Generation of a class of arbitrary two-mode field states in a cavity

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We have shown how a class of arbitrary two-mode field states can be generated by the injection of three-level Λ-type atoms into a two-mode resonator. An explicit example with SU(2) coherent states has been worked out.

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A few years back it was demonstrated that nonclassical field states can be produced by the transfer of the atomic coherence to the field in a micromaser [1]. The idea of the transfer of the atomic coherence to the field has been exploited by Vogel, Akulin, and Schleich [2], who in a recent letter gave a recipe for producing an arbitrary field state within the framework of the Jaynes-Cummings model [3]. The cavity field is initially kept in a vacuum state and the two-level atoms in a coherent superposition of the levels are injected into the cavity at such a low rate that at most a single atom is inside the cavity at a time. The superposition in each atom is selected in a specific predetermined order to drive the field toward a desired state.

In this paper we generalize this method to produce a class of arbitrary two-mode field states in a resonator. For this we consider a Raman-coupled three-level model interacting with a two-mode field in a cavity. The model is basically a cavity version of Raman scattering in which the pump and the Stokes mode interact. The anti-Stokes mode is eliminated by suitable cavity off-resonance condition. For this nature of interaction the total number of photons in both modes is a constant. The model had been studied previously in the context of collapse and revival [4], resonance fluorescence [5], and generation of two-mode trapping states [6]. For the present purpose we adopt a model where the atoms are prepared in a coherent superposition of the two lowest levels which effectively constitute the transition processes. To demonstrate the workability of the model, we have generated a specific field state, namely the two-mode SU(2) coherent field state.

It is worthwhile to note the following points regarding atomic coherence. The manipulation of atomic properties by suitable control of atomic coherence is now well known. For example, a number of authors have observed large enhancement of gain, refractive index [7], and nonlinear generation [8] in different systems by the use of appropriate superpositions of atomic states. The manipulation of the nature of the electromagnetic-field states in a cavity by controlling atomic coherence, however, is relatively new. We think that, taking into consideration experimental complications, the other variants of the model as considered in the present case or in the earlier case by Vogel, Akulin, and Schleich might be the subject of further studies for handling similar situations for the generation of other many-mode field states.

To start with we consider the energy-level diagram of the model [4–6] given in Fig. 1, where E_1, E_2 and E_3 are the energies of levels 1, 2, and 3 of the atom in the Λ configuration, respectively. The two quantized cavity modes of frequencies ω_1 and ω_2 are the pump and the Stokes modes, respectively,, with $\omega_2<\omega_1$. We assume that the cavity is tuned to an exact two-photon resonance ($E_3-E_1=\hbar\omega_1-\omega_2$), so that there is only one detuning parameter, namely Δ, defined by $\hbar\Delta=E_2-E_1-\hbar\omega_1=E_2-E_3-\hbar\omega_2$, which is assumed to be large, i.e., $\hbar\Delta \gg E_3-E_1$. The adiabatic removal of the second level results in the following effective Hamiltonian (for details see the appendix of Ref. [4]):

$$H=H_0+H_1 \; , \tag{1}$$

where the free Hamiltonian is

$$H_0=\hbar\omega_1 a_1^\dagger a_1+\hbar\omega_2 a_2^\dagger a_2+\hbar(\omega_1-\omega_2)\sigma_z/2 \; , \tag{2}$$

and the effective interaction Hamiltonian is of the form

$$H_1=\hbar g(\sigma_+ a_1 a_2^\dagger + a_2 a_1^\dagger \sigma_-) \; . \tag{3}$$

Here $\sigma_+(\sigma_-)$ represents the atomic raising (lowering) operator between levels 1 and 3. g is an effective coupling constant. There are two constants of motion, namely the free Hamiltonian H_0 and the total number of photons of the two modes.

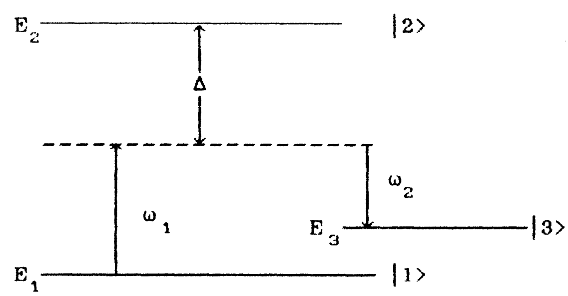

FIG. 1. Energy-level diagram of the three-level atom in the Λ configuration. The detuning Δ is large compared to (E_3-E_1). ω_1 and ω_2 are the pump and Stokes modes, respectively, and the anti-Stokes mode is eliminated by the cavity off-resonance condition.

