PHYSICAL REVIEW A VOLUME 51, NUMBER 5 MAY 1995

Homodyne correlation measurements with weak local oscillators

Werner Vogel
Universität Rostock, Fachbereich Physik, Universitätsplatz 3, Postfach 999, D-18051 Rostock, Germany
(Received 14 November 1994)

Based on proposals to measure squeezing by homodyne cross correlation [Ou, Hong, and Mandel, Phys. Rev. A **36**, 192 (1987)] and by homodyne intensity correlation measurements with a weak local oscillator [Vogel, Phys. Rev. Lett. **67**, 2450 (1991)], the method of homodyne correlation measurements is considered in detail for the case of weak local oscillators. Aside from the feasibility of measuring squeezing in low efficiency detection, it appears possible to record the correlation of the noise of two noncommuting observables, namely, the intensity and the electric field strength of the signal field. The separation of the different contributions to the measured correlations is considered and spectral correlation measurements are analyzed. Local oscillator noise, which is not balanced out in such a scheme, is found to be attenuated effectively by decreasing the local oscillator field. In particular, in the homodyne cross correlation scheme, it is of interest to use a beam splitter that significantly differs from a 50%:50% partition.

PACS number(s): 42.50.Ar

I. INTRODUCTION

Homodyne detection is a well established technique used in quantum optics for the study of phase-sensitive phenomena such as squeezing [1–6]. Usually, the signal field (SI) to be studied is superimposed with a strong, coherent local oscillator, for the theory cf. e.g. [7–11]. In a simple (unbalanced) homodyne detection scheme the superimposed light is recorded by a single detector and the statistics of the photoelectric counts or the photocurrent are analyzed for different phase shifts between the local oscillator and signal field. For a strong local oscillator the mean number of photocounts is dominated by the local oscillator field and the leading contribution to the corresponding variance is the shot noise level. The dominant deviation from the shot noise level is proportional to the normally ordered variance of the electric field strength of the signal field. Therefore, a reduction of the noise below the shot noise indicates squeezing of the signal field.

The relative noise reduction in such a measurement scheme is proportional to the overall detection efficiency, including the efficiency of the detector and the collection efficiency of the light field under study. In this manner a relative effect due to the field fluctuations of the signal field can be observed which is of the order of magnitude

$$\frac{\overline{[\Delta n(t,\Delta t)]^2} - \overline{n}(t,\Delta t)}{\overline{n}(t,\Delta t)} \cong \eta\ \Delta t|\bar{T}|^2 \times \left\langle : \left[\Delta \hat{E}_{\mathrm{SI}\,\phi}(t-x/c)\right]^2 : \right\rangle . \tag{1}$$

Here, $n(t,\Delta t)$ denotes the number of photoelectric events recorded by the detector during the time interval $[t, t+\Delta t]$, the bar denotes (classical) stochastic averaging, η is the detection efficiency, and $\bar{T}$ the (amplitude) transmission of the beam splitter combining the signal and local oscillator fields. The ": :" notation denotes normal ordering for operators, which are distinguished from c numbers by a caret. Here and in the following it is assumed that the measurement interval Δt is sufficiently short compared with the characteristic times of the radiation under study. The operator

$$\begin{aligned}\hat{E}_{\mathrm{SI}\,\phi}(t) &= \hat{E}^{(+)}_{\mathrm{SI}\,\phi}(t) + \hat{E}^{(-)}_{\mathrm{SI}\,\phi}(t) \\ &= \hat{E}^{(+)}_{\mathrm{SI}}(t)\ \exp[i(\omega_0 t + \phi)] \\ &\quad + \hat{E}^{(-)}_{\mathrm{SI}}(t)\ \exp[-i(\omega_0 t + \phi)]\end{aligned} \tag{2}$$

describes the (slowly varying) electric field strength of the signal field, the phase

$$\phi = \phi_{\mathrm{LO}} + \phi_T - \phi_R \tag{3}$$

may be controlled by the phase ϕ_{LO} of the local oscillator, ϕ_T and ϕ_R, respectively, being the phase shifts due to transmission and reflection at the beam splitter and ω_0 is the frequency of the local oscillator. Equation (1) reveals that the magnitude of the observable squeezing effect is limited by the detection efficiency.

For deriving the result of Eq. (1) it is assumed that the local oscillator is in a coherent state. In practice, small but unavoidable fluctuations of the local oscillator may introduce significant errors since the local oscillator is strong. To overcome this problem balanced homodyne detection has been proposed [12–15]. The local oscillator and the signal field are combined by a beam splitter and the two superimposed fields in the output channels are simultaneously measured. Analyzing the statistics of the difference of events recorded in the two channels, classical noise effects of the local oscillator are balanced out by subtracting their (equal) effects in the two channels. The statistics of balanced homodyne detection has also been studied for arbitrarily weak local oscillators, both for the case of ideal detectors [16] and for realistic detectors of nonunity efficiencies [17].

As long as the detection efficiencies are unity and the local oscillator is strong the full statistics of the signal field strength, namely the probability distribution

$p(E_{\mathrm{SI}\,\phi})$ of the field strength $E_{\mathrm{SI}\,\phi}$, can be measured in a balanced scheme. This quantity, known for the phase values ϕ within a π interval, contains the full information on the quantum state of the signal field [18]. For nonunity efficiencies a smoothed field strength distribution $\tilde{p}(E_{\mathrm{SI}\,\phi})$ is recorded which can be represented as the convolution of the true field strength distribution $p(E_{\mathrm{SI}\,\phi})$ with a Gaussian noise distribution $p_\eta(E)$ [17],

$$\tilde{p}(E_{\mathrm{SI}\,\phi}) = \int dE\, p(E_{\mathrm{SI}\,\phi} - E)\, p_\eta(E), \tag{4}$$

where

$$p_\eta(E) = \left(2\pi\sigma_\eta^2\right)^{-\frac{1}{2}} \exp\left(-\frac{E^2}{2\sigma_\eta^2}\right), \tag{5}$$

the variance σ_η^2 being $(1-\eta)/\eta$ times the vacuum variance of the signal field strength $E_{\mathrm{SI}\,\phi}$. Thus the determination of the true field strength distribution from the measured data requires a deconvolution that is only practicable for rather large efficiencies.

Recently, the experimental determination of field strength distributions, the construction of the Wigner functions and the corresponding density matrices has been carried out using optical homodyne tomography [19]. This method is of great interest since it yields the full quantum statistical information on the radiation field. Its application, however, is also limited by the deconvolution problem mentioned above. Some other approaches for determining the quantum state of light have been proposed which are also based on some kinds of homodyne detection schemes [20–23].

For many applications of homodyne detection methods the limitation due to nonunity efficiencies may be of less importance, especially when the light field under study is strong, highly collimated, and its spectrum is in a region where high-sensitivity detectors are available. On the other hand, it is of interest to get some insight in (phase-sensitive) field fluctuations for light fields that do not fulfil such conditions. Especially in quantum optics the noise properties of weak fields are a subject of research. A typical example is the resonance fluorescence from a single atom where already the low collection efficiency of the fluorescence makes it almost impossible to determine the fluctuations of the electric field strength from a measurement of the sub-Poissonian statistics [8] or from a smoothed field strength distribution in a balanced scheme. Moreover, in certain spectral ranges a highly efficient detection can hardly be achieved since appropriate detectors are not available. In such cases alternative schemes for homodyne detection are of interest which are not limited by the efficiency factors.

The proposal to detect squeezed light by cross correlations [24] is a technique of that type. There is one noteworthy difference between this method and balanced homodyning. Here the efficiencies appear only as multiplicative factors that do not alter the shapes of the correlation functions as functions of time. On the other hand, the efficiencies can strongly alter the shapes of the field strength distributions recorded in balanced homodyning, cf. Eqs. (4) and (5). In the homodyne cross correlation scheme, a beam splitter is used for superimposing the signal field with the local oscillator. Furthermore, it acts as the input beam splitter of a correlation device consisting of two detectors and an appropriate correlator. The cross correlation scheme was studied in the limit of a strong local oscillator. Squeezed light shows up as a positive cross correlation of the (different) fields in the two output channels. An obstacle of this approach is that the classical noise of the local oscillator is not balanced out, which makes it less attractive to detect squeezing [24]. More recently, I have proposed to use a homodyne intensity correlation scheme with a weak local oscillator for detecting squeezing in resonance fluorescence from a single trapped ion [25]. In this case the local oscillator is superimposed with the signal (fluorescence) field and the intensity correlations of the superimposed light are measured. The maximum effect due to squeezing can be observed when the local oscillator is as weak as the fluorescence of the atom. Due to the weakness of the local oscillator, additional effects are observed such as the sub-Poissonian statistics of the signal and anomalous moments containing unequal numbers of annihilation and creation operators of the signal field. These anomalous moments turn out to represent the normally ordered correlation effect between intensity and field strength fluctuations of the signal field [26] and their study is of interest for its own. The largest effects due to squeezing in resonance fluorescence are expected in such a detection scheme for a weakly driven atom and the different contributions to the homodyne intensity correlation can simply be separated via their dependences on the local oscillator phase [25].

