Atomic gravitational cavities from hollow optical fibers

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We analyze a type of cavity, or trap, for atoms: a hollow optical fiber bent into a vertical U shape. The atoms are confined by gravity and by light forces due to the evanescent wave on the fiber's interior surface. A unique feature of this cavity is its mechanical flexibility, which allows tailoring of the gravitational potential experienced by the atoms. In particular a cycloid shape gives simple harmonic motion along the fiber. It can achieve confinement times similar to the parabolic reflector type of gravitational cavity. Quantized motion and an intracavity cooling scheme are considered.

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I. INTRODUCTION

The production of trapped and cooled atoms has given rise to new kinds of experiments, ranging from high precision spectroscopy to atomic interferometry [1]. In particular "trampoline" type gravitational cavities for atoms, which utilize a parabolic reflector, have recently been demonstrated [2]. In these, a vertical cavity is formed by gravity at the top and an evanescent light atomic mirror at the bottom [3]. The light is detuned many linewidths to the blue of an atomic transition giving a repulsive dipole force. In this paper we theoretically analyze an alternative type of atomic gravitational cavity based on a hollow optical fiber. Its trapping performance is comparable to that of the trampoline cavity, and it offers interesting possibilities due to the fiber's mechanical flexibility.

Atomic cavities differ from simple traps in emphasizing the potential for multiwave interference of atomic de Broglie waves, analogously to optical cavities. They have been considered by Balykin and Letokhov *et al.* [4] and by Wilkens *et al.* [5]. Gravity, which would otherwise be a serious problem for slow atoms, is used to advantage in gravitational cavities. The first experimental demonstration was limited to two bounces [6]. However with a parabolic reflector, up to ten bounces were observed [2]. That experiment was limited by the available laser power, collisions with background gas, and stray light.

The possibility of using hollow optical fibers as coherent atomic waveguides has been investigated by Marksteiner and co-workers [7,8] and by Ol'Shanii *et al.* [9]. The configuration considered by the latter authors differs from ours because their atoms are confined by an *attractive* force due to light propagating in the fiber's hole. In contrast the only light in the holes of the fibers we consider is the evanescent field at the glass-hole interface. This light repels the atoms from the wall, creating a potential barrier and thus confining them in the transverse directions, Fig. 1. Longitudinal confinement is provided by gravity by the fiber's vertical bend, Fig. 2.

The fiber's flexibility gives the freedom to choose the cavity geometry, including the possibility of dynamic

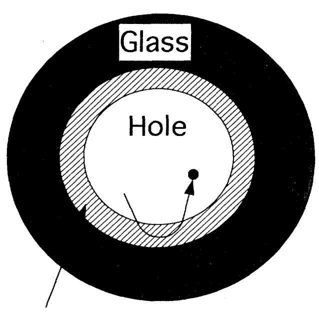

FIG. 1. Schematic cross section of a hollow optical fiber. The glass (shading) is the optical waveguide while the hole is the atomic pipe or waveguide. The atoms are confined transversely by the evanescent light field (hatching) at the glass-hole interface. A schematic trajectory of an atom (black disk) reflected by the evanescent light field is shown. As in conventional fibers the glass may be doped to form a high refractive index light guiding region adjacent to the hole.

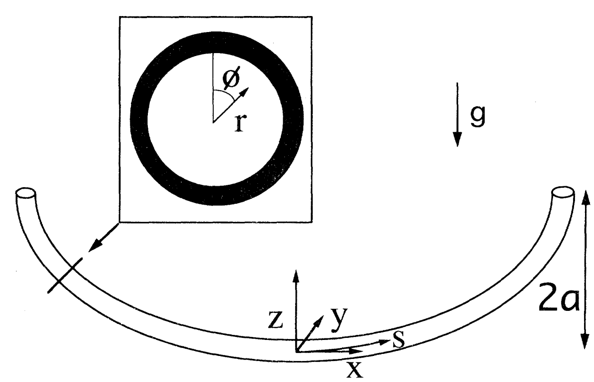

FIG. 2. Coordinate system used for cycloidal tube geometry. The longitudinal coordinate, along the fiber's axis, is s. The inset is a cross section through the fiber showing the transverse polar coordinates r and ϕ which are centered on the fiber axis. The fiber bent into a gravitational cavity is shown schematically. The height of the cycloid is $2a$.

