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Refractive index of a dilute Bose gas

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We derive the dispersion relation for the propagation of quasiresonant light with frequency ω_L in an ultracold gas of bosonic atoms in the dilute regime, i.e., for an atomic density $\rho_0 \ll (\omega_L/c)^3$. In our calculation, valid up to order 2 in density, two types of corrections to the Lorentz-Lorenz formula for the refractive index appear. The first one is due to the bosonic nature of the atoms and its contribution is related to the two-body correlation function. The second correction originates from multiple scattering of photons within pairs of close atoms, giving rise to the resonant van der Waals interaction. The temperature dependence of the refractive index gives a clear signature of quantum statistical effects, even if the degeneracy threshold for Bose-Einstein condensation is not reached.

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The physics of cold atomic gas has had a very rapid expansion during these last years. Using laser cooling and/or evaporative cooling, samples of hydrogen, alkali-metal atoms, or others have been brought to microkelvin temperatures [1]. A major goal is the achievement of strongly degenerate systems, where Bose-Einstein condensation could occur for bosonic atoms. The purpose of the present paper is to investigate the optical signature of the quantum degeneracy of the gas that a measurement of its index of refraction close to an atomic resonance would provide.

We consider a dilute gas with a density ρ_0 such that $\rho_0 \ll k_L^3$. Here $k_L = 2\pi/\lambda_L = \omega_L/c$ represents the wave vector in vacuum of the light used to probe the system. For usual optical transitions, this condition limits ρ_0 to 10^{13} atoms per cm^3, which is actually above the maximal densities that have been achieved by laser cooling techniques [2,3]. This dilute limit assumption allows us to treat multiple scattering of photons by the gas in the binary approximation, using a virial type expansion: we take into account all photon scattering processes by isolated atoms or pairs of close atoms, but we neglect processes where three atoms or more would sit within the same k_L^{-3} volume and where a photon would undergo multiple scattering inside this subsystem.

Our treatment leads to an expression of the refractive index n valid up to order 2 in ρ_0. In addition to the usual contribution to $n-1$ which is linear in ρ_0, three terms appear. The first one is the standard Lorentz-Lorenz local field correction [4]. The second one involves the quantum corrections to the two-body correlation function $\rho(\mathbf{r},\mathbf{r}')$, giving the probability for detecting simultaneously a particle in $\mathbf{r}$ and another one in $\mathbf{r}'$. The third term in ρ_0^2 contains corrections to $n-1$ originating from pairs of close atoms whose absorption frequency and linewidth have been modified by the resonant dipole-dipole interaction.

By contrast to our dilute gas limit we note that the refractive index of a dense Bose gas ($\rho_0 > k_L^3$) with condensate has been addressed in [5] and [6]. As for the polariton effect in solid state physics, a frequency gap with a width $\sim \gamma\rho_0 k_L^{-3}$ appears around the resonance frequency, where γ stands for the half width of the excited state. Inside the gap, no frequency can propagate in the medium which should be perfectly reflecting. Resonant exchanges of photons between atoms can also be included in the binary approximation [5], provided the narrow line condition $M\gamma \ll \hbar k_L^2$ is fulfilled, where M is the atomic mass. We work here in the opposite limit of a broad line $M\gamma \gg \hbar k_L^2$, which is the suitable one for alkali atoms probed on their resonance line. In addition, as shown in [5], we remark that no noticeable gap will appear in our treatment because of the low density hypothesis.

We start with the atom-field Hamiltonian written in the electric dipole approximation (length gauge) [7]:

$$H = \sum_i \frac{\mathbf{P}_i^2}{2M} + \sum_i \sum_{m=-1}^{1} \hbar(\omega_A^0 + \delta\omega_A)|i:e_m\rangle\langle i:e_m| + \int_{|\mathbf{k}|<k_M} d^3k \sum_{\boldsymbol{\varepsilon}\perp\mathbf{k}} \hbar c k a_{\mathbf{k}\boldsymbol{\varepsilon}}^\dagger a_{\mathbf{k}\boldsymbol{\varepsilon}} - \sum_i \mathbf{D}_i\cdot\mathbf{E}(\mathbf{R}_i) + \frac{1}{\varepsilon_0}\sum_i\sum_{j<i} \mathbf{D}_i\cdot\mathbf{D}_j\,\delta(\mathbf{R}_i-\mathbf{R}_j) \ . \tag{1}$$

