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ARTICLES

"Weak measurements" and the "quantum time-translation machine" in a classical system

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A simple optical experiment is discussed that implements the "quantum time-translation machine" suggested by Aharonov *et al.* [Phys. Rev. Lett. **64**, 2965 (1990)] for photons. The time-translation effect was observed experimentally as the shift of the fringe pattern in a modified Mach-Zehnder interferometer. The possibility of describing the effect classically, in terms of Maxwell's equations, suggests an alternative interpretation that does not invoke time-translation effects.

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I. INTRODUCTION

In the standard interpretation of quantum mechanics, an "ideal measurement" of an observable A on a quantum mechanical system Ψ brings the system into an eigenstate of A whose eigenvalue is the result of the measurement. This kind of measurement is ideal in the sense that it allows one to make the uncertainty arbitrarily small—at least in principle. Real measurements of course never reach this extreme limit and leave the system in a state that is not exactly equal to the eigenstate. The perturbation of the system can be kept small if the interaction with the measurement apparatus is weak enough, at the price of higher uncertainty in the measured value.

Aharonov and co-workers [1,2] call this type of measurement "weak measurement." They use it to discuss the properties of a system between two standard measurements. Their concept involves three distinct steps: the preparation of an initial state Ψ_1, a low-resolution measurement of a variable A, and a postselection measurement, which projects the states of the system onto a state Ψ_2 after the measurement. The result of such a measurement depends strongly on the scalar product $\langle \Psi_1 | \Psi_2 \rangle$ of the two states: if the states Ψ_1 and Ψ_2 are identical, the second measurement does not affect the system, and if the two states are orthogonal, the probability for a success in the second measurement is zero. In the intermediate case, however, some rather nonintuitive effects may arise. In these cases, the second measurement selects part of the total system for observation. For this part, which passes the second measurement, the expectation value for the weak measurement is $\langle A \rangle_{\text{weak}} = \langle \Psi_1 | A | \Psi_2 \rangle / \langle \Psi_1 | \Psi_2 \rangle$. The surprising property of such a measurement is that the expectation value can lie far outside the eigenvalue spectrum of A if the states Ψ_1 and Ψ_2 are almost orthogonal. One example is that the "the result of a measurement of a component of the spin of a spin-$\frac{1}{2}$ particle can turn out to be 100" [1,3].

This holds also for the case that the observable A appears in the Hamiltonian that controls the evolution of another system. A weak measurement may then find an energy that lies outside the eigenvalue spectrum of the Hamiltonian. An experimenter who prepares the system in an initial state Ψ_2 and measures the state to which it has evolved after a time τ might conclude that the system is farther from the start than it could have arrived during the measurement time if it had evolved under any of the eigenvalues of $\mathcal{H}$. Aharonov *et al.* describe a procedure that brings a quantum mechanical system into such a state by letting it evolve under a Hamiltonian that depends on an external quantum variable and subsequently performing a measurement on this external system [4,5]. They interpret this effect as a "time-translation machine."

Duck, Stevenson, and Sudarshan [3] discussed the concept of weak measurements and conclude that the prediction of Aharonov *et al.* was essentially correct and "involves nothing outside ordinary quantum mechanics." This analysis as well as the name "quantum time-translation machine" may leave the impression that this effect is a peculiarity of quantum mechanics [6]. It is the purpose of this paper to counter this view by discussing a simple optical setup that was implemented experimentally and shows the predicted effect. The theoretical analysis shows that it can be described not only quantum mechanically, but also within the classical formalism of Maxwell's equations. This allows us to relate the effect to more familiar effects in classical physics.

The paper is structured as follows: the following section summarizes the quantum mechanical description of the general situation. Section III discusses the specific experimental situation, which consists of a modified Mach-Zehnder interferometer. Section IV analyzes the situation in terms of Maxwell's equations and we conclude with a summary of the results.