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changes in shape. Such changes can, in principle, be used to cool atoms. Cooling would be necessary to achieve atomic Bose-Einstein condensation, which is a possible application of gravitational cavities [3].

In the following we investigate the dynamics of classical atoms before considering their quantized motion. The classical analysis allows a comparison with the trampoline cavity experiment.

II. THE HAMILTONIAN

By analogy with an isochronous pendulum [10] we find that a cycloidal shape for the fiber admits simple harmonic motion in the longitudinal coordinate. The cycloidal axis of the cavity is parametrized by

$$x = a(\theta + \sin\theta), \tag{1a}$$

$$z = a(1 - \cos\theta), \tag{1b}$$

where $-\pi \leq \theta \leq \pi$. The parameter a determines the length scale of the cycloid. An appropriate coordinate system (s, r, ϕ) for further calculations is given in terms of Cartesian coordinates (x, y, z) by the transformation

$$x = \frac{s}{2}S + 2a \arcsin\frac{s}{4a} - \frac{rs\cos\phi}{4a}, \tag{2a}$$

$$y = r\sin\phi, \tag{2b}$$

$$z = \frac{s^2}{8a} + rS\cos\phi, \tag{2c}$$

where we have introduced the function of s,

$$S = \sqrt{1 - \frac{s^2}{16a^2}} \quad . \tag{3}$$

The arc length, or longitudinal, coordinate is s, while r and ϕ are local transverse plane polar coordinates, see Fig. 2.

Using the Langrangian formalism, we obtain the conjugate momenta p_i and classical Hamiltonian H as

$$p_s = m\frac{ds}{dt}\left(1 - \frac{r\cos\phi}{4aS}\right)^2, \tag{4a}$$

$$p_r = m\frac{dr}{dt}, \tag{4b}$$

$$p_\phi = mr^2\frac{d\phi}{dt}, \tag{4c}$$

$$H = \frac{p_s^2}{2m}\frac{S^2}{(S - r\cos\phi/4a)^2} + \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} + mg\left(\frac{s^2}{8a} + rS\cos\phi\right), \tag{5}$$

where m is the atom's mass and g the acceleration due to gravity. For hollow optical fibers the hole radius R is much smaller than the length, $R \ll a$. In this limit, and for $|s| \ll 4a$, the Hamiltonian reduces to

$$H' = \frac{p_s^2}{2m} + \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} + \frac{1}{2}ks^2 + mgrS\cos\phi, \tag{6}$$

where we have defined the effective spring constant

$$k = \frac{mg}{4a}. \tag{7}$$

This Hamiltonian is valid for sufficiently narrow fibers everywhere except near the ends of the cycloid, $|s| \approx 4a$. So we restrict our attention to small amplitude oscillations about the bottom of the cycloid, $|s| \ll 4a$.

The approximate Hamiltonian may be written as a sum of longitudinal and transverse parts, $H' = H_l' + H_t'$, where

$$H_l' = \frac{p_s^2}{2m} + \frac{1}{2}ks^2, \tag{8}$$

$$H_t' = \frac{p_r^2}{2m} + \frac{p_\phi^2}{2mr^2} + mgrS\cos\phi. \tag{9}$$

The transverse Hamiltonian depends on the longitudinal coordinate through the potential energy term proportional to S. This term is the transverse component of the gravitational potential. Away from the ends of the cycloid, where $(s/4a)^2 \ll 1$, its dependence on s^2 can be ignored compared to that of the potential term in H_l'. Then the longitudinal and transverse parts of the Hamiltonian decouple and the atoms undergo simple harmonic motion along the fiber axis independently of their transverse motion.

III. THE CLASSICAL CAVITY

In the trampoline cavity experiment Cesium atoms were dropped as a cold cloud from a height of 2.9 mm onto a parabolic mirror [2]. They arrived at the mirror with a *mean* velocity of 0.24 m/s. An 800-mW laser detuned by 10 GHz from the D_2 transition was focused to a spot of $1/e^2$ diameter 1 mm, which could reflect atoms with velocities of up to 0.4 m/s (which allows for the velocity spread of the initial atoms).