In the present paper we consider both the homodyne cross correlation scheme and the homodyne intensity correlation scheme for an arbitrarily weak local oscillator with the aim of detecting the available information on the quantum statistics of the signal field. The separation of the sub-Poisson effect, the squeezing effect, and the correlation of intensity and field strength noise are studied for general states of the signal field. Moreover, the influence of classical local oscillator noise is considered and shown to be suppressed for weak local oscillators. Eventually, spectral homodyne correlation methods are analyzed.

The paper is organized as follows. Section II introduces the two homodyne correlation measurement schemes and gives some general relations for the observed correlation functions. The various quantum statistical effects accessible with a weak local oscillator are considered in Sec. III including the problem of their separation from each other. Section IV is devoted to the effects of local oscillator noise and in Sec. V spectral measurements are studied. A summary and some conclusions are given in Sec. VI.

II. HOMODYNE CORRELATION SCHEMES

Let us consider in the following two different schemes for homodyne correlation measurements which are suited for the simultaneous detection of squeezing, sub-Poissonian statistics and the normally ordered correla-

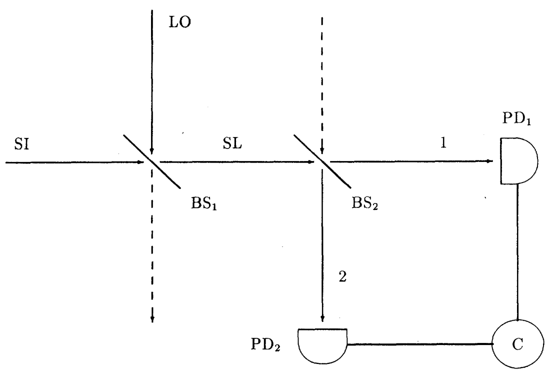

FIG. 1. Homodyne intensity correlation scheme. The signal field (SI) is superimposed by the first beam splitter (BS_1) with the local oscillator (LO), the resulting superimposed light (SL) is recorded by means of an intensity correlation device consisting of a second beam splitter (BS_2) dividing the superimposed light into two channels 1 and 2 and the intensity correlations are measured with two photodetectors (PD_1 and PD_2) and a correlator (C).

tion between the field strength and intensity fluctuations. Moreover, these schemes are of interest for the low-efficiency detection of phase-sensitive quantum statistics of light.

In the first scheme, in the following called homodyne intensity correlation scheme, the signal field is superimposed by a first beam splitter with the local oscillator. The superimposed light is then recorded by means of a typical intensity correlation device, consisting of a second beam splitter, two photodetectors, and a correlator, cf. Fig. 1. This scheme has been proposed in [25] for the simultaneous detection of squeezing and anomalous moments in resonance fluorescence. In the case of the second scheme, hereafter called homodyne cross correlation scheme, the intensity correlations between two different fields are recorded. The superpositions of the signal and the local oscillator are directly obtained at the input beam splitter of the correlation device, see Fig. 2. In this manner the two photodetectors are irradiated by differently phase shifted superpositions of signal field and local oscillator. An observational scheme of this type has been studied for a strong local oscillator in the context of the detection of squeezed light [24].

In the following we will consider the measured correlations in both schemes in more detail, with special emphasis on the situation for arbitrarily weak local oscillators.

A. Homodyne intensity correlation scheme

For analyzing the detection scheme given in Fig. 1, let us start with the events recorded by the correlation device. Based on the quantum theory of photodetection [27,28] together with the action of a beam splitter on quantized light fields we arrive at the second-order correlation of the recorded events in the form [26]

$$\overline{n_1(t_1,\Delta t_1)n_2(t_2,\Delta t_2)} = \eta_1\eta_2\,\Delta t_1\Delta t_2|\bar{T}_2|^2|\bar{R}_2|^2 \times G^{(2,2)}(t_1-x/c,t_2-y/c_2), \tag{6}$$

where

$$G^{(2,2)}(t_1-x/c,t_2-y/c) = \left\langle \hat{E}^{(-)}_{\rm SL}(t_1-x/c)\hat{E}^{(-)}_{\rm SL}(t_2-y/c)\hat{E}^{(+)}_{\rm SL}(t_2-y/c) \times\hat{E}^{(+)}_{\rm SL}(t_1-x/c)\right\rangle \tag{7}$$

is the normally ordered intensity correlation function of the superimposed light (SL) and the indices at the numbers of events and the efficiencies label the individual detectors. It is seen from Eq. (6) that the effect of small detection efficiencies on the measured correlations only consists in the factor $\eta_1\eta_2$, that is the efficiencies do not influence the measured correlation functions absent from decreasing it by a factor. This is a well known fact that allowed, for example, the detection of the intensity correlation function of the light from an atomic beam with extremely small overall efficiencies [29]. In the context of the present paper this fact is of importance since it allows homodyning in the case of low efficiencies. In contrast to this situation, in a balanced homodyne detection scheme a small efficiency smoothes out the field strength distributions to be measured [cf. Eq. (4)], which in practice also prevents the determination of the quantum state of the field based on various methods [19–23].

It is worth noting that the outgoing fields appearing in Eq. (7) behave like effectively free fields so that the positive and negative frequency operators, respectively, commute among each other [26]. Consequently, the intensity correlation function in Eq. (7) fulfils the symmetry relation

$$G^{(2,2)}(t,t+\tau) = G^{(2,2)}(t+\tau,t) \tag{8}$$

and we may confine ourselves in the following to the consideration of $G^{(2,2)}(t,t+\tau)$ for $\tau>0$. Note that Eq. (8) implies that under stationary conditions the correlation function

$$G^{(2,2)}(\tau) = \lim_{t\to\infty} G^{(2,2)}(t,t+\tau) \tag{9}$$

fulfils the relation

$$G^{(2,2)}(\tau) = G^{(2,2)}(-\tau), \tag{10}$$

so that the measured correlations are symmetric with respect to the delay time.

For comparison it is of interest to consider the corresponding decorrelated result,

$$\overline{n_1(t_1,\Delta t_1)}\;\overline{n_2(t_2,\Delta t_2)} = \eta_1\eta_2\,\Delta t_1\Delta t_2|\bar{T}_2|^2|\bar{R}_2|^2 \times\left\langle \hat{I}_{\rm SL}(t_1-x/c)\right\rangle \times\left\langle \hat{I}_{\rm SL}(t_2-y/c)\right\rangle, \tag{11}$$

which represents the product of mean numbers of events recorded by the two detectors independently of each other. In this equation we have introduced the intensity operator

$$\hat{I}_{\rm SL}(t) = \hat{E}_{\rm SL}^{(-)}(t)\, \hat{E}_{\rm SL}^{(+)}(t) \tag{12}$$

for the superimposed light.

