# Radiation pressure and coherent states of two-level atoms

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(Received 13 September 1994)

Two sets of coherent states of a two-level atom are constructed by the displacement-operator technique from the choices of the ground and excited states as the extremal state. Particular forms of both of these sets of coherent states are minimum-uncertainty states. Radiation-induced forces are determined for a two-level atom prepared in such coherent states. In general, these forces are affected by the presence of a second atom. For a minimum-uncertainty state, the force on a two-level atom is independent of the state of the field. The significance of the calculations is discussed in relation to the pure atomic state occurring in the midst of the "collapse region" of the atomic inversion in the Jaynes-Cummings model. The existence of such a pure state may provide a possible means of experimentally generating two-level atomic minimum-uncertainty states.

PACS number(s): 42.50.Vk

The use of the mechanical effects of radiation, particularly those induced by near-resonance laser light, has become a powerful technique for controlling the gross motions of neutral atoms and ions [1]. However, the possibility of preparing the atom in a coherent state has not been discussed previously in the context of radiation-induced forces, perhaps because it has not been easy to see how such states might be made in the laboratory. This is the subject of the present Brief Report, in which the radiation pressure experienced by a two-level atom prepared in a coherent state is determined and a suggestion is made as to how such a state might be experimentally realized.

In the study of the basic physics of the interaction of radiation with matter, the two-level atom, which was originally introduced by Einstein in an early paper on radiation kinetics [2], remains an important theoretical model [3]. This is particularly so in the case where the radiation field is specialized to a single mode and the dynamics are described in the rotating-wave approximation by the Jaynes-Cummings Hamiltonian [4]. As well as being exactly solvable, the Jaynes-Cummings model has been achieved experimentally with the use of Rydberg atoms in high-$Q$ microwave cavities [5]. One of the profound aspects of this model is the collapse of the atomic inversion after a short time interval, followed by further revivals and collapses [6]. This is achieved only when the radiation field is prepared in a coherent state, indicating the limitations of a semiclassical model of radiation interactions. In an interesting development of the theory—and one which may have significance in the possible experimental generation of two-level atomic coherent states—it would seem that the two-level atom returns to a pure state in the midst of the inversion-collapse region [7]. It is indicated below that this pure state is also a minimum-uncertainty state of the two-level atom.

Coherent states of a single mode of a harmonic oscillator may be defined in three equivalent ways: as minimum-uncertainty states, as the eigenstates of the mode's annihilation operator, or as the states formed by the action of the Glauber displacement operator on the mode's ground or vacuum state [8]. In the case of a two-level system, however, this freedom of choice is curtailed. Coherent states of a two-level system are constructed in terms of a complex number $\alpha = |\alpha| \exp(i\phi)$ by the action of the displacement operator

$$\hat{D}(\alpha) = \exp\left\{\alpha\hat{\pi}^\dagger - \alpha^*\hat{\pi}\right\} \tag{1}$$

on a reference or extremal state. In Eq. (1), $\hat{\pi}^\dagger = |e\rangle\langle g|$ and $\hat{\pi} = |g\rangle\langle e|$ are the raising and lowering dyadics of the two-level system, which is composed of a ground state $|g\rangle$ and an excited state $|e\rangle$. Thus a coherent state of a two-level atom is defined as

$$|\alpha, E\rangle = \hat{D}(\alpha)|E\rangle, \tag{2}$$

where the extremal state $|E\rangle$ is identified to be either $|g\rangle$ or $|e\rangle$. The displacement operator is given explicitly by the matrix

$$\hat{D}(\alpha) = \begin{bmatrix} \cos|\alpha| & \exp(i\phi)\sin|\alpha| \\ -\exp(-i\phi)\sin|\alpha| & \cos|\alpha| \end{bmatrix} \tag{3}$$

in the representation $\langle g| = (0,1)$, $\langle e| = (1,0)$, allowing the two sets of coherent states

$$|\alpha, g\rangle = \hat{D}(\alpha)|g\rangle = \cos|\alpha||g\rangle + \exp(i\phi)\sin|\alpha||e\rangle, \tag{4a}$$

$$|\alpha, e\rangle = \hat{D}(\alpha)|e\rangle = -\exp(-i\phi)\sin|\alpha||g\rangle + \cos|\alpha||e\rangle \tag{4b}$$

to be easily constructed from Eq. (2). From the unitary nature of the displacement operator (1), it is evident that these coherent states, which are of course normalized $\langle E,\alpha|\alpha,E\rangle = 1$, are complete $\sum_{E=e,g}|\alpha,E\rangle\langle E,\alpha| = 1$ and orthogonal $\langle E,\alpha|\alpha,E'\rangle = \delta_{EE'}$ in the space of $E$. It may also be verified that the states are overcomplete $\int d^2\alpha|\alpha,E\rangle\langle E,\alpha| = \pi$ and nonorthogonal $\langle E,\alpha|\alpha',E\rangle \neq 0$ in the space of $\alpha$. The set (4a) of coherent states formed from the selection of the ground state $|g\rangle$ as the extremal $|E\rangle$ has previously appeared in the literature [9].

