3$ (see Ref. [12]).

In this work we did not consider the α corrections in amplitude F and one loop corrections of the same order found in Ref. [10]. Taking into consideration the result of Ref. [10], the value of coefficient B of formula (2) shall reach the value $\tilde{B}=71$, which decreases slightly the discrepancy between the theory (2) and experiment (1). It is necessary to emphasize that the calculation of the contributions of two-loop corrections to the annihilation amplitude (11) is required in order to obtain the full value of the $O(\alpha^2)$ correction to the orthopositronium decay rate. It is possible that the interference of the zeroth-order amplitude with the second-order radiative correction to this amplitude may give an essential contribution to the coefficient $\tilde{B}$.

ACKNOWLEDGMENTS

We are extremely grateful to P. Labelle and G. P. Lepage for valuable information about the results obtained by them, and to V. V. Dvoeglazov and I. B. Khriplovich for stimulating discussions of the positronium problem. This work was carried out under the financial support of the Russian Fund of Fundamental Researches (Grant No. 93-02-3545) and the State Committee on High Education of the Russian Federation (Grant No. 94-6.7-2015).

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