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Correlation of radiation-field ground-state fluctuations in a dispersive and lossy dielectric

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The correlation of the quantum-mechanical ground-state fluctuations of the electric-field strength in a dispersive and lossy linear dielectric are studied in terms of the symmetrized autocorrelation function, with special emphasis on the optical frequency domain. Starting from the canonical quantization scheme developed by Huttner and Barnett [Phys. Rev. A **46**, 4306 (1992)], the analysis is based on a quantization of the phenomenological Maxwell theory, the effect of the medium being described by a frequency-dependent complex permittivity. In this way, the spectrally resolved ground-state autocorrelation function of the electric-field strength can be expressed in terms of the real and imaginary parts of the refractive index. Both analytical and numerical results are presented and the effects of dispersion and absorption including their dependence on the frequency interval chosen are discussed. A comparison with the vacuum fluctuations in free space is given.

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I. INTRODUCTION

As is well known, the vacuum fluctuations (ground-state fluctuations) of radiation fields play an important role in both the basic theoretical concepts of quantum electrodynamics [1] and the practical application of quantum-optical schemes for generation, processing, and detection of nonclassical light. A typical example is the spontaneous emission of light by an excited atom interacting with the vacuum radiation field. In this process vacuum fluctuations and self-reaction can be regarded as essentially contributing to the dynamics of the optically active electron [2,3]. In particular, using symmetrized correlation functions, a physically well defined separation between the two contributions can be made which from a statistical-mechanics point of view is consistent with the usual physical pictures associated with vacuum fluctuations and self-reaction [3].

When (source) light together with the vacuum passes through passive optical instruments, the effect of vacuum fluctuations on the quantum statistics of the output light requires a careful consideration. For example, dividing a signal beam by a beam splitter into two output beams, the vacuum field in the unused input channel introduces additional noise in the output beams, which may be reduced by the use of a squeezed vacuum in place of the ordinary vacuum in the unused input channel [4]. The problem of additional noise introduced by vacuum fluctuations is also observed in optical processing based on active devices, such as quantum amplifiers [5,6].

The above beam-splitter example already shows that it is necessary to take into account the presence of optical instruments when considering the quantization of radiation fields. In principle, optical instruments could be included as a part of the matter to which a radiation field is coupled and treated microscopically. However, there is a class of instruments whose action can be included phenomenologically in the quantization procedure, namely, passive macroscopic bodies that respond linearly to the radiation field under study. Moreover, if the dispersion and absorption can be disregarded, such optical instruments can be regarded as dielectric bodies, with a refractive index that may vary in space. The presence of optical instruments of this type can be taken into consideration by quantizing the radiation embedded in a dielectric with a space-dependent refractive index [7]. A canonical quantization scheme for radiation fields in linear dielectrics with a space-dependent refractive index was developed by Knöll, Vogel, and Welsch [8] and later by Glauber and Lewenstein [9] (for applications see, e.g., [7,9–12]).

If the quantum statistical properties of short-pulse light that propagates over long distances in a dielectric are desired to be studied, the effects of dispersion and absorption must be taken into consideration. A typical example is the propagation of quantum solitons in optical fibers [13]. In this context the question arises of how to quantize radiation fields in dispersive and lossy linear dielectrics to correctly describe their quantum features including the vacuum fluctuations, that is, the radiation-field fluctuations in the ground state of the coupled light-matter system. There have been a number of approaches to this problem [14–20]. Using the Hopfield model of a linear homogeneous dielectric [21] and representing the medium by a collection of interacting bosonic matter fields (a polarization field and a continuum of reservoir fields), Huttner and Barnett [18] presented a canonical quantization scheme for the electromagnetic field in the dielectric (for applications see [22,23]). It is worth noting that both the dispersion and the absorption by the medium are taken into account in a quantum-mechanically consistent way.

Starting from the Huttner-Barnett scheme, we will show that the influence of the medium can entirely be described in terms of the complex frequency-dependent permittivity, so that this scheme should also be applicable to media other than the harmonic-oscillator media considered in the derivation. In particular, introducing frequency-dependent radiation-field operators and expressing their commutators in terms of the complex permittivity, we may regard these commutation relations as a general prescription of quantization of the phenomenological Maxwell theory of radiation in a dispersive and lossy linear dielectric. To test the consistency of the quantization scheme, we show that it yields the well-known commutation relations of the field operators at equal times and that in the case of vanishing dispersion and absorption the familiar quantum theory of radiation in a dielectric with constant refractive index is recognized.

Clearly, dispersion and absorption may be expected to essentially affect the quantum statistics of radiation in a (linear) dielectric. In the present paper we analyze the effect of dispersion and absorption on the correlation of the ground-state fluctuations of the radiation field, with special emphasis on optical frequencies. Both analytical and numerical results are presented and a comparison with the correlation of the vacuum fluctuations in free space is given. Following the arguments given in Ref. [3], the analysis is based on the symmetrized correlation function of the electric-field strength, the properties of which have been studied extensively for the cases of both the free-space vacuum (for example, see [24]) and the blackbody radiation [25].

The paper is organized as follows. In Sec. II the quantization of the phenomenological Maxwell theory of radiation in a dispersive and lossy linear dielectric is performed. In Sec. III the theory is applied to the determination of the correlation of the ground-state fluctuations of the electric-field strength. Finally, a summary and some concluding remarks are given in Sec. IV.

II. QUANTIZATION OF THE PHENOMENOLOGICAL MAXWELL THEORY FOR A DISPERSIVE AND LOSSY DIELECTRIC

In the phenomenological classical Maxwell theory the propagation of radiation in a dispersive and lossy linear (homogeneous and isotropic) dielectric is frequently described by the equations

$$\mathrm{curl}\,\mathbf{E} = -\dot{\mathbf{B}}, \quad \mathrm{div}\,\mathbf{B} = 0, \tag{1}$$

$$\mathrm{curl}\,\mathbf{H} = \dot{\mathbf{D}}, \quad \mathrm{div}\,\mathbf{D} = 0, \tag{2}$$

where $\mathbf{B} = \mu_0 \mathbf{H}$ and the displacement field $\mathbf{D}(\mathbf{r},t)$ is related to the electric field $\mathbf{E}(\mathbf{r},t)$ as follows:

$$\mathbf{D}(\mathbf{r},t) = \epsilon_0 \left[\mathbf{E}(\mathbf{r},t) + \int_0^\infty d\tau\, \chi(\tau)\mathbf{E}(\mathbf{r},t-\tau)\right], \tag{3}$$

which in the Fourier space reads as

$$\underline{\mathbf{D}}(\mathbf{r},\omega) = \epsilon_0 \epsilon(\omega) \underline{\mathbf{E}}(\mathbf{r},\omega), \tag{4}$$

where

$$\epsilon(\omega) = 1 + \int_0^\infty d\tau\, e^{i\omega\tau}\chi(\tau) \tag{5}$$

is the frequency-dependent complex permittivity introduced phenomenologically. Expressing the electric and magnetic fields in terms of the vector potential,

$$\underline{\mathbf{E}}(\mathbf{r},\omega) = i\omega \underline{\mathbf{A}}(\mathbf{r},\omega), \tag{6}$$

$$\underline{\mathbf{B}}(\mathbf{r},\omega) = \mathrm{curl}\,\underline{\mathbf{A}}(\mathbf{r},\omega), \tag{7}$$

the Maxwell equations (1) and (2) are satisfied when

$$\Delta \underline{\mathbf{A}}(\mathbf{r},\omega) + \frac{\omega^2}{c^2}\epsilon(\omega)\underline{\mathbf{A}}(\mathbf{r},\omega) = \mathbf{0}. \tag{8}$$

To give a quantized version of this theory, let us start from the Huttner-Barnett (HB) model of quantization.

