Using feedback to eliminate back-action in quantum measurements

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A quantum-nondemolition (QND) measurement is one which does not disturb the quantity it measures. Any measurement can be regarded as a QND measurement followed by an additional back-action which may disturb that quantity. By feeding back the measurement result to control the system dynamics, this back-action can always be eliminated in principle. A practical device based on homodyne measurement of the x quadrature of a cavity mode is investigated. First, a $\chi^{(2)}$ crystal is inserted in the cavity so that it becomes a degenerate parametric oscillator at threshold. Then the photocurrent is used to control coherent driving of the cavity, so as to give positive feedback with unit gain. This feedback, combined with the nonlinearity, turns the homodyne measurement into a QND measurement of x. The device can also be used to measure the x quadrature of a traveling wave, and gives near-perfect QND correlations over a large bandwidth. However, the quality of the measurement is badly degraded by even slightly inefficient photodetectors.

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I. INTRODUCTION

There are two kinds of quantum measurements: those that do not disturb the quantity they measure, and those that do. This classification was made as early as 1933, by Pauli [1], who called them measurements of the first and second kind, respectively. There are various definitions that make this distinction precise, appropriate for different applications. First-kind measurements are obviously better measurements in some sense. They are also known as quantum-nondemolition (QND) or back-action-evading (BAE) measurements. The interest in such measurements was revived in 1980 by the realization that planned gravitational wave detectors would only work if a QND measurement of position could be made [2]. Since then, the application of QND measurements in quantum optics communication systems has been of considerable interest. Several theoretical schemes have been proposed [3–9], and some convincing experiments performed [10–13].

All of the optical QND schemes utilize some form of optical nonlinearity induced by the interaction of the light with crystals or atoms. The scheme I propose in this paper is no exception, but has the additional feature of feeding back the measured result onto the system. The basic idea is that a second-kind measurement can be regarded as a first-kind measurement, followed by additional back-action, which causes the measured quantity to be altered. In principle, the measured result can be fed back, altering the system dynamics so as to undo that additional back-action. It is thereby possible to turn a second-kind into a first-kind measurement. In general, this would be impractical, but the scheme I propose would enable a homodyne measurement of the x quadrature of the cavity to be turned into a QND measurement. The homodyne photocurrent is fed back to control coherent driving of the cavity, undoing the fluctuations in x introduced by the homodyne measurement. By itself, this feedback would destabilize the x quadrature. The nonlinearity (a $\chi^{(2)}$ crystal), acting as a classically driven degenerate parametric oscillator (DPO), is necessary to stabilize the x quadrature without introducing any extra noise. The combination of classical feedback and a DPO can change the measurement from QD to QND. Under ideal conditions, the device approaches a perfect broadband QND apparatus.

Rather than an immediate analysis of this proposal for using feedback to produce a QND device, Sec. II is a review of quantum measurement theory and the distinction between the two kinds of measurement. Section III builds on this, showing how feedback can, in principle, turn any measurement of the second kind into one of the first kind. This is shown to be impractical in the simple case of direct photodetection of the output of a cavity. For this reason, feedback based on homodyne detection is considered as an alternative. It is shown in Sec. IV that ideal homodyne feedback control of the driving of a cavity, combined with a DPO, can turn damping into a first-kind measurement of one quadrature of the intracavity field. In practice, it may be more useful to be able to do a QND measurement on one quadrature of a traveling wave. This can be achieved by reflecting the traveling wave off one mirror of the cavity, and doing the measurement at the other. The criteria for judging the usefulness of such a measurement are defined in Sec. V, and applied to the proposed scheme in Sec. VI.

II. QUANTUM MEASUREMENT THEORY

A. General theory

This section presents the formal structure of quantum measurement theory, and distinguishes measurements of

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the first and second kinds. The discussion will be restricted to efficient measurements. By this, I mean measurements in which the state of the system conditioned on the measurement result is pure, provided that the state before the measurement was pure also. The reason for this restriction is that the distinction between first- and second-kind measurements is only well defined for the case of efficient measurements.

The aim of quantum measurement theory is, given the initial state of the system, to be able to specify the probability of a particular measurement result and the state of the system immediately following this result. Say the measurement result is α, a random variable which will be assumed discrete for convenience. Then both the probability and the conditioned state can be found from the set of operators Ω_α, one for each possible result. These operators are arbitrary, with one condition,

$$\sum_\alpha \Omega_\alpha^\dagger \Omega_\alpha = 1, \tag{2.1}$$

where the sum is over all possible results. This is known as the completeness condition [14].

