# Simple example of nonlocality: Atoms interacting with correlated quantized fields

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We present a simple example that shows nonlocality in quantum mechanics. The basic elements are two microwave cavities equipped with correlated quantized fields that interact with two atoms. We perform state-selective measurements on these atoms that subsequently reveal a type of nonlocality recently described by Hardy [Phys. Rev. Lett. **71**, 1665 (1993)].

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## I. INTRODUCTION

Recently Hardy [1,2] has given an intriguing proof of nonlocality in quantum mechanics which has been discussed by several authors [3,4]. He has shown that a whole class of entangled spin-1/2 quantum states admits an argumentation against any local realistic interpretation of quantum theory. Like the nonlocality proof of Greenberger, Horne, and Zeilinger (GHZ) [5], he does not use a variant of Bell's inequalities [6]. Beyond that and in contrast to GHZ, Hardy needs only a two-particle state in order to formulate his *gedanken experiment* demonstrating nonlocality [7]. At the heart of his idea lies a sequence of four measurements performed on this especially designed two-particle state. The results of those measurements lead to a profound contradiction when interpreted under the assumption of a local realistic theory [3]. Other proofs of nonlocality without inequalities for two spin-1 particles have been given in Refs. [8,9]. Generalizations of these considerations can be found in Refs. [10,11].

Moreover, Hardy has proposed quantum optical realizations [12,13] of his arguments in order to test local realism. One of these experiments [12] uses correlated photon pairs from degenerate parametric down-conversion. The photon pairs are sent into two overlapping beam splitter arrangements, which serve as a two-particle interferometer. The clicks of photodetectors in the output channels demonstrate the nonlocal correlation of the two photons. In a second interferometric experiment [13] he discusses the nonlocality of a single photon [14,15].

It is the purpose of this paper to present a quantum optical arrangement that allows us to show nonlocality in a quite simple way along the lines of Ref. [2]. The setup, shown in Fig. 1, contains two microwave cavities $C_1$ and $C_2$, whose fields we correlate [16–18] using at most a single quantum transported by an excited two-level atom. We perform a readout of the cavities with the help of two more atoms. Four state-selective measurements on these atoms will reveal a chain of arguments against a local realistic interpretation of the measurement's results.

Our scheme consists of two important steps. In the first step, the preparation, we correlate the two identical microwave cavities with the help of a single quantum. Both cavities are initially in their vacuum states. This can be achieved in the following way; see Fig. 1. A two-level atom travels through both cavities in a superposition of its ground and excited state. Hence it carries at most a single excitation. The cavities are tuned on resonance with the atomic transition. After the interaction with both cavities we measure the atom in its ground state. Therefore the excitation of the atom has been transferred to the system consisting of the two cavities. This completes the preparation.

In the next step, the measurement, we perform state-selective measurements on two atoms, after each of them has passed one of these well-prepared cavities. Altogether we need four measurements on the two atoms in order to show the striking nonlocality argument which has been reported by Hardy in Ref. [2].

## II. PREPARATION

The preparation constitutes the first step. The atom, which prepares the two cavities, starts its journey in an initial superposition of the ground state $|g\rangle$ and the excited state $|e\rangle$, that is,

$$|\text{atom for preparation}\rangle = c_g|g\rangle + c_e|e\rangle \tag{1}$$

with $|c_g|^2 + |c_e|^2 = 1$. The cavities are initially in their vacuum states. We describe the resonant interaction between the atom and a single mode of each cavity using the Jaynes-Cummings Hamiltonian

$$H_I^i = \hbar g(\hat{\sigma}_+\hat{a}_i + \hat{\sigma}_-\hat{a}_i^\dagger) \tag{2}$$

in the interaction picture. The vacuum Rabi frequency $g$ determines the coupling strength between atoms and quantized cavity fields, while $\hat{\sigma}_+, \hat{\sigma}_-$ and $\hat{a}_i^\dagger, \hat{a}_i$ are the usual raising and lowering operators for the atomic transition and the number of quanta in the fields of cavity $i = 1$ and cavity $i = 2$, respectively.