A. The Huttner-Barnett quantization scheme

As mentioned, the HB quantization scheme [18] is based on a microscopic model of Hopfield type [21]. To describe the interaction between the electromagnetic field and the dielectric medium, the latter is represented by a collection of matter fields. The electromagnetic field is coupled to a polarization field which for its part is coupled to a reservoir to allow for absorption. Assuming that the polarization field is a single-frequency harmonic-oscillator field and the reservoir comprises a continuum of harmonic oscillators, the Hamiltonian of the transverse fields of the overall system can be diagonalized to obtain

$$\hat{H} = \sum_{\lambda=1,2} \int d^3k \int_0^\infty d\omega\; \hbar\omega\, \hat{C}^\dagger_\lambda(\mathbf{k},\omega)\hat{C}_\lambda(\mathbf{k},\omega), \tag{9}$$

where the operators $\hat{C}_\lambda(\mathbf{k},\omega)$ are linear combinations of the bosonic destruction and creation operators of the transverse medium excitations and the photon destruction and creation operators.

The elementary excitations described by the Hamiltonian (9) can be regarded as polaritons. Their creation and destruction operators $\hat{C}^\dagger_\lambda$ and $\hat{C}_\lambda$ satisfy the familiar boson commutation relations

$$\left[\hat{C}_\lambda(\mathbf{k},\omega), \hat{C}^\dagger_{\lambda'}(\mathbf{k}',\omega')\right] = \delta_{\lambda\lambda'}\delta(\mathbf{k}-\mathbf{k}')\delta(\omega-\omega'), \tag{10}$$

$$\left[\hat{C}_\lambda(\mathbf{k},\omega), \hat{C}_{\lambda'}(\mathbf{k}',\omega')\right] = 0, \tag{11}$$

so that in the Heisenberg picture they evolve as

$$\hat{C}_\lambda(\mathbf{k},\omega,t) = \hat{C}_\lambda(\mathbf{k},\omega,t')\, e^{-i\omega(t-t')}. \tag{12}$$

The diagonalization implies that both the medium and the electromagnetic (transverse) fields can be expressed in terms of the elementary-excitation creation and destruction operators $\hat{C}^\dagger_\lambda$ and $\hat{C}_\lambda$, respectively. From inspection of the relations given in [18] one can prove that the effect of the medium is entirely determined by the frequency-dependent complex permittivity $\epsilon(\omega)$,

$$\epsilon(\omega) = \epsilon_r(\omega) + i\epsilon_i(\omega) \tag{13}$$

($\epsilon_r = \mathrm{Re}\{\epsilon\}$, $\epsilon_i = \mathrm{Im}\{\epsilon\}$). In particular, the operators of the vector potential and the electric- and magnetic-field strengths of the radiation field may be represented as

$$\hat{\mathbf{A}}(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d^3\mathbf{k} \sqrt{\frac{\hbar}{\pi\epsilon_0}} \sum_{\lambda=1}^{2} \mathbf{e}_\lambda(\mathbf{k}) \times \int_0^\infty d\omega \left[\frac{\omega\sqrt{\epsilon_i(\omega)}}{\omega^2\epsilon(\omega) - k^2c^2} \hat{C}_\lambda(\mathbf{k},\omega) e^{i\mathbf{k}\cdot\mathbf{r}} + \mathrm{H.c.} \right], \tag{14}$$

$$\hat{\mathbf{E}}(\mathbf{r}) = \frac{i}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d^3\mathbf{k} \sqrt{\frac{\hbar}{\pi\epsilon_0}} \sum_{\lambda=1}^{2} \mathbf{e}_\lambda(\mathbf{k}) \times \int_0^\infty d\omega \left[\frac{\omega^2\sqrt{\epsilon_i(\omega)}}{\omega^2\epsilon(\omega) - k^2c^2} \hat{C}_\lambda(\mathbf{k},\omega) e^{i\mathbf{k}\cdot\mathbf{r}} - \mathrm{H.c.} \right], \tag{15}$$

$$\hat{\mathbf{B}}(\mathbf{r}) = \frac{i}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d^3\mathbf{k} \sqrt{\frac{\hbar}{\pi\epsilon_0}} \sum_{\lambda=1}^{2} \mathbf{k}\times\mathbf{e}_\lambda(\mathbf{k}) \times \int_0^\infty d\omega \left[\frac{\omega\sqrt{\epsilon_i(\omega)}}{\omega^2\epsilon(\omega) - k^2c^2} \hat{C}_\lambda(\mathbf{k},\omega) e^{i\mathbf{k}\cdot\mathbf{r}} - \mathrm{H.c.} \right] \tag{16}$$

and the displacement field reads as

$$\hat{\mathbf{D}}(\mathbf{r}) = \frac{i}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d^3\mathbf{k} \sqrt{\frac{\hbar}{\pi\epsilon_0}} \sum_{\lambda=1}^{2} \mathbf{e}_\lambda(\mathbf{k}) \times \int_0^\infty d\omega \left[\left(\epsilon_0\epsilon(\omega) \frac{\omega^2\sqrt{\epsilon_i(\omega)}}{\omega^2\epsilon(\omega) - k^2c^2} - \epsilon_0\sqrt{\epsilon_i(\omega)} \right) \times \hat{C}_\lambda(\mathbf{k},\omega) e^{i\mathbf{k}\cdot\mathbf{r}} - \mathrm{H.c.} \right]. \tag{17}$$

Note that there is no dispersion relation. In principle, for each value of ω excitations of arbitrary wave-number vectors $\mathbf{k}$ contribute to the fields. Clearly, the main contributions to the radiation field result from excitations whose $|\mathbf{k}|$ values are in the vicinity of $(\omega/c)\sqrt{\epsilon}$ [$|\mathbf{k}| \approx (\omega/c)\sqrt{\epsilon}$], as can be seen from inspection of Eqs. (14)–(16).

B. Frequency-dependent field operators

To introduce (with regard to the Maxwell equations in Fourier space) frequency-dependent field operators, from inspection of Eqs. (14)–(16) the non-Hermitian operators

$$\hat{\mathbf{a}}(\mathbf{r},\omega) = \frac{1}{\pi}\sqrt{\frac{\omega}{c}\kappa(\omega)} \times \int_{-\infty}^{\infty} d^3\mathbf{k} \sum_{\lambda=1}^{2} \mathbf{e}_\lambda(\mathbf{k}) \frac{\hat{C}_\lambda(\mathbf{k},\omega)e^{i\mathbf{k}\cdot\mathbf{r}}}{\left(\frac{\omega}{c}\right)^2\epsilon(\omega) - k^2} \tag{18}$$

and

$$\hat{\mathbf{f}}(\mathbf{r},\omega) = \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d^3\mathbf{k} \sum_{\lambda=1}^{2} \mathbf{e}_\lambda(\mathbf{k}) \hat{C}_\lambda(\mathbf{k},\omega) e^{i\mathbf{k}\cdot\mathbf{r}}, \tag{19}$$

where

$$\kappa(\omega) = \mathrm{Im}\{\sqrt{\epsilon(\omega)}\}, \tag{20}$$

$$\eta(\omega) = \mathrm{Re}\{\sqrt{\epsilon(\omega)}\}, \tag{21}$$

are suggested to be of special interest. Applying Eqs. (10) and (11), $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and $\hat{\mathbf{a}}^\dagger(\mathbf{r},\omega)$ are found to satisfy the commutation relations

$$\left[\hat{\mathbf{a}}_i(\mathbf{r},\omega), \hat{\mathbf{a}}_j^\dagger(\mathbf{r}',\omega')\right] = \delta(\omega-\omega')\triangle_{ij} \frac{\exp\left[-\frac{\omega}{c}\,\kappa(\omega)\Delta r\right]}{\frac{\omega}{c}\,\eta(\omega)\Delta r} \sin\left[\frac{\omega}{c}\,\eta(\omega)\Delta r\right], \tag{22}$$

$$[\hat{\mathbf{a}}_i(\mathbf{r},\omega), \hat{\mathbf{a}}_j(\mathbf{r}',\omega')] = 0. \tag{23}$$

