Formation of antihydrogen in the ground state and $n=2$ level

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The cross sections of antihydrogen formation in the ground state and $n=2$ level by the impact of antiprotons on the ground state of positronium have been calculated under the framework of the eikonal approximation for incident energy of 30–1000 keV. The excited-state capture cross sections are quite appreciable and are even larger than the ground-state cross sections for impact energies ≤ 75 keV. The total eikonal cross sections ($\sigma=\sigma_{1s}+\sigma_{2s}+\sigma_{2p}$) are always higher than the corresponding first-order Born approximation cross section throughout the present energy span.

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I. INTRODUCTION

The antihydrogen ($\bar{\mathrm{H}}$) is the simplest antimatter atom and its stability is greater than any other exotic atoms, such as positronium, muonium, protonium, etc. The investigations into the properties and production of antihydrogen lead one to study the fundamental questions in physics regarding antimatter-matter interaction. The symmetry on the atomic scale can be tested experimentally with the production of antimatter in the laboratory [1]. Further, $\bar{\mathrm{H}}$ could be used as a probe for the additional gravity test with antiparticles [1]. For this reason much attention is being paid by both the theoretical [2–7] and experimental workers [1,8-11] to studying the process of antihydrogen formation in antiprotons ($\bar{p}$) and positronium (Ps) collisions. Since the antiproton ($\bar{p}$) beam is now available from the Low Energy Antiproton Ring (LEAR) facility at CERN, it is now possible to produce anithydrogen in the laboratory through anitproton and positronium collisions. Although some experimental groups [1,8–11] have reported their experimental techniques for producing antihydrogen in the laboratory, until now, to our knowledge, no experimental data for $\bar{H}$ formation cross section has been available in the literature for the process: $\bar{p}+\mathrm{Ps}\rightarrow\bar{\mathrm{H}}+e$. Even very few theoretical works for the calculation of the antihydrogen formation cross sections have been performed. This demands further theoretical study of the process $\bar{p}+\mathrm{Ps}\rightarrow\bar{\mathrm{H}}+e$.

In the present paper we have calculated the cross sections of antihydrogen formation in the ground and excited $(2s,2p)$ states in antiprotons and positronium (from ground-state) collisions within the framework of eikonal approximation.

II. THEORY

The exact eikonal amplitude (post form) for $\bar{\mathrm{H}}$ formation in the nlm state for the $\bar{p}$-Ps collision process is written as [12]

$$g_{nlm}=-\frac{\mu_f}{2\pi}\int\int \exp(-i\vec{k}_f\cdot\vec{\rho})\phi^*_{nlm}(\vec{r}_2)\times\left[\frac{1}{r_1}-\frac{1}{r_{12}}\right]\phi_i(\vec{r}_{12})\times\exp(i\vec{k}_i\cdot\vec{s})\left[\frac{(r_2+r_{2z})}{(r_1+r_{1z})}\right]^{-i\eta_i}\times d\vec{r}_1 d\vec{r}_2\ , \tag{1}$$

where $\vec{r}_1$ and $\vec{r}_2$ are the position vectors of the electron and the positron, respectively, of the target Ps atom from the incident antiproton ($\bar{p}$). $\vec{r}_{12}$ denotes the coordinate of the positron with respect to the electron. $\vec{r}_{12}=\vec{r}_2-\vec{r}_1$. r_{1z} and r_{2z} are the z components of the respective vectors. $\vec{\rho}$ denotes the position vector of the electron with respect to the center of mass of the $\bar{\mathrm{H}}$ atom, and $\vec{s}$ denotes the position vector of the center of mass of the Ps atom from the antiproton with $\vec{\rho}=\vec{r}_1-a\vec{r}_2$ and $\vec{s}=\frac{1}{2}(\vec{r}_1+\vec{r}_2)$, where $a=1/(M_p+1)$, M_p being the mass of the antiproton. $\eta_i=\mu_i/k_i$; μ_i and μ_f are the reduced masses of the initial and final system, respectively; $\vec{k}_i$ and $\vec{k}_f$ are the initial and final momenta, respectively. ϕ_i and ϕ_{nlm} refer to the initial $1s$ and final ($1s$, $2s$, or $2p$) bound-state wave functions of the Ps and H atoms, respectively. We use atomic units throughout our calculations.