We may express the measured intensity correlation function of the superimposed light in terms of the local oscillator and signal fields. Due to the action of the beam splitter BS_1 these fields are combined as

$$\hat{E}_{\rm SL}^{(+)}(t) = \bar{T}_1 \hat{E}_{\rm SI}^{(+)}(t) + \bar{R}_1 \hat{E}_{\rm LO}^{(+)}(t), \tag{13}$$

where the (complex) amplitude transmission and reflection coefficients of the (symmetric) beam splitters fulfil the relations

$$\bar{T}_i = \bar{T} = |\bar{T}|\, e^{i\phi_T}, \quad \bar{R}_i = \bar{R} = |\bar{R}|\, e^{i\phi_R}, \tag{14}$$

$$|\bar{T}|^2 + |\bar{R}|^2 = 1, \tag{15}$$

$$\phi_{\bar{T}} - \phi_{\bar{R}} = \pm\tfrac{1}{2}\pi, \tag{16}$$

for $i = 1, 2$. We assume that the local oscillator is in a coherent state, that is

$$\langle \cdots \hat{E}_{\rm LO}^{(+)}(t)\rangle = \langle \cdots\rangle\, \tilde{E}_{\rm LO} \exp[-i(\omega_0 t + \phi_{\rm LO})]\,,$$

$$\langle \hat{E}_{\rm LO}^{(-)}(t) \cdots \rangle = \langle \cdots\rangle\, \tilde{E}_{\rm LO} \exp[i(\omega_0 t + \phi_{\rm LO})]\,, \tag{17}$$

where $\tilde{E}_{\rm LO}$ is the real amplitude of the local oscillator. Combining Eqs. (7), (13)–(17) and introducing slowly varying field operators according to (2) we may express the intensity correlation functions in terms of the various orders $G_i^{(2,2)}$ $(i = 0, \ldots, 4)$ with respect to the local oscillator field,

$$G^{(2,2)}(t, t+\tau) = \sum_{i=0}^{4} G_i^{(2,2)}(t, t+\tau), \tag{18}$$

where

$$G_0^{(2,2)}(t, t+\tau) = |\bar{T}|^4 \Big\langle \hat{E}_{{\rm SI}\,\phi}^{(-)}(t) \hat{E}_{{\rm SI}\,\phi}^{(-)}(t+\tau) \times \hat{E}_{{\rm SI}\,\phi}^{(+)}(t+\tau) \hat{E}_{{\rm SI}\,\phi}^{(+)}(t) \Big\rangle, \tag{19}$$

$$G_1^{(2,2)}(t, t+\tau) = 2|\bar{T}|^3 |\bar{R}| \tilde{E}_{\rm LO}\, {\rm Re}\Bigg(\Big\langle \hat{E}_{{\rm SI}\,\phi}^{(-)}(t) \times \hat{E}_{{\rm SI}\,\phi}^{(-)}(t+\tau) \hat{E}_{{\rm SI}\,\phi}^{(+)}(t+\tau) \Big\rangle + \Big\langle \hat{E}_{{\rm SI}\,\phi}^{(-)}(t) \hat{E}_{{\rm SI}\,\phi}^{(-)}(t+\tau) \hat{E}_{{\rm SI}\,\phi}^{(+)}(t) \Big\rangle \Bigg), \tag{20}$$

$$G_2^{(2,2)}(t, t+\tau) = |\bar{T}|^2 |\bar{R}|^2 \tilde{E}_{\rm LO}^2 \Bigg[2\,{\rm Re}\Bigg(\Big\langle \hat{E}_{{\rm SI}\,\phi}^{(+)}(t+\tau) \times \hat{E}_{{\rm SI}\,\phi}^{(+)}(t) \Big\rangle + \Big\langle \hat{E}_{{\rm SI}\,\phi}^{(-)}(t) \hat{E}_{{\rm SI}\,\phi}^{(+)}(t+\tau) \Big\rangle \Bigg) + \Big\langle \hat{E}_{{\rm SI}\,\phi}^{(-)}(t) \hat{E}_{{\rm SI}\,\phi}^{(+)}(t) \Big\rangle + \Big\langle \hat{E}_{{\rm SI}\,\phi}^{(-)}(t+\tau) \hat{E}_{{\rm SI}\,\phi}^{(+)}(t+\tau) \Big\rangle \Bigg], \tag{21}$$

$$G_3^{(2,2)}(t, t+\tau) = 2|\bar{T}||\bar{R}|^3 \tilde{E}_{\rm LO}^3\, {\rm Re}\Bigg(\Big\langle \hat{E}_{{\rm SI}\,\phi}^{(+)}(t) \Big\rangle + \Big\langle \hat{E}_{{\rm SI}\,\phi}^{(+)}(t+\tau) \Big\rangle \Bigg), \tag{22}$$

$$G_4^{(2,2)}(t, t+\tau) = |\bar{R}|^4 \tilde{E}_{\rm LO}^4. \tag{23}$$

With respect to the signal field, the functions $G_0^{(2,2)}$ to $G_4^{(2,2)}$ are determined by various types of correlation functions of fourth to zeroth order. Note that some of these correlation functions are not accessible in direct photocorrelation measurements.

B. Homodyne cross correlation scheme

Let us consider now the detection scheme according to Fig. 2. Compared with the first scheme, the detectors do not record the intensity correlation of a given (superimposed) light field. Instead, the second-order correlation of the events is proportional to a mixed field correlation function,

$$\overline{n_1(t_1, \Delta t_1) n_2(t_2, \Delta t_2)} = \eta_1 \eta_2\, \Delta t_1 \Delta t_2 \times \mathcal{G}^{(2,2)}(t_1 - x/c, t_2 - y/c), \tag{24}$$

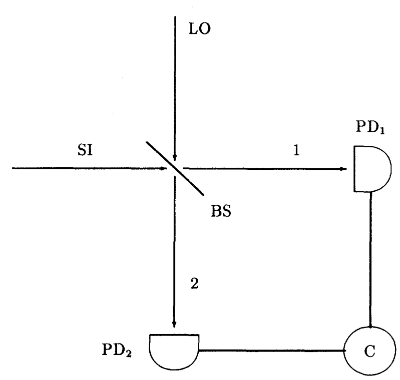

FIG. 2. Homodyne cross correlation scheme. The signal field (SI) is superimposed by a beam splitter (BS) with the local oscillator (LO), the intensity cross correlations of the resulting superimposed fields in the channels 1 and 2 are measured with two photodetectors (PD_1 and PD_2) and a correlator (C).

$$\mathcal{G}^{(2,2)}(t_1-x/c,t_2-y/c) = \left\langle \hat{E}^{(-)}_{s,1}(t_1-x/c)\hat{E}^{(-)}_{s,2}(t_2-y/c)\hat{E}^{(+)}_{s,2}(t_2-y/c) \times\hat{E}^{(+)}_{s,1}(t_1-x/c)\right\rangle, \quad (25)$$

where $\hat{E}^{(+)}_{s,1}$ and $\hat{E}^{(+)}_{s,2}$ are the two (different) superimposed fields respectively in the output channels 1 and 2 of the cross correlation scheme. Analogously to Eq. (13) they are given by

$$\hat{E}^{(+)}_{s,1}(t) = \bar{T}\hat{E}^{(+)}_{SI}(t) + \bar{R}\hat{E}^{(+)}_{LO}(t), \quad (26)$$

$$\hat{E}^{(+)}_{s,2}(t) = \bar{T}\hat{E}^{(+)}_{LO}(t) + \bar{R}\hat{E}^{(+)}_{SI}(t). \quad (27)$$

In the following we will again consider the correlation function $\mathcal{G}^{(2,2)}(t,t+\tau)$ for $\tau>0$. In order to derive the corresponding result for $\tau<0$ we may apply the relation

$$\mathcal{G}^{(2,2)}(t,t+\tau) = \overline{\mathcal{G}}^{(2,2)}(t+\tau,t), \quad (28)$$

where

$$\overline{\mathcal{G}}^{(2,2)}(t,t+\tau) = \langle\hat{E}^{(-)}_{s,2}(t)\hat{E}^{(-)}_{s,1}(t+\tau) \times\hat{E}^{(+)}_{s,1}(t+\tau)\hat{E}^{(+)}_{s,2}(t)\rangle. \quad (29)$$

Thus it is seen that the result for $\mathcal{G}^{(2,2)}(t,t+\tau)$ in the case $\tau<0$ is easily obtained from the result for $\tau>0$ by the replacements

$$t\rightarrow t-|\tau|, \quad t+\tau\rightarrow t \quad (30)$$

and, in view of Eqs. (26) and (27) together with (2), (3), and (16),

$$|\bar{T}|\leftrightarrow|\bar{R}|, \quad (31)$$

$$\hat{E}^{(\pm)}_{SI\,\phi}\rightarrow-\hat{E}^{(\pm)}_{SI\,\phi}. \quad (32)$$

In the stationary case [applying the definition in Eq. (9) for the functions $\mathcal{G}^{(2,2)}$ and $\overline{\mathcal{G}}^{(2,2)}$] we get in view of Eq. (28)

$$\mathcal{G}^{(2,2)}(\tau) = \overline{\mathcal{G}}^{(2,2)}(-\tau). \quad (33)$$

We may again consider the product of independent events recorded by the two detectors,

$$\overline{n_1(t_1,\Delta t_1)}\,\overline{n_2(t_2,\Delta t_2)} = \eta_1\eta_2\,\Delta t_1\Delta t_2\left\langle\hat{I}_{s,1}(t_1-x/c)\right\rangle \times\left\langle\hat{I}_{s,2}(t_2-y/c)\right\rangle, \quad (34)$$

with the two intensity operators defined in analogy to Eq. (12).