After the interaction with the first cavity the complete state vector reads

 

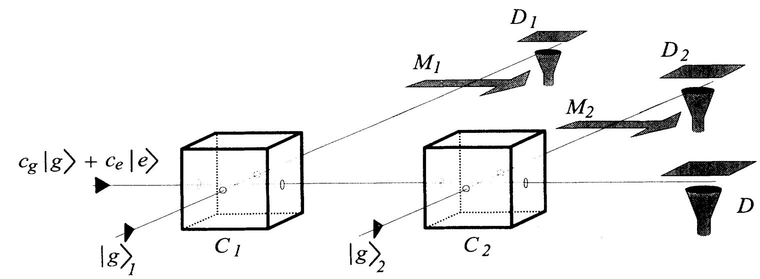


FIG. 1. Sketch of the experimental setup used to demonstrate nonlocality. In a first step an atom prepares the cavities $C_1$ and $C_2$ by traveling across them in an initial superposition $c_g|g\rangle + c_e|e\rangle$ of its ground and excited state. The ionization detector $D$ detects the ground state $|g\rangle$ of this atom. In the second step we send the atoms 1 and 2 through the cavities. We prepare both atoms in their ground states $|g\rangle_1$ and $|g\rangle_2$. After they have interacted with the cavities we perform state-selective measurements on these atoms. We achieve the state selection with the help of classical microwave fields $M_i$ and the state-selective ionization detectors $D_i$ ($i = 1, 2$).

$$\exp[-i/\hbar H_I^1 \tau]|\text{atom for preparation}\rangle|0\rangle_1|0\rangle_2$$

$$= \Big\{[c_g|g\rangle + c_e\cos(g\tau)|e\rangle]|0\rangle_1 - ic_e\sin(g\tau)|g\rangle|1\rangle_1\Big\}|0\rangle_2\,, \quad (3)$$

where $\tau$ stands for a given interaction time and $|n\rangle_i$ denotes the $n$th Fock state of the single quantized mode in cavity $i$. After cavity 2 we obtain for the state of the complete system

$$e^{-i/\hbar H_I^2\tau}e^{-i/\hbar H_I^1\tau}|\text{atom for preparation}\rangle|0\rangle_1|0\rangle_2$$

$$= \Big[c_g|0\rangle_1|0\rangle_2 - ic_e\sin(g\tau)|1\rangle_1|0\rangle_2 - ic_e\cos(g\tau)\sin(g\tau)|0\rangle_1|1\rangle_2\Big]|g\rangle + c_e\cos^2(g\tau)|0\rangle_1|0\rangle_2|e\rangle\,. \quad (4)$$

When we now find the preparing atom in the ground state $|g\rangle$ the pure field state of the cavity system reads

$$|\psi\rangle \equiv \mathcal{N}\Big[c_g|0\rangle_1|0\rangle_2 - ic_e\sin(g\tau)|1\rangle_1|0\rangle_2 - ic_e\cos(g\tau)\sin(g\tau)|0\rangle_1|1\rangle_2\Big] \quad (5)$$

with the normalization $\mathcal{N} = (1 - |c_e|^2\cos^4 g\tau)^{-1/2}$. This completes the first step. It becomes clear immediately from Eq. (5) that the single excitation carried by the superposition of the preparing atom now couples the two cavities: The state $|\psi\rangle$ represents an entangled state, which forbids us to talk of a photon that actually occupies one of the two cavities. Rather it is exactly this specific structure of $|\psi\rangle$ that gives rise to the nonlocal situation described in the next section. Our aim in the next step is to transfer the entanglement of $|\psi\rangle$ to two initially independent atoms. Subsequent state measurements on these atoms will reveal the remarkably simple example of nonlocality.

