Coulomb and screening corrections to Delbrück forward scattering

R. Solberg, K. Mork, and I. Øverbø
Department of Physics, College of Art and Science, University of Trondheim, N-7055 Dragvoll, Norway
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Coulomb and screening corrections to the amplitude for Delbrück scattering in the forward direction are calculated.

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I. INTRODUCTION

Delbrück scattering which is the elastic scattering of a photon in a Coulomb field, is of interest both as an observable high-order nonlinear process in quantum electrodynamics and as an interfering process in investigations of nuclear structure. The process has been studied by several methods, cf. the review articles [1] and the papers [2–11]. Presently known methods, either the integration of the cross section derived directly from the fourth-order vacuum polarization tensor which has been used by Papatzacos and Mork [12], or the dispersion relation approach used by De Tollis and Pistoni [13] suffice to calculate the first-order Born-approximation cross section for all relevant cases even if the numerical procedures are complicated and expensive.

However, as shown by Jarlskog *et al.* [14], Rullhusen *et al.* [6,15], Turrini, Maino, and Ventura [8], and Kasten *et al.* [16] the lowest-order cross section is not a good approximation except for very low energies and low values of the atomic number Z, and it is necessary to know higher-order effects. The first attempt to include the complete Coulomb corrections was made by Rohrlich [17] who investigated forward scattering. His results are, however, not very accurate. Later Cheng and Wu [18] calculated the Coulomb corrections in a high-energy approximation for a limited range of momentum transfers. Similar results have also been obtained by Milstein *et al.* [4,7,10] by a different method.

In the present article we calculate the Coulomb and screening corrections to the Delbrück forward amplitude for all energies of interest. Although direct measurements are not available for this case we still believe the results to be of value, both as a useful estimate of the corrections for nonforward angles and also as a checkpoint for approximate calculations.

II. THE BORN APPROXIMATION

The imaginary part of the Delbrück amplitude is more easily obtained than the real part. In the forward direction the imaginary part of the amplitude

$$D(\omega,\Theta)=(\alpha Z)^2 r_0 d(\omega,\Theta)\ , \tag{1}$$

cf. the notation in Ref. [1], is

$$\mathrm{Im}D(\omega,0)=(\omega/4\pi)\sigma_p(\omega)\ , \tag{2}$$

where ω is the photon energy, Θ the scattering angle, and $\sigma_P(\omega)$ is the pair production total cross section. This cross section is well known [19–21]. We shall calculate the real part by a dispersion relation which was established by Rohrlich and Glückstern [22],

$$\mathrm{Re}D(\omega,0)=(\omega^2/2\pi^2)\mathrm{P}\int_{2m}^{\infty} d\omega'\sigma_P(\omega')/(\omega'^2-\omega^2)\ . \tag{3}$$

Since one would expect Coulomb corrections to be minimal and screening corrections to be maximal in the forward direction compared to nonforward scattering, we shall present these corrections separately,

$$D(\omega,0)=D^B(\omega,0)+\Delta D^C(\omega,0)+\Delta D^S(\omega,0)\ , \tag{4}$$

where D^B is the Born-approximation amplitude, ΔD^C is the Coulomb correction, and ΔD^S is the screening correction to the sum $D^B+\Delta D^C$.

The Born part D^B was calculated in [22] from exact expressions for σ_P^B. We shall use a simpler form, in terms of expansions given by Maximon [21]. With ω given in units of the electron mass m we have for $\omega>4$,

$$\sigma_P^B(\omega)=\alpha Z^2 r_0^2\{28L/9-218/27+4[6L-7/2+2L^3/3-L^2-\pi^2L/3+\pi^2/6+2\zeta(3)]/\omega^2 -(3L+2)/\omega^4-(29L/36-77/216)/\omega^6\}\ ,$$

and for $\omega<4$,

$$\sigma_P^B(\omega)=\alpha Z^2 r_0^2 2\pi(\omega-2)^3(1+\varepsilon/2+23\varepsilon^2/40+37\varepsilon^3/120+61\varepsilon^4/192)/(3\omega^3)\ .$$

Here $L=\ln(2\omega)$, $\varepsilon=(\omega-2)/(\omega+2)$, and $\zeta(3)=1.202\,056\,9$. Using these expressions in (3) we obtain the Born part,

$$\mathrm{Re}d^B(\omega,0)=7\omega/18+A_1+\ln^2\omega/\omega+A_2\ln\omega/\omega+A_3/\omega+A_4/\omega^2-3/(8\omega^3)+A_5/\omega^4-29/(288\omega^5)+A_6/\omega^6\ , \tag{5}$$

TABLE I. The coefficients C in Eq. (7). Z is the atomic number.