II. QUANTUM MECHANICAL DESCRIPTION

Before discussing the specific example, we establish the notation by describing the general concept of Ref. [4]. It superimposes two states to a final state, which is, in a sense, more remote from the original state than any state it would reach within the same time without this superposition. Figure 1 illustrates the idea: we consider a system Ψ_s that undergoes two distinct evolutions described by the unitary propagators U_α, U_β, which take the initial state $\Psi_s(0)$ into the intermediate states $\Psi_\alpha(t), \Psi_\beta(t)$. The Hamiltonian that drives the two unitary evolutions depends on a quantum mechanical variable A, which does not depend on the degrees of freedom of the system. The two evolutions U_α, U_β correspond to eigenvalues α and β of an observable A. The two states Ψ_α, Ψ_β are initially identical, $\Psi_\alpha(0)=\Psi_\beta(0)=\Psi_s(0)$, but become distinct by the different evolutions

$$\Psi_\alpha(t)=U_\alpha \Psi_s(0), \quad \Psi_\beta(t)=U_\beta \Psi_s(0) \; . \tag{1}$$

The next step in the time-translation concept is the formation of a superposition of these two states by an ideal measurement of an operator A', which does not commute with A. The measurement brings the system into an eigenstate $\xi'_{\alpha'}(A')$ of the operator A', which is a linear combination of the eigenstates $\xi_\alpha(A)$ and $\xi_\beta(A)$ of the operator A,

$$\xi'_{\alpha'}(A')=c_\alpha \xi_\alpha(A)+c_\beta \xi_\beta(A) \; . \tag{2}$$

If the system before this measurement is $\Psi(t)=U_\alpha \Psi_s(0)+U_\beta \Psi_s(0)$, it becomes

$$\Psi(t+)=c_\alpha \Psi_\alpha(t)+c_\beta \Psi_\beta(t)=c_\alpha U_\alpha \Psi_S(0)+c_\beta U_\beta \Psi_s(0) \; . \tag{3}$$

Obviously, the resulting state can also be written as $\Psi(t)=U\Psi_s(0)$, where

$$U=c_\alpha U_\alpha + c_\beta U_\beta \tag{4}$$

is a "superposition of time evolutions" [4]. The not so intuitive properties of this superposition of time evolutions include the possibility that the effective time evolution U can differ significantly from the evolution of the individual systems described by U_α, U_β. In particular, it can lead to states that the system would otherwise reach only after a much longer evolution period. The authors of Refs. [4,5] compared this effect therefore to a time-translation machine.

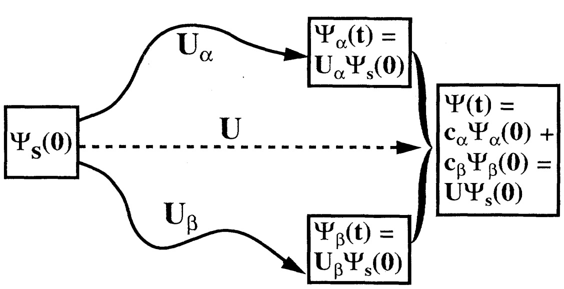

FIG. 1. Schematic representation of the superposition of time evolutions: the initial state $\Psi(0)$ evolves along two separate time evolutions U_α, U_β into the intermediate states $\Psi_\alpha(t), \Psi_\beta(t)$. The superposition of these two states, $\Psi(t)$ can be considered as having originated from the original state through the effective evolution U.

III. EXPERIMENTAL REALIZATION

We hope to clarify this seemingly counterintuitive phenomenon by discussing a simple experiment that realizes this general scheme. The "system" consists of a single mode of the radiation field and the "external" variable corresponds to the photon spin, i.e., to the polarization of the light. This is the simplest possible realization of the general concept: the system, i.e., the single mode radiation field, is a scalar, while the external variable has only two eigenstates, the smallest possible number.