## III. MEASUREMENTS

The second step starts with an atom 1 in its ground state $|g\rangle_1$ sent through cavity 1 and an atom 2 in ground state $|g\rangle_2$ sent through cavity 2. Thus the initial state of the whole system is the product state

$$|\Psi(0)\rangle = |\psi\rangle|g\rangle_1|g\rangle_2 \quad (6)$$

with the field state $|\psi\rangle$ taken from Eq. (5). It is again the Jaynes-Cummings Hamiltonian Eq. (2) that governs the interaction of each atom with the respective cavity. The atomic operators in the Hamiltonian $H_I^1$ belong now to atom 1, whereas the atomic operators in $H_I^2$ operate exclusively on atom 2. If we choose the interaction time $\tau'$ in such a way that $g\tau' = \pi/2$ is fulfilled for both atoms, we arrive at the final state

$$|\Psi(\tau')\rangle = \exp\Big[-i/\hbar(H_I^1 + H_I^2)\tau'\Big]|\Psi(0)\rangle = \mathcal{N}\Big[c_g|g\rangle_1|g\rangle_2 - c_e\sin(g\tau)|e\rangle_1|g\rangle_2 - c_e\cos g\tau\sin(g\tau)|g\rangle_1|e\rangle_2\Big]|0\rangle_1|0\rangle_2\,. \quad (7)$$

We note that atoms and quantized light fields are no longer entangled. Both cavities are left in their vacuum states and the cavity fields decouple from the atoms. This expresses the fact that both atoms will transport the photon away from the cavity whenever there exists one. This is what we call the readout of the cavities [19].

In what follows we perform state-selective measurements on the two atoms regardless what the field state is. This is possible, in correspondence with Eq. (7), since the atoms and the cavity system are no longer entangled. Hence we concentrate on the pure atomic state

$$|\text{atoms}\rangle = c_1|g\rangle_1|g\rangle_2 + c_2|e\rangle_1|g\rangle_2 + c_3|g\rangle_1|e\rangle_2 \quad (8)$$

with $c_1 \equiv \mathcal{N}c_g$, $c_2 \equiv -\mathcal{N}c_e\sin g\tau$, and $c_3 \equiv -\mathcal{N}c_e\cos g\tau\sin g\tau$. The three coefficients $c_i$ are at our disposal and, more importantly, we are able to adjust the condition $c_1c_2c_3 \neq 0$.

Let us now start with the four different measurements on the atoms. We probe the state of each atom with the help of a classical microwave field $M_i$ ($i = 1, 2$) and a following state-selective field ionization detector $D_i$. See Fig. 1. The combination of $M_i$ and $D_i$ — the analyzer $i$ — allows us to analyze an arbitrary superposition of $|g\rangle_i$ and $|e\rangle_i$ of atom $i$ [16,17].

This works as follows. When a two-level atom enters a microwave field $M$ in a state $\mu|g\rangle+\nu|e\rangle$ with $|\mu|^2+|\nu|^2=1$, it undergoes a unitary transformation to the state $|e\rangle$ due to the interaction with the classical microwave field. The values of $\mu$ and $\nu$ are adjusted via an appropriate setting of these fields. Ideal detectors $D$ ionize the excited atom and detect the appearing electron. The signal for this shall be a click at the detector. On the other hand, the atom leaves the microwave zone in the ground state $|g\rangle$ if it enters in the state orthogonal to $\mu|g\rangle+\nu|e\rangle$, that is, in the state $\nu^*|g\rangle-\mu^*|e\rangle$. In that case the detector does not answer with a click. In general the two-level atom will be in some superposition of these orthogonal states. Thus, in some cases the ideal detector will deliver a click and in some not. Whenever we register a click, we project the superposition on the state $\mu|g\rangle+\nu|e\rangle$. This completes the state-selective measurement. That is, with the help of analyzer $i$ we are able to measure observables of the form $[\mu|g\rangle_i+\nu|e\rangle_i][\mu^*{}_i\langle g|+\nu^*{}_i\langle e|]$ which represent projection operators with measurable eigenvalues 0 (no click) and 1 (click).

