PHYSICAL REVIEW A VOLUME 51, NUMBER 5 MAY 1995

Noise amplification in dispersive nonlinear media

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(Received 4 June 1993; revised manuscript received 21 November 1994)

The propagation of a partially coherent optical beam through dispersive nonlinear media is investigated theoretically by using a phase-diffusion model for the laser beam. Changes in the second-order statistical properties during beam propagation depend on whether the nonlinear medium exhibits normal or anomalous group-velocity dispersion. In the case of normal dispersion, the coherence function and the corresponding optical spectrum remain unaffected. By contrast, modulation instability is found to be responsible for noise amplification in the anomalous dispersion regime, enhancing phase fluctuations and causing spectral distortion as well as coherence degradation. Under certain conditions, phase fluctuations exhibit temporal oscillations that lead to the characteristic spectral sidebands associated with modulation instability. The nonlinear Schrödinger equation is solved numerically to study the propagation regime in which the analytic theory becomes invalid.

PACS number(s): 42.65.−k, 42.50.Ar, 42.81.Dp

I. INTRODUCTION

An optical field propagating in a nonlinear dispersive medium may exhibit an instability known as modulation instability, a phenomenon shared by many branches of physics that deal with wave propagation in nonlinear media such as fluids [1], plasma [2], and dielectric media [3]. Modulation instability in nonlinear fiber optics [4–6] refers to a process in which small perturbations from the steady state grow exponentially as a result of self-phase modulation and anomalous group-velocity dispersion (GVD) occurring in optical fibers at wavelengths beyond 1.3 μm. In most studies of modulation instability, perturbations initiating the instability are assumed to occur inside the nonlinear medium itself whereas the input field is taken to be deterministic in nature. However, all optical beams are only partially coherent in practice. The amplitude and phase fluctuations associated with a partially coherent beam can act as a seed for modulation instability and are likely to get amplified in the process, resulting in significant changes in the coherence and spectral properties of the incident optical beam.

In the present work, the objective is to study the statistical properties of a partially coherent optical beam and their modification through modulation instability during propagation in a nonlinear dispersive medium. To this end, a nonlinear Schrödinger equation [1] (NLSE) is solved analytically and numerically with stochastic initial conditions. The effect of noise on the solutions of the NLSE has been studied extensively in the past [7–11], most recently in the context of spontaneous-emission noise generated by in-line amplifiers in soliton communication systems [12]. In most of the previous work, the NLSE is solved by adding a Langevin-noise term to it by assuming an additive type of external noise. In contrast, we investigate the evolution of a fluctuating continuous-wave (cw) beam by solving the deterministic NLSE. This problem has been considered before for thermal (or chaotic) fields whose statistics can be assumed to be Gaussian [9]. In practice, the laser beams cannot be modeled as a thermal field. Our method is based on the well-known phase-diffusion model of a laser operating far above threshold [13–15]. Considering noise as represented by fluctuations (in amplitude and phase), the NLSE with stochastic initial conditions is solved analytically by using a linearization procedure similar to that used for the analysis of modulation instability [6]. The main contribution to the optical spectrum comes from phase fluctuations which are shown to be enhanced by modulation instability in the case of anomalous GVD. The phase variance exhibits a substantial increase in its magnitude that is responsible for coherence degradation together with an oscillatory structure from where the familiar symmetric sidebands arise in the spectrum.

The paper is organized as follows. In Sec. II, we describe the statistical approach and state our basic assumptions. In Sec. III, we solve the NLSE for a stochastic cw beam and determine how amplitude and phase fluctuations are affected by modulation instability. In Sec. IV, we present the results showing the influence of noise amplification on the spectrum as well as on the coherence properties of the optical field. The NLSE is solved numerically in Sec. V to study the propagation regime in which the analytic theory becomes invalid. The main results are summarized in the concluding Sec. VI.

II. MODEL AND ASSUMPTIONS

We begin by considering a linearly polarized cw beam propagating through a dispersive nonlinear medium, such

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as a single-mode optical fiber. The field amplitude $E(z,t)$ in the slowly varying envelope approximation satisfies the NLSE [6],

$$i\frac{\partial E}{\partial z}-\frac{1}{2}\beta_2\frac{\partial^2 E}{\partial t^2}+n_2k|E|^2E=0\ , \tag{1}$$

where β_2 is the GVD parameter, $k=2\pi/\lambda$ is the wave number and n_2 is the nonlinear-index coefficient. Next we suppose that noise may be represented by small fluctuations from a stationary state, and based on the fact that any real laser emits light with random fluctuations of both the amplitude and phase, we write the stochastic field as

