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Quantum jumps as an objective process of nature

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We study the time evolution of a linear superposition of two spatially separated wave packets, and we focus on the entanglement of the two distinct branches of the state vector with the environment. We focus in particular on the dynamics of a dissipative oscillator under the influence of objective processes of wave-function collapse, the continuous spontaneous localizations (CSL) recently proposed by Ghirardi *et al.* [G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A **42**, 78 (1990)]. We prove that the entanglement of the system of interest with the environment induces an accumulation of spontaneous wave-function collapses denoted by us as the environment-enhanced CSL process. This process of CSL accumulation is triggered by the same mechanism of interaction between the quantum system and the environment as that responsible for relaxation and dissipation. In agreement with the predictions of a preceding paper of our group [D. Vitali, L. Tessieri, and P. Grigolini, Phys. Rev. A **50**, 967 (1994)], the CSL processes are shown to produce negligible effects at the statistical level. However, if we assume the attitude stimulated by the recent literature on optical quantum jumps, which is forcing us to adopt individual-system pictures, we show that the single runs are characterized by processes of wave-function collapses occurring at times compatible in principle with the experimental observation.

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I. INTRODUCTION

One of the most controversial aspects of quantum mechanics stems from the linear nature of the Schrödinger equation. This means that if $|\psi_1(t)\rangle$ and $|\psi_2(t)\rangle$ are distinct solutions of the Schrödinger equation, the wave function $|\psi(t)\rangle=|\psi_1(t)\rangle+|\psi_2(t)\rangle$ is also a solution of it with an acceptable physical meaning. In the special case where $|\psi_1(t)\rangle$ and $|\psi_2(t)\rangle$ are states of a macroscopic system corresponding to two distinct trajectories, this property leads to a striking conflict with classical mechanics and with our direct perception of dynamical processes, thereby making it difficult to recover classical from quantum mechanics. The problem of the proper interpretation of quantum mechanics is still the subject of research and passionate debate [1–4] and we are not in a position to give here a fair and complete illustration of the wide gamut of interpretations proposed. We limit ourselves [5] to roughly dividing the researchers of this field of investigation into two major groups. The authors of the former group [6–10] rest on the belief that it is possible to settle all the paradoxical aspects of quantum mechanics without changing it, but only considering the important fact that no isolated system exists in nature and that the interaction between the system of interest and its environment always has to be properly considered. For the sake of simplicity, we choose Zurek [6,7] as the representative of this former group.

The authors of the latter group [1,4,11–16] maintain that the problems raised by the superposition of distinct macroscopic positions can only be solved if quantum mechanics is adequately changed in such a way as to derive a unified representation of both macroscopic and microscopic processes [11]. This extension of quantum mechanics must be made in such a way as to leave the dynamics of microscopic systems essentially coincident with the predictions of ordinary quantum mechanics. For macroscopic systems, on the other hand, the deviations from ordinary quantum mechanics must become significant for the new dynamical rules to prevent the superposition of macroscopically distinct states with no need to use the influence of the environment to account for the destruction of quantum-mechanical coherence. We essentially refer to the theoretical proposal made by Ghirardi, Rimini, and Weber (GRW) [11]. According to these authors, the wave-function collapses already take place at the level of microscopic systems without resulting, though, in significant dynamical corrections to the prescriptions of ordinary quantum mechanics. This is so for the following reason. Let us image that $|\psi(t)\rangle=|\psi_1(t)\rangle+|\psi_2(t)\rangle$ refers to a microscopic body, e.g., a proton, and that the distance between the two distinct positions is ΔQ. The process of direct collapse affecting an individual microscopic constituent is characterized by the two key parameters λ and $1/\sqrt{\alpha}$. The first is the rate of occurrence of the process of wave-function collapse, and its value is set [11] at $\lambda=10^{-16}\ \text{sec}^{-1}$. The value of the second parameter is assumed [11] to be $1/\sqrt{\alpha}=10^{-5}$ cm and represents the distance beyond which two distinct spatial components of the same state must be found for a collapse to be effective. In other words, if $\Delta Q \ll 1/\sqrt{\alpha}$, the collapse into one of the two distinct positions never occurs and if $\Delta Q\approx 1/\sqrt{\alpha}$, or larger, it occurs in 10^8 years. It is evident that with these new dynamical rules, the dynamics of a microscopic body fits almost exactly the predictions of ordinary quantum mechanics.

Let us consider now a macroscopic system of N particles. In the GRW scheme, the evolution of the statistical operator is dictated by the Liouville–von Neumann equation, corrected by the occurrence of the processes of spontaneous localizations [11]:

$$\frac{\partial\hat{\rho}}{\partial t}=-\frac{i}{\hbar}[\hat{H},\hat{\rho}]+\sum_{i=0}^{N}\lambda_i(T_i[\hat{\rho}]-\hat{\rho}) , \tag{1.1}$$

where

$$T_i[\hat{\rho}]=\left[\frac{\alpha}{\pi}\right]^{3/2}\int d^3x \exp\left[-\frac{\alpha}{2}(\hat{\mathbf{q}}_i-\mathbf{x})^2\right]\hat{\rho}\times\exp\left[-\frac{\alpha}{2}(\hat{\mathbf{q}}_i-\mathbf{x})^2\right] .$$

Typically, the center of mass Q and the internal degrees of freedom $\{r\}$ are decoupled, since one has $H=H_Q+H_r$. This decoupling is present also in the GRW version of quantum mechanics. In fact, it is easy to prove that the density matrix for the center of mass $\rho_Q=\mathrm{Tr}_r\{\rho\}$ obeys the evolution equation

$$\frac{\partial\hat{\rho}_Q}{\partial t}=-\frac{i}{\hbar}[\hat{H}_Q,\hat{\rho}_Q]+\left[\sum_{i=0}^{N}\lambda_i\right](T_Q[\hat{\rho}_Q]-\hat{\rho}_Q) , \tag{1.2}$$

that is, the center of mass of a macroscopic system behaves as a single particle, which is now subject to a strong localization process, with an enhanced rate $\lambda=\sum_{i=1}^{N}\lambda_i$. This implies that for a body of 1 g, a linear superposition of two wave packets at a distance larger than 10^{-5} cm is reduced into one of its components in a time of the order of 10^{-7} sec. This means that the quantum-mechanical coherence survives for times much shorter than the time required for us to perceive the motion of macroscopic bodies, and that this motion is virtually indistinguishable from that predicted by ordinary classical mechanics. In the formalism of the statistical operator, this collapse accumulation manifests itself in the quick diagonlization of the density matrix in the coordinate representation. However, the processes of spontaneous localization correspond to a real collapse of the wave function and an equivalent individual-system picture is now available [12,13], describing the evolution of the state vector in terms of a stochastic and nonlinear Schrödinger equation. In spite of the fact that throughout this paper we shall be adopting the statistical formalism of (1.1), we shall keep in mind this individual-system picture and, adopting the terminology of Ref. [13], we shall refer to these processes as processes of continuous spontaneous localization (CSL).