As a first measurement (i) we determine whether the atoms are both in their excited states and the appropriate observables are

$$\hat{E}_1=|e\rangle_{11}\langle e|,\quad \hat{E}_2=|e\rangle_{22}\langle e|\,. \tag{9}$$

Thus we do not need the two microwave zones for this first measurement. According to the atomic state Eq. (8), we will always find

$$\text{(i)}\qquad E_1E_2=0 \tag{10}$$

for the measured values $E_i$. We will never register a click on $D_1$ *and* a click on $D_2$: There is at most only one quantum in the whole system and therefore Eq. (8) does not contain a term of the form $|e\rangle_1|e\rangle_2$, which would provide the possibility of a double click.

In the second measurement (ii) we analyze atom 1 in the superposition

$$\hat{S}_1=|\eta_1|^2\left[|g\rangle_1-\frac{c_1^*}{c_2^*}|e\rangle_1\right]\left[{}_1\langle g|-\frac{c_1}{c_2}{}_1\langle e|\right], \tag{11}$$

where $|\eta_1|^2=(1+|c_1/c_2|^2)^{-1}$ ensures normalization. Now we need the microwave field $M_1$ in order to adjust the measurement of this special superposition. The coefficients $c_i$ are taken from the atomic state Eq. (8). If we measure $\hat{S}_1$ on atom 1, we find for atom 2 the reduced pure state

$$\hat{\rho}_2=\frac{\mathrm{tr}_1(\hat{S}_1|\text{atoms}\rangle\langle\text{atoms}|)}{\mathrm{tr}_{12}(\hat{S}_1|\text{atoms}\rangle\langle\text{atoms}|)}=|e\rangle_{22}\langle e|\,, \tag{12}$$

where $\mathrm{tr}_1$ denotes the trace over atom 1 and $\mathrm{tr}_{12}$ denotes the trace over atom 1 and atom 2 keeping the reduced state normalized. Consequently, if we now measure the observable $\hat{E}_2$ on atom 2, we will obtain with certainty the result $E_2=1$, that is,

$$\text{(ii)}\qquad \text{if}\quad S_1=1\quad \text{then}\quad E_2=1\,. \tag{13}$$

Exactly the same arguments apply when we analyze in the third measurement (iii) the observable

$$\hat{S}_2=|\eta_2|^2\left[|g\rangle_2-\frac{c_1^*}{c_3^*}|e\rangle_2\right]\left[{}_2\langle g|-\frac{c_1}{c_3}{}_2\langle e|\right] \tag{14}$$

on atom 2 and $\hat{E}_1$ on atom 1. The factor $|\eta_2|^2=(1+|c_1/c_3|^2)^{-1}$ denotes again the normalization constant. By symmetry the result will be

$$\text{(iii)}\qquad \text{if}\quad S_2=1\quad \text{then}\quad E_1=1\,. \tag{15}$$

The last measurement (iv) aims towards the combination of $\hat{S}_1$ and $\hat{S}_2$ with two appropriate microwave settings. We derive from Eq. (8) the result $S_1S_2=1$, that is, $S_1=1$ and $S_2=1$ with a probability

$$\mathrm{tr}_{12}(\hat{S}_1\hat{S}_2|\text{atoms}\rangle\langle\text{atoms}|)=|\eta_1\eta_2c_1|^2\,. \tag{16}$$

Therefore we obtain as the result of our last measurement

$$\text{(iv)}\qquad S_1S_2=1\quad \text{with probability}\quad |\eta_1\eta_2c_1|^2\,. \tag{17}$$

Hardy has now proven in Ref. [2] that the combination of the four predictions (i)–(iv) makes a local realistic interpretation of quantum mechanics impossible. We are not going to repeat his proof. The aim of the present paper is just to provide this simple physical system, which gives rise to the nonlocality argument brought up in Ref. [2].