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The density operator for the atom-field system evolves as

$$\rho(t)=U(t)\rho(0)U^{\dagger}(t) , \quad (4)$$

where the evolution operator $U(t)$ in terms of an unperturbed U_0 and perturbed U_1 is given by

$$U(t)=\exp(-iHt/\hbar)=U_0(t)U_1(t) . \quad (5)$$

Working in the atomic basis, expansion and simplification of $U_1(t)$ yield

$$U_1(t)=\begin{vmatrix} \cos(g\hat{\mu}t) & -ia_1a_2^{\dagger}\sin(g\hat{\mu}'t)/\hat{\mu}' \\ -ia_2a_1^{\dagger}\sin(g\hat{\mu}t)/\hat{\mu} & \cos(q\hat{\mu}'t) \end{vmatrix} , \quad (6)$$

where $\hat{\mu}$ and $\hat{\mu}'$ are two operators

$$\hat{\mu}=\{a_2^{\dagger}a_2(a_1^{\dagger}a_1+1)\}^{1/2} ,$$
$$\hat{\mu}'=\{a_1^{\dagger}a_1(a_2^{\dagger}a_2+1)\}^{1/2} . \quad (7)$$

For the problem of interest here we assume that the atom is initially in a linear superposition of its two nondegenerate ground states as

$$|\psi_k(0)\rangle_A=|3\rangle+\varepsilon_k|1\rangle , \quad (8)$$

where ε_k is the complex amplitude coefficient for the kth atom. The initial state of the field can be expressed as

$$|\psi(0)\rangle_F=\sum_{n=0}^{N}\phi_{n,N-n}|n,N-n\rangle , \quad (9)$$

where N is the total number of photons of the fields, and the ket$|n,N-n\rangle$ $(=|n\rangle_1|N-n\rangle_2)$ implies that the first mode contains n photons while the second mode has $N-n$ photons. Here $\phi_{n,N-n}$ is an expansion coefficient associated with the number state $|n,N-n\rangle$.

We now consider a series of three-level Λ-type atoms interacting with a two-mode field in a cavity via the interaction as stated above. We assume that initially the first mode is in vacuum and the second mode contains N photons. We also assume that after the passage of the $(k-1)$th atom and just before the injection of the kth atom, the cavity field is in a state

$$|\phi^{(k-1)}\rangle=\sum_n\phi_n^{(k-1)}|n,N-n\rangle . \quad (10)$$

The kth atom enters the cavity in the superposed state $(|3\rangle+i\varepsilon_k|1\rangle)$. After the interaction, when the atom has left the cavity, the state of the combined atom-field system is given by

$$|\psi^{(k)}(t)\rangle=\exp(-iH_1t/\hbar)|\psi(0)\rangle=U_1(t)|\psi(0)\rangle , \quad (11)$$

where

$$|\psi(0)\rangle=\begin{vmatrix} 1 \\ i\varepsilon_k \end{vmatrix}\otimes\sum_{n=0}^{N}\phi_{n,N-n}^{(k-1)}|n,N-n\rangle . \quad (12)$$

We then have

$$|\psi^{(k)}\rangle=\sum_n\phi_n^{(k-1)}[C_n^{(k)}|n,N-n,3\rangle-iS_n^{(k)}|n+1,N-(n+1),1\rangle$$
$$+i\varepsilon_kC_{n-1}^{(k)}|n,N-n,1\rangle+\varepsilon_kS_{n-1}^{(k)}|n-1,N-(n-1),3\rangle] , \quad (13)$$

where

$$C_n^{(k)}=\cos(g\mu_{n,N-n}\tau_k) , \quad S_n^{(k)}=\sin(g\mu_{n,N-n}\tau_k) , \quad (14)$$

with $\mu_{n,N-n}=\sqrt{[n(N-n+1)]}$. Here τ_k is the interaction time of the kth atom.

As soon as an atom exits the cavity one detects whether the atom is in the excited state $(|3\rangle)$ or the ground state $(|1\rangle)$. If the atom is in the ground state then one has to continue the process for more transfer of energy from the atom to the field to obtain the desired state. On the other hand, if the atom is found in the excited state then one has to repeat process [2]. Now let the field state after the exit of the kth atom be

$$|\phi^{(k)}\rangle=\sum_{n=0}^{k}\phi_n^{(k)}|n,N-n\rangle . \quad (15)$$

So from Eq. (13) the new coefficients $\phi_n^{(k)}$'s are given in terms of the old coefficients $\phi_n^{(k-1)}$ according to the following recurrence relation:

$$\phi_n^{(k)}=S_{n-1}^{(k)}\phi_{n-1}^{(k-1)}-\varepsilon_kC_{n-1}^{(k)}\phi_n^{(k-1)} . \quad (16)$$

Each injected atom increases the photon number by one in the first mode by destroying one photon in the second mode. Now our objective is to prepare a desired field state of the form

$$|\phi\rangle=\sum_{n=0}^{N}\phi_n^{(N)}|n,N-n\rangle . \quad (17)$$

Following Ref. [2], we then construct the characteristic polynomial equation for ε_n of order N. We solve it and choose the lowest value of ε_N out of N roots. Having obtained ε_N we obtain a set of $\phi_n^{(N-1)}$'s. In the next step we calculate $\phi_n^{(N-2)}$ in the same way. We continue this until we reach the state corresponding $\phi_0^{(0)}$, i.e., the initial field state.