Combining Eqs. (2), (14)–(17), and (25)–(27) the correlation function measured in the cross correlation scheme ($\tau>0$) can also be expressed in terms of its orders $\mathcal{G}^{(2,2)}_i(t,t+\tau)$ with respect to the local oscillator field. We arrive at

$$\mathcal{G}^{(2,2)}(t,t+\tau) = \sum_{i=0}^{4}\mathcal{G}^{(2,2)}_i(t,t+\tau), \quad (35)$$

where

$$\mathcal{G}^{(2,2)}_0(t,t+\tau) = |\bar{R}|^2|\bar{T}|^2\left\langle\hat{E}^{(-)}_{SI\,\phi}(t)\hat{E}^{(-)}_{SI\,\phi}(t+\tau)\hat{E}^{(+)}_{SI\,\phi}(t+\tau)\hat{E}^{(+)}_{SI\,\phi}(t)\right\rangle, \quad (36)$$

$$\mathcal{G}^{(2,2)}_1(t,t+\tau) = 2|\bar{T}||\bar{R}|\tilde{E}_{LO}\,\mathrm{Re}\left(|\bar{R}|^2\left\langle\hat{E}^{(-)}_{SI\,\phi}(t)\hat{E}^{(-)}_{SI\,\phi}(t+\tau)\hat{E}^{(+)}_{SI\,\phi}(t+\tau)\right\rangle-|\bar{T}|^2\left\langle\hat{E}^{(-)}_{SI\,\phi}(t)\hat{E}^{(-)}_{SI\,\phi}(t+\tau)\hat{E}^{(+)}_{SI\,\phi}(t)\right\rangle\right), \quad (37)$$

$$\mathcal{G}^{(2,2)}_2(t,t+\tau) = -|\bar{T}|^2|\bar{R}|^2\tilde{E}^2_{LO}\,2\,\mathrm{Re}\left(\left\langle\hat{E}^{(+)}_{SI\,\phi}(t+\tau)\hat{E}^{(+)}_{SI\,\phi}(t)\right\rangle+\left\langle\hat{E}^{(-)}_{SI\,\phi}(t)\hat{E}^{(+)}_{SI\,\phi}(t+\tau)\right\rangle\right) +|\bar{T}|^4\tilde{E}^2_{LO}\left\langle\hat{E}^{(-)}_{SI\,\phi}(t)\hat{E}^{(+)}_{SI\,\phi}(t)\right\rangle+|\bar{R}|^4\tilde{E}^2_{LO}\left\langle\hat{E}^{(-)}_{SI\,\phi}(t+\tau)\hat{E}^{(+)}_{SI\,\phi}(t+\tau)\right\rangle, \quad (38)$$

$$\mathcal{G}^{(2,2)}_3(t,t+\tau) = 2|\bar{T}|^3|\bar{R}|\tilde{E}^3_{LO}\,\mathrm{Re}\left(\left\langle\hat{E}^{(+)}_{SI\,\phi}(t)\right\rangle\right)-2|\bar{T}||\bar{R}|^3\tilde{E}^3_{LO}\,\mathrm{Re}\left(\left\langle\hat{E}^{(+)}_{SI\,\phi}(t+\tau)\right\rangle\right), \quad (39)$$

$$\mathcal{G}^{(2,2)}_4(t,t+\tau) = |\bar{T}|^2|\bar{R}|^2\tilde{E}^4_{LO}. \quad (40)$$

Two differences are observed when we compare the results for the correlation function $\mathcal{G}^{(2,2)}$ observed in the homodyne cross correlation scheme with the corresponding expressions for $G^{(2,2)}$ recorded in the homodyne intensity correlation scheme. The first, less important one is that some reflection and transmission factors are exchanged in various contributions to the correlation functions. Of more importance is the second difference consisting in the changes of the signs of some contributions, which appear due to the different phase shifts in the two output channels of the cross correlation scheme. This difference will influence the details of the determination of the various correlation effects.

III. OBSERVED EFFECTS AND THEIR SEPARATION

In this section we consider the effects observable in the homodyne correlation schemes introduced above. According to Sec. II the basic quantities observed in both the measurement schemes are the second-order correlations of events $\overline{n_1(t_1,\Delta t_1)n_2(t_2,\Delta t_2)}$ and the corresponding product $\overline{n_1(t_1,\Delta t_1)}\ \overline{n_2(t_2,\Delta t_2)}$ of events recorded by the two detectors independently of each other. Both kinds of quantities have been related to field correlation functions, the proportionality factor containing the detection efficiencies in the same manner for both cases. Thus we may restrict the following consideration to the corresponding field correlation functions.

Since any physical correlation function decorrelates for sufficiently large time delays it is advantageous to compare the short-time value of the correlation function with its long-time value. In this sense we may introduce for the two detection schemes the following measures for photon pair correlations:

$$\Delta G^{(2,2)}(t) = G^{(2,2)}(t,t) - \lim_{\tau\to\infty} G^{(2,2)}(t,t+\tau) \qquad (41)$$

for the homodyne intensity correlation scheme and correspondingly

$$\Delta \mathcal{G}^{(2,2)}(t) = \mathcal{G}^{(2,2)}(t,t) - \lim_{\tau\to\infty} \mathcal{G}^{(2,2)}(t,t+\tau) \qquad (42)$$

for the homodyne cross correlation scheme. Physically these quantities characterize the (equal time) pair correlations of the events recorded in the two output channels of the correlation devices. Note that the relations

$$\lim_{\tau\to\infty} G^{(2,2)}(t,t+\tau) = \left\langle \hat{I}_{\rm SL}(t)\right\rangle \left\langle \hat{I}_{\rm SL}(\infty)\right\rangle \qquad (43)$$

and

$$\lim_{\tau\to\infty} \mathcal{G}^{(2,2)}(t,t+\tau) = \left\langle \hat{I}_{\rm s,1}(t)\right\rangle \left\langle \hat{I}_{\rm s,2}(\infty)\right\rangle \qquad (44)$$

are fulfilled, so that these expressions are related to the uncorrelated events according to Eqs. (11) and (34).

Based on Eqs. (18) and (35), respectively, we write

$$\Delta G^{(2,2)}(t) = \sum_{i=0}^{4} \Delta G_i^{(2,2)}(t) \qquad (45)$$

and

$$\Delta \mathcal{G}^{(2,2)}(t) = \sum_{i=0}^{4} \Delta \mathcal{G}_i^{(2,2)}(t), \qquad (46)$$

where the subscript i again denotes the order with respect to the local oscillator field strength. From Eqs. (23) and (40) we easily deduce

$$\Delta G_4^{(2,2)}(t) = \Delta \mathcal{G}_4^{(2,2)}(t) = 0. \qquad (47)$$

The physical effects represented by the other terms are briefly discussed below.

For convenience we will introduce in the following the operator of the signal field intensity:

$$\hat{I}_{\rm SI}(t) = \hat{E}^{(-)}_{{\rm SI}\,\phi}(t)\hat{E}^{(+)}_{{\rm SI}\,\phi}(t), \qquad (48)$$

the standard notation :: for normally ordering of field operators, and the notation $\Delta\hat{A}(t) = \hat{A}(t) - \langle\hat{A}(t)\rangle$, the latter should be distinguished from the definitions in Eqs. (41) and (42) applying to correlation functions rather than operators.

A. Sub-Poissonian statistics

Let us first consider the effects in zeroth order in the local oscillator field. In the homodyne intensity correlation scheme we derive from Eq. (41) together with (19)

$$\Delta G_0^{(2,2)}(t) = |\bar{T}|^4(\langle : [\hat{I}_{\rm SI}(t)]^2 :\rangle - \langle\hat{I}_{\rm SI}(t)\rangle\langle\hat{I}_{\rm SI}(\infty)\rangle). \qquad (49)$$

In the same way we get from Eqs. (42) and (36) for the homodyne cross correlation scheme

$$\Delta \mathcal{G}_0^{(2,2)}(t) = |\bar{T}|^2|\bar{R}|^2(\langle : [\hat{I}_{\rm SI}(t)]^2 :\rangle - \langle\hat{I}_{\rm SI}(t)\rangle\langle\hat{I}_{\rm SI}(\infty)\rangle). \qquad (50)$$

Under stationary conditions these expressions may be further simplified (in the steady-state case time arguments are throughout omitted in the following)

$$\Delta G_0^{(2,2)} = |\bar{T}|^4\left\langle : \left(\Delta\hat{I}_{\rm SI}\right)^2 : \right\rangle, \qquad (51)$$

$$\Delta \mathcal{G}_0^{(2,2)} = |\bar{T}|^2|\bar{R}|^2\left\langle : \left(\Delta\hat{I}_{\rm SI}\right)^2 : \right\rangle. \qquad (52)$$

It is seen that in both kinds of homodyne correlation schemes the contribution to the measured correlations which is independent of the local oscillator field is proportional to the normally ordered variance of the intensity of the signal field. Therefore, negative contributions of this term indicate a sub-Poissonian statistics of the signal field.