The quantity $\mathbf{E}$ is the electric field operator, which represents the electric displacement vector (up to a factor ε_0) in the length gauge used in (1), and $a_{\mathbf{k}\boldsymbol{\varepsilon}}$, $a_{\mathbf{k}\boldsymbol{\varepsilon}}^\dagger$ are the photon annihilation and creation operators for a photon with wave vector $\mathbf{k}$ and polarization $\boldsymbol{\varepsilon}$ in vacuum. The atoms are assumed to be identical, with a "bare" resonance frequency ω_A^0; $\mathbf{R}_i$, $\mathbf{P}_i$ represent the position and momentum operators for the center of mass of the ith atom, and $\mathbf{D}_i$ is the electric dipole operator of the ith atom. For simplicity we consider here an atomic transition between a ground state g with angular momentum $J_g = 0$ and an excited state with $J_e = 1$; m denotes the magnetic quantum number for e. We therefore restrict the present discussion to the bosonic case. Also we note d the reduced dipole moment for this transition ($\gamma = d^2\omega_A^3/6\pi\varepsilon_0\hbar c^3$), and k_M the cutoff parameter in reciprocal space [7]. The correction $\delta\omega_A = d^2k_M^3/9\pi^2\varepsilon_0$ to the atomic frequency ω_A^0 corresponds to the dipole self-energy, which arises when one uses the length gauge.

In order to derive the refractive index of the gas, we have to determine how the propagation of a probe field in a coherent state $\mathbf{E}_L(\mathbf{r})e^{-i\omega_L t}$ + c.c. is modified by the presence of the dipoles. We suppose that the atoms are moving very slowly so that Doppler shifts can be neglected, in comparison with γ. For atoms cooled at the recoil limit, i.e., with a rms velocity of a few recoil velocities $\hbar k_L/M$, this requires the broad line condition $M\gamma \gg \hbar k_L^2$. The quantity $M\gamma/\hbar k_L^2$ typically ranges between 10^2 and 10^3 for alkali-metal atoms so that our assumption is quite reasonable in this case. A direct consequence is a decoupling between the atomic internal and external dynamics; we can average over the field and atomic internal states, while keeping the operator character of the external atomic motion. The average of any dipole operator $\langle \mathbf{D}_i\rangle_{\mathrm{F+Int}}$ over field and atomic internal state reaches a forced regime $\mathbf{d}_i e^{-i\omega_L t}$ + H.c., where $\mathbf{d}_i$ is still an operator with respect to external atomic degrees of freedom. The corresponding average of the electric field operator is then $\mathbf{e}(\mathbf{r})e^{-i\omega_L t}$ + H.c. with

$$\mathbf{e}(\mathbf{r}) = \mathbf{E}_L(\mathbf{r}) + \frac{1}{\varepsilon_0}\sum_i \mathbf{d}_i\,\delta(\mathbf{r}-\mathbf{R}_i) + \frac{1}{\varepsilon_0}\sum_i [g(\mathbf{r}-\mathbf{R}_i)]\,\mathbf{d}_i \quad (2)$$

where $[g(\mathbf{r})]$ is the 3×3 matrix

$$g_{\alpha\beta}(\mathbf{r}) = -\int \frac{d^3k}{2(2\pi)^3}\sum_{\boldsymbol{\varepsilon}\perp\mathbf{k}} ck\varepsilon_\alpha\varepsilon_\beta e^{i\mathbf{k}\cdot\mathbf{r}}\left(\frac{1}{\omega_L - ck + i\eta} - \frac{1}{\omega_L + ck + i\eta}\right) - \delta_{\alpha\beta}\delta(\mathbf{r}) \quad (3)$$

$$= -\frac{k_L^3}{4\pi}\frac{e^{ik_L r}}{k_L r}\left[\left(1 + \frac{3i}{k_L r} - \frac{3}{k_L^2 r^2}\right)\frac{r_\alpha r_\beta}{r^2} - \left(1 + \frac{i}{k_L r} - \frac{1}{k_L^2 r^2}\right)\delta_{\alpha\beta}\right] - \frac{1}{3}\delta_{\alpha\beta}\delta(\mathbf{r})\ . \quad (4)$$

In Eq. (4) we recover the well-known electric field radiated by an oscillating dipole [8,9]. This field varies as $1/r$ at long distances ($k_L r \gg 1$). At short distances ($k_L r \ll 1$), the $1/r^3$ term is dominant and it generates the resonant van der Waals interaction between an excited and a ground state atom.