$$E(z,t)=[A+\delta A(z,t)]\exp\{i[\Phi+\delta\Phi(z,t)]\}\ , \tag{2}$$

where $A=\sqrt{I_0}$ and $\Phi=n_2kI_0z$ are the stationary values for the amplitude and phase for a field with the average beam intensity I_0, obtained from a full nonlinear analysis of Eq. (1). Here, δA and $\delta\Phi$ represent small fluctuations from the average values A and Φ, respectively, such that $\delta A \ll A$ and $\delta\Phi \ll \Phi$. We are essentially using a well-known model [10] of the laser in which the optical field is represented as the sum of a constant phasor and a weak Gaussian-noise phasor whose phase varies randomly over the entire 2π range. In that case, the amplitude and phase fluctuations of the total field, δA and $\delta\Phi$, can be shown [12] to represent real Gaussian random processes with zero average ($\langle\delta A\rangle=\langle\delta\Phi\rangle=0$, where angle brackets denote the ensemble average). The correlation functions of δA and $\delta\Phi$ depend on the statistical properties of the input beam. In the phase-diffusion model $\delta\Phi$, besides being a Gaussian process, is also assumed to be Markoffian such that frequency fluctuations represent white noise whereas phase fluctuations have a variance that grows linearly with time.

We are interested in the second-order statistical properties of the optical field after it has propagated an arbitrary distance z inside the fiber. To this end we define the field autocorrelation function or the mutual coherence function $\Gamma(z,t)$ as

$$\Gamma(z,t)=\langle E^*(z,0)E(z,t)\rangle \tag{3}$$

together with its Fourier transform

$$S(z,\omega)=\int_{-\infty}^{\infty}\Gamma(z,t)\exp(i\omega t)dt\ . \tag{4}$$

By substituting Eq. (2) into Eq. (3) and introducing the phase-shift variable $\Delta\Phi=\delta\Phi(z,t)-\delta\Phi(z,0)$, one arrives at the following result:

$$\Gamma(z,t)=I_0\langle e^{i\Delta\Phi}\rangle+\sqrt{I_0}\langle[\delta A(0,t)+\delta A(z,t)]e^{i\Delta\Phi}\rangle +O((\delta A)^2)\ , \tag{5}$$

where the last term is negligible compared with the others. It is clear that the strongest contribution to the outgoing spectrum comes from the first term consisting of pure phase fluctuations. The second term represents coupling between amplitude and phase fluctuations, but for simplicity of discussion we consider here only the first term. It is known [14,16] that the contribution of the second term leads to a slight asymmetry in the spectrum. Providing the amplitude and phase fluctuations remain small compared with the average beam intensity, the system although nonlinear behaves in a linear fashion with respect to the fluctuations. In this case the phase-shift variable $\Delta\Phi$ keeps the Gaussian character on propagation so that the average in Eq. (5) may be rewritten as

$$\Gamma(z,t)=I_0\exp[-\tfrac{1}{2}\langle(\Delta\Phi)^2(t)\rangle]\ , \tag{6}$$

and the problem is reduced to evaluating the phase variance $\langle(\Delta\Phi)^2(t)\rangle$. This is done in the next section.

III. NOISE AMPLIFICATION

To evaluate the phase variance we linearize Eq. (1) by substituting Eq. (2) into it and retaining only the terms linear in δA and $\delta\Phi$. The result is

$$\frac{1}{2}\beta_2\frac{\partial^2\delta A}{\partial t^2}+\sqrt{I_0}\frac{\partial\delta\Phi}{\partial z}-2n_2kI_0\delta A=0\ , \tag{7a}$$

$$\frac{1}{2}\beta_2\sqrt{I_0}\frac{\partial^2\delta\Phi}{\partial t^2}-\frac{\partial\delta A}{\partial z}=0\ . \tag{7b}$$

The set of Eqs. (7) is readily solved by the Fourier method. By introducing the Fourier transforms $\delta a(z,\Omega)$ and $\delta\phi(z,\Omega)$

$$\delta A(z,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\delta a(z,\Omega)\exp(-i\Omega t)d\Omega\ , \tag{8a}$$

$$\delta\Phi(z,t)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\delta\phi(z,\Omega)\exp(-i\Omega t)d\Omega\ , \tag{8b}$$

and using them in Eqs. (7), we obtain

$$\frac{\partial^2\delta a}{\partial z^2}+K^2\delta a=0\ , \tag{9a}$$

$$\frac{\partial^2\delta\phi}{\partial z^2}+K^2\delta\phi=0\ , \tag{9b}$$

where K represents the wave number and is given by

$$K^2=\frac{\beta_2\Omega^2}{4}(\beta_2\Omega^2+4n_2kI_0)\ . \tag{10}$$

In a medium with normal dispersion ($\beta_2>0$) the wave number K is always real. In this case, the noise component at the frequency Ω propagating inside the medium experiences a phase shift Kz, but its amplitude remains unaffected. As a result, the statistical properties of the input beam remains nearly unaffected during propagation. The situation changes drastically in a medium with anomalous dispersion ($\beta_2<0$) since K becomes imaginary within the frequency range $-\Omega_c<\Omega<\Omega_c$, where