Here the abbreviation

$$\Delta r = |\mathbf{r}-\mathbf{r}'| \tag{24}$$

is used and the action of $\triangle_{ij}$ on an arbitrary function of space $F(\mathbf{r})$ is defined by

$$\triangle_{ij}F(\mathbf{r}) = \int_{-\infty}^{\infty} d^3\mathbf{r}'\, \delta_{ij}^{\perp}(\mathbf{r}-\mathbf{r}')F(\mathbf{r}'), \tag{25}$$

where

$$\delta_{ij}^{\perp}(\mathbf{r}) = \frac{1}{(2\pi)^3} \int_{-\infty}^{\infty} d^3\mathbf{k} \left(\delta_{ij} - \frac{k_ik_j}{k^2}\right) e^{i\mathbf{k}\cdot\mathbf{r}} \tag{26}$$

is the transverse δ function. The commutation relation (22) is closely related to the correlation of the ground-state fluctuations of the radiation field in a dispersive and lossy dielectric [Eq. (59) in Sec. III]. Using Eq. (19) and the commutation relations (10) and (11), we deduce that, on recalling Eq. (25), the operators $\hat{\mathbf{f}}(\mathbf{r},\omega)$ and $\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)$ satisfy the commutation relations

$$\left[\hat{\mathbf{f}}_i(\mathbf{r},\omega), \hat{\mathbf{f}}_j^\dagger(\mathbf{r}',\omega')\right] = \delta(\omega-\omega')\triangle_{ij}\delta^3(\mathbf{r}-\mathbf{r}') = \delta(\omega-\omega')\delta_{ij}^{\perp}(\mathbf{r}-\mathbf{r}'), \tag{27}$$

$$\left[\hat{\mathbf{f}}_i(\mathbf{r},\omega), \hat{\mathbf{f}}_j(\mathbf{r}',\omega')\right] = 0. \tag{28}$$

Finally, the commutation relations between $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and $\hat{\mathbf{f}}(\mathbf{r},\omega), \hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)$ are found from Eqs. (18) and (19) together with Eqs. (10), (11), and (25):

$$\left[\hat{a}_i(\mathbf{r}',\omega'), \hat{f}_j^\dagger(\mathbf{r},\omega)\right] = -\delta(\omega-\omega')\sqrt{\frac{\omega}{2\pi c}\kappa(\omega)}\,\Delta_{ij}\frac{1}{\Delta r}\exp\left[i\frac{\omega}{c}\sqrt{\epsilon(\omega)}\,\Delta r\right], \tag{29}$$

$$\left[\hat{a}_i(\mathbf{r}',\omega'), \hat{f}_j(\mathbf{r},\omega)\right] = 0. \tag{30}$$

The field commutators considered above reflect the properties of the medium only through the real and imaginary parts of the permittivity $\epsilon(\omega)$. Hence, for an arbitrary dielectric [with $\epsilon(\omega)$ being given phenomenologically] the commutation relations (22), (23), and (27)–(30) may be regarded as the definitions of the operators $\hat{\mathbf{a}}(\mathbf{r},\omega)$, $[\hat{\mathbf{a}}^\dagger(\mathbf{r},\omega)]$ and $\hat{\mathbf{f}}(\mathbf{r},\omega)$ $[\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)]$.

Applying Eq. (25), straightforward calculation yields the commutation relations in the following explicit forms:

$$\begin{aligned}\left[\hat{a}_i(\mathbf{r},\omega), \hat{a}_j^\dagger(\mathbf{r}',\omega')\right] = {} & \frac{1}{\beta}\delta(\omega-\omega')e^{-\gamma\Delta r}\Bigg\{\delta_{mn}\Bigg[\frac{\sin\beta\Delta r}{\Delta r} + \frac{1}{(\Delta r)^2}\Bigg(\frac{\beta\cos\beta\Delta r + \gamma\sin\beta\Delta r}{\beta^2+\gamma^2} \\ & + \frac{(\gamma^2-\beta^2)\sin\beta\Delta r + 2\gamma\beta(\cos\beta\Delta r - e^{\gamma\Delta r})}{\Delta r(\beta^2+\gamma^2)^2}\Bigg)\Bigg] \\ & - \frac{\Delta r_m \Delta r_n}{(\Delta r)^2}\Bigg[\frac{\sin\beta\Delta r}{\Delta r} + \frac{3}{(\Delta r)^2}\Bigg(\frac{\beta\cos\beta\Delta r + \gamma\sin\beta\Delta r}{\beta^2+\gamma^2} \\ & + \frac{(\gamma^2-\beta^2)\sin\beta\Delta r + 2\gamma\beta(\cos\beta\Delta r - e^{\gamma\Delta r})}{\Delta r(\beta^2+\gamma^2)^2}\Bigg)\Bigg]\Bigg\},\end{aligned} \tag{31}$$

$$\begin{aligned}[\hat{a}_i(\mathbf{r}',\omega'), \hat{f}_j^\dagger(\mathbf{r},\omega)] = {} & -\sqrt{\frac{\gamma}{2\pi}}\delta(\omega-\omega')\exp[i(\beta+i\gamma)\Delta r][(\beta^2-\gamma^2+2i\beta\gamma)(\Delta r)^3]^{-1} \\ & \times\Bigg[\delta_{ij}((\beta^2-\gamma^2+2i\beta\gamma)(\Delta r)^2 + i(\beta+i\gamma)\Delta r + \exp[-i(\beta+i\gamma)\Delta r] - 1) \\ & - \frac{\Delta r_i \Delta r_j}{(\Delta r)^2}((\beta^2-\gamma^2+2i\beta\gamma)(\Delta r)^2 + 3i(\beta+i\gamma)\Delta r + 3\{\exp[-i(\beta+i\gamma)\Delta r] - 1\})\Bigg],\end{aligned} \tag{32}$$

where the quantities

$$\beta = \frac{\omega}{c}\eta(\omega), \tag{33}$$

$$\gamma = \frac{\omega}{c}\kappa(\omega) \tag{34}$$

are introduced. Note that in the limit $\Delta r \to 0$ the commutator $[\hat{a}_i(\mathbf{r},\omega), \hat{a}_j^\dagger(\mathbf{r}',\omega')]$ tends to $(2/3)\delta_{ij}\delta(\omega-\omega')$.

C. Phenomenological field quantization

We now express the operators of the vector potential and the electric-field strength in terms of the operators $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and $\hat{\mathbf{a}}^\dagger(\mathbf{r},\omega)$. For this purpose we combine Eqs. (14)–(16) and (18) to obtain

$$\hat{\mathbf{A}}(\mathbf{r}) = \int_0^\infty d\omega\, \mathcal{A}(\omega)\hat{\mathbf{a}}(\mathbf{r},\omega) + \text{H.c.}, \tag{35}$$

$$\hat{\mathbf{E}}(\mathbf{r}) = i\int_0^\infty d\omega\, \omega\mathcal{A}(\omega)\hat{\mathbf{a}}(\mathbf{r},\omega) + \text{H.c.}, \tag{36}$$

$$\hat{\mathbf{B}}(\mathbf{r}) = \text{curl}\,\hat{\mathbf{A}}(\mathbf{r}), \tag{37}$$

where

$$\mathcal{A}(\omega) = \frac{1}{2\pi c}\sqrt{\frac{\hbar\omega}{\epsilon_0 c}\eta(\omega)}\,. \tag{38}$$

Further, from Eqs. (17) and (19) we see that the displacement field $\hat{\mathbf{D}}(\mathbf{r})$ can be expressed in terms of $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and $\hat{\mathbf{f}}(\mathbf{r},\omega)$ as

$$\hat{\mathbf{D}}(\mathbf{r}) = i\int_0^\infty d\omega\,\left[\omega\epsilon_0\epsilon(\omega)\mathcal{A}(\omega)\hat{\mathbf{a}}(\mathbf{r},\omega) - \epsilon_0\mathcal{F}(\omega)\hat{\mathbf{f}}(\mathbf{r},\omega)\right] + \text{H.c.}, \tag{39}$$

where

$$\mathcal{F}(\omega) = 2c\sqrt{\frac{2\pi c}{\omega}\kappa(\omega)}\,\mathcal{A}(\omega). \tag{40}$$