We now neglect counter-rotating terms [rotating wave approximation (RWA)] and we assume a weak, nonsaturating probe field, so that we can also neglect excited state populations in comparison with ground state ones. Each dipole $\mathbf{d}_i$ is driven by the field $\mathbf{e}(\mathbf{R}_i)$, which is evaluated from (2) and which can be split into three terms: (i) the probe field $\mathbf{E}_L(\mathbf{r})$, (ii) the field radiated by the atoms $j \neq i$, and (iii) the field radiated by the ith atom at its own location. To calculate this last contribution we evaluate the limits of $g(\mathbf{r})$ and $\delta(\mathbf{r})$ when $\mathbf{r} \to \mathbf{0}$ using the cutoff parameter k_M. A term proportional to k_M^3 compensates for the frequency shift $\delta\omega_A$. The remaining terms lead to radiative corrections to ω_A^0 giving the "true" resonance frequency ω_A, and to a damping term proportional to γ. We obtain the following linear response of the dipole $\mathbf{d}_i$:

$$\mathbf{d}_i = \alpha\varepsilon_0\mathbf{E}_L(\mathbf{R}_i) + \alpha\sum_{j\neq i}[g(\mathbf{R}_i - \mathbf{R}_j)]\,\mathbf{d}_j\ . \quad (5)$$

The coefficient α is the polarizability of the atom at the probe frequency:

$$\alpha = -\frac{d^2}{\hbar\varepsilon_0}\frac{1}{\delta + i\gamma}\ , \quad (6)$$

where $\delta = \omega_L - \omega_A$ is the atom-laser detuning.

The set of equations (5), written for each atom, contains all the physics governing the propagation of the probe wave in the gas. In a different context, it has also been taken as a starting point in the work of Mazur and Mandel [10] (see also [11]). In [10], the frequency of the light field was supposed to be much smaller than the resonance frequency of the scatterers. One could then solve (5) by a perturbative expansion in $\alpha g(\mathbf{r})$, but one had to keep track at the same time of the modification of the atomic polarizability due to the ground state–ground state interactions. The physical problem considered in the present paper is quite different. On one hand, it involves resonant scattering of photons by the gas so that we have to solve (5) at any order in $\alpha g(\mathbf{r})$, by taking into account recurrent scattering within any given pair of close atoms. On the other hand, the ground state–ground state interactions, which we have left out in the RWA, have a negligible contribution since the typical interatomic distances are much larger than the range of these interactions.[1]

We now transform (5) into a hierarchy of equations relating quantities averaged over the external quantum states of the dipoles. We start with the three following functions: average density $\rho(\mathbf{r})$, average dipole $\boldsymbol{\mathcal{D}}(\mathbf{r})$, and average electric field $\boldsymbol{\mathcal{E}}(\mathbf{r})$:

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[1] From our results, it is possible to recover the prediction of [10] for the refractive index up to order 2 in ρ_0 by expanding the expression for C in (24) up to order 2 in $\alpha\tilde{g}$, and by replacing α in (23) by an effective polarizability $\bar{\alpha}$ accounting for the ground state–ground state interactions.

$$\rho(\mathbf{r}) = \left\langle \sum_i \delta(\mathbf{r}-\mathbf{R}_i) \right\rangle , \tag{7}$$

$$\mathcal{D}(\mathbf{r}) = \frac{1}{\rho(\mathbf{r})} \left\langle \sum_i \delta(\mathbf{r}-\mathbf{R}_i)\mathbf{d}_i \right\rangle , \tag{8}$$

$$\mathcal{E}(\mathbf{r}) = \langle \mathbf{e}(\mathbf{r}) \rangle - \frac{1}{\varepsilon_0}\rho(\mathbf{r})\mathcal{D}(\mathbf{r}) . \tag{9}$$

The first relation between these quantities is simply obtained by averaging (2) over the external atomic state:

$$\mathcal{E}(\mathbf{r}) = \mathbf{E}_L(\mathbf{r}) + \frac{1}{\varepsilon_0}\int d^3r'[g(\mathbf{r}-\mathbf{r}')]\mathcal{D}(\mathbf{r}')\rho(\mathbf{r}') . \tag{10}$$