The center of gravity is an illuminating example of macroscopic variable subject to a significant process of accumulation of the individual spontaneous localizations. However, in the field of condensed matter we cannot ignore the fluctuation-dissipation processes as a possible source of accumulation of the CSL processes. This is so because in condensed matter a microscopic constituent interacts with infinitely many other constituents, and since each of them undergoes an individual CSL process, we might reasonably argue that this interaction process is responsible for an accumulation of collapses as well as for the birth of fluctuation-dissipation properties. In an earlier paper [17], we have assessed that an ordinary process of quantum-mechanical fluctuation-dissipation is left essentially unchanged by the occurrence of collapses affecting the bath particles. However, this result, albeit incontrovertible, was based on a statistical picture and for this reason left essentially unsolved the question of whether or not an individual-system representation of the system of interest would lead to jumps occurring at times compatible with experimental observation. This question is of some interest if we make the optimistic assumption that the technology of the individual-system observation [18] might become so advanced as to go beyond the ambiguities of the so-called Zeno effect [19,20].

We think that the conclusion of the previous paper [17] confirms that the GRW physics is statistically equivalent to ordinary quantum mechanics. However, we want to prove here that this statistical equivalence does not conflict with a significant enhancement of the processes of spontaneous collapses and that, in addition to that which was pointed out by the authors of the GRW and CSL theories, concerning the center of gravity of a rigid body, an additional process of enhancement of significant interest for quantum statistical mechanics might exist. This enhancement process is triggered by the same physical mechanism as that behind the environment-induced decoherence phenomena. To give an intuitive illustration of it, let us illustrate first the process of environment-induced decoherence. As pointed out earlier, the environment-induced decoherence is, according to Zurek [6,7], the key ingredient to use to explain why in classical mechanics the superpositions of distinct spatial components are forbidden. We shall refer to this point of view as decoherence theory (DT) [21]. To make the basic aspects of the DT clear, let us study the dynamics of a dissipative quantum oscillator, initially placed in the following state:

$$|\psi_s(0)\rangle=\frac{1}{\sqrt{2}}(|\varphi\rangle+|-\varphi\rangle) , \tag{1.3}$$

where $|\varphi\rangle$ is a coherent state shifted with respect to the equilibrium position by the quantity $\Delta Q/2$ in the positive direction and $|-\varphi\rangle$ the coherent state shifted by the same quantity in the opposite direction. The representation in terms of coherent states is supposed to be the most convenient to recover, in the proper limit, classical physics from quantum physics [22]. The larger we make the quantum numbers, the closer quantum mechanics should be to the predictions of classical physics. However, the larger $|\varphi|$ is, the wider the separation ΔQ between the spatial positions of these two states, thereby generating a paradoxical condition, referred to in literature as the Schrödinger cat [3,23]. The DT provides the recipe to settle this paradoxical problem by adopting the following two key ingredients.

(i) Influence of the environment. A macroscopic system cannot be regarded as being fully isolated from its environment. Therefore, the initial condition (1.3) must be replaced by a proper initial condition concerning both the quantum oscillator, which is expected to become "classical," and its environment. The choice made for the initial state of the total wave function of the "universe," $|\psi_T(0)\rangle$, is

$$|\psi_T(0)\rangle=\frac{1}{\sqrt{2}}(|\varphi\rangle+|-\varphi\rangle)|E\rangle\ , \tag{1.4}$$

where $|E\rangle$ represents the initial quantum state of the environment. The evolution of the whole system is driven by ordinary quantum mechanics.

(ii) The orthodox definition of quantum mean value of an observable $\hat{A}$. According to the orthodox interpretation [22], the quantum mean value of an observable $\hat{A}$, denoted by $\langle\hat{A}\rangle$, is the result of an average over an infinite number of measurements repeated under the same conditions or, equivalently, of an average over a Gibbs ensemble of identical systems. Thus, even when we are studying a pure state, rather than a statistical mixture, the correct way of expressing $\langle\hat{A}\rangle$ is given by

$$\langle\hat{A}\rangle=\mathrm{Tr}\rho_T\hat{A}\ , \tag{1.5}$$

where ρ_T, in accordance with (i), must be the density matrix of the "whole universe." Definition (1.5) implies a statistical interpretation of quantum mechanics resting only on ensembles of systems, with no place left for a description in terms of individual systems.

Let us see how, by using these two ingredients, the DT "settles" the problem posed by the Schrödinger cat. First of all, due to the interaction with the oscillator of interest, the environment rearranges itself very quickly and within a very short time τ_D the initial state (1.4) is changed into

$$|\psi_T(0)\rangle=\frac{1}{\sqrt{2}}(|\varphi\rangle|E_\varphi\rangle+|-\varphi\rangle|E_{-\varphi}\rangle)\ , \tag{1.6}$$

where the states $|E_\varphi\rangle$ and $|E_{-\varphi}\rangle$ fulfill the orthogonality condition

$$\langle E_{-\varphi}|E_\varphi\rangle=0 \tag{1.7}$$

due to the macroscopic character of the environment. At this stage, ingredient (ii) enters into play. If we measure an observable concerning the oscillator, namely, the subsystem of interest of the whole universe, the adoption of the definition of quantum mean value (1.5) implies

$$\langle\hat{A}\rangle=\mathrm{Tr}(\rho_T\hat{A})=\mathrm{Tr}_s(\rho_S\hat{A})\ , \tag{1.8}$$

where ρ_S is the reduced density matrix defined by

$$\rho_S\equiv\mathrm{Tr}_e\rho_T\ . \tag{1.9}$$

Using (1.7), it is immediately seen that at times $t\approx\tau_D$, the reduced density matrix of the system of interest is given by

$$\rho_S=\tfrac{1}{2}(|\varphi\rangle\langle\varphi|+|-\varphi\rangle\langle-\varphi|)\ , \tag{1.10}$$

which is indistinguishable from a statistical distribution of oscillators, half of them located at the position $Q=\Delta Q/2$ and half of them at the position $Q=-\Delta Q/2$.

Note that the decoherence time coincides with the time necessary for the states $|E_\varphi(t)\rangle$ and $|E_{-\varphi}(t)\rangle$ to become orthogonal. This is described by a unitary time evolution of the whole universe and does not have the important ingredient of irreversibility necessary to establish a complete equivalence between the dephasing process and the occurrence of a real collapse, which is irreversible by definition. This proves that the problem of recovering classical mechanics is subtly related to the microscopic derivation of irreversibility and decoherence itself as an irreversible process. We think that this aspect is more conveniently dealt with from within the CSL physics, which is irreversible and becomes approximately reversible when it is expected to be almost equivalent to either quantum or classical mechanics. This aspect will be discussed somewhere else. For the time being, let us follow the conventional approaches to decoherence and irreversibility, including the subjective assumptions necessary to make these approaches satisfactory [3].