We choose our coordinate system such that the scattering takes place in the xz plane and the direction of the incident antiproton is along the z axis. The ground-state wave function of the Ps atom is

$$\phi_i(\vec{r}_{12})=C_0\exp(-\lambda_i r_{12})\ ,$$

with $C_0=1/\sqrt{(8\pi)}$ and $\lambda_i=0.5$. The ground- and excited-state $(2s,2p)$ wave functions [13] of the $\bar{\mathrm{H}}$ atom are given as follows:

$$\phi_{1s}(\vec{r}_2)=C_0'\exp(-\lambda_f r_2)\ ,$$

with $C_0'=\frac{1}{\sqrt{\pi}}$ and $\lambda_f=1$;

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$$\phi_{2s}(\vec{r}_2)=C_1'(1-\tfrac{1}{2}r_2)\exp(-\lambda_f r_2)\ ,$$

with $C_1'=1/2\sqrt{(2\pi)}$ and $\lambda_f=0.5$;

$$\phi_{2p_x}(\vec{r}_2)=C_2' r_2\exp(-\lambda_f r_2)\sin\theta\cos\varphi\ ;$$

$$\phi_{2p_y}(\vec{r}_2)=C_2' r_2\exp(-\lambda_f r_2)\sin\theta\sin\varphi\ ;$$

$$\phi_{2p_z}(\vec{r}_2)=C_2' r_2\exp(-\lambda_f r_2)\cos\theta\ ,$$

with $C_2'=1/4\sqrt{(2\pi)}$ and $\lambda_f=0.5$.

The matrix elements g_{nlm} in Eq. (1) for the degenerate final $2p$ states corresponding to $m=-1, 0, +1$ are denoted as g_{-1}, g_0, and g_{+1}, respectively. It can be shown that

$$g_0=g_{2p_z}\ ,\quad g_{\pm1}=(g_{2p_x}\pm ig_{2p_y})/\sqrt{2}\ , \tag{1a}$$

whre g_{2p_x}, g_{2p_y}, and g_{2p_z} are the x, y, and z components of the $2p$ capture amplitude, respectively. In order to evaluate Eq. (1), we first start with the parent integral

$$J=\int\int \exp(-i\vec{k}_f\cdot\vec{\rho})\exp(-\lambda_f r_2)\exp(i\vec{\varepsilon}\cdot\vec{r}_2)\times\left[\frac{1}{r_1}-\frac{1}{r_{12}}\right]\exp(-\lambda_i r_{12})\exp(i\vec{k}_i\cdot\vec{s})\times\left[\frac{(r_2+r_{2z})}{(r_1+r_{1z})}\right]^{-i\eta_i} d\vec{r}_1 d\vec{r}_2\ , \tag{2}$$

where $\vec{\varepsilon}$ is an arbitrary vector introduced in order to generate g_{2p_x}, g_{2p_y}, and g_{2p_z} through the relation

$$J_{x,y,z}=-i\lim_{\varepsilon_{x,y,z}\to 0}\frac{\partial J}{\partial\varepsilon_{x,y,z}}\ . \tag{3}$$

However, for capture in the $1s$ or $2s$ state, we put $\vec{\varepsilon}=0$ in (2).

The ($1s$-$1s$) capture amplitude is, therefore, given by

$$g_{1s}=-\frac{\mu_f C}{2\pi}[J]_{\varepsilon=0}\ , \tag{4}$$

where $C=C_0C_0'$. The ($1s$-$2s$) capture amplitude is given by

$$g_{2s}=-\frac{\mu_f C_0}{2\pi}\left[C_1'J+\frac{C_1'}{2}\frac{\partial J}{\partial\lambda_f}\right]_{\varepsilon=0}\ . \tag{5}$$

The x and the z components of the ($1s$-$2p$) capture amplitude are

$$g_{2p_x}=-\frac{\mu_f C_0 C_2'}{2\pi}J_x\ , \tag{6}$$

$$g_{2p_z}=-\frac{\mu_f C_0 C_2'}{2\pi}J_z\ . \tag{7}$$

Due to the choice of our coordinate system as mentioned before, $g_{2p_y}=0$.