We now multiply (5) by $\delta(\mathbf{r}-\mathbf{R}_i)$, sum over i, and average over the external atomic state; we get

$$\mathcal{D}(\mathbf{r}) = \alpha\varepsilon_0\mathbf{E}_L(\mathbf{r}) + \alpha\int d^3r'[g(\mathbf{r}-\mathbf{r}')] \times\mathcal{D}(\mathbf{r}'/\mathbf{r})\frac{\rho(\mathbf{r},\mathbf{r}')}{\rho(\mathbf{r})} . \tag{11}$$

We had to introduce at this stage two other functions: the two-body correlation function $\rho(\mathbf{r},\mathbf{r}')$ and the conditional average dipole $\mathcal{D}(\mathbf{r}'/\mathbf{r})$, giving the average dipole in $\mathbf{r}'$, knowing that an atom is sitting in $\mathbf{r}$:

$$\rho(\mathbf{r},\mathbf{r}') = \left\langle \sum_i \sum_{j\neq i} \delta(\mathbf{r}-\mathbf{R}_i)\delta(\mathbf{r}'-\mathbf{R}_j) \right\rangle , \tag{12}$$

$$\mathcal{D}(\mathbf{r}'/\mathbf{r}) = \frac{1}{\rho(\mathbf{r},\mathbf{r}')} \left\langle \sum_i \sum_{j\neq i} \delta(\mathbf{r}-\mathbf{R}_i)\delta(\mathbf{r}'-\mathbf{R}_j)\mathbf{d}_j \right\rangle . \tag{13}$$

Continuing this procedure, we can derive an infinite hierarchy of equations between more and more complex conditional average dipoles: $\mathcal{D}(\mathbf{r}''/\mathbf{r},\mathbf{r}'),\ldots$. To get an explicit expression for the refractive index, one has to break this hierarchy at some stage.

Before doing this, we specialize Eqs. (10) and (11) to the geometry appropriate for the derivation of the refractive index n. To avoid technical problems due to diffraction and multiple reflected waves, we assume that the atoms fill the half space $z > 0$ with a uniform density $\rho(\mathbf{r}) = \rho_0$. The two-body density inside the medium is cast in the form

$$\rho(\mathbf{r},\mathbf{r}') = \rho_0^2[1+\varphi(\mathbf{r}-\mathbf{r}')] , \tag{14}$$

where the pair correlation function φ depends on the bosonic nature of the atoms. We choose an incident probe field $\mathbf{E}_L(\mathbf{r})$ varying as $\boldsymbol{\varepsilon}_x e^{ik_Lz}$; inside the medium, neglecting border effects, we look for a solution for $\mathcal{E}(\mathbf{r})$ and $\mathcal{D}(\mathbf{r})$ proportional to $\boldsymbol{\varepsilon}_x e^{ikz}$. The real part of the complex number k gives the refractive index, and the imaginary part leads to the scattering cross section. We apply to (10) the operator $\Delta_\mathbf{r} + k_L^2$; the contribution of $\mathbf{E}_L$ cancels out. We use

$$(\Delta_\mathbf{r} + k_L^2)g_{\alpha\beta}(\mathbf{r}) = \left(-k_L^2\delta_{\alpha\beta} - \frac{\partial}{\partial r_\alpha}\frac{\partial}{\partial r_\beta}\right)\delta(\mathbf{r}) \tag{15}$$

in which only the term in $\delta_{\alpha\beta}$ contributes to (10), and we get inside the medium:

$$\rho_0\mathcal{D}(z) = \left(\frac{k^2}{k_L^2} - 1\right)\varepsilon_0\mathcal{E}(z) . \tag{16}$$

We now perform a linear combination of (10) and (11) canceling the probe field, and we eliminate the electric field from (16), to get

$$\mathcal{D}(z)\left[1 - \frac{\alpha\rho_0}{k^2/k_L^2 - 1}\right] = \alpha\rho_0\int d^3r'[g(\mathbf{r}-\mathbf{r}')]\{\mathcal{D}(\mathbf{r}'/\mathbf{r})[1+\varphi(\mathbf{r}-\mathbf{r}')] - \mathcal{D}(\mathbf{r}')\} . \tag{17}$$