In the fiber cavity case, the laser power can be coupled into a smaller area and hence the intensity incident on the internal glass-hole interface can be much higher than for the trampoline cavity. Due to the curvature of the fiber, atoms bounce from the walls at a glancing angle and the maximum transverse velocity is not as high as for the trampoline case with normal incidence. The combination of these effects suggests that less laser power will be required for similar confinement times. Alternatively the atoms might be confined with increased detuning and smaller losses.

To demonstrate this we calculate the power required to confine cesium atoms in a fiber with hole diameter $2R$=1 mm and glass thickness 0.1 mm. Such a fiber can be thought of as a capillary tube. The hole diameter is chosen to correspond to that of the trampoline experiment while the glass thickness is rather arbitrary. The atoms are transversely confined by a potential barrier with height U which in the limit of large atom-laser detuning Δ is given by [1],

$$U \approx \hbar\frac{\Omega^2}{4(2\pi\Delta)}, \tag{10}$$

where Δ has the units of Hz (not rad/s). Ω is the Rabi frequency of the evanescent field at the glass-hole interface,

$$\Omega = \frac{dE_0}{\hbar}, \tag{11}$$

where d is the atomic dipole moment, and E_0 is the electric field strength, which is related to the light intensity I in the glass by $I = 0.5c\epsilon_0 nE_0^2$, with n the glass refractive index (we use n=1.5). If an atom has sufficient transverse kinetic energy to surmount this barrier it will collide with the wall of the fiber and, we assume, be lost.

To investigate the transverse velocities with which the atoms hit the wall requires computer simulations. For simplicity a number of approximations are made in the simulations. First, we ignore the y dimension and hence only consider motion in two spatial dimensions. Second, the atoms are assumed to reflect off an infinite potential barrier at the walls. Finally we approximate the cycloid by a parabola. This is accurate for small amplitude oscillations.

Simulations were run for parameters corresponding to a cycloid with $a = 7.6$ mm, so that the period of longitudinal oscillation was 0.35 s. Dropping the atoms from a height of 2.9 mm we observed about 20 bounces per oscillation. The maximum transverse velocity in the fiber was found to be 0.18 m/s. Equating the corresponding transverse kinetic energy to the height of the potential barrier we find that 100 mW of guided power at a detuning of Δ=10 GHz will confine cesium atoms in this fiber. This is less than the 800 mW required for the trampoline cavity [2] due to the combined advantages of a reduced optical area, a uniform, instead of Gaussian, intensity distribution and the fact that atoms make glancing collisions with the walls. Our 100-mW power estimate assumes that the intensity at the glass-hole interface equals the average intensity in the glass. Note that lower powers result if the glass thickness is reduced.

The major losses in the trampoline cavity experiment were due to stray light from the mirror beam, background gas collisions, and photon absorption during reflection. The first two of these are technical problems, but the absorption loss is more fundamental. The average number of photon absorptions per atom per reflection, n_p, may be estimated by assuming that the atom's motion is classical and that the excitation probability is $(\Omega/4\pi\Delta)^2$, where Ω is the Rabi frequency corresponding to the field at the atom's location. This gives [2]

$$n_p = \Gamma mv/\alpha\hbar\Delta, \tag{12}$$

where Γ is the natural linewidth of the atomic transition, v is the transverse speed of the atom on entering the evanescent field, and $1/\alpha$ is the characteristic distance of the exponential decay of the evanescent field (0.21 μm for [2]). Using the atomic parameters of Table I and the mean incident velocity gives for the trampoline experiment n_p=0.06. With a period of 0.05 s this produces an average loss rate of about 1 per second per atom.

Our simulations give us the transverse velocity at each wall bounce and hence allow us to estimate the average loss rate in the fiber, from Eq. (12), as 1.5 per second per atom. According to Eqs. (10) and (11) increasing the guided power by a factor of 5, from 100 mW to 500 mW, allows the detuning to be increased from 10 GHz to 50 GHz without lowering the confining potential barrier. The fiber loss rate is then about 0.3 per second per atom, about 30% of that for the trampoline, corresponding to an absorption limited confinement time three times longer. However, it is expected that trampoline cavities will be improved by techniques such as enhancement of the evanescent field using a surface waveguide [11].