C \ Z	13	53	82	92
C_1	9.618×10^{-4}	1.452×10^{-2}	2.953×10^{-2}	3.425×10^{-2}
C_2	-1.423×10^{-2}	-2.115×10^{-1}	-4.246×10^{-1}	-4.924×10^{-1}
C_3	3.294×10^{-2}	4.794×10^{-1}	9.275×10^{-1}	1.056
C_4	-3.203×10^{-2}	-4.045×10^{-1}	-5.375×10^{-1}	-4.465×10^{-1}
C_5	3.589×10^{-2}	3.303×10^{-1}	-1.544×10^{-1}	-7.231×10^{-1}
C_6	-7.754×10^{-2}	-7.136×10^{-1}	3.336×10^{-1}	1.562
C_7	6.128×10^{-3}	4.546×10^{-2}	-9.435×10^{-2}	-2.334×10^{-1}
C_8	9.554×10^{-2}	8.793×10^{-1}	-4.110×10^{-1}	-1.925
C_9	-5.880×10^{-2}	-4.441×10^{-1}	8.347×10^{-1}	2.121
C_{10}	-6.793×10^{-3}	7.300×10^{-4}	2.373×10^{-1}	4.180×10^{-1}

with $A_1=-2.2512$, $A_2=0.38629$, $A_3=2.7873$, $A_4=-3.5098$, $A_5=0.77$, $A_6=3.6910$. Formula (5) gives accuracy better than 1% for $\omega>6$ and is only 14% off at $\omega=5$. For low energies one may use

$$\mathrm{Re}\,d^B(\omega,0)=B_1\omega^2+B_2\omega^4+B_3\omega^6 \tag{6}$$

with $B_1=3.1735\times10^{-2}$ $B_2=3.1610\times10^{-4}$, $B_3=1.4790\times10^{-5}$. Formula (6) is better than 1% for $\omega<2$ and is only 10% off at $\omega=4$.

III. COULOMB AND SCREENING CORRECTIONS

In order to compute the Coulomb corrections we use the Coulomb corrections $\Delta\sigma_P^C$ to σ_P^B found by Øverbø [23]. He gives an analytic formula valid for $\omega>3.5$; for lower ω we have to interpolate from numbers given in the tables. We present our results in terms of a high-energy formula, valid with accuracy better than 1% for $\omega>5$, the error increasing to 2% for $\omega=4$,

$$\begin{aligned}\mathrm{Re}\Delta d^C(\omega,0)=&C_1\ln^3\omega+C_2\ln^2\omega+C_3\ln\omega\\&+C_4+C_5\ln^2\omega/\omega+C_6\ln\omega/\omega\\&+C_7\ln\omega/\omega^2+C_8/\omega+C_9/\omega^2+C_{10}/\omega^4\ ,\end{aligned} \tag{7}$$

where the Z-dependent coefficients C are given in Table I, and a low-energy formula, valid with accuracy of 0.2% at $\omega=0.3$ and 18% at $\omega=1$,

$$\mathrm{Re}\Delta d^C(\omega,0)=D_1\omega^2+D_2\omega^4+D_3\omega^6\ , \tag{8}$$

where the D's are given in Table II. The results are also shown in Fig. 1 where $\mathrm{Re}\Delta d^C(\omega,0)$ is given in percent of $\mathrm{Re}\,d^B(\omega,0)$, and $\mathrm{Im}\Delta d^C(\omega,0)$ is given in percent of $\mathrm{Im}\,d^B(\omega,0)$ as functions of ω for three values of Z. We note that the Coulomb corrections to the real part are considerable even at $\omega=1$ MeV. Our results differ substantially from those given earlier by Rohrlich [17], the reason being his use of inaccurate values for the pair cross section.

We obtain the screening correction Δd^S using screening corrections σ_P^S to the pair cross section given by Øverbø [24]. These corrections are expressed partly by analytic formulas and partly by the tables, and we use numerical interpolation and integration to get $\mathrm{Re}\Delta d^S(\omega,0)$ from the dispersion relation (3). The results are shown in Fig. 2. We note that screening corrections are most important for the real part in contrast to the Coulomb corrections which are largest for the imaginary part. For high energies the following formula may be used,

$$\begin{aligned}\mathrm{Re}\Delta d^S(\omega,0)=&-7\omega/18+S_1\ln^2\omega+S_2\ln\omega+S_3\\&+S_4\ln^2\omega/\omega+S_5\ln\omega/\omega+S_6/\omega+S_7/\omega^2\\&+S_8/\omega^3+S_9/\omega^4+S_{10}/\omega^5\ ,\end{aligned} \tag{9}$$

where the coefficients S are given in Table III. Equation (9) is accurate for $\omega>50$, the error for $\omega=50$ is 0.2%, it increases to 2.3% for $\omega=30$ and to 17% for $\omega=20$. For low energies,

$$\mathrm{Re}\Delta d^S(\omega,0)=T_1\omega^2+T_2\omega^4+T_3\omega^6\ , \tag{10}$$

where the T's are given in Table IV. This formula is good for photon energies between 0.4 and and 1 MeV, the error here is less than 5%. Note that our screening corrections also include the small combined Coulomb and screening correction.