$$\Omega_c=\left[\frac{4}{L_{\mathrm{NL}}|\beta_2|}\right]^{1/2} \tag{11}$$

and we have introduced a nonlinear length scale defined as $L_{\mathrm{NL}}=(n_2kI_0)^{-1}$. The noise components at the frequency Ω lying inside this range grow exponentially as

$$\delta a(z,\Omega)=\delta a(0,\Omega)\exp[g(\Omega)z/2]\ , \tag{12a}$$

$$\delta\phi(z,\Omega)=\delta\phi(0,\Omega)\exp[g(\Omega)z/2]\ , \tag{12b}$$

where the gain $g(\Omega)$ is defined as $g=2\text{Im}(K)$ or

$$g(\Omega)=|\beta_2\Omega|\sqrt{\Omega_c^2-\Omega^2} \quad \text{for } |\Omega|<\Omega_c \tag{13}$$

and $g(\Omega)=0$ for $|\Omega|>\Omega_c$.

The phase difference $\Delta\Phi(z,t)$ is now obtained by using Eqs. (8) and (12) and is given by

$$\Delta\Phi=\frac{1}{\sqrt{2\pi}}\int_0^\infty \delta\phi(0,\Omega)\exp(iKz) \times[\exp(-i\Omega t)-1]d\Omega+\text{c.c.} , \tag{14}$$

where c.c. means that we have to add the complex conjugate. Using the well-known property of a stochastic process

$$\langle\delta\phi^*(0,\Omega)\delta\phi(0,\Omega')\rangle=S_\Phi(0,\Omega)\delta(\Omega-\Omega') , \tag{15}$$

where $S_\Phi(0,\Omega)$ is the power spectrum of the input process $\delta\Phi(0,t)$, we finally obtain the following expression for the phase variance:

$$\langle(\Delta\Phi)^2(z,t)\rangle=\frac{2}{\pi}\int_0^\infty S_\Phi(0,\Omega)(1-\cos\Omega t) \times\exp[g(\Omega)z]d\Omega . \tag{16}$$

Equation (15) may be rewritten in the illustrative form

$$\langle(\Delta\Phi)^2(z,t)\rangle=\langle(\Delta\Phi)^2(0,t)\rangle +\frac{2}{\pi}\int_0^{\Omega_c} S_\Phi(0,\Omega)(1-\cos\Omega t) \times\{\exp[g(\Omega)z]-1\}d\Omega , \tag{17}$$

where one clearly sees how the selective amplification of noise components in the frequency range $|\Omega|<\Omega_c$ leads to an increase in the phase variance.

The spectral line shape of most lasers can be assumed to have a Lorentzian profile [10,11]. The corresponding frequency noise has a constant spectral density (white noise). For such lasers the spectral density of phase fluctuations can be written as $S_\Phi(0,\Omega)=2\pi\Delta\nu/\Omega^2$ together with $\langle(\Delta\Phi)^2(0,t)\rangle=2\pi\Delta\nu t$, where $\Delta\nu$ is the full width at half maximum (FWHM) of the Lorentzian spectral line shape [11]. In this case, the phase variance at a length L by using Eq. (17) is given by

$$\langle(\Delta\Phi)^2(L,\tau)\rangle=2\pi\frac{\Delta\nu}{\nu_c}\left[\tau+\frac{1}{\pi^2}\int_0^1[1-\cos(2\pi x\tau)] \times\{\exp[g(x)L]-1\} \times\frac{dx}{x^2}\right] , \tag{18}$$

where we have introduced a dimensionless time $\tau=\nu_c t$ with $\nu_c=\Omega_c/2\pi$. Equation (18) also shows that the dimensionless ratio $\Delta\nu/\nu_c$ plays an important role in governing the noise amplification process. This is what one might expect if we note that the linewidth $\Delta\nu$ is a measure of the degree of coherence of the input beam (it is inversely related to the coherence time) whereas ν_c is a characteristic frequency associated with the nonlinear dispersive medium.