It is worth noting that in the case of the homodyne intensity correlation scheme the sub-Poissonian statistics of the signal field directly contributes to the sub-Poissonian statistics of the superimposed light. In this case, from Eqs. (7) and (41) under stationary conditions we arrive at

$$\Delta G^{(2,2)} = \left\langle : \left(\Delta\hat{I}_{\rm SL}\right)^2 : \right\rangle. \qquad (53)$$

On the other hand, in the cross correlation scheme there does not exist a single superimposed light. Therefore, nonclassical effects of the signal field cannot be interpreted as contributions to some overall nonclassical effect in this scheme.

B. Correlation of intensity and field strength noise

Making use of Eqs. (20), (41) and (37), (42), respectively, we get for the first-order terms in the local oscillator field the results

$$\Delta G_1^{(2,2)}(t) = |\bar{T}|^3|\bar{R}|\tilde{E}_{\rm LO}[2\langle :\hat{E}_{{\rm SI}\,\phi}(t)\hat{I}_{\rm SI}(t):\rangle - \langle \hat{E}_{{\rm SI}\,\phi}(\infty)\rangle\langle \hat{I}_{\rm SI}(t)\rangle - \langle \hat{E}_{{\rm SI}\,\phi}(t)\rangle\langle \hat{I}_{\rm SI}(\infty)\rangle], \quad (54)$$

and

$$\Delta \mathcal{G}_1^{(2,2)}(t) = |\bar{T}||\bar{R}|\left(|\bar{R}|^2 - |\bar{T}|^2\right)\tilde{E}_{\rm LO}\left\langle :\hat{E}_{{\rm SI}\,\phi}(t)\hat{I}_{\rm SI}(t):\right\rangle - |\bar{T}||\bar{R}|^3\tilde{E}_{\rm LO}\left\langle \hat{E}_{{\rm SI}\,\phi}(t)\right\rangle\left\langle \hat{I}_{\rm SI}(\infty)\right\rangle + |\bar{T}|^3|\bar{R}|\tilde{E}_{\rm LO}\left\langle \hat{E}_{{\rm SI}\,\phi}(\infty)\right\rangle\left\langle \hat{I}_{\rm SI}(t)\right\rangle. \quad (55)$$

For stationary light fields these expressions simplify according to

$$\Delta G_1^{(2,2)} = 2|\bar{T}|^3|\bar{R}|\tilde{E}_{\rm LO}\left\langle :\Delta\hat{E}_{{\rm SI}\,\phi}\Delta\hat{I}_{\rm SI}:\right\rangle, \quad (56)$$

$$\Delta \mathcal{G}_1^{(2,2)} = |\bar{T}||\bar{R}|\left(|\bar{R}|^2 - |\bar{T}|^2\right)\tilde{E}_{\rm LO}\left\langle :\Delta\hat{E}_{{\rm SI}\,\phi}\Delta\hat{I}_{\rm SI}:\right\rangle. \quad (57)$$

The contributions given in Eqs. (56) and (57) represent the (normally ordered) correlation of field strength and intensity fluctuations of the signal field. This quantum statistical moment contains two noncommuting observables. Such kinds of correlations are usually not observed in direct detection, therefore, these contributions have been denoted as anomalous moments [25]. In the homodyne intensity correlation scheme, from Eqs. (45), (56), and (53) it is obvious that negative values of the normally ordered correlation of intensity and field strength of the signal field also increase the tendency of the superimposed light to become sub-Poissonian. As already noted above, in the homodyne cross correlation scheme a similar interpretation of the same correlation term fails.

Another difference between the first-order terms in the two measurement schemes should be emphasized. The correlations of intensity and field strength fluctuation can be measured in the homodyne intensity correlation scheme with 50%:50% beam splitters whereas unequal partitions are required to observe this term in the homodyne cross correlation scheme. Therefore, it appears to be useful to perform the cross correlation experiment by involving a beam splitter that significantly differs from a 50%:50% partition. A beam splitter close to a 14%:86% partition (with respect to the intensities) would be the optimum for observing the effect given in Eq. (57).

The question may arise for what kinds of radiation field states is the normally ordered correlation of fluctuations in the electric field strength and the intensity of relevance. For some fundamental fields in quantum optics, such as coherent states, photon number states, or thermal fields, such correlations do not exist. An example for the existence of correlations of this type is the resonance fluorescence from a single atom as has been demonstrated in [25]. Moreover, it is straightforward to prove that a squeezed coherent state [30] shows such correlations. A deeper understanding of these correlations, however, requires further research.

C. Squeezing

The second-order term with respect to the local oscillator field strength is readily derived from Eqs. (21) and (41) for the homodyne intensity correlation scheme in the form

$$\Delta G_2^{(2,2)}(t) = |\bar{T}|^2|\bar{R}|^2\tilde{E}_{\rm LO}^2[\langle :[\hat{E}_{{\rm SI}\,\phi}(t)]^2:\rangle - \langle \hat{E}_{{\rm SI}\,\phi}(t)\rangle\langle \hat{E}_{{\rm SI}\,\phi}(\infty)\rangle + \langle \hat{I}_{\rm SI}(t)\rangle - \langle \hat{I}_{\rm SI}(\infty)\rangle] \quad (58)$$

and from Eqs. (38) and (42) we get

$$\Delta \mathcal{G}_2^{(2,2)}(t) = -|\bar{T}|^2|\bar{R}|^2\tilde{E}_{\rm LO}^2\{\langle :[\hat{E}_{{\rm SI}\,\phi}(t)]^2:\rangle - \langle \hat{E}_{{\rm SI}\,\phi}(t)\rangle\rangle\hat{E}_{{\rm SI}\,\phi}(\infty)\rangle\} + |\bar{R}|^4\tilde{E}_{\rm LO}^2[\langle \hat{I}_{\rm SI}(t)\rangle - \langle \hat{I}_{\rm SI}(\infty)\rangle], \quad (59)$$

for the homodyne cross correlation scheme. Under steady-state conditions we eventually arrive at

$$\Delta G_2^{(2,2)} = |\bar{T}|^2|\bar{R}|^2\tilde{E}_{\rm LO}^2\left\langle :\left(\Delta\hat{E}_{{\rm SI}\,\phi}\right)^2:\right\rangle, \quad (60)$$

$$\Delta \mathcal{G}_2^{(2,2)} = -|\bar{T}|^2|\bar{R}|^2\tilde{E}_{\rm LO}^2\left\langle :\left(\Delta\hat{E}_{{\rm SI}\,\phi}\right)^2:\right\rangle. \quad (61)$$

It is seen from this result that both the measurement schemes are suited to record the normally ordered variance of the electric field strength and, therefore, squeezing can be detected. However, whereas in the homodyne intensity correlation measurement the squeezing effect of the signal gives a negative contribution to the photon pair correlation, in the cross correlation scheme the inverse is true: squeezing gives rise to a positive contribution to the correlations, cf. also [24]. In other words, in the latter scheme negative correlations can be obtained when the noise of the field strength of the signal field significantly exceeds the corresponding vacuum noise level. We already mentioned that the negative correlations in this scheme are no signature of a nonclassical effect so that this result is not surprising.