In general, for $E = g$ or $e$, the coherent states (4) allow the variances

$$\langle E,\alpha|(\Delta X)^2|\alpha,E\rangle = (1/4) - \cos^2|\alpha|\sin^2|\alpha|\cos^2\phi, \tag{5a}$$

$$\langle E,\alpha|(\Delta Y)^2|\alpha,E\rangle = (1/4) - \cos^2|\alpha|\sin^2|\alpha|\sin^2\phi \tag{5b}$$

 

of the quadrature operators

$$\hat{X} = (1/2)\left\{\hat{\pi} + \hat{\pi}^{\dagger}\right\}, \tag{6a}$$

$$\hat{Y} = -(i/2)\left\{\hat{\pi} - \hat{\pi}^{\dagger}\right\} \tag{6b}$$

to be determined, along with the relationship

$$|\langle E,\alpha|[\hat{X},\hat{Y}]|\alpha,E\rangle|^2 = \frac{1}{4}\cos^2(2|\alpha|). \tag{7}$$

The right-hand sides of Eqs. (5) and (7) are $E$ independent but $\alpha$ dependent. These are important properties of the quadratures' variances and commutator for two-dimensional coherent states; they are not all present in the equivalent quantities evaluated on coherent states of higher state-space dimensions [10]. Mindful of the uncertainty relationship

$$\langle(\Delta X)^2\rangle\langle(\Delta Y)^2\rangle \geq \frac{1}{4}\left|\langle[\hat{X},\hat{Y}]\rangle\right|^2, \tag{8}$$

one can see that (4) form intelligent states, that is, states for which the inequality (8) is saturated, for all values of $\phi$ if $|\alpha| = 0$ or $\pi$. (The angular range is restricted to values between 0 and $2\pi$.) On the other hand, intelligent states may also be formed for all values of $|\alpha|$ if $\phi = 0,\pi/2,\pi,3\pi/2$ and, further, if $|\alpha| = \pi/4,3\pi/4$, then these states become minimum-uncertainty states and the right-hand side of (8) forms a local minimum. These conditions for intelligent and minimum-uncertainty states have been previously reported for the set (4a) formed from the choice of $E = g$ [9]. The states $|\alpha,E\rangle$ give the same physics: ignoring the phase angle $\phi$, the states $|\alpha,e\rangle$ and $|\alpha,g\rangle$ are equivalent when $|\alpha|$ is replaced by $|\alpha|-\pi/2$, consistent with the fact that these states lie at opposite points on the Bloch sphere ($\phi = 0$).

All pure states can be called coherent states [11] and, apart from a global phase factor, an arbitrary state for a two-level system may be written as $\cos(\beta)|g\rangle + \exp(i\theta)\sin(\beta)|e\rangle$. In particular, the minimum-uncertainty states of a two-level system are the same pure states, to within a phase factor, to which a two-level system, when driven by a single coherent mode, evolves at a time within the region of collapse of the inversion [7]. This may provide a possible means of generating such minimum-uncertainty states.

Coherent states play a unique role in quantum dynamics since they form nonvanishing expectation values with the appropriate particle creation and annihilation operators. In the present case of two-level atomic coherent states, the expectation values of $\hat{\pi}$ and $\hat{\pi}^{\dagger}$ constructed out of the states $|\alpha,E\rangle$, $E = e$ or $g$, are proportional to $\sin(2|\alpha|)$. The effect of this is to allow the dynamical variations within one atom to influence another nearby atom through changes in the interaction photon field.

In the case of radiation pressure, this interatomic influence produces a nontrivial contribution to the force on any one atom. To illustrate this, two two-level atoms, each distinguished by an index $\mu = 1,2$, with a common energy separation of $\hbar\Omega$ between their respective ground $|g_\mu\rangle$ and excited $|e_\mu\rangle$ states, are considered to interact with a single-mode $\mathbf{k}$ of a quantized radiation field $\hat{\mathbf{A}}(\mathbf{r}) = \sqrt{\hbar/2\omega\varepsilon_0 V}\boldsymbol{\epsilon}\hat{a}\exp(i\mathbf{k}\cdot\mathbf{r}) + \text{H.c.}$ This field, written in terms of a unit polarization vector $\boldsymbol{\epsilon}$ and the usual Boson operators $\hat{a},\hat{a}^{\dagger}$, is quantized throughout a cavity volume $V$. It is supposed that the interatomic separation is sufficiently great that dipole-dipole effects may be ignored and that the Coulomb fields of each atom are localized about their respective centers of mass $\mathbf{R}_\mu$. The system's total Hamiltonian