It should be noted that when amplitude fluctuations are neglected, Eqs. (7) show that phase fluctuations do not undergo any change on propagation. Therefore, although amplitude fluctuations are neglected in Eq. (5), they play an important role on the process of noise amplification since without them there would be no amplification at all. As indicated in Eq. (12a) amplitude fluctuations also grow exponentially in the presence of modulation instability. Their neglect in Eq. (5) is justified as long as they remain well below the average intensity level. The results obtained here are likely to remain valid for lengths L such that $L\sim L_{\rm NL}$. For $L\gg L_{\rm NL}$ the amplitude fluctuations can become comparable to I_0. The linearization procedure used here is then not applicable, and fluctuations are unlikely to remain Gaussian. This regime can only be studied by solving the NLSE numerically. The numerical results are presented in Sec. V.

IV. ANALYTIC RESULTS

To illustrate how modulation instability modifies the coherence and the spectral properties of the partially coherent optical field through noise amplification, we calculate numerically the phase variance given by Eq. (18).

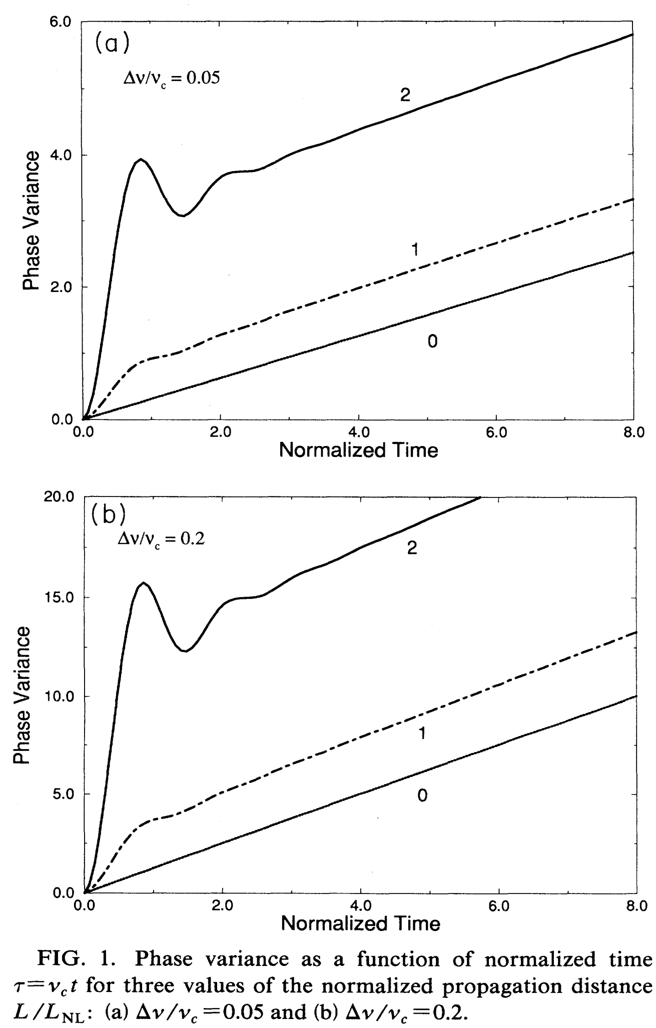

FIG. 1. Phase variance as a function of normalized time $\tau=\nu_c t$ for three values of the normalized propagation distance $L/L_{\rm NL}$: (a) $\Delta\nu/\nu_c=0.05$ and (b) $\Delta\nu/\nu_c=0.2$.

The phase variance is plotted as a function of τ in Fig. 1(a) for three values of the normalized propagation distance L/L_{NL} by choosing $\Delta\nu/\nu_c = 0.05$. Figure 1(b) is drawn under the identical conditions except for a larger bandwidth of the input field such that $\Delta\nu/\nu_c = 0.2$. In both cases, the effect of nonlinear propagation is not only to increase the overall variance but also to add an oscillatory structure to it. It will be seen later that these oscillations are responsible for spectral sidebands induced by modulation instability.

The enhanced phase fluctuations are expected to degrade the coherence of the transmitted light. Figure 2 shows the coherence functions corresponding to the phase variance of Fig. 1 by using Eq. (6). We find that the coherence time, defined as the time at which the value of the correlation function $\Gamma(z,t)$ is reduced by a factor of 2 compared with its value at $t=0$, is drastically diminished even for $L/L_{\mathrm{NL}}=2$. These plots show that the extent to which coherence degradation occurs depends on the bandwidth of the input light and on the propagation distance inside the nonlinear dispersive medium.