D. Transient electric field

In view of Eq. (47) the only remaining effect is that in the third order of the local oscillator field. From Eqs. (22), (41), (39), and (42) we derive for the schemes under study

$$\Delta G_3^{(2,2)}(t) = |\bar{T}||\bar{R}|^3\tilde{E}_{\rm LO}^3\left[\left\langle \hat{E}_{{\rm SI}\,\phi}(t)\right\rangle - \left\langle \hat{E}_{{\rm SI}\,\phi}(\infty)\right\rangle\right] \quad (62)$$

and

$$\Delta \mathcal{G}_3^{(2,2)}(t) = -|\bar{T}||\bar{R}|^3\tilde{E}_{\rm LO}^3\left[\left\langle \hat{E}_{{\rm SI}\,\phi}(t)\right\rangle - \left\langle \hat{E}_{{\rm SI}\,\phi}(\infty)\right\rangle\right]. \quad (63)$$

The observed effects consist in the deviation of the mean electric field strength from its stationary value. Consequently, under steady-state conditions there is no effect in the third order with respect to the local oscillator field strength, that is

$$\Delta G_3^{(2,2)} = \Delta \mathcal{G}_3^{(2,2)} = 0. \quad (64)$$

E. Separation of effects

The simultaneous detection of various effects with a given experimental setup may be of interest as long as a unique way exists to distinguish the different effects from each other. For the resonance fluorescence of a weakly driven two-level atom this problem has been discussed already for the homodyne intensity correlation scheme. This special light source allows a separation of the relevant terms under stationary conditions. The intensity fluctuations, the squeezing effect, and the intensity-field strength correlation may simply be distinguished by their different periodicities with respect to the local oscillator phase [25]. In the general case, where an arbitrary signal field is observed this is no longer true. In particular, the squeezing effect in general consists of the combination of a π periodic part with respect to the local oscillator phase and a part which is phase insensitive. Therefore, in general the concept for separating the different effects from each other needs some modification.

Let us consider the stationary situation in more detail since it yields well defined effects of the signal field. Suppose that the quantities $\Delta G^{(2,2)}$ and $\Delta \mathcal{G}^{(2,2)}$, respectively, consisting of the contributions according to Eqs. (51), (56), (60) and (52), (57), (61), have been measured. The intensity fluctuation terms are simply determined in the two schemes by blocking the local oscillator inputs. Therefore, the difference of the full correlation function and that obtained with the local oscillator being blocked leaves us with a combination of the squeezing term and the correlation between intensity and field strength fluctuations,

$$\Delta G^{(2,2)} - \Delta G^{(2,2)}\Big|_{\tilde{E}_{\mathrm{LO}}=0} = \Delta G_1^{(2,2)} + \Delta G_2^{(2,2)} \quad (65)$$

and

$$\Delta \mathcal{G}^{(2,2)} - \Delta \mathcal{G}^{(2,2)}\Big|_{\tilde{E}_{\mathrm{LO}}=0} = \Delta \mathcal{G}_1^{(2,2)} + \Delta \mathcal{G}_2^{(2,2)}. \quad (66)$$

One way of separating the remaining terms could be the variation of the local oscillator field strength. Moreover, the terms $\Delta G_1^{(2,2)}$ and $\Delta \mathcal{G}_1^{(2,2)}$, respectively, are 2π periodic with respect to the local oscillator field as can be seen from Eqs. (56) and (57) together with (2). Since the squeezing term ($\Delta G_2^{(2,2)}$ or $\Delta \mathcal{G}_2^{(2,2)}$) does not contain a 2π periodic contribution it can be separated from the correlations of intensity and field strength noise after constructing the expressions given in Eq. (65) or (66).

In the homodyne cross correlation scheme one may easily determine the squeezing effect of the signal field by making use of Eq. (66) together with (57). In this case the correlations between the fluctuations of the intensity and the field strength of the signal are simply suppressed by using a 50%:50% beam splitter, see Eq. (57). Repeating the measurements with a beam splitter significantly different from a 50%:50% partition one could obtain both the contributions of Eq. (66).

IV. ATTENUATION OF LOCAL OSCILLATOR NOISE

Analyzing the homodyne cross correlation scheme for a strong local oscillator [24], the authors have pointed out that the fact that the local oscillator noise is not balanced out makes the method less attractive for the detection of squeezed light. However, it will be shown in this section that this disadvantage can almost be eliminated in the case of a weak local oscillator. For this purpose let us consider stationary Gaussian amplitude fluctuations of the local oscillator [31], where

$$\tilde{E}_{\mathrm{LO}}(t) = E_0 + \delta E(t), \quad (67)$$

$$\overline{\delta E(t)} = 0 \quad (68)$$

and

$$\overline{\delta E(0)\delta E(\tau)} = \overline{(\delta E)^2}\, \exp(-\gamma|\tau|), \quad (69)$$

γ and $\overline{(\delta E)^2}$, respectively, are the correlation time and the variance of the amplitude fluctuations, the bar denotes stochastic averaging over the classical laser fluctuations. We will not consider phase fluctuations of the local oscillator for the following reason. In homodyne experiments of the type we are interested in usually the phase diffusions of the local oscillator and the signal field are correlated so that only a small difference diffusion of the corresponding phases could be relevant. We may, therefore, assume that the effective linewidth of this small effect is much smaller than the characteristic linewidth of the signal field.

Calculating the effects of the laser noise we may again consider the various powers in the local oscillator amplitude E_0 (which corresponds to $\tilde{E}_{\mathrm{LO}}$ in the above considerations where the local oscillator noise has been disregarded). Let us consider, for example, a typical contribution of the local oscillator noise effects to the measured correlations which may be regarded as the direct competition to the squeezing effect. We only deal with the homodyne cross correlation scheme since the situation in the homodyne intensity correlation scheme is essentially the same. Calculating the fourth-order terms in $\tilde{E}_{\mathrm{LO}}$ in the presence of amplitude fluctuations according to Eqs. (67)–(69) by considering averaged quantities $\overline{\mathcal{G}_4^{(2,2)}}$, we get in place of Eq. (47)

$$\overline{\Delta \mathcal{G}_4^{(2,2)}} = |\bar{T}|^2|\bar{R}|^2 \left\{ 4E_0^2\, \overline{(\delta E)^2} + 2\left[\overline{(\delta E)^2}\right]^2 \right\} \approx 4|\bar{T}|^2|\bar{R}|^2 E_0^2\, \overline{(\delta E)^2}, \quad (70)$$

where we made use of the suitable assumption that the relative amplitude noise of the local oscillator is small,

$$\frac{\overline{(\delta E)^2}}{E_0^2} = \epsilon \ll 1. \tag{71}$$

Now we compare the effect due to local oscillator noise given in Eq. (70) with the squeezing effect of the signal field in the measurement scheme under study. For this reason we replace in Eq. (61) $\tilde{E}_{\rm LO}$ by E_0 and arrive at a signal-to-noise ratio

$$\left|\frac{\Delta \mathcal{G}_2^{(2,2)}}{\overline{\Delta \mathcal{G}_4^{(2,2)}}}\right| = \frac{|\langle : (\Delta \hat{E}_{\mathrm{SI}\,\phi})^2 : \rangle|}{4\,\overline{(\delta E)^2}}. \tag{72}$$

This result might suggest that the signal-to-noise ratio is insensitive to the amplitude of the local oscillator. However, the classical fluctuations of the local oscillator can be attenuated in the same manner as its amplitude, for example by the use of an appropriate beam splitter. That is, the ratio ϵ defined in Eq. (71) may be regarded as a constant when the amplitude of the local oscillator is changed. Thus it is more instructive to rewrite the signal-to-noise ratio in the form

$$\left|\frac{\Delta \mathcal{G}_2^{(2,2)}}{\overline{\Delta \mathcal{G}_4^{(2,2)}}}\right| = \frac{|\langle : (\Delta \hat{E}_{\mathrm{SI}\,\phi})^2 : \rangle|}{4\,\epsilon E_0^2}. \tag{73}$$

It is easily seen that the usually preferred strong local oscillator may become crucial in the homodyne correlation scheme since the classical noise of the local oscillator, although it is assumed to be rather small, may prevent the detection of the quantum noise of the signal field we are interested in. On the other hand, when the local oscillator is sufficiently weak the situation may be changed. Let us assume that the strength of the local oscillator is comparable to the quantum noise of the signal field as has been proposed for the detection of squeezing in resonance fluorescence [25], that is

$$E_0^2 \approx |\langle : (\Delta \hat{E}_{\mathrm{SI}\,\phi})^2 : \rangle|. \tag{74}$$

Consequently, the order of magnitude of the signal-to-noise ratio is essentially determined by the inverse of the relative amplitude noise of the local oscillator,

$$\left|\frac{\Delta \mathcal{G}_2^{(2,2)}}{\overline{\Delta \mathcal{G}_4^{(2,2)}}}\right| \propto \frac{1}{\epsilon} \gg 1. \tag{75}$$

We may conclude that the influence of the (classical) amplitude fluctuations of the local oscillator in a homodyne correlation measurement scheme is almost suppressed provided the local oscillator is sufficiently weak. It should be noted that the situation is similar for the other contributions of the classical amplitude noise to the measured correlation functions, so that we renounce their detailed consideration here.