The decoherence time τ_D has been evaluated by Unruh and Zurek [24] to be

$$\tau_D=\frac{\hbar^2}{2\Gamma Mk_BT(\Delta Q)^2}\ , \tag{1.11}$$

where $1/\Gamma$ is the longitudinal relaxation time of the oscillator. This is a dephasing time and it turns out to be extremely short, thereby ensuring a quick destruction of the Schrödinger cat (1.4). This result coincides with that of a preceding work by Zurek [25] and Caldeira and Leggett [26] and the philosophy behind it is essentially shared by Joos and Zeh [9].

Let us see now how this process serves also the purpose of triggering a sensitive enhancement of the spontaneous collapses. Let us consider the case, explicitly studied in this paper, where the environment is represented by an infinitely large number of harmonic oscillators, linearly coupled to the oscillator of interest. The entangled condition (1.6) means that the individual ith oscillator of the bath, with frequency ω_i and coupling strength ε_i, tends to set itself in a state which is a linear superposition of two wave packets localized around two distinct spatial positions at a distance

$$\Delta L_i=\frac{\varepsilon_i\Delta Q}{m_i\omega_i^2} \tag{1.12}$$

from one another. This is so because the coupling between the oscillator of interest and ith bath oscillator shifts the origin of the harmonic potential of the latter oscillator by the quantity (1.12) in the positive or in the negative direction, according to whether the oscillator of interest is shifted by the quantity ΔQ in the positive or the negative direction, respectively.

The direct CSL process acting on this "bath" oscillator causes a reduction of its wave function and hence of the whole entangled state (1.6) with increasing efficiency as ΔL_i widens, until it fulfills the condition

$$\frac{1}{\sqrt{\alpha}}\ll\Delta L_i\ , \tag{1.13}$$

which ensures the complete efficiency of the CSL reduction process. Notice that we shall assume for the bath oscillators an Ohmic distribution of frequencies. Thus, the large ΔQ, the more numerous the bath oscillators satisfying (1.13), thereby resulting in an increased rate for the collapse of the wave function of the central oscillator. In a certain sense, the central oscillator collects collapses from the bath oscillators, resulting in a cumulative effect, which we refer to as the environment-enhanced CSL effect (EECSL).

Notice that the EECSL mechanism of collapse accumulation is well distinct from that taking place on the center of gravity of a macroscopic body, as we shall discuss in more detail later on in this paper. A substantial difference between the two processes, however, already emerges from the sketchy description of the mechanisms outlined above. The collapse accumulation on the center of gravity refers to the dynamics of a macroscopic variable, and the GRW and CSL theories have been tailored for the specific purpose of recovering in this case the prescriptions of ordinary classical mechanics. The environment-enhanced CSL effect, on the other hand, refers to both macroscopic and microscopic variables. This is so because it is closely connected to the processes of fluctuation and dissipation, and these, in turn, are not limited to macroscopic variables, but can also be extended to a microscopic variable, with the only proviso that this microscopic variable is coupled to infinitely many degrees of freedom.

We shall not be giving the individual-system picture now available to describe the CSL processes [12,13]. However, the reduced master equation that will be built up in the following sections of this paper will turn out to be the sum of a decoherence term corresponding to the DT and one, much smaller so as to fulfill the conditions of statistical agreement with ordinary quantum mechanics, corresponding to the EECSL effect. Within an individual-system picture, the dynamics described by the reduced master equation must be perceived as the time evolution of the wave function of the system of interest expanded over distinct spatial components. The EECSL decoherence process corresponds to real collapses of the wave function into these components. What would the individual-system time evolution corresponding to the DT be like? Within the CSL perspective, the DT motion corresponds to the phases of the expansion coefficients over the distinct spatial components undergoing a sort of Brownian motion triggered by the interaction with the environment and with no real collapse into any of these components.

The paper is organized as follows. Section II is devoted to the mathematical derivation of the EECSL effect on the basis of the mathematical results of [17]. The physical nature of this process and its connection with the fluctuation-dissipation process are illustrated in Sec. III. With the concluding remarks of Sec. IV, we discuss in more detail the consequences of these results on an individual-system representation.

II. THEORETICAL DERIVATION OF THE EECSL EFFECT

This section is devoted to the theoretical derivation of the rate of the EECSL processes in the special case of a linear oscillator, either stable or unstable, interacting with an environment of harmonic oscillators. The calculations of this section rely heavily on the formal development carried out in the preceding work [17]. For the sake of clarity, we shall shortly review the treatment of [17], and we shall adapt it to the specific purpose of this paper.

We study the following model Hamiltonian:

$$\hat{H}=\frac{\hat{P}^2}{2M}\pm\frac{1}{2}M\Omega^2\hat{\Omega}^2+\sum_{i=1}^{N}\frac{\hat{p}_i^2}{2m_i}+\frac{1}{2}m_i\omega_i^2\times\left[\hat{q}_i+\frac{\varepsilon_i}{m_i\omega_i^2}\hat{Q}\right]^2, \tag{2.1}$$

where the double sign $\pm$ means that we do not limit ourselves to studying the case of the stable oscillator of Ref. 17 (positive sign) and that our result can also be applied to the case of the unstable oscillator (negative sign). The $N+1$ particles of the system are assumed to be distinguishable.

Due to the harmonic nature of the system under study, it is convenient to adopt the Wigner picture [27]. We describe the state of the whole system (in a statistical sense) in terms of the Wigner quasiprobability distribution

$$\rho_W(\{q_i,p_i\};t)=\frac{1}{(\pi\hbar)^{N+1}}\int\prod_{i=0}^{N}dy_i\exp\left[\frac{2i}{\hbar}\sum_{i=0}^{N}p_iy_i\right]\langle\{q_i-y_i\}|\hat{\rho}|\{q_i+y_i\}\rangle\ , \tag{2.2}$$

whose evolution in the presence of the CSL processes acting on *each* oscillator, is given by [17]

$$\frac{\partial\rho_W}{\partial t}=\{H,\rho_W\}_{PB}+\sum_{i=0}^{N}\lambda_i\left[\exp\left[\frac{\alpha\hbar^2}{4}\frac{\partial^2}{\partial p_i^2}\right]-1\right]\rho_W\ , \tag{2.3}$$

where the label $i=0$ denotes the oscillator of interest, i.e., $\hat{q}_0\equiv\hat{Q}$ and $\hat{p}_0\equiv\hat{P}$.

The dynamical contribution of ordinary quantum mechanics is given by the first term on the rhs of Eq. (2.3) and it is equal to the classical Poisson brackets because in the linear case under discussion, the differential operator generating the quantum evolution of the Wigner quasiprobability is proved [27] to coincide with the classical evolution generator.

The second term on the rhs of (2.3), a non-Gaussian diffusionlike operator, expresses the effect that the CSL processes have on the Wigner quasiprobability. Notice that in the preceding paper [17] this same term was used to express the dynamical correction that, according to the GRW theory [11], must be added to the standard quantum evolution. In that specific case, the parameters λ_i denote the mean frequency of the process of spontaneous localization for the ith oscillator, and the "decoherence parameter" α is defined by the spatial length $1/\sqrt{\alpha}$, which represents the distance after which a linear superposition is transformed into a statistical mixture. Within the CSL theory [12,13] the standard Schrödinger equation is supplemented by the inclusion of a nonlinear and stochastic correction, with a strength γ that is related to the parameters α and λ by $\gamma=\lambda(\alpha/4\pi)^{-3/2}$. The parameter α retains the original meaning. Under the assumption that the particles are distinguishable, the GRW and the CSL lead to the same evolution equation for the statistical density matrix [12,13].