IV. CONCLUSIONS

We have given the Coulomb and screening corrections to the Delbrück forward-scattering amplitude. We may in addition also estimate the effects of radiative correc-

TABLE II. The coefficients D in Eq. (8).

D \ Z	13	53	82	92
D_1	7.095×10^{-6}	1.266×10^{-4}	8.510×10^{-5}	-1.064×10^{-4}
D_2	3.585×10^{-6}	5.438×10^{-6}	1.141×10^{-4}	1.324×10^{-4}
D_3	1.625×10^{-6}	2.402×10^{-5}	4.849×10^{-5}	5.625×10^{-5}

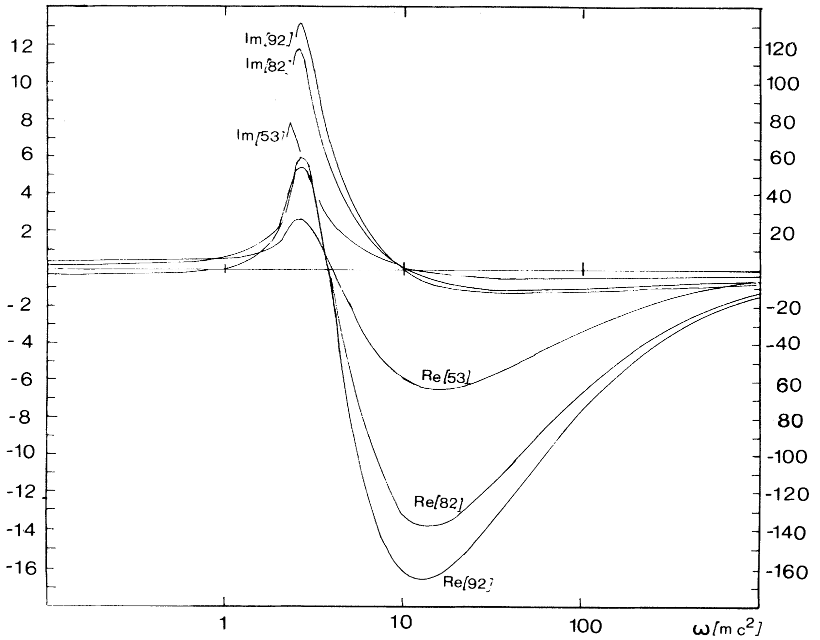

FIG. 1. The quantity $\mathrm{Re}\Delta d^{C}(\omega,0)$ given in percent of $\mathrm{Re}d^{B}(\omega,0)$, and $\mathrm{Im}\Delta d^{C}(\omega,0)$ in percent of $\mathrm{Im}d^{B}(\omega,0)$. These are the Coulomb corrections to the real and imaginary parts of the forward Delbrück amplitude relative to the Born parts as a function of the photon energy ω in units of the electron mass. The curves are marked Re (Im) for the corrections to the real (imaginary) parts, and the actual atomic numbers are given. The scale on the left is valid for the corrections to the real parts while the scale on the right hand side is valid for the corresponding imaginary parts.