Let us examine first the simple decorrelation approximation $\mathcal{D}(\mathbf{r}'/\mathbf{r}) = \mathcal{D}(\mathbf{r}')$. In this case Eq. (17) is straightforwardly solved, using $\mathcal{D}(\mathbf{r}') = \exp[ik(z'-z)]\mathcal{D}(\mathbf{r})$, and it leads to the dispersion relation

$$\frac{k^2}{k_L^2} = 1 + \alpha\rho_0\left(1 - \alpha\rho_0\int d^3r\, g_{xx}(\mathbf{r})e^{-ikz}\varphi(\mathbf{r})\right)^{-1} . \tag{18}$$

For $\varphi \equiv 0$, we recover the well-known result for the refractive index of a dilute perfect classical gas [4]. For a perfect Bose gas we expect $\varphi \neq 0$, so that (18) already involves quantum statistical corrections, varying as ρ_0^2. Note that, at order 2 in ρ_0, we can replace in the integral term of (18) e^{-ikz} by e^{-ik_Lz}.

Actually, other corrections in ρ_0^2 are expected when one goes beyond the approximation $\mathcal{D}(\mathbf{r}'/\mathbf{r}) = \mathcal{D}(\mathbf{r})$ and we now have to evaluate them for consistency. We derive from (5) the next equation in the hierarchy:

$$\mathcal{D}(\mathbf{r}/\mathbf{r}'') = \alpha\varepsilon_0\mathbf{E}_L(\mathbf{r}) + \alpha[g(\mathbf{r}-\mathbf{r}'')]\mathcal{D}(\mathbf{r}''/\mathbf{r}) + \int d^3r'\alpha[g(\mathbf{r}-\mathbf{r}')]\mathcal{D}(\mathbf{r}'/\mathbf{r},\mathbf{r}'')\frac{\rho(\mathbf{r},\mathbf{r}',\mathbf{r}'')}{\rho(\mathbf{r},\mathbf{r}'')} . \tag{19}$$

We break the hierarchy at this level, assuming first that $\mathbf{r}$ and $\mathbf{r}''$ are distant by more than $1/k_L$. In this case, since the function $g(\mathbf{r}-\mathbf{r}')$ in the integral of (19) favors points $\mathbf{r}'$ close to $\mathbf{r}$, we can consider that $\mathcal{D}(\mathbf{r}'/\mathbf{r},\mathbf{r}'')$, average dipole in $\mathbf{r}'$ knowing that atoms are sitting in $\mathbf{r}$ and $\mathbf{r}''$, is approximately equal to $\mathcal{D}(\mathbf{r}'/\mathbf{r})$. For simplicity, we also neglect three-body correlations $[\rho(\mathbf{r},\mathbf{r}',\mathbf{r}'')/\rho(\mathbf{r},\mathbf{r}'') \simeq \rho(\mathbf{r},\mathbf{r}')/\rho(\mathbf{r})]$, which could contribute in the ultralow temperature limit. Subtracting (11) from (19), we then obtain

$$\mathcal{D}(\mathbf{r}/\mathbf{r}'') - \alpha[g(\mathbf{r}-\mathbf{r}'')]\mathcal{D}(\mathbf{r}''/\mathbf{r}) = \mathcal{D}(\mathbf{r})\ . \tag{20}$$

Exchanging $\mathbf{r}$ and $\mathbf{r}''$ in (20), we get a 2×2 system leading to

$$\mathcal{D}(\mathbf{r}''/\mathbf{r}) = [1-\alpha^2 g^2(\mathbf{r}-\mathbf{r}'')]^{-1} \times\{\mathcal{D}(\mathbf{r}'') + \alpha[g(\mathbf{r}-\mathbf{r}'')]\mathcal{D}(\mathbf{r})\}\ . \tag{21}$$

If $\mathbf{r}$ and $\mathbf{r}''$ are close to each other, extra corrections should be included in (20) and (21), originating from points in the integral of (19) where $\mathbf{r}$, $\mathbf{r}'$, and $\mathbf{r}''$ lie within the same $1/k_L^3$ volume. Fortunately these configurations have a negligible weight in the final result if $\rho_0 \ll k_L^3$, as it can be shown more rigorously by a systematic expansion of the electromagnetic field in powers of ρ_0/k_L^3.