We write the initial state of the radiation field as $\Psi_s(0)$ and the two polarization states as $|\alpha\rangle, |\beta\rangle$. If the light is initially polarized 45° with respect to the polarization α, the total initial wave function is the product state $\Psi(0)=(1/\sqrt{2})[\Psi_\alpha(0)+\Psi_\beta(0)]$, where $\Psi_\alpha(0)=\Psi_s(0)|\alpha\rangle$. The two substates evolve as

$$\Psi_\alpha(t)=U_\alpha \Psi_\alpha(0), \quad \Psi_\beta(t)=U_\beta \Psi_\beta(0) \; , \tag{5}$$

where the evolution operators are complex numbers with modulus one. The choice of a one-dimensional system ascertains that the propagators are always well defined quantities with a single parameter. This parameter—the phase—plays the role of the time.

The final superposition of the two states is obtained by projecting the external state onto a state that is a superposition of the two eigenstates $|\alpha\rangle, |\beta\rangle$, with coefficients that are different from those of the initial condition. The most interesting situation arises if we project the external system onto a polarization state $|\alpha'\rangle=\cos(\theta)|\alpha\rangle+\sin(\theta)|\beta\rangle$ which represents a linear polarization rotated by an angle θ from the direction α. We then obtain

$$\Psi_s(t;\theta)=(1/\sqrt{2})[\cos(\theta)U_\alpha+\sin(\theta)U_\beta]\Psi_s(0) \; . \tag{6}$$

In our experimental implementation, a birefringent element in the beam path distinguishes the evolution of the two polarization states. At location r behind the birefringent element and time t, the field has evolved into a state described by the propagators

$$U_\alpha=e^{i\omega t-\mathbf{k}\cdot\mathbf{r})}, \quad U_\beta=e^{i(\omega t-\mathbf{k}\cdot\mathbf{r}-k\Delta n l}=U_\alpha e^{i\delta} \; , \tag{7}$$

where $\delta=-k\Delta n l$ is the additional phase acquired by polarization β, $\mathbf{k}$ is the wave vector, Δn is the refractive index difference, and l is the length of the birefringent element. The projection onto a state of the external system is accomplished by a polarizer that selects a linear polarization state determined by its orientation θ from the direction $|\alpha\rangle$. The effective propagator for the projected state is then

$$\begin{aligned}\sqrt{2}U&=U_\alpha[\cos\theta+\sin(\theta)e^{i\delta}]\\ &=U_\alpha[\cos\theta+\sin(\theta)(\cos\delta+i\sin\delta)]=U_\alpha a e^{i\delta} \; ,\end{aligned} \tag{8}$$

with the overall phase

$$\phi=\tan^{-1}\left[\frac{\sin\theta\sin\delta}{\cos\theta+\sin\theta\cos\delta}\right] . \tag{9}$$

Standard optical techniques can measure this phase. As an example, Fig. 2 shows a possibility that uses a Mach-Zehnder interferometer. This setup eliminates the time-dependent term $e^{i\omega t}$, and the reference phase can be adjusted to eliminate also the propagation effect described by the term $e^{i\mathbf{k}\cdot\mathbf{r}}$ contained in U_α. If the polarizer is oriented in the direction of $|\alpha\rangle$ ($|\beta\rangle$), corresponding to $\theta=0$ ($\pi/2$), the phase ϕ of the meter beam behind the analyzer becomes 0 (δ), as expected. In the range $0<\theta<\pi/2$, the overall phase falls within the range [0, δ] spanned by the phases acquired by the two basis states Ψ_α, Ψ_β, but it falls outside the range for $\pi/2<\theta<\pi$. This is therefore the time-translation regime.