To obtain the spectral line shape, we take the Fourier transform of the coherence function numerically [see Eq. (4)]. The results are shown in Figs. 3(a) and 3(b) corresponding to the coherence functions of Figs. 2(a) and 2(b), respectively. The spectral features can be understood by noting that phase fluctuations of the input beam provide a seed for the modulation instability. Maximum amplification of noise occurs for the frequency components in the vicinity of $|\nu| = \nu_c\sqrt{2}$ since the gain of modulation instability is largest at that frequency [6]. For $\Delta\nu/\nu_c = 0.05$, the instability sidebands are clearly seen for $L/L_{\mathrm{NL}}=2$. For larger values of $\Delta\nu/\nu_c$, the sidebands are not easily resolved since the central line becomes so broad that it begins to merge with the sidebands. This feature is evident in Fig. 3(b) drawn $\Delta\nu/\nu_c = 0.2$. Since the analytic theory is unlikely to remain valid for $L \gg L_{\mathrm{NL}}$, one may question the validity of the $L/L_{\mathrm{NL}}=2$ curve in Fig. 3. The next section presents the numerical results that are valid for all fiber lengths, compares them with the analytic results of this section, and discusses the validity region of the analysis.

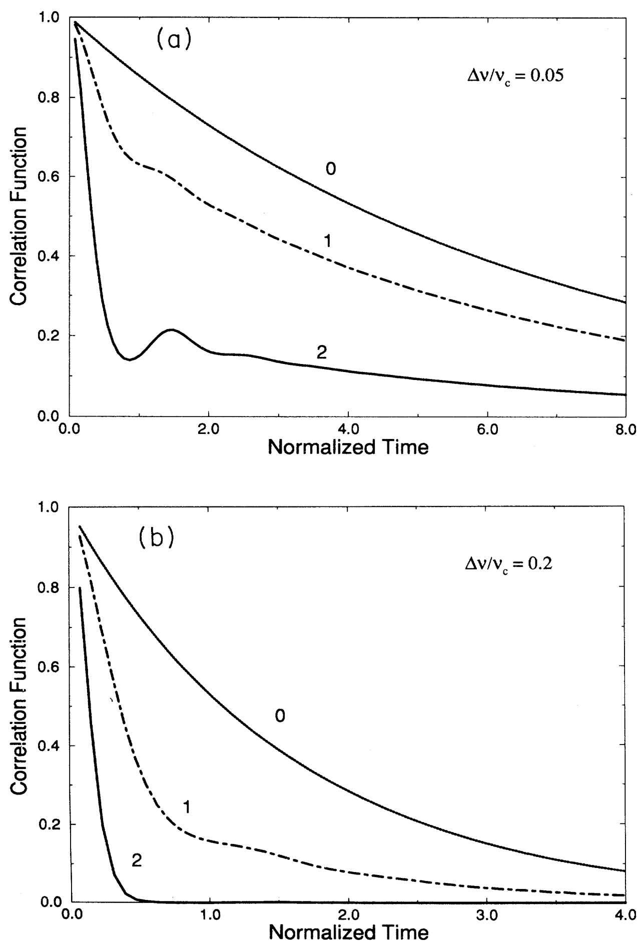

FIG. 2. Coherence degradation induced by modulation instability. Second-order coherence function (normalized to 1) is plotted as a function of normalized time τ for three values of L/L_{NL} under conditions identical to those of Fig 1: (a) $\Delta\nu/\nu_c = 0.05$ and (b) $\Delta\nu/\nu_c = 0.2$.