We finally inject the relation (21) in (17) and we find that the dispersion relation (18) is changed to

$$\frac{k^2}{k_L^2} = 1 + \alpha\rho_0\left\{1 - \alpha\rho_0\int d^3r\, g_{xx}(\mathbf{r})\varphi(\mathbf{r})e^{-ik_L z} - \rho_0\int d^3r\left[\frac{\alpha^2 g^2(\mathbf{r}) + \alpha^3 g^3(\mathbf{r})e^{-ik_L z}}{1-\alpha^2 g^2(\mathbf{r})}\right]_{xx}[1+\varphi(\mathbf{r})]\right\}^{-1}\ . \tag{22}$$

To get more physical insight, we split the tensor $g_{\alpha\beta}(\mathbf{r})$ given in (4) as $\tilde{g}_{\alpha\beta}(\mathbf{r}) - \delta_{\alpha\beta}\delta(\mathbf{r})/3$. The $\delta(\mathbf{r})$ function in (22) is regularized using a cutoff in $\mathbf{p}$ space. Its respective contributions to the two integrals of (22) are $\alpha\rho_0\varphi(\mathbf{0})/3$ and $-\alpha\rho_0[1+\varphi(\mathbf{0})]/3$ so that the sum does not depend on the pair correlation function φ. We are left with

$$\frac{k^2}{k_L^2} = 1 + \frac{\alpha\rho_0}{1+C} \tag{23}$$

where the quantity C is given by

$$C = -\frac{1}{3}\alpha\rho_0 - \alpha\rho_0\int d^3r\,\tilde{g}_{xx}(\mathbf{r})\varphi(\mathbf{r})e^{-ik_L z} - \rho_0\int d^3r\left[\frac{\alpha^2\tilde{g}^2(\mathbf{r}) + \alpha^3\tilde{g}^3(\mathbf{r})e^{-ik_L z}}{1-\alpha^2\tilde{g}^2(\mathbf{r})}\right]_{xx}[1+\varphi(\mathbf{r})]\ . \tag{24}$$

The standard expression for the refractive index of the gas (order 1 in ρ_0) corresponds to the approximation $C = 0$ in (23). Our expression (24) includes all the corrections to k/k_L up to second order in ρ_0.

We now discuss, in the case of an ideal Bose gas, the three terms arising in (24). The first one is a purely classical local field correction; if taken alone, it would lead to the well-known Lorentz-Lorenz formula [4]. Note that it originates from the contact term $-\delta(\mathbf{r})/3$ present in the field (4) radiated by a dipole, so that our derivation of the Lorentz-Lorenz term does not rely on the excluded volume conventionally introduced around the atoms in the literature.

The second term in (24), which already appears in (18), is related to a modification of the photon scattering cross section due to the quantum statistics of the atoms. Consider for simplicity the situation $|\delta| \gg \gamma$, which we can treat within the Born approximation, neglecting multiple scattering events. In this case, when an atom with initial momentum $\mathbf{p}_i$ scatters a photon from $\mathbf{k}_i$ to $\mathbf{k}_f$, the cross section is enhanced if the final momentum class $\mathbf{p}_i + \hbar(\mathbf{k}_i - \mathbf{k}_f)$ is already populated. After integration over the initial momentum class $\mathbf{p}_i$ and over the scattered photon direction $\mathbf{k}_f$ in order to get the total cross section, one recovers the contribution to n'' of the second term in (24).

The last correction in (24) corresponds to the modifications of the refractive index induced by the resonant van der Waals interaction. A pair of atoms separated by a distance r with $k_L r < 1$ has two pairs of resonant absorption frequencies $\omega_\pm$ such that $\omega_\pm - \omega_A \propto \pm\gamma/(k_L r)^3$. Also the width of the resonance can be larger or lower than γ, depending on whether the excited state has a superradiant or subradiant character. These two effects arise in the last term of (24); for instance for $|\delta| \gg \gamma$, the zeros of the real part of the denominator $1-\alpha^2\tilde{g}^2(\mathbf{r})$ are found to occur for values of the detuning δ equal to the resonant van der Waals shift. This last term of (24) can also be recovered in a scattering theory approach. One has to sum all diagrams in which two given atoms resonantly exchange a photon; in that case, the analog of the approximation $\mathcal{D}(\mathbf{r}'/\mathbf{r},\mathbf{r}'') \simeq \mathcal{D}(\mathbf{r}'/\mathbf{r})$ leading to (20) consists in neglecting diagrams where a photon would undergo multiple scattering within a triplet of atoms. This type of correction is well known from the physics of electromagnetic wave propagation in dielectric media [12,13]. More recently, similar corrections have been considered for the more complex problem of light diffusion and transport in random media [14].