In the experimental implementation, a HeNe laser beam propagates through a Mach-Zehnder interferometer. In one of the two arms, an adjustable retardation plate (Soleil-Babinet compensator) provides the differential evolution of the two polarization states of the meter beam. The axis of the compensator, which defines the polarization direction α, was oriented at $\pi/4$ from the polarization of the incident laser beam. The two polarization states Ψ_α, Ψ_β therefore have equal weights. The meter and reference beams were recombined on the output beam splitter of the interferometer. A rotatable polarization analyzer in the path of the combined laser beams allowed the selection of the final polarization state. The two beams were slightly misaligned, such that a fringe pattern appeared on a detection screen. A phase shift of the meter beam could then be measured as a shift of the position of this fringe pattern. The resulting data were recorded with a photodiode array, digitized, and transferred to a computer, where the phase was extracted by a least squares fitting procedure.

Figure 3 shows a quantitative comparison of the theoretical and experimental phases as a function of the analyzer orientation. The line represents the theoretically expected behavior for a relative phase of $\delta=0.3$, while the circles represent the experimentally observed values. The agreement between theoretical and experimental values lies within the experimental uncertainty. The experimental data were corrected for variations in the optical path length associated with the rotation of the polarizer by subtracting the corresponding data from a run with $\delta=0$. The shaded area at the bottom of the figure marks the range of phases [0, δ] that fall within the range spanned by the two basis states Ψ_α, Ψ_β. The remaining part outside this range includes the majority of the points. In the terminology of Aharonov *et al.*, it demonstrates the time-translation effect.

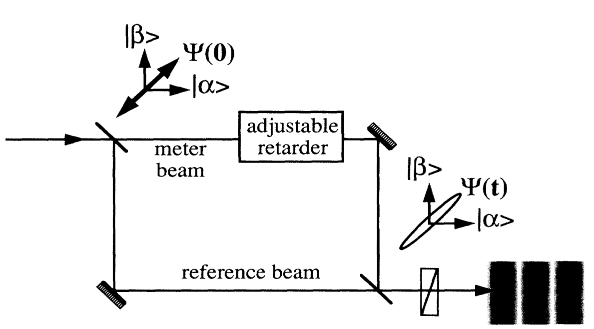

FIG. 2. Schematic representation of the experimental setup.

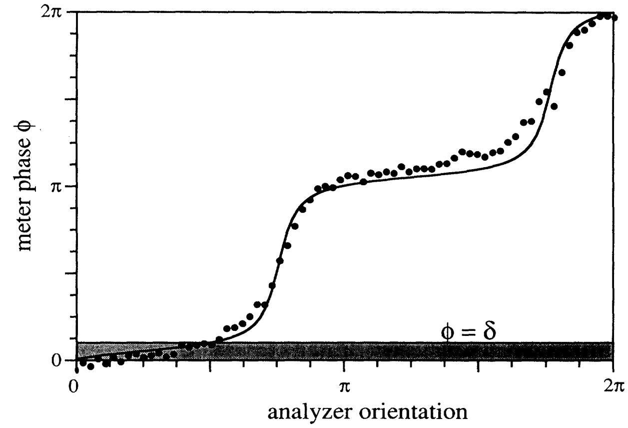

FIG. 3. Comparison of experimental and theoretical phases for $\delta=0.3$ as a function of analyzer orientation. The shaded region at the bottom extends from $\phi=0$ to $\phi=\delta$.

IV. CLASSICAL INTERPRETATION

Obviously, the experiment described here does not require a quantum mechanical analysis—Maxwell's equations predict the same result. Classical optics describes the light after the retardation plate as elliptically polarized: the electric field vector at a given point rotates while its amplitude changes. To assign a unique phase value to this field, a reduction to a one-dimensional system is necessary. The question of how the phase of an elliptically polarized field can be defined was discussed by Pancharatnam [7], who chose the direction of the major axis of the polarization ellipsis to define a linearly polarized component whose phase defines the phase of the elliptically polarized wave.