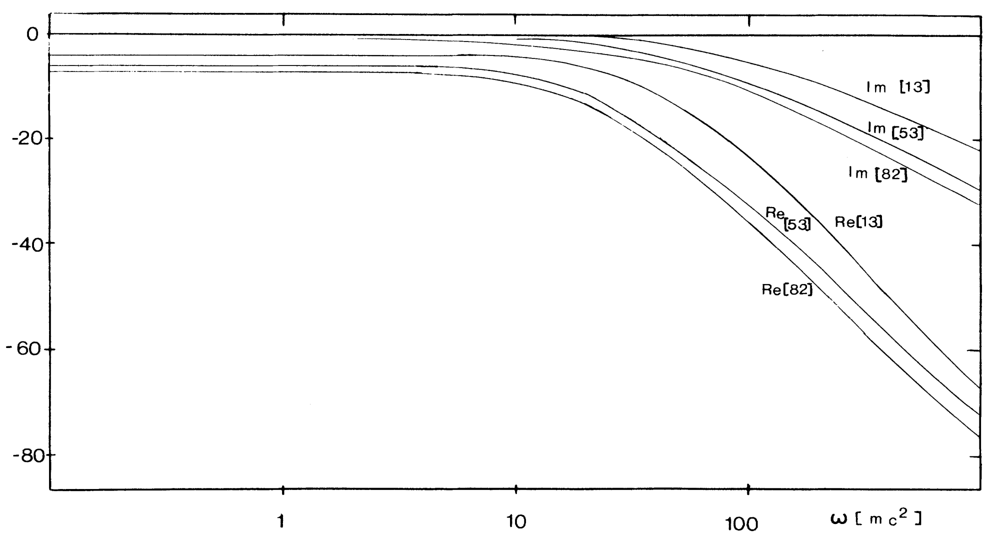

FIG. 2. The quantity $\mathrm{Re}\Delta d^{S}(\omega,0)$ given in percent of $\mathrm{Re}d^{B}(\omega,0)$ and $\mathrm{Im}\Delta d^{S}(\omega,0)$ in percent of $\mathrm{Im}d^{B}(\omega,0)$. These are the screening corrections to the real and imaginary parts of the forward Delbrück amplitudes relative to the Born parts as a function of the photon energy in units of the electron mass. The curves are marked Re (Im) for the corrections to the real (imaginary) parts, and the actual atomic numbers are given.

TABLE III. The coefficients S in Eq. (9).

S \ Z	13	53	82	92
S_1	8.776	6.018	4.966	4.964
S_2	-6.197×10^{1}	-3.831×10	-2.966×10	-3.001×10
S_3	1.461×10^{2}	8.486×10	6.432×10	6.566×10
S_4	-7.071×10^{2}	-2.166×10^{2}	-1.567×10^{2}	-1.534×10^{2}
S_5	5.889×10^{3}	1.449×10^{3}	1.003×10^{3}	9.859×10^{3}
S_6	-1.424×10^{4}	-3.045×10^{3}	-2.051×10^{3}	-2.040×10^{3}
S_7	4.586×10^{4}	5.654×10^{3}	3.712×10^{3}	3.725×10^{3}
S_8	3.750×10^{-1}	3.750×10^{-1}	3.750×10^{-1}	3.750×10^{-1}
S_9	5.4604×10^{6}	8.617×10^{4}	6.897×10^{4}	5.084×10^{4}
S_{10}	1.007×10^{-1}	1.007×10^{-1}	1.007×10^{-1}	1.007×10^{-1}

TABLE IV. The coefficients T in Eq. (10).

T \ Z	13	53	82	92
T_1	-1.044×10^{-3}	-1.816×10^{-3}	-2.115×10^{-3}	-2.265×10^{-1}
T_2	-2.247×10^{-7}	-4.984×10^{-7}	3.269×10^{-6}	-6.292×10^{-6}
T_3	1.134×10^{-7}	2.123×10^{-7}	8.793×10^{-7}	1.324×10^{-6}

tions to be small, of order 1%, since the pair production corrections at high energies $\omega \gg 1$ are known from Mork and Olsen [25] to be close to 1%, and we expect the corrections to be small at low energies. We mention that our results already have been used by Kasten *et al.* [26] in their experimental investigations of Coulomb corrections to Delbrück scattering.

Finally we shall add some comments on the use of one-variable dispersion relations for calculating the real part of the Delbrück amplitude. As mentioned before it is comparatively easy to find the imaginary part also for nonforward angles. For instance, one may apply the generalized optical theorem which was first used by Kessler [27] and later applied by Ehlotzky and Sheppey [28]. This method involves available pair-production amplitudes in the physical region, and works quite well for calculating the imaginary part to the Born-approximation amplitude. We have extended the calculations of [28] and find that the method is superior to the one of Papatzacos and Mork [12] for large scattering angles and not too high energies, but is inferior for small angles. We also believe that it will be possible to calculate the imaginary part to the Coulomb corrections by using exact Coulomb wave functions in the relevant pair-production amplitudes. We plan to do these calculations.

From the imaginary part one may obtain the real part by a dispersion relation. Writing a relation in ω for constant Θ, however, one finds that two cuts are needed, one starting at $\omega=2$ and another at $\omega=i/\sin(\Theta/2)$. The last one means that one has to know the imaginary amplitude for imaginary ω, which will involve nontrivial analytic continuations.