We now investigate the result of (23) and (24) more explicitly. The pair correlation function reflecting the quantum statistics of the ideal Bose gas $\varphi(\mathbf{r}-\mathbf{r}')$ differs from 0 for $|\mathbf{r}-\mathbf{r}'|$ smaller than a typical correlation length Λ. For a weakly degenerate gas, Λ is on the order of the de Broglie thermal wavelength $h/\sqrt{2\pi M k_B T}$ and $\varphi(0) = 1$. Around the Bose-Einstein condensation, Λ is related to the atomic density $\Lambda \sim \rho_0^{-1/3}$. When the temperature is further decreased, Λ increases but $\varphi(0)$ decreases. At $T = 0$, all particles accumulate into the ground state of the system and $\varphi \equiv 0$. As an example, we have plotted in Fig. 1 with a solid line the variations with temperature of the real part n' and imaginary part n'' of the refractive index ($n' + in'' = k/k_L$), for a fixed density and a fixed detuning δ. Since Doppler shifts are very small and have been ignored, the only temperature dependence of n' and n'' in Fig. 1 originates from the one in φ. At high temperature ($\Lambda \to 0$) and at low temperature [$\varphi(r) \to 0$], we recover the results for classical particles ($\varphi \equiv 0$), which is represented in Fig. 1 as a solid horizontal line.

Also we have plotted in Fig. 1 with a dotted line the refractive index obtained by keeping only the first two terms in (24); the dotted curve stands for the Bose gas and the dotted horizontal line stands for the classical gas. The comparison to the corresponding curves in solid line shows that the dipole-dipole corrections have only a small contribution to the result. We can in particular use the dotted line to estimate analytically the maximum deviation of n' and n'' from their classical values; it is obtained for a temperature on the order of the condensation temperature, and scales approximately as $\rho_0^{5/3}$ with density.

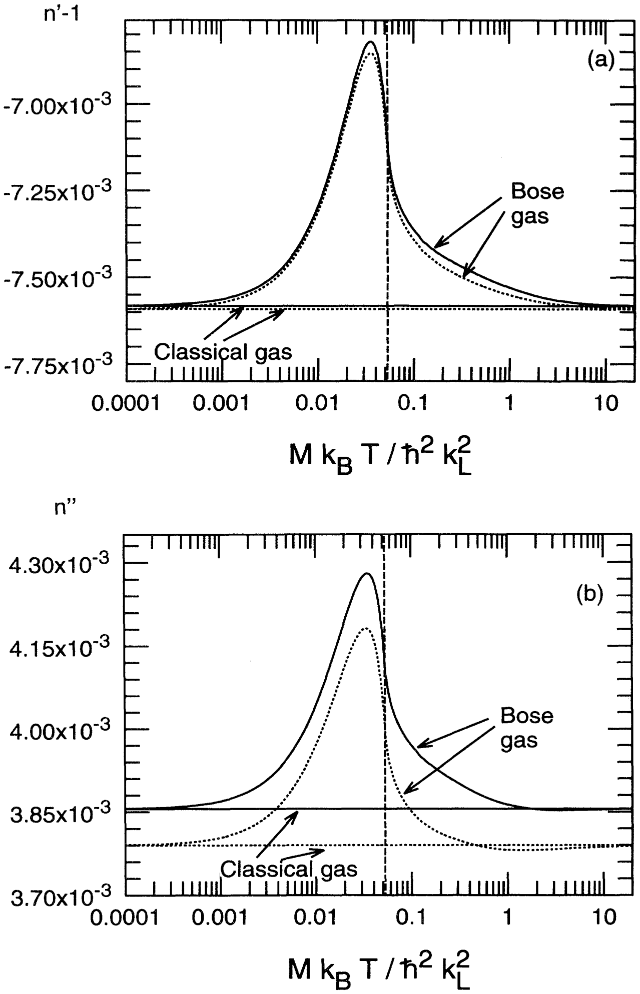

FIG. 1. Real part n' (a) and imaginary part n'' (b) of the complex refractive index as a function of the temperature. The solid lines give the result including all the terms of (24) for bosons and for classical particles ($\varphi \equiv 0$). The dotted lines give the result, also for bosons and classical particles, obtained by keeping only the first two terms of (24), i.e., by disregarding the effect of multiple scattering within pairs of close atoms. The atom-laser detuning is $\delta = 2\gamma$ and the atomic density is $\rho_0 = k_L^3/16\pi^3$. The vertical dashed line indicates the threshold for Bose-Einstein condensation at this density.