V. SPECTRAL MEASUREMENTS

So far we have considered two homodyne correlation measurement schemes with the aim to determine effects such as sub-Poissonian statistics, squeezing, and the normally ordered correlation of the fluctuations of intensity and electric field strength of a given signal field. In practice, however, effects such as squeezing may appear only in a given spectral range and a total squeezing effect as considered above may be absent. In such cases a more careful, spectral analysis of the measured data is required, as given for the cross correlation scheme in the strong local oscillator limit in [24]. In this section we are especially interested in spectral properties that can additionally be observed by applying a weak local oscillator.

Let us assume that the events recorded by the detectors of the correlation device are amplified and we may describe this amplification simply as a deterministic multiplication process so that we may write for the stochastic current [32,26]

$$i(t) = g\,e\,\frac{n(t,\Delta t)}{\Delta t}, \tag{76}$$

g being the corresponding gain factor. Thus we get for the mean photocurrent and the second-order correlation function of the currents i_1 and i_2 measured in the correlation schemes under study

$$\overline{i}\,(t) = \frac{g\,e}{\Delta t}\,\overline{n}(t,\Delta t), \tag{77}$$

$$\overline{i_1(t_1) i_2(t_2)} = \frac{g_1\,g_2\,e^2}{\Delta t_1 \Delta t_2}\,\overline{n_1(t_1,\Delta t_1) n_2(t_2,\Delta t_2)}. \tag{78}$$

According to this result together with Eqs. (6), (7) and (24), (25) for the two correlation schemes the statistical properties of the macroscopic photocurrent are directly related to the quantum correlations of the signal light field as considered above.

The classical photocurrent can be spectrally analyzed, which may simply be described by Fourier methods,

$$\tilde{i}(\omega) = \frac{1}{2\pi}\int dt\, e^{i\omega t}\, i(t). \tag{79}$$

Confining our consideration to stationary fields we readily derive for the spectral correlations of the current

$$\overline{\tilde{i}_1(\omega_1)\tilde{i}_2(\omega_2)} = \delta(\omega_1+\omega_2)\, C(\omega_2), \tag{80}$$

where the (second-order) current spectrum $C(\omega)$ is given by

$$C(\omega) = \frac{1}{2\pi}\int d\tau\, e^{i\omega\tau}\, \overline{i_1(0) i_2(\tau)}. \tag{81}$$

Correspondingly, for the stationary current noise spectrum we get

$$\Delta C(\omega) = \frac{1}{2\pi}\int d\tau\, e^{i\omega\tau}\, \overline{\Delta i_1(0) \Delta i_2(\tau)}, \tag{82}$$

where

$$\Delta i(t) = i(t) - \bar{i}. \tag{83}$$

It is straightforward to relate the fluctuation spectrum of the current to the corresponding spectrum of the light we are interested in. Combining Eqs. (6), (11), (77), (78), (82), and (83) we derive for the homodyne intensity correlation scheme

$$\Delta C(\omega) = g_1 g_2 \eta_1 \eta_2 e^2 |\bar{T}|^2 |\bar{R}|^2 S^{(2,2)}(\omega), \tag{84}$$

where $S^{(2,2)}(\omega)$ is the intensity fluctuation spectrum of the superimposed light according to

$$S^{(2,2)}(\omega) = \frac{1}{2\pi} \int d\tau \, e^{i\omega\tau} \langle : \Delta\hat{I}_{\rm SL}(0) \Delta\hat{I}_{\rm SL}(\tau) : \rangle. \tag{85}$$

In the same manner we arrive in the case of the homodyne cross correlation scheme by applying Eqs. (24) and (25) at

$$\Delta \mathcal{C}(\omega) = g_1 g_2 \eta_1 \eta_2 e^2 \mathcal{S}^{(2,2)}(\omega), \tag{86}$$

where the intensity cross correlation spectrum $\mathcal{S}^{(2,2)}(\omega)$ is given by

$$\mathcal{S}^{(2,2)}(\omega) = \frac{1}{2\pi} \int d\tau \, e^{i\omega\tau} \langle : \Delta\hat{I}_{s,1}(0) \Delta\hat{I}_{s,2}(\tau) : \rangle. \tag{87}$$

Whereas the spectrum $S^{(2,2)}(\omega)$ recorded in the homodyne intensity correlation scheme is readily proved to be real, this is in general not the case for $\mathcal{S}^{(2,2)}(\omega)$ derived in the cross correlation scheme. This difference is directly related to the different symmetry properties of the observed correlation functions with respect to the delay time as given in Eqs. (10) and (33).

It is worth noting that there is another way to determine the spectra given in Eqs. (85) and (87). The measured correlation functions can directly be Fourier transformed instead of analyzing the spectral properties of the photocurrents. Especially in the homodyne cross correlation scheme this requires one to record the correlation functions for positive and negative delay times.

We may now decompose the spectra with respect to the orders in the local oscillator amplitude as already done for the corresponding total effects. Let us pay attention to the phase-sensitive spectra, namely the squeezing spectrum and the spectrum of the correlation between intensity and field strength fluctuations. For the intensity correlation scheme these spectra are derived from Eq. (85) together with the correlation functions in (20) and (21), substracting the corresponding factored versions. Analogously, in the cross correlation scheme Eqs. (87), (37), and (38) are used.

In the second order with respect to the local oscillator field we find for the intensity correlation scheme

$$S_2^{(2,2)}(\omega) = |\bar{T}|^2 |\bar{R}|^2 \tilde{E}_{\rm LO}^2 S_{\rm SQ}(\omega), \tag{88}$$

which can be related to corresponding result for the cross correlation scheme via

$$\mathcal{S}_2^{(2,2)}(\omega) = -S_2^{(2,2)}(\omega). \tag{89}$$

The quantity

$$S_{\rm SQ}(\omega) = \frac{1}{2\pi} \int d\tau \, e^{i\omega\tau} \left\langle : \Delta\hat{E}_{{\rm SI}\,\phi}(0) \Delta\hat{E}_{{\rm SI}\,\phi}(\tau) : \right\rangle \tag{90}$$

is the well known squeezing spectrum [33] which can be recorded in both measurement schemes. Integrating over the full frequency range yields the total (stationary) squeezing effect,

$$\int d\omega \, S_{\rm SQ}(\omega) = \left\langle : \left(\Delta\hat{E}_{{\rm SI}\,\phi} \right)^2 : \right\rangle. \tag{91}$$

Let consider now the first-order contribution in the local oscillator field. In the homodyne intensity correlation scheme we derive from Eq. (85) by using the correlation function (20), its factored version, and the symmetry relation (8)

$$S_1^{(2,2)}(\omega) = |\bar{T}|^3 |\bar{R}| \tilde{E}_{\rm LO} \left[S_{I/E}(\omega) + S_{E/I}(\omega) \right]. \tag{92}$$

The spectra $S_{I/E}(\omega)$ and $S_{E/I}(\omega)$ correspondingly are defined by

$$S_{I/E}(\omega) = \frac{1}{2\pi} \int d\tau \, e^{i\omega\tau} \left\langle : \Delta\hat{E}_{{\rm SI}\,\phi}(0) \Delta\hat{I}_{\rm SI}(\tau) : \right\rangle \tag{93}$$

and

$$S_{E/I}(\omega) = \frac{1}{2\pi} \int d\tau \, e^{i\omega\tau} \left\langle : \Delta\hat{E}_{{\rm SI}\,\phi}(\tau) \Delta\hat{I}_{\rm SI}(0) : \right\rangle. \tag{94}$$

They fulfil the relation

$$S_{E/I}(\omega) = \left[S_{I/E}(\omega) \right]^*, \tag{95}$$

so that Eq. (92) can be written as

$$S_1^{(2,2)}(\omega) = 2|\bar{T}|^3 |\bar{R}| \tilde{E}_{\rm LO} {\rm Re} \left[S_{I/E}(\omega) \right]. \tag{96}$$

Applying Eq. (87) together with the correlation function (37) and the relations (30)–(32) for extending this function to negative τ values, we get for the homodyne cross correlation scheme

$$\mathcal{S}_1^{(2,2)}(\omega) = |\bar{T}||\bar{R}|\tilde{E}_{\rm LO}\{(|\bar{R}|^2 - |\bar{T}|^2){\rm Re}[S_{I/E}(\omega)] + i\,{\rm Im}[S_{I/E}(\omega)]\}. \tag{97}$$