We now use our technique [12,14] of evaluation of the exact eikonal amplitude to reduce J into a single-dimensional integral. For this pupose, at first we use the following contour integral representation for the eikonal phase term occurring in Eq. (2):

$$y^{-i\eta}=-\frac{1}{2i\sin(\pi i\eta)\Gamma(i\eta)}\int_c(-\lambda)^{i\eta-1}e^{-\lambda y}d\lambda\ . \tag{8}$$

Thus, in view of Eq. (8), Eq. (2) reduces to

$$J\approx\int_{\Gamma_1}\int_{\Gamma_2}\int\int \exp(-i\vec{k}_f\cdot\vec{\rho})\exp(-\lambda_f r_2)\exp(i\vec{\varepsilon}\cdot\vec{r}_2)\left[\frac{1}{r_1}-\frac{1}{r_{12}}\right]\times\exp(-\lambda_i r_{12})\exp(i\vec{k}_i\cdot\vec{s})p_1(\lambda_1,\vec{r}_1)p_2(\lambda_2,\vec{r}_2)d\vec{r}_1 d\vec{r}_2 d\lambda_1 d\lambda_2\ , \tag{9}$$

where

$$p_1(\lambda_1,\vec{r}_1)=e^{-\lambda_1(r_1+r_{1z})}(-\lambda_1)^{-i\eta_i-1}$$

and

$$p_2(\lambda_2,\vec{r}_2)=e^{-\lambda_2(r_2+r_{2z})}(-\lambda_2)^{i\eta_i-1}\ .$$

Using Fourier transform techniques, the space integrations (over $\vec{r}_1$ and $\vec{r}_2$) in Eq. (9) can be performed analytically. The actual space integral occurring in Eq. (9) can be generated [12] by parametric differentiations of the function

$$J_0=2\pi^2\int_0^\infty\frac{dx}{\alpha x^2+2\beta x+\gamma}\ . \tag{10}$$

where α, β, and γ are functions [12] of $\vec{k}_i$, $\vec{k}_f$, $\vec{\varepsilon}$, λ_i, and λ_f. The product $\alpha\gamma$ is split in such a manner [15] that both the functions α and γ are individually linear functions of the integration variables λ_1 and λ_2. By virtue of this choice we can perform the λ_1 and λ_2 integrations analytically.

We now proceed to carry out the λ_1 and λ_2 integrations analytically following the method developed by Sinha *et al.* [12], leaving behind the x integration occurring in Eq. (10). The result of integration (I_0) with respect to λ_1 and λ_2 can be expressed in terms of the Gauss hypergeometric function (${}_2F_1$):

$$I_0=-\frac{4\pi^2}{a}\left[\frac{b}{a}\right]^{i\eta_i}\left[\frac{c}{a}\right]^{-i\eta_i}\left[\frac{ad}{bc}\right]^{i\eta_i}\times{}_2F_1(1-i\eta_i,-i\eta_i,1;z)\ , \tag{11}$$

with $z=1-(bc)/(ad)$; a, b, c, and d being functions of the variable x, and the parameters $\vec{k}_i$, $\vec{k}_f$, $\vec{\varepsilon}$, λ_i, and λ_f.

Thus, we are finally left with a one-dimensional real integral over the variable x which has been carried out numerically by the Gaussian quadrature method. We have

TABLE I. $\bar{H}$ formation cross sections in the ground state (σ_{1s}) and $n=2$ level (σ_{2s} and σ_{2p}) for the process $\bar{p}+\mathrm{Ps}(1s)\rightarrow\bar{H}$ ($1s$, $2s$, or $2p$)$+e$ (in units of 10^{-16} cm^2). The numbers in square brackets indicate the power of 10 by which the entry is to be multiplied.