Recalling Eq. (12), from Eqs. (18) and (19) we easily see that in the Heisenberg picture $\hat{\mathbf{a}}(\mathbf{r},\omega,t)$ and $\hat{\mathbf{f}}(\mathbf{r},\omega,t)$ evolve as

$$\hat{\mathbf{a}}(\mathbf{r},\omega,t) = \hat{\mathbf{a}}(\mathbf{r},\omega,t')\,e^{-i\omega(t-t')}, \tag{41}$$

$$\hat{\mathbf{f}}(\mathbf{r},\omega,t) = \hat{\mathbf{f}}(\mathbf{r},\omega,t')\,e^{-i\omega(t-t')}. \tag{42}$$

Hence, complementing in Eq. (35) $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and $\hat{\mathbf{a}}^\dagger(\mathbf{r},\omega)$ by $\exp(-i\omega t)$ and $\exp(i\omega t)$, respectively, we obtain a Fourier representation of the (Heisenberg) operator of the vector potential $\hat{\mathbf{A}}(\mathbf{r},t)$:

$$\hat{\mathbf{A}}(\mathbf{r},t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega), \tag{43}$$

where

$$\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega) = \mathcal{A}(\omega)\hat{\mathbf{a}}(\mathbf{r},\omega) \tag{44}$$

[note that $\underline{\hat{\mathbf{A}}}(\mathbf{r},-\omega) = \underline{\hat{\mathbf{A}}}^\dagger(\mathbf{r},\omega)$]. Equation (44) reveals that [apart from $\mathcal{A}(\omega)$] $\hat{\mathbf{a}}(\mathbf{r},\omega)$ corresponds to the Fourier transform of the operator of the vector potential $\hat{\mathbf{A}}(\mathbf{r},t)$. Similarly, the Heisenberg operators of the electric- and magnetic-field strengths can be represented as, according to Eqs. (36) and (37),

$$\hat{\mathbf{E}}(\mathbf{r},t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega), \tag{45}$$

$$\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega) = i\omega\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega) = i\omega\mathcal{A}(\omega)\hat{\mathbf{a}}(\mathbf{r},\omega), \tag{46}$$

and

$$\hat{\mathbf{B}}(\mathbf{r},t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega), \tag{47}$$

$$\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega) = \mathrm{curl}\,\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega) = \mathcal{A}(\omega)\,\mathrm{curl}\,\hat{\mathbf{a}}(\mathbf{r},\omega). \tag{48}$$

Finally, Eq. (39) implies that

$$\hat{\mathbf{D}}(\mathbf{r},t) = \int_{-\infty}^{\infty} d\omega\, e^{-i\omega t}\underline{\hat{\mathbf{D}}}(\mathbf{r},\omega), \tag{49}$$

$$\underline{\hat{\mathbf{D}}}(\mathbf{r},\omega) = \epsilon_0\epsilon(\omega)\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega) - \epsilon_0\mathcal{F}(\omega)\hat{\mathbf{f}}(\mathbf{r},\omega). \tag{50}$$

Substituting in the phenomenological Maxwell equations (1) and (2) for the fields $\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)$, $\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega)$, and $\underline{\hat{\mathbf{D}}}(\mathbf{r},\omega)$ the results of Eqs. (46), (48), and (50), respectively, we find that

$$\Delta\hat{\mathbf{a}}(\mathbf{r},\omega) + \frac{\omega^2}{c^2}\epsilon(\omega)\hat{\mathbf{a}}(\mathbf{r},\omega) = 4\pi\sqrt{\frac{\omega}{2\pi c}\,\kappa(\omega)}\,\hat{\mathbf{f}}(\mathbf{r},\omega), \tag{51}$$

which is of course consistent with the definitions of $\hat{\mathbf{a}}(\mathbf{r},\omega)$, Eq. (18), and $\hat{\mathbf{f}}(\mathbf{r},\omega)$, Eq. (19).

Equations (43)–(51) together with the commutation relations (22), (23) and (27), (28) may be regarded as the quantum-theoretical version of the classical phenomenological field theory [Eqs. (4)–(8)]. The microscopic model primarily used does not explicitly enter into the equations and the effect of the medium is fully taken into account by the real and imaginary parts of the frequency-dependent permittivity $\epsilon(\omega)$ satisfying the Kramers-Kronig relation.

The quantized theory must of course be consistent with the canonical commutation relations of the fields at equal times. Indeed, noting that from the analytic properties of a given $\epsilon(\omega)$ the relation

$$\int_0^{\infty} d\omega\,\omega\exp\left[-\frac{\omega}{c}\kappa(\omega)\Delta r\right]\sin\left[\frac{\omega}{c}\eta(\omega)\Delta r\right] = -c^2\pi\delta'(\Delta r) \tag{52}$$

can be proved (see the Appendix), the familiar equal-time commutation relations can easily be derived. In particular, we obtain

$$\left[\hat{\mathbf{A}}_i(\mathbf{r},t),\hat{\mathbf{E}}_j(\mathbf{r}',t)\right] = -\frac{i\hbar}{\epsilon_0}\delta_{ij}^{\perp}(\Delta\mathbf{r}). \tag{53}$$

Another test of the consistency of the theory is the behavior in the limit $\epsilon(\omega)\to 1$ [$\epsilon_r(\omega)\to 1$, $\epsilon_i(\omega)\to 0$]. In this limit, from Eq. (22) the operators

$$\hat{a}_\lambda(\mathbf{k}) = \frac{2\sqrt{\pi}}{c}\int_0^{\infty} d\omega\,\sqrt{|\mathbf{k}|\omega\eta(\omega)} \times \frac{1}{(2\pi)^3}\int_{-\infty}^{\infty} d^3\mathbf{r}\, e^{-i\mathbf{k}\cdot\mathbf{r}}\mathbf{e}_\lambda(\mathbf{k})\cdot\hat{\mathbf{a}}(\mathbf{r},\omega) \tag{54}$$

are found to satisfy the photonic commutation relations

$$\left[\hat{a}_\lambda(\mathbf{k}),\hat{a}_\lambda^\dagger(\mathbf{k})\right] = \delta_{\lambda\lambda'}\,\delta(\mathbf{k}-\mathbf{k}') \tag{55}$$

and the vector potential $\hat{\mathbf{A}}$, Eq. (35), takes the familiar form

$$\hat{\mathbf{A}}(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}}\sum_{\lambda=1}^{2}\int_{-\infty}^{\infty} d^3\mathbf{k}\,\sqrt{\frac{\hbar}{2\epsilon_0 c|\mathbf{k}|}}\,\mathbf{e}_\lambda(\mathbf{k}) \times\left[\hat{a}_\lambda(\mathbf{k})e^{i\mathbf{k}\cdot\mathbf{r}} + \mathrm{H.c.}\right]. \tag{56}$$

It is worth noting that there is an essential difference between the classical and quantum-theoretical descriptions. Comparing Eq. (50) with Eq. (4), we see that in the quantum constitutive equation (50) an additional term $\propto\sqrt{\kappa(\omega)}\,\hat{\mathbf{f}}(\mathbf{r},\omega)$ appears which gives rise to the inhomogeneous wave equation (51) in place of the homogeneous equation (8). This term is obviously required to correctly describe the additional (quantum) noise introduced by the absorption of light by the medium. Only in the case when the losses are disregarded [$\kappa(\omega)=0$] can the classical constitutive equation be directly transferred to quantum theory. Clearly, in this case $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and $\hat{\mathbf{a}}^\dagger(\mathbf{r},\omega)$ can be regarded as basic-field operators. They satisfy the homogeneous wave equation [Eq. (51) with $\kappa(\omega)=0$] and their commutation relations are given by Eqs. (22) and (23).