This result reveals that both the real and the imaginary parts of the spectra defined in Eqs. (93) and (94) can be determined, where the real part requires the application of a beam splitter different from a 50%:50% partition. When the spectrum is derived from the correlation function $\mathcal{G}_1^{(2,2)}(\tau)$ it may be useful to construct the symmetric and the antisymmetric part of this function, respectively, which directly yield the real and imaginary part of $S_{I/E}(\omega)$ by means of Fourier transforms. Integrating the real part over the full spectral range yields the corresponding overall effect of the correlation of field strength and intensity fluctuations according to

$$\int d\omega \, {\rm Re} \left[S_{I/E}(\omega) \right] = \left\langle : \Delta\hat{E}_{{\rm SI}\,\phi} \Delta\hat{I}_{\rm SI} : \right\rangle, \tag{98}$$

whereas the overall effect of the imaginary part is zero,

$$\int d\omega \, {\rm Im} \left[S_{I/E}(\omega) \right] = 0. \tag{99}$$

These results show that both homodyne correlation schemes render it possible to detect phase-sensitive spectral properties such as spectral squeezing and the spectral correlation of intensity and field strength noise.

VI. SUMMARY AND CONCLUSIONS

In the present paper two kinds of homodyne correlation schemes have been studied, a homodyne intensity correlation scheme and a homodyne cross correlation scheme. These schemes are of particular interest in cases where low overall efficiencies prevent the detection of phase-sensitive effects of quantized light fields by means of standard balanced or unbalanced homodyning. This advantage of the correlation techniques is due to the fact that small efficiencies act on the measured correlation functions only as reduction by a factor, without altering their time-dependent shapes. Contrary to this situation, in balanced homodyne detection small overall efficiencies smooth out the measured field strength distributions and prevent the detection of the light effects in which we are interested.

In standard homodyne detection schemes usually a strong local oscillator is used. In this case one may derive from the measured correlation functions in the two homodyne correlation schemes the normally ordered variance of the field strength operator of the signal field, negative values of this quantities indicating squeezing. However, the relative squeezing effect observed with a strong local oscillator is very small. Its order of magnitude is given by the ratio of the normally ordered field strength variance of the signal to the intensity of the (strong) local oscillator. Moreover, local oscillator noise is not balanced out in the homodyne correlation schemes. Consequently, small classical excess noise of the local oscillator is effectively amplified when the local oscillator is strong, which makes the method less attractive for the detection of squeezed light.

To overcome these disadvantages of the measurement principle, we have studied the case of the local oscillator field being arbitrarily weak. In particular, it is advantageous to use a local oscillator being of the same strength as the signal effects under study. This leads to substantial modifications of the homodyne correlation methods. First, the observable (relative) squeezing effect becomes much larger. Second, new phase-sensitive effects become accessible. When the local oscillator is weak, three different statistical effects can be simultaneously observed: the normally ordered intensity variance, the normally ordered field strength variance, and the normally ordered correlation of the intensity and field strength fluctuations. Third, decreasing the strength of the local oscillator effectively leads to a deamplification of the disturbing effects of its classical noise. This resolves the problem that the local oscillator noise is not balanced out in the homodyne correlation schemes.

The observation of three different effects in one measurement scheme is useful as long as a unique way of their separation exists. It has been shown that this separation is possible in general by making use of the different dependences of the effects on the amplitude and the phase of the local oscillator. In the particular case of the homodyne cross correlation scheme it turns out that the detection of the correlation of intensity and field strength fluctuations requires an input beam splitter of the correlation device that significantly differs from a 50%:50% partition.

Besides the direct measurement of the mentioned effects, the corresponding spectral properties can be recorded by the homodyne correlation schemes. Thus three kinds of spectra are accessible, the spectrum of intensity fluctuations, the squeezing spectrum and last but not least the correlation spectrum of field strength and intensity fluctuations. Especially spectra of the latter type, which give the spectral correlation of two noncommuting observables, to our knowledge have not been studied so far and their further investigation should be of some interest.

ACKNOWLEDGMENT

This research was supported by the Deutsche Forschungsgemeinschaft.

[1] R.E. Slusher, L.W. Hollberg, B. Yurke, J.C. Mertz, and J.F. Valley, Phys. Rev. Lett. **55**, 2409 (1985).
[2] L.-A. Wu, H.J. Kimble, J.L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986).
[3] Some papers on squeezed light experiments are given in the special issue *Squeezed States of the Electromagnetic Field*, J. Opt. Soc. Am. B **4**, Oct. 1987.
[4] M. Xiao, L.-A. Wu, and H.J. Kimble, Phys. Rev. Lett. **59**, 278 (1987).
[5] P. Grangier, R.E. Slusher, B. Yurke, and A. La Porta, Phys. Rev. Lett. **59**, 2153 (1987).
[6] E.S. Polzik, J. Carri, and H.J. Kimble, Phys. Rev. Lett. **68**, 3020 (1992).
[7] J.H. Shapiro and H.P. Yuen, IEEE Trans. Inf. Theory, IT-**25**, 179 (1979).
[8] L. Mandel, Phys. Rev. Lett. **49**, 136 (1982).
[9] B.L. Schumaker, Opt. Lett. **9**, 189 (1984).
[10] B. Yurke, Phys. Rev. A **32**, 311 (1985).
[11] H.J. Carmichael, J. Opt. Soc. Am B **4**, 1588 (1987).
[12] H.P. Yuen and V.W.S. Chan, Opt. Lett. **8**, 177 (1983); **8**, 345(E) (1983).
[13] G.L. Abbas, V.W.S. Chan, and T.K. Yee, Opt. Lett. **8**, 419 (1983).
[14] B.L. Schumaker, Opt. Lett. **8**, 189 (1983).
[15] J.H. Shapiro, IEEE J. Quantum Electron. **QE-21**, 237 (1985).
[16] S.L. Braunstein, Phys. Rev. A **42**, 474 (1990).
[17] W. Vogel and J. Grabow, Phys. Rev. A **47**, 4227 (1993).
[18] K. Vogel and H. Risken, Phys. Rev. A **40**, 2847 (1989).
[19] D.T. Smithey, M. Beck, M.G. Raymer, and A. Faridani,

Phys. Rev. Lett. **70**, 1244 (1993).
[20] M. Freyberger, K. Vogel, and W. Schleich, Quantum Opt. **5**, 65 (1993); Phys. Lett. A **176**, 41 (1993).
[21] U. Leonhardt and H. Paul, Phys. Rev. A **48**, 4598 (1993).
[22] G.S. Agarwal and S. Chaturvedi, Phys. Rev. A **49**, R665 (1994).
[23] H. Kühn, D.-G. Welsch, and W. Vogel, J. Mod. Opt. **41**, 1607 (1994).
[24] Z.Y. Ou, C.K. Hong, and L. Mandel, Phys. Rev. A **36**, 192 (1987).
[25] W. Vogel, Phys. Rev. Lett. **67**, 2450 (1991).
[26] W. Vogel and D.-G. Welsch, *Lectures on Quantum Optics* (Akademie Verlag GmbH, Berlin / VCH Publishers, Inc., New York, 1994).
[27] P.L. Kelley and W.H. Kleiner, Phys. Rev. **136**, A316 (1964).
[28] R.J. Glauber, in *Quantum Optics and Electronics*, edited by C. DeWitt, A. Blandin, and C. Cohen-Tannoudji (Gordon and Breach, New York, 1965), p. 144.
[29] H.J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977).
[30] D. Stoler, Phys. Rev. D **1**, 3217 (1970); **4**, 1925 (1971); H.P. Yuen, Phys. Rev. A **13**, 2226 (1976).
[31] For the model of laser fluctuations considered and its relation to laser theory see M. Schubert, K.-E. Süsse, W. Vogel, and D.-G. Welsch, Opt. Quantum Electron. **12**, 65 (1980); **13**, 301 (1981).
[32] For example, see H.J. Carmichael, J. Opt. Soc. Am. B **4**, 1588 (1987); for a more detailed stochastic description of the multiplication process cf. H. Kühn and D.-G. Welsch, Phys. Rev. Lett. **67**, 580 (1991).
[33] C.W. Gardiner and C.M. Savage, Opt. Commun. **50**, 173 (1984); M.J. Collet, D.F. Walls, and P. Zoller, Opt. Commun. **52**, 145 (1984).