TABLE I. Atomic parameters for cesium and metastable helium.

Atom	m (kg)	λ (μm)	d (Asm)	Γ (MHz)
Cs	2.3×10^{-25}	0.852	2.5×10^{-29}	5.2
He*	6.6×10^{-27}	1.083	2.0×10^{-29}	1.6

We next consider a fiber cavity constructed from a genuine hollow optical fiber, rather than from a capillary tube. Hollow fibers with doped high index cores and 2-μm-diameter holes have been manufactured. For the $2\ ^3P_1 \leftrightarrow 2\ ^3S_1$ transition of metastable helium with wavelength 1.083 μm, the hollow fiber described in Ref. [12] supports only two optical modes. Since these modes are straightforward to calculate we can make accurate estimates of the guided powers required for confinement [8].

Using the atomic parameters of Table I we find that metastable helium requires only 25 μW of laser power at a detuning of 10 GHz to confine the atoms. However computer simulations predict a corresponding loss rate of about 6 per second per atom. This figure is high because the atoms hit the wall more frequently in a narrow hole. Laser power can be traded for detuning to obtain a loss rate of 0.2 per second per atom with 0.75 mW of laser power at a detuning of 300 GHz.

IV. THE QUANTUM CAVITY

So far we have considered the atomic motion to be classical. We next consider the limit in which the atoms behave like de Broglie waves and hence obey a Schrödinger equation. Naively quantizing the approximate Hamiltonian Eq. (6) gives,

$$\hat{H}' = -\frac{\hbar^2}{2m}\left[\frac{\partial^2}{\partial s^2}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial\phi^2}\right] + \frac{1}{2}ks^2 mgrS\cos\phi. \tag{13}$$

Canonically quantizing the full Hamiltonian Eq. (5) in Cartesian coordinates and then using $R \ll a$ and $|s| \ll 4a$ gives the same result.

The quantum regime is relevant at low energies where the atomic de Broglie wavelength is of the order of the tube diameter. In the following we assume a hole diameter of 2 μm [12]. A cesium atom with a 2 μm de Broglie wavelength is subrecoil cooled (the recoil cooled wavelength equals the transition wavelength of 0.852 μm) and has a kinetic energy of 2.4×10^{-31} J.

In the limit of motion characterized by small quantum numbers the atom is confined to the bottom of the fiber where $(s/4a)^2 \ll 1$. Then, as in Sec. II, the Hamiltonian Eq. (13) separates into decoupled longitudinal and transverse parts

$$\hat{H}'_l = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial s^2} + \frac{1}{2}ks^2, \tag{14}$$

$$\hat{H}''_t = -\frac{\hbar^2}{2m}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2}\right] + mgr\cos\phi. \tag{15}$$

The longitudinal Hamiltonian $\hat{H}'_l$ is just a harmonic oscillator and hence has energy eigenstates [13]

$$\Psi_n^{(l)}(s) = C_n H_n(s/s_0)\exp\left[-(s/s_0)^2\right], \tag{16a}$$

$$s_0 = \sqrt{\frac{\hbar}{m\omega}}, \tag{16b}$$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{4a}}, \tag{16c}$$

where H_n is the nth Hermite polynomial and C_n is a normalization constant. The energy eigenvalues are

$$E_n^{(l)} = (n + \tfrac{1}{2})\hbar\omega. \tag{17}$$

For cesium atoms, we have $s_0 \approx (1.7 \times 10^{-5})a^{1/4}$m and an energy level splitting of $\hbar\omega \approx (1.7 \times 10^{-34})a^{-1/2}$ J, with a in meters. For a cavity with length scale a =7 mm, $s_0 \approx 5$ μm, which is the scale of the ground state, and $\hbar\omega \approx 2 \times 10^{-33}$ J, which corresponds to a temperature of approximately 0.3 nK.