The probability of finding all the outcoming N atoms in the ground state is

$$P_N=\prod_{k=1}^{N}P_g^{(k)} , \quad (18)$$

where

TABLE I. Atomic superposition $|3\rangle + i\varepsilon_k|1\rangle$ of the kth atom needed to obtain the two-mode SU(2) coherent field state for the interaction parameter $g\tau = 0.28\pi$. $P_g^{(k)}$ is the probability of finding the kth atom in the ground state $|1\rangle$ after its interaction with the cavity fields provided that all earlier atoms will be detected in the ground state $|1\rangle$. The probability P_5 to find all the atoms in the ground state, $P_5 = 0.026$.

k	ε_k	$\lvert\varepsilon_k\rvert$	$P_g^{(k)}$
1	$0.53 - 0.41\,i$	0.67	0.897
2	$12.63 - 6.83\,i$	14.36	0.423
3	$-0.48 + 0.72\,i$	0.87	0.664
4	$0.25 + 0.14\,i$	0.29	0.634
5	$-0.07 + 1.91\,i$	1.91	0.165

$$P_g^{(k)} = \frac{1}{(1+|\varepsilon_k|^2)} \sum_{n=0}^{k} |S_{n-1}^{(k)} \phi_{n-1}^{(k-1)} - \varepsilon_k C_{n-1}^{(k)} \phi_n^{(k-1)}|^2 \tag{19}$$

is the probability of finding the kth atom (as it leaves the cavity) in the ground state provided that all the earlier atoms are detected in the ground state. This is an important quantity since this is directly related to the experimental process of probing the cavity field by the detection of atoms by the field-ionization method.

To get an idea of the workability of the two-mode scheme, we now show how this can be realized for the generation of a typical two-mode field state, e.g., an SU(2) coherent state which can be expanded in terms of the two-mode number states as

$$|\xi j\rangle = (1+|\xi|^2)^{-N/2} \sum_{k=0}^{N} ({}^N C_k)^{1/2} \xi^k |k, N-k\rangle , \tag{20}$$

where ξ is in general complex and $j = N/2$. The statistical properties of these states have been discussed in details by Buzek and Quang [9]. Gerry and Kiefer [10] have considered the dynamical interaction of this field with two-level atoms.

Since the SU(2) coherent state is our desired two-mode state, we write

$$|\phi\rangle = (1+|\xi|^2)^{-N/2} \sum_{k=0}^{N} ({}^N C_k)^{1/2} \xi^k |k, N-k\rangle . \tag{21}$$

For this particular example we have chosen $N = 5$. In Table I we have tabulated the calculated values of $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_5$ for a constant interaction parameter $g\tau_k = g\tau = 0.28\pi$. In Fig. 2 we have shown the plot of P_5, the probability of finding all five outcoming atoms in the ground state, as a function of the interaction time $g\tau$. It is evident that for trapping states one obtains the vanishing probability in P_5.

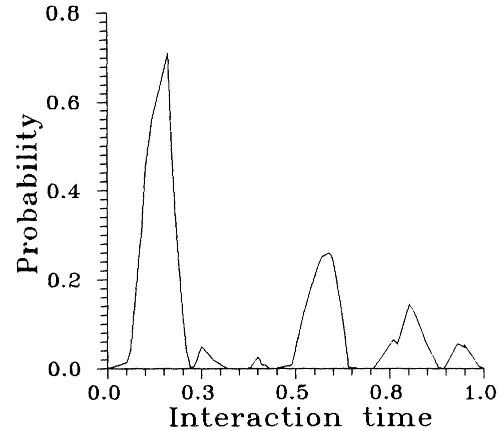

FIG 2. Probability (P_5) of finding all five atoms in the ground state is plotted with the dimensionless interaction time ($g\tau/\pi$) for the SU(2) coherent field state. Here we have chosen the maximum absolute value of the roots of ε_k. The occurrence of the trapping states are at $g\tau = \pi/\sqrt{5}$, $\pi/\sqrt{8}$, $\pi/3$, $2\pi/\sqrt{5}$, $\pi/2$, $2\pi/3$, and π, where the probability P_5 vanishes.

In conclusion, we have proposed a scheme for the generation of a class of two-mode field states in a cavity by the transfer of atomic coherence to the cavity field. The more general two-mode distribution is somewhat restricted in the present model by the constancy of the total number of quanta of the two modes. We hope that this may be removed by adopting suitable three-level systems with two modes where all three levels participate appropriately in the transition processes. We note in passing that although specificity of a transition process in an atom-field interaction depicted in a typical model restricts the nature of the final state, the method of transfer of atomic coherence into the field allows us to engineer the nature of final state of the field as desired by us in an arbitrary way while still remaining within the framework of the same model.

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