Incident $\bar{p}$ energy (E) in keV	σ_{1s} FBA	σ_{1s} eikonal	σ_{2s} FBA	σ_{2s} eikonal	σ_{2p} FBA	σ_{2p} eikonal
30	4.67[−1]	4.34[−1]	1.12[−1]	9.46[−2]	4.86[−1]	3.62
40	2.32[−1]	2.44[−1]	5.41[−2]	4.11[−2]	1.64[−1]	1.20
50	1.25[−1]	1.44[−1]	2.86[−2]	2.33[−2]	6.46[−2]	4.57[−1]
75	3.46[−2]	4.53[−2]	7.41[−3]	7.69[−3]	9.86[−3]	6.33[−2]
100	1.22[−2]	1.72[−2]	2.46[−3]	2.96[−3]	2.28[−3]	1.35[−2]
125	5.10[−3]	7.48[−3]	9.73[−4]	1.28[−3]	6.87[−4]	3.84[−3]
150	2.39[−3]	3.61[−3]	4.36−4]	6.07[−4]	2.48[−4]	1.32[−3]
175	1.22[−3]	1.89[−3]	2.16[−4]	3.12[−4]	1.02[−4]	5.26[−4]
200	6.72[−4]	1.06[−3]	1.15[−4]	1.71[−4]	4.68[−5]	2.33[−4]
225	3.91[−4]	6.22[−4]	6.52[−5]	9.92[−5]	2.32[−5]	1.13[−4]
250	2.38[−4]	3.83[−4]	3.88[−5]	6.02[−5]	1.23[−5]	5.85[−5]
275	1.51[−4]	2.45[−4]	2.41[−5]	3.79[−5]	6.87[−6]	3.21[−5]
300	9.88[−5]	1.61[−4]	1.56[−5]	2.47[−5]	4.02[−6]	1.85[−5]
325	6.66[−5]	1.09[−4]	1.03[−5]	1.66[−5]	2.45[−6]	1.11[−5]
350	4.61[−5]	7.59[−5]	7.06[−6]	1.14[−5]	1.54[−6]	6.88[−6]
375	3.26[−5]	5.39[−5]	4.93[−6]	8.01[−6]	9.98[−7]	4.40[−6]
400	2.35[−5]	3.90[−5]	3.52[−6]	5.74[−6]	6.64[−7]	2.90[−6]
425	1.72[−5]	2.86[−5]	2.56[−6]	4.19[−6]	4.52[−7]	1.95[−6]
450	1.28[−5]	2.14[−5]	1.89[−6]	3.11[−6]	3.14[−7]	1.34[−6]
475	9.69[−6]	1.62[−5]	1.42[−6]	2.34[−6]	2.22[−7]	9.41[−7]
500	7.41[−6]	1.24[−5]	1.08[−6]	1.78[−6]	1.60[−7]	6.72[−7]
750	8.54[−7]	1.44[−6]	1.18[−7]	1.99[−7]	1.14[−8]	4.52[−8]
1000	1.76[−7]	2.98[−7]	2.38[−8]	4.02[−8]	1.70[−9]	6.50[−9]

made a general computer code to calculate the ${}_2F_1$ function over the entire complex plane of the argument, making use of its proper analytic continuations. The functions containing branch cuts in Eq. (11) are also computed with proper care. The convergence of the final one-dimensional integral has been tested by increasing the number of Gaussian quadrature points.

In view of the above calculations, we can calculate the differential cross sections through the relations

$$\frac{d\sigma_{1s}}{d\Omega}=\frac{\mu_i k_f}{\mu_f k_i}|g_{1s}|^2 , \tag{12}$$

$$\frac{d\sigma_{2s}}{d\Omega}=\frac{\mu_i k_f}{\mu_f k_i}|g_{2s}|^2 , \tag{13}$$

and

$$\frac{d\sigma_{2p}}{d\Omega}=\frac{\mu_i k_f}{\mu_f k_i}(|g_0|^2+|g_{+1}|^2+|g_{-1}|^2)$$

$$\equiv\frac{\mu_i k_f}{\mu_f k_i}(|g_{2p_x}|^2+|g_{2p_z}|^2) . \tag{14}$$

The corresponding integrated cross sections are calculated using the formula

$$\sigma=\int\frac{d\sigma}{d\Omega}d\Omega . \tag{15}$$

III. RESULTS AND DISCUSSION

We have computed the integrated cross sections of antihydrogen formation in the ground and excited ($2s$,$2p$)