In Ref. [17] we derived an exact expression of the contracted Wigner quasiprobability $\rho_W(Q,P;t)$, which is obtained from the total distribution (2.2) by integration over the coordinates of the environment oscillators. The result of this derivation can be expressed in a simple form in terms of the Fourier transform of $\rho_W(Q,P;t)$, i.e., the characteristic function $F(\chi_0,\pi_0;t)$ defined by

$$F(\chi_0,\pi_0;t)\equiv\int dQ\,dP\exp\left[\frac{i}{\hbar}(Q\chi_0+P\pi_0)\right]\rho_W(Q,P;t)$$

$$=\mathrm{Tr}\left\{\exp\left[\frac{i}{\hbar}(\hat{Q}\chi_0+\hat{P}\pi_0)\right]\hat{\rho}(t)\right\}. \tag{2.4}$$

It was proved [17] that the characteristic function $F(\chi_0,\pi_0;t)$, corresponding to a given initial condition for the *whole* system, $F_0(\{\chi_i,\pi_i\})$, can be written as follows:

$$F(\chi_0,\pi_0;t)=\exp\left[\sum_{i=0}^{N}\lambda_i\int_0^t d\tau\left\{\exp\left[-\frac{\alpha}{4}\left[\dot{A}_{i0}(\tau)\pi_0+\frac{A_{i0}(\tau)}{m_0}\chi_0\right]^2\right]-1\right\}\right]$$
$$\times F_0\left[\left\{m_i\ddot{A}_{i0}(t)\pi_0+\frac{m_i}{m_0}\dot{A}_{i0}(t)\chi_0,\ \dot{A}_{i0}(t)\pi_0+A_{i0}(t)\frac{\chi_0}{m_0}\right\}\right] \tag{2.5}$$

$(m_0=M)$, namely, as the product of two factors, of which the first provides the effects of the CSL processes and the second describes the standard quantum evolution. The dynamical quantities $A_{i0}(t)$ appearing in (2.5) are given by

$$A_{00}(t)=A(t)=\sum_{k=0}^{N}R_{0k}^2\frac{\sin(z_k t)}{z_k}, \tag{2.6a}$$

where

$$R_{0k}^2=\left[1+\sum_{i=1}^{N}\frac{\varepsilon_i^2}{m_iM}\frac{1}{(z_k^2-\omega_i^2)^2}\right]^{-1} \tag{2.6b}$$

and

$$A_{i0}(t)=-\frac{\varepsilon_i}{\omega_i\sqrt{Mm_i}}\int_0^t d\tau\,A(t)\sin[\omega_i(t-\tau)],\quad i\neq0. \tag{2.6c}$$

The "frequencies" z_k are the normal-mode frequencies of the model Hamiltonian (2.1) and are the $N+1$ solutions of the eigenvalue equation

$$z^2=\pm\Omega^2-\sum_{i=1}^{N}\frac{\varepsilon_i^2}{m_iM\omega_i^2}\frac{z^2}{\omega_i^2-z^2}. \tag{2.7}$$

Note that in the stable case [when in (2.7) the positive sign applies] all the $N+1$ frequencies z_k are real. In the unstable case [when in (2.7) the negative sign applies] this eigenvalue equation results in N real frequencies and in one imaginary, thereby yielding exponentially diverging expressions for the functions $A_{i0}(t)$.

In Ref. [17] the general solution (2.5) was analyzed in detail in the case of a particular class of initial conditions, the "constrained equilibrium initial condition," with the whole system in a state of canonical equilibrium at temperature T, except for the mean values of the central oscillator, $\langle P(0)\rangle$ and $\langle Q(0)\rangle$. We need now to adapt the approach of Ref. [17] to different initial conditions in order to discuss the processes of decoherence and collapse of the linear superposition (1.3). For this reason, we have to consider the *factorized* initial states of the form (1.4), which are not contained in the class of constrained equilibrium initial conditions. To this end, we shall consider the class of factorized initial conditions of the form

$$\hat{\rho}(0)=\hat{\rho}_0\hat{\rho}_B, \tag{2.8a}$$

$$\hat{\rho}_B=\frac{\exp(-\hat{H}_B/k_BT)}{\mathrm{Tr}[\exp(-\hat{H}_B/k_BT)]},$$

$$\hat{H}_B=\sum_{i=1}^{N}\frac{\hat{p}_i^2}{2m_i}+\tfrac{1}{2}m_i\omega_i^2\hat{q}_i^2, \tag{2.8b}$$

describing the oscillator of interest in a generic state with density matrix $\hat{\rho}_0$ and the environment, decoupled from the central oscillator, with the canonical equilibrium distribution corresponding to the temperature T. If we apply the general solution (2.5) to this case, we get (see also Ref. [28])

$$F(\chi_0,\pi_0;t)=\exp\left[\sum_{i=0}^{N}\lambda_i\int_0^t d\tau\left\{\exp\left[-\frac{\alpha}{4}\left[\dot{A}_{i0}(\tau)\pi_0+\frac{A_{i0}(\tau)}{M}\chi_0\right]^2\right]-1\right\}\right]$$
$$\times\Phi_0\left[M\ddot{A}(t)\pi_0+\dot{A}(t)\chi_0,\dot{A}(t)\pi_0+A(t)\frac{\chi_0}{M}\right]\exp\{-\tfrac{1}{2}[X(t)\chi_0^2+Y(t)\pi_0^2+M\dot{X}(t)\chi_0\pi_0]\}\;, \tag{2.9}$$

where $\Phi_0(\chi_0,\pi_0)$ is the characteristic function associated with [according to definition (2.4)] the initial condition $\hat{\rho}_0$ of the central oscillator and the functions $X(t)$ and $Y(t)$ are expressed in terms of the $A_{i0}(t)$ in the following way [28]:

$$X(t)=\frac{\hbar}{2M^2}\sum_{n=1}^{N}m_n\omega_n\coth\left[\frac{\hbar\omega_n}{2k_BT}\right]\times\left[A_{n0}^2(t)+\frac{\dot{A}_{n0}^2(t)}{\omega_n^2}\right], \tag{2.10a}$$

$$Y(t)=\frac{\hbar}{2}\sum_{n=1}^{N}m_n\omega_n\coth\left[\frac{\hbar\omega_n}{2k_BT}\right]\left[\dot{A}_{n0}^2(t)+\frac{\ddot{A}_{n0}^2(t)}{\omega_n^2}\right]. \tag{2.10b}$$

Although the exact time evolution for the reduced quasiprobability distribution provides a complete description of the dynamics of the central oscillator, a more direct approach to the EECSL processes and to the evaluation of the mean collapse time τ_C associated with them can be derived from the explicit expression of the master equation. As shown in Refs. [24,29,30], when the CSL processes are switched off, the off-diagonal density matrix elements are made to quickly vanish in a preferred basis *(pointer basis)* by the diffusion term of the master equation. When the CSL processes on each oscillator of the bath are switched on, there appears an additional contribution to the decoherence of the system of interest, which, does not, however, have anything to do with a standard decoherence process. This increase in the decoherence rate is caused by the occurrence of real wave-function collapses whose frequency is significantly increased, with respect to the case of an isolated oscillator, by the same entanglement process as that causing the decoherence of the DT [6,7].