$$\hat{H} = \sum_{\mu=1,2}\left\{\frac{\hat{\mathbf{P}}_\mu^2}{2m_\mu} + \hbar\Omega\hat{\pi}_\mu^{\dagger}\hat{\pi}_\mu\right\} + \hbar\omega\hat{a}^{\dagger}\hat{a} + \hat{H}_{\text{int}}, \tag{9a}$$

$$\hat{H}_{\text{int}} = \sum_{\mu=1,2} i\hbar\omega^{\frac{1}{2}}\gamma_\mu\hat{\pi}_\mu^{\dagger}\hat{a}\exp(i\mathbf{k}\cdot\mathbf{R}_\mu) + \text{H.c.} \tag{9b}$$

in the rotating-wave approximation is expressed in terms of the individual atomic lowering and raising dyadics $\hat{\pi}_\mu = |g_\mu\rangle\langle e_\mu|$ and $\hat{\pi}_\mu^{\dagger} = |e_\mu\rangle\langle g_\mu|$, as well as the atomic gross momenta $\hat{\mathbf{P}}_\mu = -i\hbar\{\partial/\partial\mathbf{R}_\mu\}$. The ladder operators of the Hamiltonian form the equal-time commutators

$$\left[\hat{\pi}_\mu,\hat{\pi}_{\mu'}^{\dagger}\right] = \delta_{\mu\mu'}\left\{1 - 2\hat{\pi}_\mu^{\dagger}\hat{\pi}_\mu\right\}, \tag{10a}$$

$$\left[\hat{a},\hat{a}^{\dagger}\right] = 1 \tag{10b}$$

and various constants are grouped under two parameters

$$\gamma_\mu = e\left[\frac{1}{2\varepsilon_0\hbar V}\right]^{\frac{1}{2}}\langle g_\mu|\boldsymbol{\epsilon}\cdot\hat{\boldsymbol{D}}_\mu|e_\mu\rangle, \tag{11}$$

written in terms of off-diagonal elements $-e\langle g_\mu|\hat{\boldsymbol{D}}_\mu|e_\mu\rangle$ of the atomic dipole moments. The modifications to the Thomas-Reiche-Kuhn sum rule necessary for determining the matrix-element component of (11) have appeared recently in the literature [12].

The expectation value of the radiation-induced force

$$\langle\hat{\mathbf{F}}_1(t)\rangle = \langle\{\hbar\omega^{\frac{1}{2}}\gamma_1\mathbf{k}\hat{\pi}_1^{\dagger}(t)\hat{a}(t)\exp[i\mathbf{k}\cdot\mathbf{R}_1(t)] + \text{H.c.}\}\rangle \tag{12}$$

on atom 1 at time $t$ follows from Ehrenfest's theorem in the form $\langle\hat{\mathbf{F}}_1\rangle = -\langle\partial\hat{H}/\partial\mathbf{R}_1\rangle$. The time dependence is shown explicitly in (12) as a reminder that in the Heisenberg representation, quantum-mechanical operators must be reexpressed in terms of their values at time $t = 0$ if an expectation value involving initial states is to be determined. In the present situation, this is achieved by means of the temporal variations

$$\begin{aligned}\hat{a}(t) = {} & \hat{a}(0)\exp[-i\omega t] \\ & - \sum_{\mu=1,2}\gamma_\mu\exp[-i\{\omega t + i\mathbf{k}\cdot\mathbf{R}_\mu(0)\}] \\ & \times\int_0^t dt'\ \hat{\pi}'_\mu(t')\exp[i\{\omega - \mathbf{k}\cdot\mathbf{V}_\mu\}t'],\end{aligned} \tag{13a}$$

$$\begin{aligned}\hat{\pi}_\mu(t) = {} & \hat{\pi}_\mu(0)\exp[-i\omega t] \\ & + \gamma_\mu\left[\hat{\pi}_\mu(0),\hat{\pi}_\mu^{\dagger}(0)\right]\exp[-i\{\Omega t - \mathbf{k}\cdot\mathbf{R}_\mu(0)\}] \\ & \times\int_0^t dt'\ \hat{a}(t')\exp[i\{\Omega + \mathbf{k}\cdot\mathbf{V}_\mu\}t'],\end{aligned} \tag{13b}$$

together with the equivalent equations for the Hermitian conjugates $\hat{a}^\dagger(t)$,$\hat{\pi}^\dagger_\mu(t)$, consistent with Eqs. (10) and the Heisenberg formalism $\dot{\hat{O}} = (i/\hbar)[\hat{H},\hat{O}]$. The temporal variations of the centers of mass have been accounted for in (13) by writing

$$\mathbf{R}_\mu(t) = \mathbf{R}_\mu(0) + \mathbf{V}_\mu t, \tag{14}$$

where $\mathbf{V}_\mu$ is the velocity of the $\mu$ atom, sufficient to provide a final determination of the expectation values of the force correct to the second order in $\gamma_\mu$.