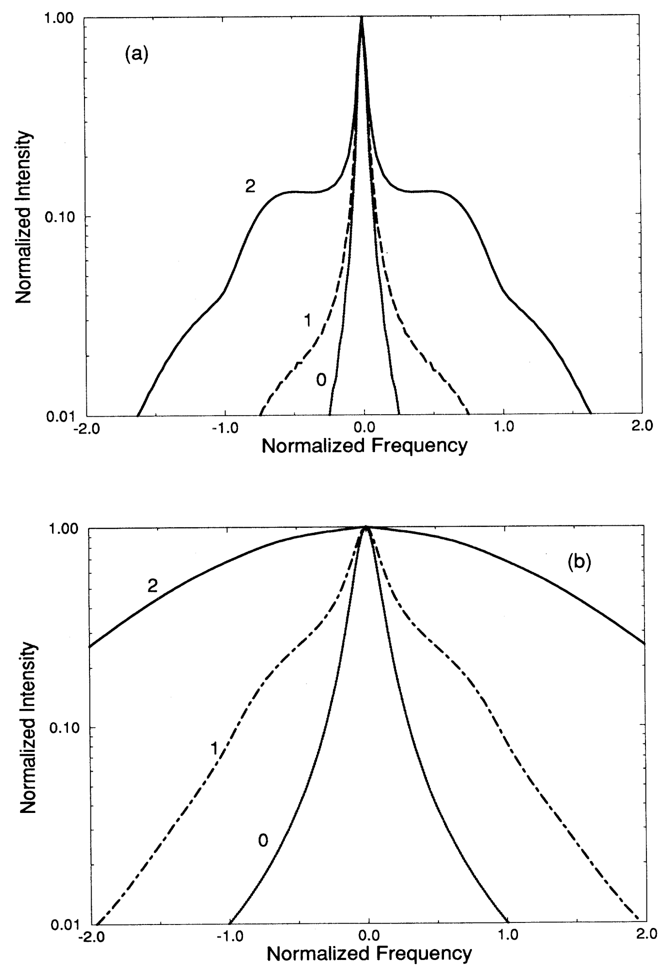

FIG. 3. Spectral line shapes corresponding to the coherence functions shown in Fig. 2: (a) $\Delta\nu/\nu_c = 0.05$ and (b) $\Delta\nu/\nu_c = 0.2$.

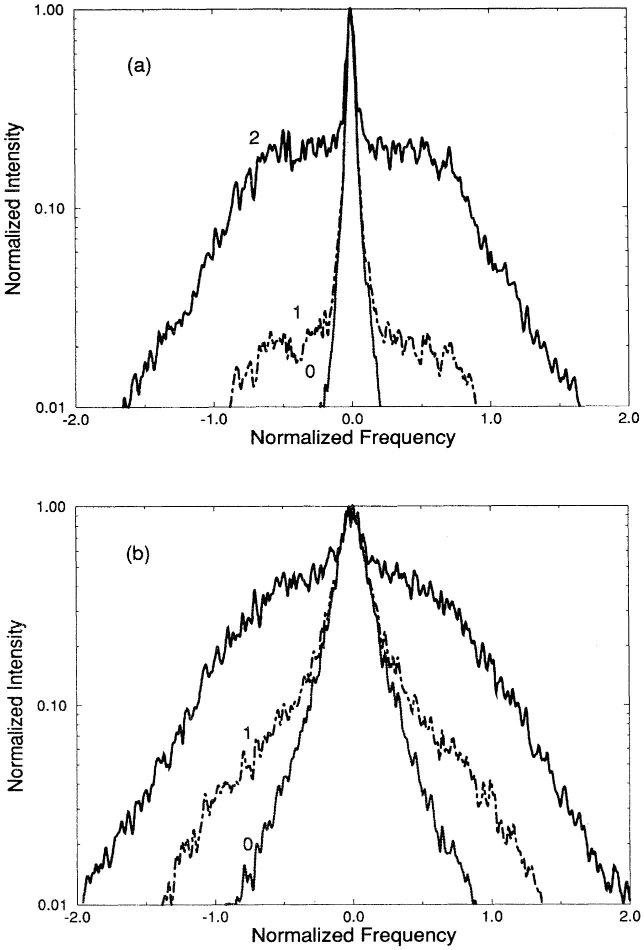

FIG. 4. Numerically simulated field spectra under conditions identical to those of Fig. 3: (a) $\Delta\nu/\nu_c = 0.05$ and (b) $\Delta/\nu_c = 0.2$.

V. NUMERICAL SIMULATIONS

To study the propagation regime $L > L_{\mathrm{NL}}$ more accurately, we solve the NLSE [Eq. (1)] numerically by using the split-step Fourier method [6] for a stochastic input field $E(0,t)$ of the form given by Eq. (2). The random processes $\delta A(0,t)$ and $\delta\Phi(0,t)$ are generated numerically. Phase fluctuations are assumed to follow the phase-diffusion model appropriate for lasers operating far above threshold [13]. Specifically, the spectral line shape of the input laser field is taken to be Lorentzian with a FWHM $\Delta\nu$ so that $\langle(\Delta\Phi)^2(0,\tau)\rangle = 2\pi\Delta\nu\tau$, where $\Delta\Phi(0,\tau) = \delta\Phi(0,t+\tau) - \delta\Phi(0,t)$ is the fluctuating phase difference after a time delay τ. In order to avoid problems resulting from a finite temporal simulation window, the average input amplitude A in Eq. (2) is replaced by a Gaussian pulse whose width is much larger than the time scale of fluctuations. The NLSE is solved repeatedly a large number of times (100 times for the results shown here) to simulate different realizations of the random process over which an ensemble average can be performed to obtain the temporal and spectral intensities observable in the laboratory experiments.

Figure 4 shows the numerically simulated spectra under conditions identical to those of Fig. 3. A comparison of Figs. 3 and 4 shows that the analytical results based on Eq. (18) are reasonably accurate for $L/L_{\mathrm{NL}} \leq 1$. For propagation distances such that $L/L_{\mathrm{NL}} > 2$, numerical results differ substantially from the predictions based on Eq. (18), indicating that the approximations made in the analytical model are no longer valid. This is easily understood by noting that the linearized equations for amplitude and phase fluctuations [Eq. (7)] become invalid when fluctuations become as large as 10% of the central peak, a situation occurring for $L/L_{\mathrm{NL}} > 2$.