Therefore, let us derive the explicit expression for the master equation. By differentiating the exact expression (2.9), after some straightforward algebra we derive the non-Markovian master equation for the Wigner quasiprobability $\rho_W(Q,P;t)$ of the central oscillator. Its exact expression in the case of factorized initial conditions of the form (2.8) is

$$\frac{\partial\rho_W(Q,P;t)}{\partial t}=[L_{\mathrm{HR}}(t)+L_{\mathrm{CSL}}(t)]\rho_W(Q,P;t)\;. \tag{2.11}$$

The first term describes the standard quantum evolution of the damped oscillator, and its expression, first derived by Haake and Reibold [28], reads

$$L_{\mathrm{HR}}(t)\equiv-\frac{P}{M}\frac{\partial}{\partial Q}+M\tilde{\Omega}^2(t)Q\frac{\partial}{\partial P}+\gamma(t)\frac{\partial}{\partial P}P+D(t)\frac{\partial^2}{\partial P^2}-f(t)\frac{\partial^2}{\partial P\partial Q}\;, \tag{2.12}$$

with

$$\tilde{\Omega}^2(t)=\frac{\ddot{A}(t)^2-\dot{A}(t)\dddot{A}(t)}{\dot{A}^2(t)-A(t)\ddot{A}(t)}\;, \tag{2.13a}$$

$$\gamma(t)=\frac{A(t)\dddot{A}(t)-\dot{A}(t)\ddot{A}(t)}{\dot{A}^2(t)-A(t)\ddot{A}(t)}\;, \tag{2.13b}$$

$$D(t)=\tfrac{1}{2}\dot{Y}(t)+\gamma(t)Y(t)+\tfrac{1}{2}M^2\tilde{\Omega}^2(t)\dot{X}(t)\;, \tag{2.13c}$$

$$f(t)=\frac{Y(t)}{M}-\tfrac{1}{2}M\gamma(t)\dot{X}(t)-M\tilde{\Omega}^2(t)X(t)-\tfrac{1}{2}M\ddot{X}(t)\;. \tag{2.13d}$$

The second term in (2.11) is the crucial correction to the standard quantum dynamics resulting from the CSL processes. It is convenient to write this contribution by isolating the term corresponding to the bare continuous spontaneous localization of the central oscillator, i.e., the CSL process that the central oscillator would undergo in the case of no interaction with the environment. We thus obtain

$$L_{\mathrm{CSL}}(t)=L_{\mathrm{CSL}}^b+L_{\mathrm{CSL}}^d(t)\;, \tag{2.14a}$$

$$L_{\mathrm{CSL}}^b=\lambda_0\left[\exp\left[\frac{\alpha\hbar^2}{4}\frac{\partial^2}{\partial P^2}\right]-1\right], \tag{2.14b}$$

$$L_{\mathrm{CSL}}^d(t)=\sum_{i=0}^{N}\frac{\lambda_i\alpha\hbar^2}{2}\int_0^t d\tau[\ddot{A}_{i0}(\tau)+\gamma(t)\dot{A}_{i0}(\tau)+\tilde{\Omega}^2(t)A_{i0}(\tau)]\left[\dot{A}_{i0}(\tau)\frac{\partial^2}{\partial P^2}+\frac{A_{i0}(\tau)}{M}\frac{\partial^2}{\partial Q\partial P}\right]\times\exp\left[\frac{\alpha\hbar^2}{4}\left[\dot{A}_{i0}(\tau)\frac{\partial}{\partial P}+\frac{A_{i0}(\tau)}{M}\frac{\partial}{\partial Q}\right]^2\right]. \tag{2.14c}$$

We want to prove that the CSL processes are enhanced by the occurrence of dissipation. From a formal point of view, dissipation is obtained when the bath frequencies have a continuous distribution. The continuum limit is obtained by replacing discrete sums with integrals and by using the spectral density [31]

$$J(\omega)=\frac{\pi}{2}\sum_{i=1}^{N}\frac{\varepsilon_i^2}{m_i\omega_i}\delta(\omega-\omega_i) . \tag{2.15}$$

We consider in particular the Ohmic spectral density with a Lorentzian cutoff function, namely,

$$J(\omega)=\eta\omega\frac{\omega_c^2}{\omega^2+\omega_c^2} , \tag{2.16}$$

where η is the friction coefficient and ω_c is the frequency cutoff of the environment. In the limiting case of an extremely fast thermal bath, which means $\omega_c \to \infty$, we get [28]

$$\gamma(t)=\frac{\eta}{M}\equiv 2\Gamma,\quad \tilde{\Omega}^2(t)=\pm\Omega^2 , \tag{2.17}$$

where Γ is the damping coefficient of the velocity of the oscillator of interest. The negative sign in (2.17) refers to the unstable oscillator and the positive to the stable oscillator. The interesting aspect of the infinitely fast bath case is that because of (2.17), the first factor in the integral of Eq. (2.14c) for $i=0$ identically vanishes, that is,

$$\ddot{A}_{00}(\tau)+\gamma(t)\dot{A}_{00}(\tau)+\tilde{\Omega}^2(t)A_{00}(\tau)=0 \tag{2.18}$$

for both the unstable and the stable oscillator. This implies that we can rewrite the contribution $L^d_{\mathrm{CSL}}(t)$ in (2.14c) as

$$L^d_{\mathrm{CSL}}(t)=\sum_{i=1}^{N}\frac{\lambda_i\alpha\hbar^2}{2}\int_0^t d\tau[\ddot{A}_{i0}(\tau)+2\Gamma\dot{A}_{i0}(\tau)\pm\Omega^2A_{i0}(\tau)]\left[\dot{A}_{i0}(\tau)\frac{\partial^2}{\partial P^2}+\frac{A_{i0}(\tau)}{M}\frac{\partial^2}{\partial Q\partial P}\right]$$
$$\times\exp\left[\frac{\alpha\hbar^2}{4}\left[\dot{A}_{i0}(\tau)\frac{\partial}{\partial P}+\frac{A_{i0}(\tau)}{M}\frac{\partial}{\partial Q}\right]^2\right] , \tag{2.19}$$

where now the sum runs from $i=1$ to $i=N$. This means that in the $\omega_c\to\infty$ limit, expression (2.14a) corresponds to an unambiguous separation between the direct and the indirect CSL processes. The term L^b_{CSL} in (2.14a) describes the effect of the spontaneous localizations acting directly on the oscillator of interest. The second term, $L^d_{\mathrm{CSL}}(t)$, describes the collapses that the localizations acting on the bath oscillators produce on the wave function of the central oscillator because of the entangled structure of the state vector (1.6) induced by the interaction with the environment. The reader can realize that this is plausible by observing that the functions A_{i0} (with $i>0$), in (2.14c) vanish when the interaction strengths ε_i are made to vanish [cf. (2.6c)], thereby implying that the term L^d_{CSL} of (2.14b) is reduced to zero if the interaction between the system of interest and the environment is switched off.