Papatzacos and Mork [12] suggested a dispersion relation in the standard kinematic variables s and t, with constant t. However, De Tollis, Lusignoli, and Pistoni [29] pointed out that this equation was incorrect. De Tollis *et al.* [5,13,29] have successfully applied a dispersion relation in the variables $d=\omega\sin(\Theta/2)$ and $p=\omega\cos(\Theta/2)$ for constant d to Delbrück scattering. We now correct the equation given in [12] by adding a subtraction constant,

$$\mathrm{Re}\,d(s,t)=\mathrm{Re}\,d(0,t)+(s/\pi)\mathrm{P}\int_{4m^2}^{\infty}ds'\,\mathrm{Im}\,d(s',t)/[s'(s'-s)]\ .$$

We have tested this equation using the method of [27] to obtain the imaginary part of $d(s,t)$. Unfortunately there is no simple way to obtain the unphysical $d(0,t)$, and we therefore calculated the difference $d(s_1,t)-d(s_2,t)$ which is independent of $d(0,t)$. The results were in agreement with known results within numerical errors of a few percent. However, we must conclude that this method is not very convenient for calculating the real part, the cost being higher than with the method of [12]. The reason is that in the dispersion integral above the main contribution always comes from values of $s'\gg t$, and in this region it is time-consuming to calculate $\mathrm{Im}\,d(s',t)$.

[1] P. Papatzacos and K. Mork, Phys. Rep. **21C**, 81 (1975); A. I. Milstein and M. Schumacher, Phys. Rep. **243**, 183 (1994).
[2] T. Bar-Noy and S. Kahane, Nucl. Phys. A **288**, 132 (1977).
[3] H. Cheng, E. C. Tsai, and X. Zhu, Phys. Rev. D **26**, 908 (1982).
[4] A. I. Milstein and V. M. Strakhovenko, Phys. Lett. A **95**, 135 (1983); Zh. Eksp. Teor. Fiz. **85**, 14 (1983) [Sov. Phys. JETP **58**, 8 (1983)].
[5] B. De Tollis and L. Luminari, Nuovo Cimento A **81**, 633 (1984).
[6] P. Rullhusen *et al.*, Phys. Rev. D **36**, 733 (1987).
[7] A. I. Milstein and R. Zh. Shaisultanov, J. Phys. A: Math. Gen. **21**, 2941 (1988).
[8] S. Turrini, G. Maino, and A. Ventura, Phys. Rev. C **39**, 824 (1989).
[9] R. Nolte *et al.*, Phys. Rev. C **40**, 1175 (1989).
[10] A. I. Milstein, P. Rullhusen, and M. Schumacher, Phys. Lett. B **247**, 481 (1990).
[11] H. Falkenberg *et al.*, At. Data Nucl. Data Tables **50**, 1 (1992).
[12] P. Papatzacos and K. Mork, Phys. Rev. D **12**, 206 (1975).
[13] B. De Tollis and G. Pistoni, Nuovo Cimento A **42**, 499 (1977).
[14] G. Jarlskog *et al.*, Phys. Rev. D **8**, 3813 (1973).
[15] P. Rullhusen *et al.*, Phys. Rev. C **27**, 559 (1983).
[16] B. Kasten *et al.*, Phys. Rev. C **33**, 1606 (1986).
[17] F. Rohrlich, Phys. Rev. **108**, 169 (1957).
[18] M. Cheng and T. T. Wu, Phys. Rev. D **5**, 3077 (1972).
[19] I. Øverbø, K. Mork, and H. A. Olsen, Phys. Rev. A **8**, 668 (1973).
[20] H. Davies, H. A. Bethe, and L. C. Maximon, Phys. Rev. **93**, 788 (1954).
[21] L. C. Maximon, J. Res. Natl. Bur. Stand. B **72**, 79 (1968).
[22] F. Rohrlich and R. L. Glückstern, Phys. Rev. **86**, 1 (1952).
[23] I. Øverbø, Phys. Lett. B **71**, 412 (1977).
[24] I. Øverbø, Nuovo Cimento **47**, 43 (1978); Phys. Scr. **19**, 299 (1979).
[25] K. Mork and Haakon Olsen, Phys. Rev. **140**, B1661 (1965); **166**, 1862 (1968).
[26] B. Kasten *et al.*, Phys. Rev. C **33**, 1606 (1986).
[27] P. Kessler, J. Phys. Rad. **19**, 739 (1958).
[28] F. Ehlotzky and S. C. Sheppey, Nuovo Cimento **33**, 1185 (1964).
[29] B. De Tollis, M. Lusignoli, and G. Pistoni, Nuovo Cimento A **32**, 227 (1976).