Figures 5 and 6 show the evolution of the optical spec-

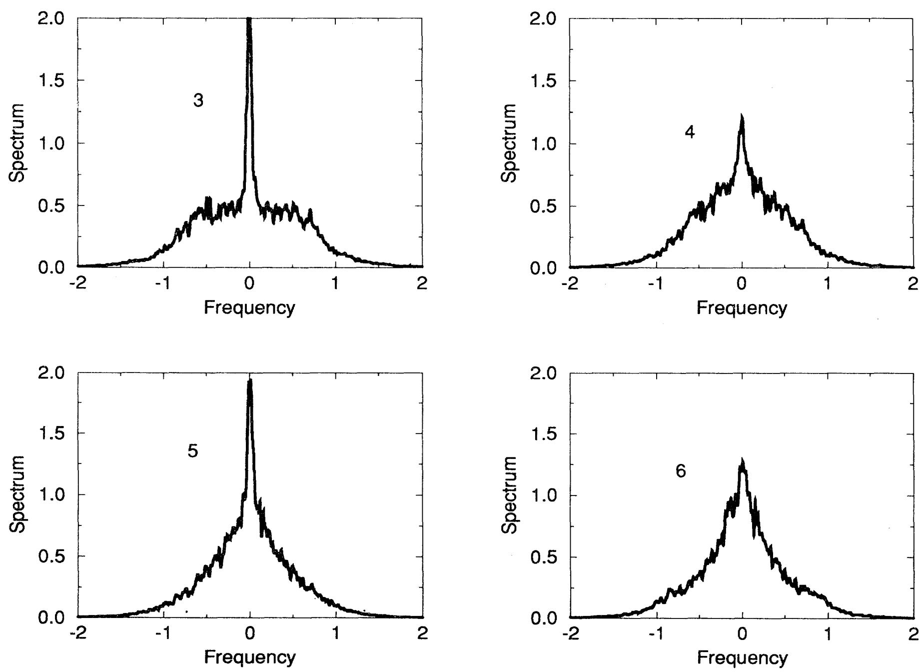

FIG. 5. Numerically simulated spectra for values of $L/L_{\mathrm{NL}} = 3$–6 for the case $\Delta\nu/\nu_c = 0.05$.

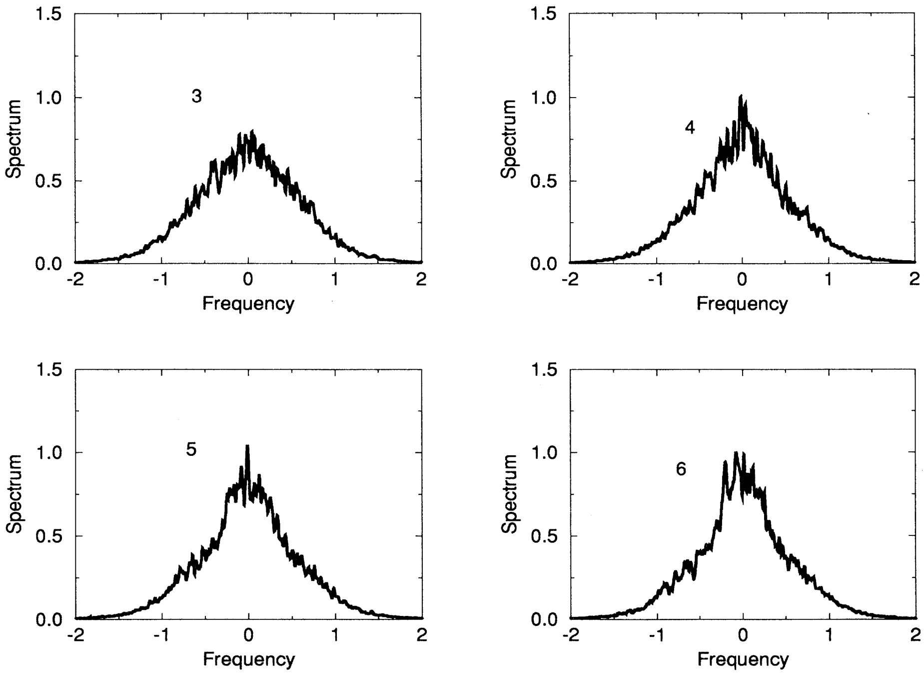

FIG. 6. Numerically simulated spectra for values of $L/L_{\mathrm{NL}}=3-6$ for the case $\Delta\nu/\nu_c=0.2$.