To make it possible for us to derive a simple expression for the collapse time τ_C associated with the indirect CSL process, we focus on the term $L^d_{\mathrm{CSL}}(t)$ and we make a weak-coupling assumption. This assumption requires some comments. As noticed in Ref. [17], the presence of CSL processes introduces non-Gaussian corrections to the usual Gaussian time evolution of the damped quantum oscillator, thereby making it rather involute. Here we can recover Gaussian properties, and with them a significantly simplified picture, by making the weak-coupling assumption: in the limiting case of very weak ε_i's, we see from (2.19) that the derivatives of order higher than the second can be safely disregarded, thereby obtaining

$$L^d_{\mathrm{CSL}}(t)=D_{\mathrm{CSL}}(t)\frac{\partial^2}{\partial P^2}-f_{\mathrm{CSL}}(t)\frac{\partial^2}{\partial Q\partial P} , \tag{2.20}$$

where

$$D_{\mathrm{CSL}}(t)=\sum_{i=1}^{N}\frac{\lambda_i\alpha\hbar^2}{2}\int_0^t d\tau\, \dot{A}_{i0}(\tau)[\ddot{A}_{i0}(\tau)+2\Gamma\dot{A}_{i0}(\tau)\pm\Omega^2A_{i0}(\tau)] \tag{2.21a}$$

and

$$f_{\mathrm{CSL}}(t)=-\sum_{i=1}^{N}\frac{\lambda_i\alpha\hbar^2}{2M}\int_0^t d\tau\, A_{i0}(\tau)[\ddot{A}_{i0}(\tau)+2\Gamma\dot{A}_{i0}(\tau)\pm\Omega^2A_{i0}(\tau)] . \tag{2.21b}$$

Let us now evaluate the explicit form of these diffusion coefficients in the continuum limit. As done in Ref. [17], we make the plausible assumption that each oscillator is subject to the same localization process, i.e., $\lambda_i=\lambda$, $i=1,2,\ldots,N$. The continuum limit of (2.21) thus exists and, after tedious calculations, in the infinitely fast bath limit $\omega_c\to\infty$ we get for the CSL contribution to the diffusion coefficients the following simple expressions, valid for both the stable and the unstable oscillator:

$$D_{\mathrm{CSL}}(t)=\frac{\lambda\alpha\hbar^2\Gamma}{2},\quad f_{\mathrm{CSL}}(t)=0 . \tag{2.22}$$

Henceforth, we shall focus on the decoherence process and, consequently, on the diffusion terms of the master equation, which are the main cause of decoherence. By retaining only the second-order differential operators in the momentum P, the master equation (2.11) for the Wigner quasiprobability is made to read

$$\frac{\partial \rho_W(Q,P;t)}{\partial t} \approx [D_q + D_{\rm CSL}(t)] \frac{\partial^2}{\partial P^2} \rho_W(Q,P;t) \ , \quad (2.23)$$

where $D_{\rm CSL}(t)$ is given by Eq. (2.22) and D_q is the high-temperature limit of the standard diffusion coefficient $D(t)$ of Eq. (2.13c), i.e.,

$$D(t) \approx D_q = 2M\Gamma k_B T \ , \quad (2.24)$$

which is valid for both the stable and the unstable oscillators. Therefore, we see that in the weak-coupling limit, the CSL processes on the environment degrees of freedom have only the effect of renormalizing the momentum diffusion coefficient by adding a correction term linear in time. As will be made more apparent later, this correction does not have significant effects from a statistical point of view, since the correction term is extremely small compared to the standard diffusion term of the DT [6,7]. However, from the point of view of the individual systems the consequences are striking. This is so because this contribution to decoherence is actually the manifestation of a real collapse of the wave function, which takes place at a time scale that in principle does not conflict with the possibility of an experimental control. This is the EECSL effect mentioned in Sec. I.

Let us evaluate the collapse time of the EECSL effect. Equation (2.23) corresponds to the following equation for the density matrix in the coordinate representation $\rho(Q,Q';t)$:

$$\frac{\partial \rho(Q,Q',t)}{\partial t} = -\left[\frac{2M\Gamma k_B T}{\hbar^2} + \frac{\lambda\alpha\Gamma t}{2}\right](Q-Q')^2 \times \rho(Q,Q',t) \ , \quad (2.25)$$

whose formal solution is

$$\rho(Q,Q',t) = \exp\left[-\frac{2M\Gamma k_B T(Q-Q')^2 t}{\hbar^2} - \frac{\lambda\alpha\Gamma(Q-Q')^2 t^2}{4}\right] \rho(Q,Q',0) \ . \quad (2.26)$$

If we consider an initial linear superposition of two localized wave packets spatially separated by a distance ΔQ, as, for example, the initial state (1.3), the corresponding initial density matrix in the coordinate representation shows two distinct symmetric off-diagonal bumps (one centered at $Q=\Delta Q/2$, $Q'=-\Delta Q/2$ and the other one at $Q=-\Delta Q/2$, $Q'=\Delta Q/2$) describing quantum interference between the two wave packets. Therefore, if we set $(Q-Q')=\Delta Q^2$, Eq. (2.26) yields the quick suppression of quantum coherence by the joint effect of two independent decoherence mechanisms. The former, corresponding to the first term in the exponent of (2.26), is the standard process of decoherence resulting from the dissipative coupling of the system with the environment, studied by Caldeira and Leggett [26], Joos and Zeh [9], Zurek and co-workers [24,30], and Hu, Paz, and Zhang [29]. The latter is the explicit expression of the contribution to decoherence of the EECSL process. The decoherence rate of these two independent processes can be estimated by setting each separated term in the exponent of Eq. (2.26) equal to one, thereby getting for the former process

$$\frac{1}{\tau_D} = \frac{2M\Gamma k_B T \Delta Q^2}{\hbar^2} \ , \quad (2.27)$$

which is the usual prediction for the decoherence rate [24,25], and for the latter process

$$\frac{1}{\tau_C} = \frac{\sqrt{\lambda\alpha\Gamma\Delta Q^2}}{2} \ , \quad (2.28)$$

which is the rate of the EECSL process. It is remarkable that the EECSL collapse time τ_C is made shorter and shorter by increasing the dissipation rate Γ.

III. MORE ON THE PHYSICAL MEANING OF THE EECSL PROCESS

In this section, we shall shed further light on the physical meaning of the EECSL process derived in Sec. II, by comparing with the direct process of collapse and with the collapse accumulation discussed in [11–13]. We shall also provide further physical arguments to establish a more apparent connection between the EECSL process and the DT [6,7].