In the general case of dispersion and absorption (related to each other by the Kramers-Kronig relation) both the operators $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and $\hat{\mathbf{f}}(\mathbf{r},\omega)$ appear. It should be pointed out that $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and $\hat{\mathbf{f}}(\mathbf{r},\omega)$ are not independent of each other. Regarding Eq. (51) as the spatial-evolution equation for $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and the term $\propto\sqrt{\kappa(\omega)}\,\hat{\mathbf{f}}(\mathbf{r},\omega)$ as the source field, we easily see that Eq. (51) is solved by

$$\hat{\mathbf{a}}(\mathbf{r},\omega) = -\sqrt{\frac{\omega}{2\pi c}\kappa(\omega)} \times \int_{-\infty}^{\infty} d^3\mathbf{r}' \, \frac{\hat{\mathbf{f}}(\mathbf{r}',\omega)\exp\left[i\frac{\omega}{c}\sqrt{\epsilon(\omega)}\,|\mathbf{r}'-\mathbf{r}|\right]}{|\mathbf{r}'-\mathbf{r}|}. \tag{57}$$

Although this $\hat{\mathbf{a}}(\mathbf{r},\omega)$ looks like a particular solution of Eq. (51), we are not free to add a solution of the homogeneous equation. Recalling the commutation relations for the $\hat{\mathbf{f}}(\mathbf{r},\omega)$ and $\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)$ [Eqs. (27) and (28)], one can prove that Eq. (57) yields the correct commutation relations (22) and (23) for $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and $\hat{\mathbf{a}}^\dagger(\mathbf{r},\omega)$ as well as the correct mixed commutation relations (29) and (30). Note that Eq. (57) is in agreement with Eqs. (18) and (19). Expressing, according to Eq. (19), the operators $\hat{C}_\lambda(\mathbf{k},\omega)$ in terms of $\hat{\mathbf{f}}(\mathbf{r},\omega)$ and inserting the result in Eq. (18), we just obtain the representation of $\hat{\mathbf{a}}(\mathbf{r},\omega)$ given in Eq. (57).

We see that in the general case of a dispersive and lossy dielectric being considered $\hat{\mathbf{f}}(\mathbf{r},\omega)$ and $\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)$ [in place of $\hat{\mathbf{a}}(\mathbf{r},\omega)$ and $\hat{\mathbf{a}}^\dagger(\mathbf{r},\omega)$] may be regarded as basic-field operators whose commutation relations are given by Eqs. (27) and (28). All the other fields, such as the vector potential and the electric field, can uniquely be obtained from $\hat{\mathbf{f}}(\mathbf{r},\omega)$ and $\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)$. It is worth noting that in this quantization scheme $\hat{\mathbf{f}}(\mathbf{r},\omega)$ is not a Langevin (white-)noise operator, frequently introduced in a (Markovian) relaxational treatment of losses. In particular, the average of $\hat{\mathbf{f}}(\mathbf{r},\omega)$ cannot be zero in general because, according to Eq. (57), this average determines quantities, such as the average of the electric-field strength, that essentially depend on the coherence properties of the light.

III. SYMMETRIC GROUND-STATE CORRELATION FUNCTION OF THE ELECTRIC-FIELD STRENGTH

Applying the results of Sec. II, we now turn to the study of the correlation of the radiation-field ground-state fluctuations in a dispersive and lossy linear dielectric characterized by a frequency-dependent complex permittivity.

A. Basic equations

Following the approach frequently used in quantum electrodynamics [24,26], let us consider the symmetric correlation function of the electric-field strength,

$$K_{mn}(\Delta\mathbf{r},\tau) = \tfrac{1}{2}\langle 0|\hat{E}_m(\mathbf{r},t+\tau)\hat{E}_n(\mathbf{r}+\Delta\mathbf{r},t) + \hat{E}_n(\mathbf{r}+\Delta\mathbf{r},t)\hat{E}_m(\mathbf{r},t+\tau)|0\rangle. \tag{58}$$

Using Eqs. (45) and (46) and recalling that $\langle 0|\hat{\mathbf{a}}^\dagger(\mathbf{r},\omega) = 0 = \hat{\mathbf{a}}(\mathbf{r},\omega)|0\rangle$, we may rewrite Eq. (58) as

$$K_{mn}(\Delta\mathbf{r},\tau) = \frac{1}{2}\int_0^\infty d\omega \int_0^\infty d\omega' \mathcal{A}(\omega)\mathcal{A}(\omega')\omega\omega' \times\{[\hat{\mathbf{a}}_m(\mathbf{r},\omega),\hat{\mathbf{a}}_n^\dagger(\mathbf{r}+\Delta\mathbf{r},\omega')]\,e^{-i[\omega(t+\tau)-\omega' t]} + [\hat{\mathbf{a}}_n(\mathbf{r}+\Delta\mathbf{r},\omega'),\hat{\mathbf{a}}_m^\dagger(\mathbf{r},\omega)]\,e^{i[\omega(t+\tau)-\omega' t]}\}, \tag{59}$$

Applying the commutation relation (31) yields

$$\begin{aligned} K_{mn}(\Delta\mathbf{r},\tau) = &-\frac{\hbar}{4\pi^2c^2\epsilon_0}\frac{d^2}{d\tau^2}\int_0^\infty d\omega\, e^{-\gamma\Delta r}\cos\omega\tau \\ &\times\Bigg\{\delta_{mn}\left[\frac{\sin\beta\Delta r}{\Delta r} + \frac{1}{(\Delta r)^2}\left(\frac{\beta\cos\beta\Delta r+\gamma\sin\beta\Delta r}{\beta^2+\gamma^2} + \frac{(\gamma^2-\beta^2)\sin\beta\Delta r + 2\gamma\beta\left(\cos\beta\Delta r - e^{\gamma\Delta r}\right)}{\Delta r(\beta^2+\gamma^2)^2}\right)\right] \\ &-\frac{\Delta r_m\Delta r_n}{(\Delta r)^2}\left[\frac{\sin\beta\Delta r}{\Delta r} + \frac{3}{(\Delta r)^2}\left(\frac{\beta\cos\beta\Delta r+\gamma\sin\beta\Delta r}{\beta^2+\gamma^2}\right.\right. \\ &\left.\left.+\frac{(\gamma^2-\beta^2)\sin\beta\Delta r + 2\gamma\beta\left(\cos\beta\Delta r - e^{\gamma\Delta r}\right)}{\Delta r(\beta^2+\gamma^2)^2}\right)\right]\Bigg\}. \end{aligned} \tag{60}$$

So far, Eq. (60) is exact. To perform the ω integral, knowledge of the dependence on frequency of the permittivity of the medium actually considered is required. In general, the calculation of this integral is hardly expected to yield a closed solution.

B. Optical region

To illustrate the influence of a dielectric on the motion of the vacuum fluctuations, we perform the ω integral in Eq. (60) approximately. Restricting attention to optical frequencies within an interval of width $2\Delta\omega$,

$$\omega_0 - \Delta\omega < \omega < \omega_0 + \Delta\omega, \tag{61}$$

$$\frac{\Delta\omega}{\omega_0} \ll 1, \tag{62}$$

where ω_0 is an appropriately chosen center frequency, and assuming that dispersion and absorption are small on a length scale of β^{-1} and a time scale of ω^{-1}, we may let

$$(\beta\,\Delta r)^{-1} \ll 1, \tag{63a}$$

$$(\omega\,\tau)^{-1} \ll 1. \tag{63b}$$

Hence, in Eq. (60) in each set of large square brackets we may keep only the first term to obtain

$$K_{mn}(\Delta\mathbf{r},\tau) \approx -k_{mn}(\Delta\mathbf{r}) \times \frac{d^2}{d\tau^2}\int_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega} d\omega\, e^{-\gamma\Delta r}\cos\omega\tau\,\sin\beta\Delta r, \tag{64}$$

where the abbreviation

$$k_{mn}(\Delta\mathbf{r}) = \frac{\hbar}{4\pi^2c^2\epsilon_0}\frac{1}{\Delta r}\left[\delta_{mn} - \frac{\Delta r_m\Delta r_n}{(\Delta r)^2}\right] \tag{65}$$

is used.