Suppose that the atoms are prepared in coherent states

$$|\alpha_\mu, g\rangle = \cos|\alpha_\mu||g_\mu\rangle + \exp(i\phi_\alpha)\sin|\alpha_\mu||e_\mu\rangle, \tag{15a}$$

$$|\alpha_\mu, e\rangle = -\exp(-i\phi_\alpha)\sin|\alpha_\mu||g_\mu\rangle + \cos|\alpha_\mu||e_\mu\rangle, \tag{15b}$$

where $\alpha_\mu = |\alpha_\mu|\exp(i\phi_\mu)$, while the radiation field is prepared in the number state $|n\rangle$. The total system is therefore prepared in one of four possible tensor-product states:

$$|\Psi_{E_1E_2}\rangle = |n\rangle \otimes |\alpha_1, E_1\rangle \otimes |\alpha_2, E_2\rangle, \tag{16}$$

with $E_\mu$ assuming the labels $g_\mu$ and $e_\mu$. If the initial conditions are chosen such that

$$\phi_1 - \phi_2 = \mathbf{k}\cdot\mathbf{R}_1(0) - \mathbf{k}\cdot\mathbf{R}_2(0), \tag{17a}$$

$$\mathbf{V}_1 = \mathbf{V}_2 = \mathbf{V}, \tag{17b}$$

say, and it is assumed then the radiation polarization vector $\boldsymbol{\epsilon}$ is real, then the expectation values of the forces on atom 1 at time $t$ take the simple forms

$$\langle\Psi_{E_1E_2}|\hat{\mathbf{F}}_1|\Psi_{E_1E_2}\rangle = 2\hbar\omega\mathbf{k}\gamma_1^2\left\{n - (1+2n)C_{E_1}^2\right\}\frac{\sin\Delta t}{\Delta} - \hbar\omega\mathbf{k}\frac{\gamma_1\gamma_2}{2}\sin 2|\alpha_1|\,\sin 2|\alpha_2|\frac{\sin\Delta t}{\Delta}, \tag{18}$$

correct to second order in the parameters (11), where

$$\Delta = \omega - \Omega - \mathbf{k}\cdot\mathbf{V} \tag{19}$$

is the detuning, corrected for the Doppler effect, and

$$C_{g_1} = \sin|\alpha_1|, \qquad C_{e_1} = \cos|\alpha_1|. \tag{20}$$

The first term of Eq. (18) is the radiation-induced force experienced by an isolated atom and the factors (20) depend only on the coherent state in which atom 1 is prepared. It should be noted that the second term of Eq. (18), which arises only if atom 2 is present, is independent of the particular settings of $E_1$,$E_2$ and would vanish if one or both atoms were prepared in an eigenstate. The effect of the initial conditions (17) is to reduce to the particular form of Eq. (19), the argument of the sine function appearing in the second term of the force (18). Perhaps the simplest way to satisfy the conditions (17) is to prepare the atoms in the same coherent states, with their velocities perpendicular to $\mathbf{k}$.

If the atoms are now prepared in minimum-uncertainty states, such that $|\alpha_1| = |\alpha_2| = \pi/4$, then Eq. (18) becomes independent of the number of photons in the mode, viz.,

$$\langle\Psi_{E_1E_2}|\hat{\mathbf{F}}_1|\Psi_{E_1E_2}\rangle = -\hbar\omega\mathbf{k}\left\{\gamma_1^2 + \frac{\gamma_1\gamma_2}{2}\right\}\frac{\sin\Delta t}{\Delta}. \tag{21}$$

For a two-level atom prepared in such a state, the probability of it radiating a photon is the same as the probability of it absorbing a photon; consequently, the state of the field does not influence the radiation pressure. In this situation, the second term in Eq. (21), which is due solely to the presence of the second atom, is of the same order of magnitude as the first term.

In principle, coherent states of a two-level atom, in the form of minimum-uncertainty states, may be prepared by flying such atoms through a micromaser cavity with an appropriate time of flight or, more easily, by applying an appropriate microwave pulse [7]. If such an atom, emerging from the micromaser cavity, were to interact with a second photon field, then the radiation-induced force would be independent of the field's intensity. This may provide a possible experimental verification of the existence of a pure state in the midst of the collapse region of the Jaynes-Cummings model.

I thank R. Loudon and S. J. D. Phoenix for encouragement and enlightening discussions.

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