The classical nature of the optical response of a Bose gas at $T = 0$ is also found in the case of an inhomogeneous sample since the two-body distribution still factorizes in this case: $\rho(\mathbf{r},\mathbf{r}') = \rho(\mathbf{r})\rho(\mathbf{r}')$. For instance, several authors have considered the scattering of light by a cloud of size $a \gg \lambda_L$ with N atoms in a Bose condensate, either in the cw regime [15,16] or in the pulsed regime [17]. In particular, for $N \gg (k_L a)^2$, they have shown that the total cross section $\sigma(\omega_L)$ has a very broad component, with a width $\gamma N/(k_L a)^2$ instead of γ for one atom. In fact, such a cooperative feature is not specific of Bose statistics and can be obtained from a continuous classical distribution of dipoles [18].

We have ignored here the contribution of atomic motion to the dispersion relation. As stated above, for isolated atoms cooled at the recoil limit, this is valid in the broad line limit. For a pair of atoms separated by a distance $r < k_L^{-1}$ and excited resonantly by the probe light, we have also to check that the acceleration by the resonant van der Waals potential $\sim \pm\gamma/(k_L r)^3$ leads to a negligible displacement of the atoms during the lifetime of the excited state. A simple classical calculation of this displacement, following the lines of [19], shows that this pair of atoms is still resonant with the light after a time γ^{-1} if we restrict to sufficiently small detunings: $|\delta|/\gamma < (M\gamma/\hbar k_L^2)^{3/8}$. For alkali-metal atoms, this imposes detunings not larger than a few linewidths. For ultralow temperatures, where atomic delocalization is much larger than the wavelength, a more quantitative treatment of this radiative heating process would require a quantum approach such as the one developed recently in [20].

Our treatment can be extended to more complex atomic transitions with a nonzero angular momentum J_g in the ground state. The three components of a given $\mathbf{d}_i$ have to be replaced by the $(2J_e+1)\times(2J_g+1)$ quantities $d_i(m_e, m_g) = \langle\, |i : e, m_e\rangle\langle i : g, m_g|\, \rangle_{\mathrm{F+Int}}$, with $-J_g \leq m_g \leq J_g$ and $-J_e \leq m_e \leq J_e$. Also the ground state matrix elements $G_i(m_g, m_g') = \langle\, |i : g, m_g\rangle\langle i : g, m_g'|\, \rangle_{\mathrm{F+Int}}$ are new dynamical variables. For a weak nonsaturating probe, two damping time constants then appear in the evolution of the averaged atomic operators. The first one is the optical pumping time T_p, scaling as the inverse of the probe intensity. It corresponds to the average delay between two successive photon scattering events by a given atom, and it gives the characteristic time required for the $G_i(m_g, m_g')$'s to reach a steady state. The second time scale, γ^{-1}, is much shorter and it gives the time scale for the $d_i(m_e, m_g)$'s to adjust to a given value of the set $\{G_i(m_g, m_g')\}$. For a measurement of the refractive index of a gas cooled at the recoil limit, we require that the probing time t is much smaller than T_p, in order to minimize the heating due to the emission of fluorescence photons. If the detuning δ is not larger than a few linewidths, the light shifts of the various ground state sublevels are comparable to $\hbar/T_p$ and we can therefore neglect any evolution of the $G_i(m_g, m_g')$'s during t. Consequently, for $\gamma^{-1} \ll t \ll T_p$ a set of equations with the same structure as (2) and (5) is obtained, where the $\mathbf{d}_i$ are replaced by the $d_i(m_e, m_g)$ and where the scalar polarizability α is replaced by polarization tensors depending on the ground state averages $G_i(m_g, m_g')$.

To summarize, we have emphasized in this paper that the symmetrization principle manifests itself in the radiated mean field when quantum pair correlations are nonzero. The contribution of these quantum correlations has been explicitly derived in the case of the index of refraction of an homogeneous gas. As indicated in Fig. 1, this should allow an observation of quantum statistical effects before the threshold for degeneracy is obtained.

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