For a transparent medium, such as a fiber, it may further be justified to approximate (in the frequency interval chosen) the refractive index and absorption coefficients $\eta(\omega)$ and $\kappa(\omega)$, respectively, as follows:

$$\eta(\omega) \approx \eta_0 + \eta_1\frac{\omega}{\omega_0}, \tag{66}$$

$$\kappa(\omega) \approx \kappa(\omega_0) \equiv \kappa_0. \tag{67}$$

Substituting in Eqs. (33) and (34) for $\eta(\omega)$ and $\kappa(\omega)$ the (approximate) expressions (66) and (67), respectively, Eq. (64) may be written as, on changing the integration variable,

$$K_{mn}(\Delta\mathbf{r},\tau) = K'_{mn}(\Delta\mathbf{r},\tau) + K'_{mn}(\Delta\mathbf{r},-\tau), \tag{68}$$

$$K'_{mn}(\Delta\mathbf{r},\tau) \approx -\tfrac{1}{2}k_{mn}(\Delta\mathbf{r})\frac{d^2}{d\tau^2} \times \int_{-\Delta\omega}^{+\Delta\omega} d\omega\,\exp\left[-\frac{\Delta r}{c}\kappa_0(\omega_0+\omega)\right] \times \sin\left(\omega_0\tau_1 + \omega\tau_2 + \eta_1\frac{\omega^2}{\omega_0}\frac{\Delta r}{c}\right), \tag{69}$$

where

$$\tau_1 = (\eta_0 + \eta_1)\frac{\Delta r}{c} - \tau, \tag{70}$$

$$\tau_2 = (\eta_0 + 2\eta_1)\frac{\Delta r}{c} - \tau. \tag{71}$$

In general, Eqs. (66) and (67) imply that (in the frequency interval chosen) small losses are observed, so that

$$\kappa_0 \ll \eta_0 + \eta_1. \tag{72}$$

In this case one can go on to simplify Eq. (69). Recalling the conditions (72), (63a), (63b), and (62), we easily see that, compared with $\sin[\omega_0\tau_1 + \omega\tau_2 + \eta_1(\omega^2/\omega_0)(\Delta r/c)]$, the exponential function $\exp[-(\Delta r/c)\kappa_0(\omega_0 + \omega)]$ is slowly varying in ω and may therefore be removed from the ω integral in Eq. (69). Further, replacing $d^2/d\tau^2$ by $-\omega_0^2$ (note that $\Delta\omega/\omega_0 \ll 1$), we obtain

$$K'_{mn}(\Delta\mathbf{r},\tau) \approx \frac{1}{2i}\omega_0^2k_{mn}(\Delta\mathbf{r}) \times \exp\left(-\frac{\Delta r}{c}\kappa_0\omega_0\right)\exp(i\omega_0\tau_1)\,I(\Delta r,\tau) + \text{c.c.}, \tag{73}$$

where

$$I(\Delta r,\tau) = \tfrac{1}{2}\int_{-\Delta\omega}^{+\Delta\omega} d\omega\,\exp\left[i\left(\omega\tau_2 + \eta_1\frac{\omega^2}{\omega_0}\frac{\Delta r}{c}\right)\right]. \tag{74}$$

Introducing the (slowly varying) amplitude

$$\tilde{K}_{mn}(\Delta\mathbf{r},\tau) = \omega_0^2k_{mn}(\Delta\mathbf{r}) \times \exp\left(-\frac{\Delta r}{c}\omega_0\kappa_0\right)|I(\Delta r,\tau)|, \tag{75}$$

Eq. (73) may be rewritten as

$$K'_{mn}(\Delta\mathbf{r},\tau) \approx \tilde{K}_{mn}(\Delta\mathbf{r},\tau)\sin[\omega_0\tau_1 + \varphi_I(\Delta r,\tau)], \tag{76}$$

where $\varphi_I(\Delta r,\tau)$ is the phase of $I(\Delta r,\tau)$.

For the sake of transparency it may be convenient to introduce the real phase and group velocities $v_{\rm ph}(\omega)$ and $v_{\rm gr}(\omega)$, respectively,

$$v_{\rm ph}(\omega) = \frac{\omega}{\beta} = \frac{c}{\eta_0 + \eta_1\frac{\omega}{\omega_0}}, \tag{77}$$

$$v_{\rm gr}(\omega) = \frac{d\omega}{d\beta} = \frac{c}{\eta_0 + 2\eta_1\frac{\omega}{\omega_0}} \tag{78}$$

[cf. Eqs. (33) and (66)]. It should be noted that in classical optics complex wave numbers $k = \beta + i\gamma = (\omega/c)[\eta(\omega)+i\kappa(\omega)]$ are frequently introduced and complex phase and group velocities ω/k and $d\omega/dk$ are defined. As long as the dependence on frequency of the (small) absorption coefficient may be neglected [cf. Eqs. (67) and (72)], the real part of k essentially reflects the medium dispersion [cf. Eq. (66)]. In this case the effect on light propagation of the real part of the complex wave number β can well be distinguished from the effect of the imaginary part of the wave number γ and a description in terms of the real phase and group velocities (77) and (78) and the (real) absorption coefficient may be more illustrative than the use of complex quantities.

Recalling Eqs. (70) and (71), we see that

$$\tau_1 = \frac{\Delta r}{v_{\rm ph}(\omega_0)} - \tau, \tag{79}$$

$$\tau_2 = \frac{\Delta r}{v_{\rm gr}(\omega_0)} - \tau \tag{80}$$

and Eq. (74) reads as

$$I(\Delta r,\tau)=\tfrac{1}{2}\int_{-\Delta\omega}^{+\Delta\omega}d\omega\,\exp\left[i\left(\frac{\Delta r}{v_{\rm gr}(\omega_0+\frac{1}{2}\omega)}-\tau\right)\omega\right]. \tag{81}$$

From inspection of Eq. (76) together with Eqs. (75) and (81) we see that (under the assumptions made) the dielectric affects the slowly varying part of the correlation function through the absorption coefficient and the group velocity including its dispersion, whereas the effect of the medium on the rapidly varying part is given by the phase velocity and a (space-time dependent) phase shift owing to the group-velocity dispersion.

In particular, Eq. (81) reveals that with an increasing value of $\Delta\omega$ the effect of the dispersion of the group velocity needs a careful consideration. It may be disregarded when the frequency interval is sufficiently small, so that $(v'_{\rm gr}\Delta\omega)/v_{\rm gr}\ll 1$, which means that $\eta_1(\Delta\omega/\omega_0)\ll 1$ [cf. Eq. (74)]. In this case we may let $v_{\rm gr}(\omega_0+\frac{1}{2}\Delta\omega)\approx v_{\rm gr}(\omega_0)$ and Eq. (81) simply reduces to

$$I(\Delta r,\tau)\approx\left[\frac{\Delta r}{v_{\rm gr}(\omega_0)}-\tau\right]^{-1}\sin\left[\left(\frac{\Delta r}{v_{\rm gr}(\omega_0)}-\tau\right)\Delta\omega\right]. \tag{82}$$