Nevertheless, for the sake of completeness, we give a short reasoning why the four predictions (i)–(iv) of our experiment sound like a contradiction when considered under the assumption of a local realistic viewpoint. Suppose two observers 1 and 2 operating and reading the analyzers 1 and 2. Note that these analyzers might be far away from each other. Let us consider a run of the experiment, where observer 1 decides to detect $\hat{S}_1$ and observer 2 decides to detect $\hat{S}_2$. According to prediction (iv) there is a nonvanishing probability [20] that they read the values $S_1=1$ *and* $S_2=1$, provided they have chosen suitable microwave field zones. Observer 1 can point now to prediction (ii) and deduce that observer 2 finds his initially unexcited atom in the excited state, if he decides to measure $\hat{E}_2$. The other way around observer 2 points to prediction (iii) and deduces that observer 1 will find his atom excited, if he decides to measure the observable $\hat{E}_1$. However, we know, that there is — due to the preparing atom — at most only one quantum in the whole system. Hence both observers cannot be right and we have already seen — prediction (i) — that quantum theory indeed forbids the situation of two excited atoms. The arguments of the two observers lead to this evident contradiction because they both argue based on locality assumptions. Both observers think that they obtain their measurement results regardless of what the other is actually measuring. However, the results that can be obtained by observer 1 depend on the observable that observer 2 decides to detect and vice versa.

## IV. CONCLUSIONS

We have seen how nonlocality arises in the interaction of two-level atoms with the quantized modes of two correlated microwave cavities. It is a simple system that consists only of basic elements. A first atom that passes both cavities was used for the preparation of the system. This atom itself was initially prepared in a superposition of its ground and excited state before it enters the cavities. This superposition is crucial for the presented arguments. In order to get prediction (iv) with a nonvanishing probability Eq. (16), the coefficient $c_1 = \mathcal{N} c_g$ must not vanish. Thus, the ground state has to be present in the superposition Eq. (1). After this preparation we used two more atoms, each interacting with a single cavity, in order to perform the readout of the cavities. A properly chosen interaction time has allowed us to transfer the prepared correlation of the two cavities to an entanglement between the two atoms. We have demonstrated how state-selective measurements on these atoms reveal their nonlocal correlations in a remarkably simple way.

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[19] The field state of the cavity system Eq. (5) has been transferred to the atoms Eq. (7). In both cases it is this special kind of entangled state where one component is missing: the component $|1\rangle_1|1\rangle_2$ in the field state and $|e\rangle_1|e\rangle_2$ in the atomic state. In his original proposal [1] Hardy avoids such type of components via a $e^+$-$e^-$ annihilation. In our scheme the $|1\rangle_1|1\rangle_2$ and $|e\rangle_1|e\rangle_2$ components cannot occur from the very beginning: The preparing atom carries at most a single quantum. This does not necessarily mean that our preparation is easier to perform: a very sensitive point of our scheme certainly is the correct adjustment of the interaction parameter $g\tau' = \pi/2$.
[20] The nonlocal effect reaches a maximum when this probability Eq. (17) is maximized. Hardy [2] has calculated the maximum $|\eta_1\eta_2c_1|^2 = 0.09$, which would require, in our notation, the coefficients $|c_1|^2 = (\sqrt{5}-1)/(\sqrt{5}+3)$ and $|c_2|^2 = |c_3|^2 = 2/(\sqrt{5}+3)$; see also the connection to the *golden mean* in Ref. [3]. In our case the $c_i$ depend on the interaction parameter $g\tau$ and the coefficients $c_g, c_e$ of the initial atomic superposition. In particular we cannot adjust the relation $|c_2|^2 = |c_3|^2$, which would be necessary to reach the above mentioned absolute maximum. Nevertheless we can almost reach it: If we choose, for example, $|c_g/c_e|^2 = 0.1$, we obtain the maximal probability $|\eta_1\eta_2c_1|^2 = 0.089$ for an interaction parameter $g\tau = 0.43$. In any case it will be difficult experimentally to distinguish an event with a probability of 9% from any kind of noise in the measurement process.