In the preceding section, we have seen that, in the infinitely fast bath limit and in the weak-coupling approximation, the CSL contribution to the effective Liouvillian stemming from the contraction over the bath, is

$$L_{\rm CSL}(t) = \lambda_0\left[\exp\left[\frac{\alpha\hbar^2}{4}\frac{\partial^2}{\partial P^2}\right] - 1\right] + \frac{\lambda\alpha\hbar^2\Gamma t}{2}\frac{\partial^2}{\partial P^2} \ . \quad (3.1)$$

This simple expression clearly shows the separation of the CSL contribution into the direct process [first term in the rhs of (3.1)] and the indirect one [second term in the rhs of (3.1)] working on the central oscillator as a consequence of its interaction with the bath.

If we focus only on the indirect contribution, as we already did in Sec. II, we obtain the following expression for the time evolution of the statistical operator in the coordinate representation:

$$\rho(Q,Q',t) = \exp\left[-\frac{\Gamma\lambda\alpha(Q-Q')^2 t^2}{4}\right] \rho(Q,Q',0) \ . \quad (3.2)$$

A direct CSL process with localization rate Λ would instead read

$$\rho(Q,Q',t) = \exp\left\{-\Lambda\left[1-\exp\left[-\frac{\alpha(Q-Q')^2}{4}\right]\right]t\right\} \times \rho(Q,Q',0) \ . \quad (3.3)$$

Comparing Eqs. (3.2) and (3.3), we see that the two processes take a similar, although not identical, form in the limit

$$|Q-Q'| \ll \frac{1}{\sqrt{\alpha}} \sim 10^{-5} \text{ cm} \tag{3.4}$$

(where α has been assigned the value suggested in Ref. [11]). Roughly speaking, the indirect process can then be compared to a direct one with a localization rate linearly increasing in time, as if the localization rate Λ, instead of being time independent, were given by

$$\Lambda = \lambda \Gamma t \ . \tag{3.5}$$

The different nature of the two processes is stressed by the opposite limiting condition,

$$\frac{1}{\sqrt{\alpha}} \ll |Q-Q'| \ . \tag{3.6}$$

In this case, the direct process of Eq. (3.3) becomes

$$\rho(Q,Q',t) = \exp(-\Lambda t)\rho(Q,Q',0) \ , \tag{3.7}$$

with no dependence on the distance $|Q-Q'|$ left, while the indirect one (3.2) still retains this dependence.

The mathematically different forms of Eqs. (3.2) and (3.7) mirror the distinct character of the two underlying physical processes. A direct CSL process realizes the reduction of a superposition of two spatially separated wave packets with a rate Λ independent of the distance ΔQ between them, whenever ΔQ is larger than $1/\sqrt{\alpha}$. The indirect process (3.2), on the other hand, always depends on ΔQ, becoming faster and faster as ΔQ grows larger. As pointed out in Sec. I, where we have already given an intuitive explanation of the EECSL effect, this effect is triggered by the same mechanism, the entanglement between system and environment, as that responsible for the environment decoherence. This is so because the entangled condition (1.6) means that the individual ith oscillator of the bath tends to set itself in a state which is a linear superposition of two wave packets localized around two distinct spatial positions at the distance (1.2).

Notice that in the case of the CSL accumulation concerning the center of gravity, the non-Hamiltonian term in the evolution equation of the center of mass retains its form with respect to the case of a single particle [11–13]. This is not the case for the EECSL process, as made manifest by the comparison between the two terms on the rhs of Eq. (3.1): the form of the first one in no way can be reduced to that of the second, stemming from the indirect process. The indirect CSL process illustrated in Sec. II is environment induced, and is triggered by the same entanglement effect as that producing the dephasing process of the DT [6,7].

We shed further light on the physical meaning of Eq. (2.25) and, consequently, on our main result (2.28), by remarking that the CSL theory, due to the appearance of non-Hamiltonian terms in the evolution equation, implies the energy conservation principle to be broken [11]. In fact, spontaneous localizations produce a steady increase of the mean kinetic energy of a physical system, which, for an individual particle of mass M, can be estimated to be [11]

$$\Delta E = \frac{\lambda \alpha \hbar^2 t}{4M} \ . \tag{3.8}$$

This energy increase can be equivalently seen as a linear growth of the temperature of the central oscillator as well as that of its bath, $\Delta T = \Delta E / k_B$. If we now adopt the approximate expression (2.24) of the diffusion coefficient, $D(t) \approx D_q$, and we take into account this temperature drift, we get *exactly* the renormalized diffusion coefficient of Eq. (2.23) and therefore Eqs. (2.25)–(2.28). It is then clear that from a statistical point of view the detection of the EECSL effect would be equivalent to the experimental determination of this steady temperature increase. The new constants of nature of the CSL theory, λ and α, however, are given the values $\lambda \approx 10^{-16}$ sec^{-1}, $1/\sqrt{\alpha} \approx 10^{-5}$ cm [11] so as to make this temperature increase completely negligible; in Ref. [13] it has been estimated that $\Delta T / t = 10^{-13}$ K per year. This makes immediately evident why any statistical manifestation of the EECSL process is quite negligible with respect to the standard decoherence mechanism.

Using (2.27) and (2.28) we can assess that in the ordinary physical conditions the following inequality,

$$\tau_D \ll \tau_C \ , \tag{3.9}$$

holds true. Actually, the new collapse time τ_C is also larger than the typical relaxation time of the oscillator $\tau_R \approx 1/\Gamma$, for any experimentally feasible value of the wave-packet separation ΔQ. However, it is of crucial importance to stress that, differently from the "bare" CSL reduction time of $1/\lambda \approx 10^{16}$ sec, the collapse time τ_C is not astronomically large, but usually is within the range of experimental observation: for example, for $\Delta Q = 1$ cm and $\Gamma = 10^9$ sec^{-1}, we get $\tau_C \approx 10$ sec.

In conclusion, if we carry out an observation that implies and average over an ensemble of Gibbs systems, it is virtually impossible to detect corrections to the decoherence rate (2.27) predicted by Caldeira and Leggett [26] and by Unruh and Zurek [24].

IV. CONCLUDING REMARKS

At first sight, the conclusions of this paper seem to conflict with those of the preceding work of our group [17], where we proved that the CSL processes lead to statistical predictions virtually equivalent to those of ordinary quantum mechanics, thereby giving the impression that no accumulation of the CSL processes on the system of interest can take place. Actually, the conclusions of [17] only rule out the accumulation processes of the same kind as those concerning the center of gravity of a macroscopic body [11]. As shown in this paper, the statistical equivalence of the new physics with ordinary quantum mechanics does not prevent an accumulation of spontaneous wave-function collapses, the EECSL effect, from occurring. Since this objective process of wave-function collapse does not produce detectable statistical effects, the main conclusion of Ref. [17] still holds true. The detection of the EECSL by means of real experiments must refer to the observation of an individual system.