Achieving such low atomic temperatures will be a formidable problem, although it is in principle possible, for example, by velocity selective coherent population trapping (VSCPT) techniques. Two-dimensional cooling to temperatures of 300 nK using VSCPT has been reported [14]. At such low temperatures various heating mechanisms such as those associated with the acoustic vibrations of the fiber will have to be understood and controlled [15].

The energy level spacing between the lowest eigenstates of $\hat{H}''_t$ can be estimated by ignoring its gravitational part and using the boundary condition that the eigenstates are zero on the fiber walls. This gives eigenstates that are Bessel functions in the radial coordinate and which have energy spacings of about $7\hbar^2/mR^2 \approx 3 \times 10^{-31}$ J for cesium [7]. This is about two orders of magnitude greater than the spacing of the longitudinal energy eigenvalues. Hence it should be possible to populate only the lowest transverse eigenstate.

Calculating the loss rate from an eigenstate must take account of the quantized motion of the atoms and of the spatial variation of the confining evanescent field. We have not done this. However, the *straight* fiber case has been considered by Marksteiner *et al.* [8], who predict loss rates of about 0.01 per second per atom for recoil cooled cesium. They also considered the lowering of the effective potential barrier by the Casimir-Polder interaction of the atoms with the glass walls.

V. DISCUSSION

We next discuss a potential cooling scheme using a hollow optical fiber gravitational cavity. We also consider the experimental feasibility of such cavities.

Cooling might be achieved using an adiabatic potential change such as discussed by Zaugg *et al.* [16]. The requisite slow change in potential could be made by slowly flattening out the fiber. This would adiabatically increase the length scale a and hence decrease the energy scale $\hbar\omega$.

The extremely low energy splitting of the longitudinal eigenstates of practical fiber waveguides prompts us to consider ways to increase it. This would facilitate population of the ground state. To increase the energy splitting the effective acceleration g in Eq. (16c) could be increased. This could be done by using ions instead of atoms and applying a uniform vertical electric field. Field strengths that change the acceleration by many orders of magnitude are possible. Ramping the field would provide another method for adiabatic cooling.

Although our work has identified potential advantages of hollow fiber gravitational cavities, many difficulties remain to be explored. Light in the fiber's hole can be avoided by suitable optical coupling, or by making the fiber sufficiently long that the end into which light is coupled is far from the end into which atoms are coupled. Light in the hole will then have completely leaked out.

The quality of the vacuum in the fiber's hole will be important for reducing collisions with background gas. One technique for evacuating the fiber's hole would be to flush it with helium. Its solubility in glass will allow the helium to permeate out through the walls of the fiber. This evacuation technique has the advantage of being relatively insensitive to the fiber's length [17].

A problem common to both classical and de Broglie regimes is detection of the atoms. However, laser induced fluorescence detection through the glass fiber appears feasible.

We believe that our work shows that hollow fiber atomic gravitational cavities are in principle competitive with trampoline type gravitational cavities. Their advantages arise first from their mechanical flexibility and second from their optical properties. We hope that this is just one of many potential applications of hollow optical fiber atomic waveguides.

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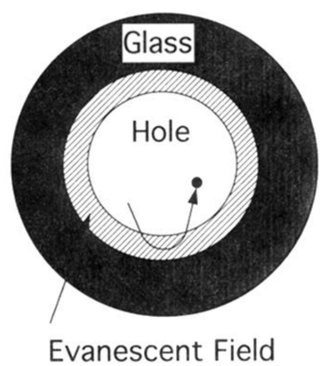

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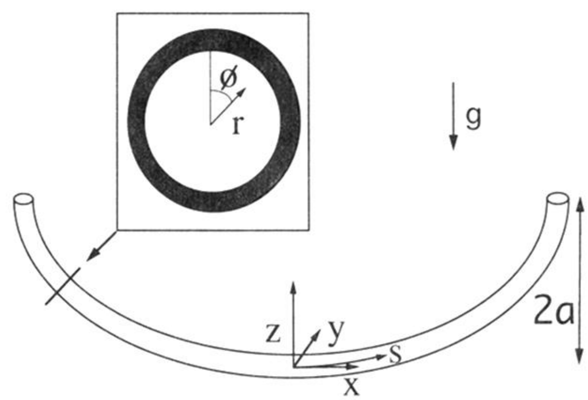

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