Clearly, for any finite value of $\Delta\omega$ the dispersion of the group velocity becomes observable when the distance Δr is sufficiently large (and the absorption is small enough). The exact values of $I(\Delta r,\tau)$ may be found by expressing the integral in Eq. (74) in terms of Fresnel integrals $S(z)=(2/\sqrt{2\pi})\int_0^z dx\,\sin x^2$ and $C(z)=(2/\sqrt{2\pi})\int_0^z dx\,\cos x^2$:

$$\begin{aligned}I(\Delta r,\tau)=\frac{\sqrt{2\pi}}{4}\sqrt{\frac{\omega_0 c}{\eta_1\Delta r}}\exp\left(-i\frac{\tau_2^2 c\omega_0}{4\eta_1\Delta\omega}\right)\Bigg\{&C\left(\frac{1}{2}\sqrt{\frac{\omega_0 c}{\eta_1\Delta r}}\left(\tau_2+2\eta_1\frac{\Delta\omega}{\omega_0}\frac{\Delta r}{c}\right)\right)+iS\left(\frac{1}{2}\sqrt{\frac{\omega_0 c}{\eta_1\Delta r}}\left(\tau_2+2\eta_1\frac{\Delta\omega}{\omega_0}\frac{\Delta r}{c}\right)\right)\\&-C\left(\frac{1}{2}\sqrt{\frac{\omega_0 c}{\eta_1\Delta r}}\left(\tau_2-2\eta_1\frac{\Delta\omega}{\omega_0}\frac{\Delta r}{c}\right)\right)-iS\left(\frac{1}{2}\sqrt{\frac{\omega_0 c}{\eta_1\Delta r}}\left(\tau_2-2\eta_1\frac{\Delta\omega}{\omega_0}\frac{\Delta r}{c}\right)\right)\Bigg\},\end{aligned} \tag{83}$$

where τ_2 is given in Eq. (80) [or Eq. (71)]. Note that

$$\tau_2\pm 2\eta_1\frac{\Delta\omega}{\omega_0}\frac{\Delta r}{c}=\frac{\Delta r}{v_{\rm gr}(\omega_0\pm\Delta\omega)}-\tau. \tag{84}$$

Typical examples of the correlation of (optical) radiation-field ground-state fluctuations are shown in Figs. 1–3. Whereas in the vacuum ($\eta=1$, $\kappa=0$) the fluctuations are strongly correlated to each other at the space-time points on the light cone $\Delta r/c\approx\pm\tau$ (Fig. 1), in the case of a dispersive medium the range of strong correlation is shifted towards the space-time points linked by the group velocity $v_{\rm gr}(\omega_0)$, i.e., $\Delta r/v_{\rm gr}(\omega_0)\approx\pm\tau$ (Figs. 2 and 3). In a dispersive and lossy dielectric an additional spatial exponential decay of the correlation of the field fluctuations is observed, the characteristic length $c/(\kappa_0\omega_0)$ being in agreement with the classical absorption length (Fig. 3). Similar to the case of classical light propagation, a radiation-field ground-state fluctuation that has been created randomly at time t and space $\mathbf{r}$ and

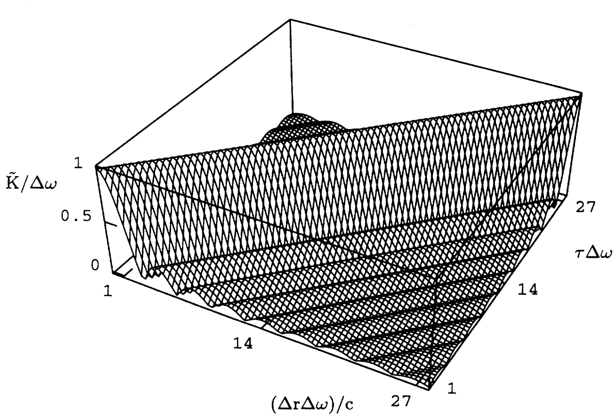

FIG. 1. The slowly varying amplitude of the symmetrized ground-state autocorrelation function of the electric-field strength $\tilde{K}(\Delta\mathbf{r},\tau)=\tilde{K}_{mn}(\Delta\mathbf{r},\tau)/[\omega_0^2 k_{mn}(\Delta\mathbf{r})]$ of the radiation field in free space is shown for a frequency interval of relative width $\Delta\omega/\omega_0=0.3$.

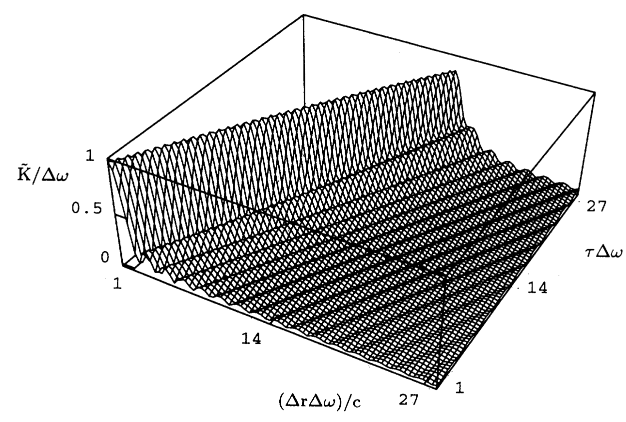

FIG. 2. The slowly varying amplitude of the symmetrized ground-state autocorrelation function of the electric-field strength $\tilde{K}(\Delta\mathbf{r},\tau)=\tilde{K}_{mn}(\Delta\mathbf{r},\tau)/[\omega_0^2 k_{mn}(\Delta\mathbf{r})]$ of the radiation field in a dispersive dielectric is shown for a frequency interval of relative width $\Delta\omega/\omega_0=0.3$. The values of the refractive index and the absorption coefficient are $\eta(\omega_0)=\eta_0+\eta_1=1.46$ ($\eta_0=1.1$, $\eta_1=0.36$) and $\kappa_0=0$, respectively.

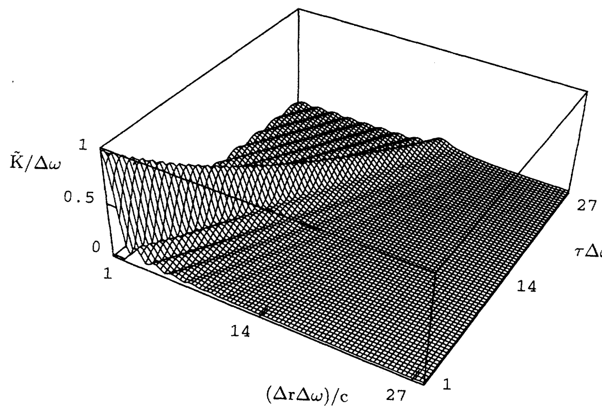

FIG. 3. The slowly varying amplitude of the symmetrized ground-state autocorrelation function of the electric-field strength $\tilde{K}(\Delta\mathbf{r},\tau) = \tilde{K}_{mn}(\Delta\mathbf{r},\tau)/[\omega_0^2 k_{mn}(\Delta\mathbf{r})]$ of the radiation field in a dispersive and lossy dielectric is shown for a frequency interval of relative width $\Delta\omega/\omega_0 = 0.3$. The values of the refractive index and the absorption coefficient are $\eta(\omega_0) = \eta_0 + \eta_1 = 1.46$ ($\eta_0 = 1.1$, $\eta_1 = 0.36$) and $\kappa_0 = 0.04$, respectively.

propagates through the dielectric $[\Delta r/v_{\rm gr}(\omega_0) \approx \pm\tau]$ can be destroyed by absorption by the medium, the probability being increased with distance Δr. Clearly, the absorption does not "remove" the fluctuations from the field, which are a consequence of Heisenberg's uncertainty principle, but it destroys the correlation of the fluctuations at different space-time points. Two contrary tendencies in the behavior of the correlation of the field fluctuations are observed when the value of $\Delta\omega$ is increased. First, the range of correlation is more restricted to the vicinity of space-time points that satisfy the condition $\Delta r/v_{\rm gr}(\omega_0) = \pm\tau$ (which in vacuum corresponds to the light-cone condition). Second, the effect of group-velocity dispersion implies that the range of strong correlation is broadened with increasing distance (Fig.2).

IV. SUMMARY AND CONCLUSIONS

Starting from the HB quantization scheme for a microscopic (Hopfield) model of a dispersive and lossy linear dielectric, we have represented the radiation field operators in terms of frequency-dependent basic-field operators whose commutators at different space-frequency points depend on the medium only through the complex permittivity $\epsilon(\omega)$. This representation may therefore be regarded as a general prescription of quantization of the phenomenological Maxwell theory for a dispersive and lossy linear dielectric. In particular, in the limit when $\epsilon(\omega) \to 1$ the familiar quantum theory of radiation in free space is recognized.