We are tempted to state that whenever the entanglement of the system with its environment occurs, the EECSL process if also activated. This might have remarkable consequences in the field of quantum chaos. It is well known [3] that the effect of classical chaos is to increase quantum uncertainty so quickly as to make essentially quantum also those systems that would fulfill the conditions set by the correspondence principle for a quantum system to be considered classical. Zurek and Paz recently pointed out [32] that the interaction with the environment restores the statistical equivalence with classical physics through a mechanism of entanglement of the same kind as that of (1.6). Notice that the theoretical development of Zurek and Paz [32] rests on an unstable oscillator interacting with its environment, namely, the same system as (2.1) with the negative sign. The present paper suggests that, in addition to ensuring the statistical equivalence between classical and quantum mechanics as shown in Ref. [32], the interaction between system and environment could also induce real wave-function collapses, thereby contributing to the recovery, for an individual system having a classically chaotic counterpart, of the concept itself of classical trajectory.

The EECSL processes are attractive because they seem to make much wider the processes of accumulation of wave-function collapses discussed by Ghirardi, Pearle, and Rimini in Ref. [13], insofar as they are triggered by the same interaction mechanisms as those responsible for dissipation and irreversibility. Thus, they apply to a wider set of physical conditions, namely, to microscopic and mesoscopic dissipative systems as well as the macroscopic isolated systems of Refs. [11–13].

We think that an important field of research to apply the techniques used in this paper is that of direct observations of individual systems, the optical quantum jumps being an especially appealing example of this field of investigation [18]. It must be pointed out that this is obliging the researchers, and also those who are using ordinary quantum mechanics, to adopt individual-system pictures. A remarkable example of this kind is given by the work of Gisin and Percival [33]. These authors developed an individual-system picture with the constraint of an exact equivalence with ordinary quantum mechanics. The theory of Gisin and Percival, used as an individual-system counterpart of the DT, would lead, for the physical process studied in this paper, to erratic trajectories for the expansion coefficients on the states $|\varphi\rangle$ and $|-\varphi\rangle$, characterized by abrupt jumps at times of the order of τ_D. We think that within the CSL perspective, the time duration of these erratic fluctuations with no jumps would be much more extended and would last for the time $\tau_C \gg \tau_D$. The single runs provided by the CSL theory would be qualitatively similar to those of the Gisin and Percival theory, but they would be dilated over a much more extended time scale. However, in the academic example here under discussion, the collapse time τ_C of Eq. (2.28) assumes physically significant values only when the distance ΔQ is so large that Eq. (2.28) loses much of its concrete meaning. For distances of experimental significance, the two branches would be reunited much earlier by the longitudinal process itself. However, research work by our group to apply the theoretical results to more realistic physical conditions, mimicking the main features of the processes of optical quantum jumps, is in progress.

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[1] R. Penrose, *Shadows of the Mind* (Oxford University Press, Oxford, 1994).
[2] R. Omnès, Rev. Mod. Phys. **64**, 339 (1992).
[3] P. Grigolini, *Quantum Irreversibility and Measurement* (World Scientific, Singapore, 1993).
[4] K. R. W. Jones, Phys. Rev. A **50**, 1062 (1994).
[5] A satisfactory illustration of the debate among the authors who think that quantum mechanics only needs an improved interpretation [2] and those who maintain that quantum mechanics must be suitably generalized is beyond the scope of this paper. The interested reader can find a concise and illuminating illustration of this debate in Ref. [4].
[6] W. H. Zurek, Phys. Rev. D **24**, 1516 (1981); **26**, 1862 (1982).
[7] W. H. Zurek, Phys. Today **44**(10), 36 (1991).
[8] H. D. Zeh, Found. Phys. **1**, 69 (1970).
[9] E. Joos and H. D. Zeh, Z. Phys. B **5**, 223 (1985).
[10] M. Gell-Mann and J. B. Hartle, Phys. Rev. D **47**, 3345 (1993).
[11] G. C. Ghirardi, A. Rimini, and T. Weber, Phys. Rev. D **34**, 470 (1986).
[12] P. Pearle, Phys. Rev. A **39**, 2277 (1989).
[13] G. C. Ghirardi, P. Pearle, and A. Rimini, Phys. Rev. A **42**, 78 (1990); G. C. Ghirardi, and A. Rimini, in *Sixty-two Years of Uncertainty,* edited by A. I. Miller (Plenum, New York, 1990), p. 167.
[14] L. Diòsi, Phys. Rev. A **40**, 1165 (1989).
[15] J. Ellis, S. Mohanty, and D. Nanopoulos, Phys. Lett. B **221**, 113 (1989).
[16] G. J. Milburn, Phys. Rev. A **44**, 5401 (1991).
[17] D. Vitali, L. Tessieri, and P. Grigolini, Phys. Rev. A **50**, 967 (1994).
[18] R. J. Cook, *Progress in Optics XXVIII* (Elsevier, New York, 1990), p. 361.
[19] B. Misra and E. C. G. Sudarshan, J. Math. Phys. **18**, 756 (1976).
[20] It must be pointed out that the Zeno effect, and the observation of optical quantum jumps as well, is an indirect manifestation of the wave-function collapses induced by the measurement process, and a single jump is the result of many much faster collapses. However, we cannot rule out, in principle, the possibility that the individual-system observation might be made in future with a technique

more direct than the Zeno effect method.

[21] As we shall see, Zurek's approach rests on the key role of the "decoherence." While the name decoherence is due to Zurek, the idea is much older; it can be found in clearly recognizable form in the work of some authors in the early 1950s [see, e.g., N. G. Van Kampen, Physica **20**, 603 (1954)].

[22] C. Cohen-Tannoudji, F. Laloë, *Quantum Mechanics* (Wiley, New York, 1977).

[23] E. Schrödinger, Naturwissenschaften **23**, 807 (1935); **23**, 823 (1935); **23**, 844 (1935).

[24] W. G. Unruh and W. H. Zurek, Phys. Rev. D **40**, 1071 (1989).

[25] W. H. Zurek, in *Frontiers of Nonequilibrium Statistical Physics,* Vol. 135 of *NATO Advanced Study Institute Series B: Physics,* edited by G. T. Moore and M. O. Scully (Plenum, New York, 1986), p. 145.

[26] A. O. Caldeira and A. J. Leggett, Phys. Rev. A **31**, 1059 (1985).

[27] M. Hillery, R. F. O'Connell, M. O. Scully, and E. P. Wigner, Phys. Rep. **106**, 122 (1984).

[28] F. Haake and R. Reibold, Phys. Rev. A **32**, 2462 (1985).

[29] B. L. Hu, J. P. Paz, and Y. Zhang, Phys. Rev. D **45**, 2843 (1992).

[30] J. P. Paz, S. Habib, and W. H. Zurek, Phys. Rev. D **47**, 488 (1993).

[31] A. O. Caldeira and A. J. Leggett, Ann. Phys (N.Y.) **149**, 274 (1983).

[32] W. H. Zurek and J. P. Paz, Phys. Rev. Lett. **72**, 2508 (1994).

[33] N. Gisin, and I. C. Percival, J. Phys. A **25**, 5677 (1992); Phys. Lett. A **167**, 315 (1992); J. Phys. A **26**, 2233 (1993); **26**, 2245 (1993).