This reduction to a single dimension discards the information about the component in the orthogonal direction. A complete description, which is adapted to the present experiment, considers the phase of the field vector in an arbitrary direction

$$\phi(\theta)=\tan^{-1}\left[\frac{\sin\theta\sin\delta}{\cos\theta+\sin\theta\cos\delta}\right], \tag{10}$$

which is of course identical to Eq. (9) derived by the quantum mechanical analysis. Similarly, the field amplitude depends on the orientation θ as

$$a(\theta)=\sqrt{1+\sin(2\theta)\cos\delta}\ . \tag{11}$$

Both the phase and the amplitude of the linearly polarized components are thus functions of the orientation and can assume a range of values that varies from 0 to 2π for the phase and from $1-\cos\delta$ to $1+\cos\delta$ for the amplitude. Taking all orientations into account, the probability to measure a particular phase ϕ is

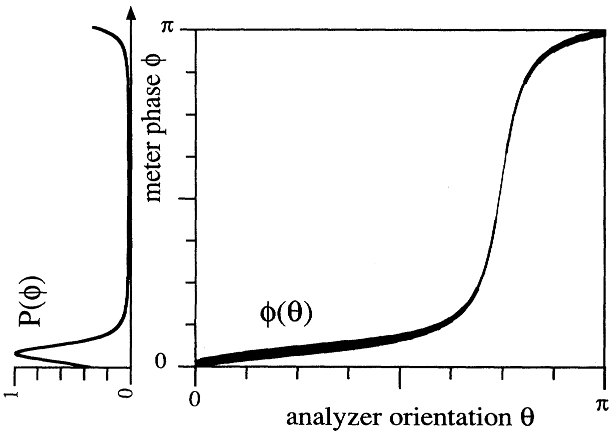

FIG. 4. Phase of a linearly polarized wave that results from projection of the elliptically polarized wave on a given direction as a function of orientation. The thickness of the line indicates the field amplitude for the corresponding orientation. The function plotted along the vertical axis indicates the weighted probability $P(\phi)$ for finding a phase ϕ.

$$P(\phi)=\int_0^{2\pi} d\theta\, a(\theta) \left[\frac{d\phi(\theta)}{d\theta} \right]^{-1} . \qquad (12)$$

This function may be interpreted as a probability distribution for the phase of an elliptically polarized wave, averaged over all orientations and weighted with the field amplitude.

Figure 4 summarizes this dependence: It shows the phase $\phi(\theta)$ of the linearly polarized component as a function of the orientation from zero to π. The width of the line is proportional to the amplitude $a(\theta)$ in this direction. The region where the line is wide corresponds to the orientation along the major axis of the polarization ellipsis. Here, the variation of the phase with orientation is slow. The region where the amplitude is small, which corresponds to the minor axis of the ellipsis, shows a fast variation of the optical phase.

The phase distribution $P(\theta)$ is plotted along the vertical axis. It shows that the phase of the elliptically polarized wave is not a unique number, which would correspond to a δ function, but a distribution whose width depends on the ellipticity. For the case of circularly polarized light, this function becomes a constant, no particular phase is preferred. Pancharatnam's definition corresponds to the maximum of this function, which is also its average. It represents thus a natural choice for a reduction of the distribution to a single number.

V. DISCUSSION

The setup described here realizes the time-translation effect of Refs. [4,5] but can be described classically, using Maxwell's equations. We conclude therefore that the effect is not specific to quantum mechanics, but occurs also in classical field theories. It results directly from the linearity of the theory (the Schrödinger equation in one case, the Maxwell equations in the other). The superposition of two states $\Psi_{\alpha,\beta}$ (in the quantum mechanical terminology) or waves (in the classical case) with different phase and polarization leads to a state Ψ whose phase is not a fixed quantity, but a distribution that depends on the direction of the field vector and on the phase difference δ. For identical phases, $\delta=0$, the combined wave becomes linearly polarized and its phase is well defined, independent of the analyzer orientation. A nonvanishing phase difference δ establishes a correlation between orientation θ and optical phase $\phi(\theta)$, which becomes complete when $\delta=\pi/2$. The dependence between the two variables means that a measurement of the optical phase in one direction is a biased measurement that selects a specific value out of this distribution. That such selective measurements yield results that depend on the selection taken and can assume any value within the total distribution is well known from classical physics and not a specifically quantum mechanical effect.