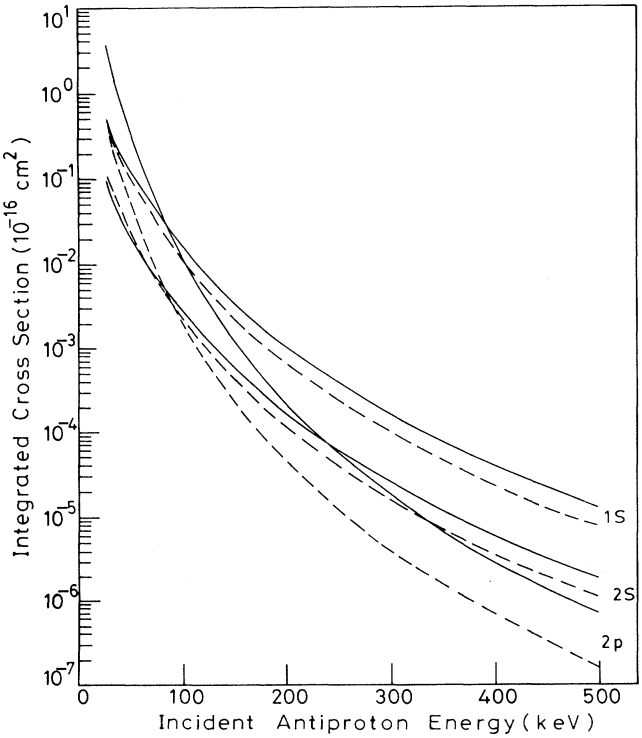

FIG. 1. The integrated cross sections for antihydrogen formation in the ground-state and $n=2$ level from the ground-state Ps atom as a function of incident antiproton energy. —— indicates present eikonal results and — — — indicates our FBA results.

states for the process $\bar{p}+\mathrm{Ps}(1s)\rightarrow\bar{\mathrm{H}}(1s,2s,\text{ or }2p)+e$ in the framework of eikonal approximation for different incident antiproton ($\bar{p}$) energies.

Table I displays the integrated cross-section results for the antihydrogen formation in $1s$, $2s$, and $2p$ states from the ground state of Ps for the incident energy range 30–1000 keV. Both the eikonal and first-order Born approximation (FBA) results are presented. The quoted FBA values (post form) are derived by putting $\eta_i=0$ in our same computer program and they show good agreement with those of Nahar and Wadehra [6], who have studied the same process in the first Born approximation. However, it should be mentioned here that the present results in Table I are obtained using the post form of interaction, while the results of Ref. [6] refer to the prior form of interaction.

Figure 1 gives a visual comparison of the data quoted in Table I. It is apparent from Fig. 1 that both the FBA and the eikonal individual cross sections ($1s$, $2s$, and $2p$) decreases gradually with increasing energy. The $2p$ eikonal cross sections are always higher than the corresponding FBA results, while the $1s$ and $2s$ eikonal results are higher than the FBA cross sections for energies ≥ 30 keV and ≥ 60 keV, respectively. It is also evident from Table I and Fig. 1 that for eikonal approximation, the capture into the $2p$ state is higher than into the $1s$ and $2s$ states at impact energies ≤ 75 keV for $1s$ and ≤ 250 keV for $2s$. The same condition is also noted for the FBA results but for different energy limits. This feature is in conformity with the earlier theoretical [7] findings, which studied the same process in the low-energy region.

It is evident from Table I that below 75 keV, $\sigma_2(\sigma_2=\sigma_{2s}+\sigma_{2p})>\sigma_{1s}$, i.e., in this energy region, capture into excited states is higher than that in the ground state, while from 75 keV and onward, the ground-state capture cross section dominates. It may also be noted from Table I that the total eikonal cross section ($\sigma=\sigma_{1s}+\sigma_{2s}+\sigma_{2p}$)is always higher than the corresponding FBA value throughout the energy region considerd in this work.

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