It is worth noting that the quantization requires an operator constitutive equation (in Fourier space) that differs from the classical one in an additional term proportional to the square root of the imaginary part of the permittivity describing the losses. This term is required to correctly describe the additional noise introduced by absorption. In consequence of the modified constitutive equation the operator of the vector potential satisfies an inhomogeneous wave equation (in Fourier space) in place of the familiar homogeneous equation in classical theory. The inhomogeneous term $\propto \sqrt{{\rm Im}[\epsilon(\omega)]}\,\hat{\mathbf{f}}(\mathbf{r},\omega)$ only disappears when the losses are ignored.

Although the inhomogeneous term looks like a Langevin random noise operator, this resemblance is a formal one because the solution of the inhomogeneous wave equation that satisfies the correct commutation relations is fully determined by $\hat{\mathbf{f}}(\mathbf{r},\omega)$. It should be pointed out that in the limit of vanishing losses $({\rm Im}[\epsilon(\omega)] \to 0)$ this solution satisfies the homogeneous wave equation, with $\epsilon(\omega)$ real. Since all field operators can uniquely be obtained from $\hat{\mathbf{f}}(\mathbf{r},\omega)$, this field may be regarded as a basic field in the general case of a dispersive and lossy linear dielectric being considered. Clearly, the average of $\hat{\mathbf{f}}(\mathbf{r},\omega)$ cannot vanish in general.

The commutation relations of the radiation-field operators are closely related to the field fluctuations in the ground state, which in the case of free space are the familiar vacuum fluctuations. Using the commutation relations valid in the case when the radiation field is embedded in a dispersive and lossy linear dielectric, we have calculated the symmetrized autocorrelation function of the ground-state electric-field strength, with special emphasis on the optical region. Restricting attention to a transparent medium, we have studied the influence of absorption, phase and group velocities, and group-velocity dispersion on the dynamics of the field fluctuations within a frequency interval small compared with the center frequency.

As expected, the absorption is responsible for a spatial decay of the correlation of the field fluctuations. Further, the light cone of strong correlation, which in empty space is determined by the velocity of light in vacuum, is now given by the group velocity in the medium, provided that the spatial distance is not too large. With increasing distance the range of strong correlation is smoothed owing to the dispersion of the group velocity. Compared with the case of free space, the effect of the dielectric on the rapidly varying part of the correlation function can be described by substituting for the velocity of light in vacuum the phase velocity in the medium.

We finally mention that the theory may be extended to describe the quantum noise associated with the propagation of arbitrary light pulses in dispersive and lossy linear dielectrics.

APPENDIX: PROOF OF EQ. (52)

To prove the relation

$$\int_0^\infty d\omega\omega \exp\Big[-\frac{\omega}{c}\kappa(\omega)\Delta r\Big]\sin\Big[\frac{\omega}{c}\eta(\omega)\Delta r\Big] = -c^2\pi\delta'(\Delta r), \tag{A1}$$

we first recall some properties of the dielectric function $\epsilon(\omega) = \epsilon^*(-\omega)$, given in [27]. It is an analytical function of the complex frequency Ω in the upper complex half plane without zeros and $\epsilon(\Omega) \to 1$ for $\Omega \to \infty$. In this half plane the imaginary part of $\epsilon(\Omega)$ is positive (negative) for the positive (negative) real part of Ω. Further, along the imaginary frequency axis $\epsilon(\Omega)$ is real, where its lower bond is given by unity. These properties imply that for positive real values of k and c the equation

$$\Omega^2 \epsilon(\Omega) - k^2 c^2 = 0 \tag{A2}$$

cannot be satisfied when Ω is in the upper half plane. To show this, we decompose Ω and ϵ in real (Ω_r, ϵ_r) and imaginary (Ω_i, ϵ_i) parts:

$$\left(\Omega_r^2 - \Omega_i^2\right) \epsilon_r(\Omega) - 2\Omega_r \Omega_i \epsilon_i(\Omega) - k^2 c^2 = 0, \tag{A3}$$

$$\left(\Omega_r^2 - \Omega_i^2\right) \epsilon_i(\Omega) + 2\Omega_r \Omega_i \epsilon_r(\Omega) = 0. \tag{A4}$$

Since

$$\mathrm{sgn}\{\epsilon_i(\Omega)\} = \mathrm{sgn}\{\Omega_r\}, \quad \mathrm{sgn}\{\Omega_i\} = 1, \tag{A5}$$

Eq. (A4) reveals that

$$\mathrm{sgn}\{\epsilon_r(\Omega)\} = -\mathrm{sgn}\{\Omega_r^2 - \Omega_i^2\}. \tag{A6}$$

Hence the expression on the left-hand side in Eq. (A3) cannot be zero. Note that this result is also valid for the real and imaginary axes and the axis $\Omega_r^2 = \Omega_i^2$.

Using the integral relation

$$\int_{-\infty}^{\infty} d^3\mathbf{k}\, e^{i\mathbf{k}\cdot\Delta\mathbf{r}} \left[\frac{\omega^2}{c^2}\epsilon(\omega) - k^2\right]^{-1} = -\frac{2\pi^2}{\Delta r} \exp\left[i\frac{\omega}{c}\sqrt{\epsilon(\omega)}\right], \tag{A7}$$

we may write

$$\int_0^{\infty} d\omega\, \omega \exp\left[-\frac{\omega}{c}\kappa(\omega)\Delta r\right] \sin\left[\frac{\omega}{c}\eta(\omega)\Delta r\right] = \int_0^{\infty} d\omega\, \frac{\omega}{2i} \left\{\exp\left[i\frac{\omega}{c}\sqrt{\epsilon(\omega)}\Delta r\right] - \text{c.c.}\right\}$$
$$= \frac{1}{2i}\int_0^{\infty} d\omega\, \omega \left(-\frac{\Delta r}{2\pi^2}\right) \int_{-\infty}^{\infty} d^3\mathbf{k}\, e^{i\mathbf{k}\cdot\Delta\mathbf{r}} \left\{\left[\frac{\omega^2}{c^2}\epsilon(\omega) - k^2\right]^{-1} - \text{c.c.}\right\}. \tag{A8}$$

To evaluate the integral

$$\frac{1}{2i}\int_0^{\infty} d\omega\, \omega \left\{\left[\frac{\omega^2}{c^2}\epsilon(\omega) - k^2\right]^{-1} - \text{c.c.}\right\} = \frac{1}{2i}\int_{-\infty}^{\infty} d\omega\, \omega \left[\frac{\omega^2}{c^2}\epsilon(\omega) - k^2\right]^{-1}, \tag{A9}$$

we recall that $\Omega[(\Omega^2/c^2)\epsilon(\Omega) - k^2]^{-1}$ has no poles in the upper complex half plane, so that

$$\frac{1}{2i}\int_{-\infty}^{\infty} d\omega\, \omega \left[\frac{\omega^2}{c^2}\epsilon(\omega) - k^2\right]^{-1} = -\frac{1}{2i}\lim_{R\to\infty} c^2 \int_0^{\pi} id\theta\, \frac{R^2 e^{2i\theta}}{R^2 e^{2i\theta}\epsilon(Re^{i\theta}) - k^2c^2} = -\tfrac{1}{2}\pi c^2. \tag{A10}$$

Combining Eqs. (A8) and (A10), we finally obtain

$$\int_0^{\infty} d\omega\, \omega \exp\left[-\frac{\omega}{c}\kappa(\omega)\Delta r\right] \sin\left[\frac{\omega}{c}\eta(\omega)\Delta r\right] = c^2\frac{\Delta r}{4\pi}\int_{-\infty}^{\infty} d^3\mathbf{k}\, e^{i\mathbf{k}\cdot\Delta\mathbf{r}} = c^2\frac{\Delta r}{4\pi}(2\pi)^3\delta(\Delta\mathbf{r}) = c^2\pi\frac{\delta(\Delta r)}{\Delta r} = -c^2\pi\delta'(\Delta r). \tag{A11}$$

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