trum for $\Delta\nu/\nu_c=0.5$ and 0.2, respectively, in the nonlinear regime in which the analytic theory is not valid by considering propagation distances L/L_{NL} in the range of 3–6. In both cases, the spectral line shape eventually broadens so much that the sidebands are no longer resolved. In fact, the line shape for $L/L_{\mathrm{NL}} \gg 1$ becomes independent of the linewidth of the input signal. This feature is shown in Fig. 7 where the spectral line shape at $L/L_{\mathrm{NL}}=6$ is compared for three values of $\Delta\nu/\nu_c=0.02$ (dotted), 0.05 (dashed), and 0.2 (solid). The three curves nearly coincide even though the linewidth of the input signal varies by a factor of 10. This behavior is easily understood by noting that spectral broadening seen in Fig. 7 is a consequence of modulation instability whose gain is

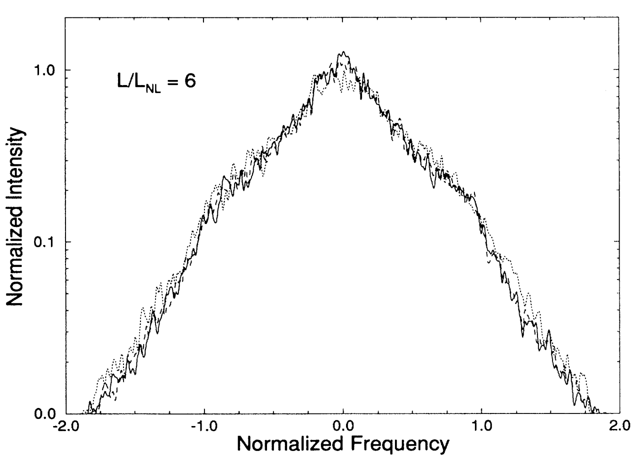

FIG. 7. Comparison of field spectra for $\Delta\nu/\nu_c=0.02$ (dotted), 0.05 (dashed), and 0.2 (solid) for a large propagation distance such that $L/L_{\mathrm{NL}}=6$.

independent of the input linewidth. The Lorentzian line shape of the input field simply provides a seed for the growth of modulation instability. Its linewidth governs the initial growth pattern such as the development of the sidebands, but eventually both the central line and the sidebands broaden so much that they overlap and merge together. For this reason, the final line shape obtained for large propagation distances does not depend on the input linewidth.

Note also that for a deterministic input field, the output spectrum is known to exhibit a multipeak structure resulting from self-phase modulation (SPM) [6]. Our results show that such a structure is washed out by the amplification of noise fluctuations inherent in the input field. This is easily understood by noting that the spectral line shape of the input field sets the ultimate resolution that can be realized in practice. Since modulation instability enhances phase fluctuations, the linewidth becomes so large that the SPM-induced multipeak structure cannot be solved.

VI. CONCLUSIONS

The propagation of a partially coherent optical beam through dispersive nonlinear media is investigated theoretically by using a phase-diffusion model for the laser beam. The approach uses the deterministic NLSE to propagate a stochastic cw beam whose statistical properties are known at the input. Its basic assumption is that the input field consists of an intense time-independent average field on which noise appears as small fluctuations in amplitude and phase from the average value. Changes in the second-order statistical properties during beam propagation depend on whether the nonlinear medium exhibits normal or anomalous group-velocity dispersion. In the case of normal dispersion, the coherence function and the corresponding optical spectrum remain unaffected. By contrast, modulation instability is found to be responsible for noise amplification in the anomalous dispersion regime, enhancing phase fluctuations and causing spectral distortion as well as coherence degradation. Under certain conditions phase fluctuations exhibit temporal oscillations that lead to the characteristic spectral sidebands associated with the modulation instability.

An important conclusion of our analysis is that fiber nonlinearity and dispersion can lead to large spectral broadening if a partially coherent cw beam propagates in the anomalous region of an optical fiber, whereas the spectrum will not change significantly in the case of normal dispersion. This kind of behavior has been observed experimentally in optical communication systems where the noise produced by in-line amplifiers led to a large spectral broadening in the case of anomalous dispersion [17]. A better experimental test of our theoretical results will consist of using a cw or quasi-cw laser beam whose spectrum has been broadened in the GHz range so that stimulated Brillouin scattering does not occur inside the optical fiber acting as a nonlinear dispersive medium. Although the results are obtained here in the context of optical fibers, they represent general features of nonlinear dispersive media and are applicable to other branches of physics where modulation instability is likely to occur.

ACKNOWLEDGMENTS

One of us (S.B. Cavalcanti) gratefully acknowledges the financial support of the Brazilian agency, Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq). The research was supported by the Army Research Office through the University Research Initiative.

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