This interpretation of the effect as a biased measurement can easily be transferred to the quantum mechanical description: the combined system is initially in a product state of the system and external variable. The evolution under the coupling Hamiltonian establishes a correlation between them—the resulting state is no longer separable, system and external variable become Einstein-Podolsky-Rosen (EPR) correlated. A measurement performed on the external variable preselects then the value of the system observable.

A more detailed quantum mechanical analysis depends on the interpretation of the quantum mechanical formalism. In a purely statistical interpretation of quantum mechanics [8], there is no significant difference from the classical viewpoint, but an interpretation at the level of individual particles leads to rather different conclusions. The Copenhagen interpretation, e.g., implies that in those rare cases where the measurement of the variable A' yields the value α', the whole system makes a transition into a state with advanced or retarded phase. In a causal interpretation, like that of Bohm [9–11], we would conclude that the measurement on the external variable A' modifies the quantum potential in such a way that only those particles whose initial conditions are consistent with the advanced or retarded arrival at the detector can reach the detector at all. How close the analogy between the classical and the quantum mechanical description is depends therefore to some degree on the interpretation of quantum mechanics that one prefers.

In conclusion, we have demonstrated an optical implementation of the quantum time-translation machine by superposition of time evolutions suggested by Aharonov *et al.* and Vaidman [4,5]. This implementation shows all the features that the general concept predicts and also allows, besides the quantum mechanical, a classical description. We conclude that the observed effects are not specific to quantum mechanics but are well known in classical field theories. The time-translation effect can be reinterpreted as a selective measurement: the coupled evolution of system and external variable establishes correlations between them that make it possible to select a specific value of the system variable by an appropriate setting of the external variable. This interpretation is applicable in the classical as well as in the quantum mechanical regime.

ACKNOWLEDGMENTS

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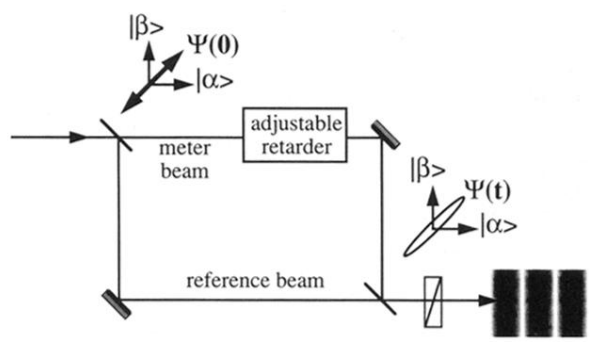

FIG. 2. Schematic representation of the experimental setup.

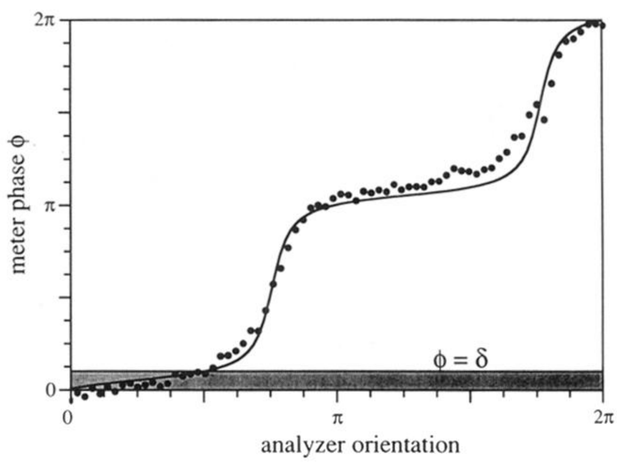

FIG. 3. Comparison of experimental and theoretical phases for $\delta=0.3$ as a function of analyzer orientation. The shaded region at the bottom extends from $\phi=0$ to $\phi=\delta$.