The probability for obtaining a particular result α is found from the measurement operator by

$$P_\alpha = \mathrm{Tr}[\tilde{\rho}'_\alpha], \tag{2.2}$$

where

$$\tilde{\rho}'_\alpha = \Omega_\alpha \rho \, \Omega_\alpha^\dagger \tag{2.3}$$

is an unnormalized density operator, where ρ is the density operator immediately before the measurement. The state of the system conditioned on the result α is simply given by

$$\rho'_\alpha = \tilde{\rho}'_\alpha / P_\alpha. \tag{2.4}$$

This economy of theory is a consequence of a more fundamental notion of probability relating to projectors in Hilbert space [15]. If the initial state of the system is pure ($\rho = |\psi\rangle\langle\psi|$), then the unnormalized conditioned state is obviously

$$|\tilde{\psi}'_\alpha\rangle = \Omega_\alpha |\psi\rangle. \tag{2.5}$$

If the measurement is performed, but the result ignored, then the new state of the system will be mixed in general and cannot be represented by a state vector. This unconditioned state is denoted simply by ρ'. It is equal to the sum of the conditioned density operators (2.4), weighted by the probabilities (2.2)

$$\begin{aligned} \rho' &= \sum_\alpha P_\alpha \rho'_\alpha \\ &= \sum_\alpha \Omega_\alpha \rho \, \Omega_\alpha^\dagger . \end{aligned} \tag{2.6}$$

It is easy to verify from the completeness condition (2.1) that $\mathrm{Tr}[\rho'] = 1$, as required by conservation of probability.

The probability for obtaining the result α (2.2) can also be written

$$P_\alpha = \mathrm{Tr}[\rho W_\alpha], \tag{2.7}$$

where

$$W_\alpha = \Omega_\alpha^\dagger \Omega_\alpha \tag{2.8}$$

is a Hermitian operator. Note that many different sets of operators Ω_α may have the same set of probability generating operators W_α. That is to say, a measurement is not completely specified by the probabilities of obtaining the results. What would be missing would be a further specification of the *back-action* of the apparatus on the system. This can be seen specifically in the case where all of the W_α are bounded operators [15,16]. Then the operators Ω_α can be written

$$\Omega_\alpha = U_\alpha V_\alpha, \tag{2.9}$$

where

$$V_\alpha = W_\alpha^{1/2} = V_\alpha^\dagger \,, \quad U_\alpha^\dagger = U_\alpha^{-1}. \tag{2.10}$$

That is, Ω_α can be written as the product of a unitary and a Hermitian operator. Assuming a pure initial state, the unnormalized conditioned state vector is thus written

$$|\tilde{\psi}'_\alpha\rangle = U_\alpha V_\alpha |\psi\rangle. \tag{2.11}$$

The action of V_α produces the minimum change in the system, required by Heisenberg's relation, to be consistent with a measurement giving the information about the state specified by the probabilities (2.7). The action of U_α represents additional back-action, an unnecessary perturbation of the system.

A back-action-evading measurement is reasonably defined by the requirement that, for all α, U_α equals unity (up to a phase factor that can be ignored without loss of generality). This is equivalent to the requirement that all Ω_α be Hermitian. One criticism of this definition is that it disallows any Hamiltonian evolution of the system during the measurement. For an interaction time T, such evolution would contribute a unitary $\exp[-iHT]$ to *all* measurement operators Ω_α. This evolution can thus be removed from all of the U_α by making the unitary transformation into the interaction picture. Thus, a better requirement for back-action evasion is that all U_α be unity in the interaction picture. This is the definition that I will use to distinguish first-kind measurements from all other (second-kind) measurements.

B. Continuous measurement theory

A special case of quantum measurement theory that is of considerable importance is continuous measurement theory. That is, a constant measurement interaction allows successive measurements, the duration of each being infinitesimal. If the state matrix at time t is $\rho(t)$, then the unnormalized conditioned density operator after the measurement in the interval $(t, t+dt)$ is denoted

$$\tilde{\rho}_\alpha(t+dt) = \Omega_\alpha(dt)\rho(t)\,\Omega_\alpha^\dagger(dt), \tag{2.12}$$

where the operators $\Omega_\alpha(dt)$ are arbitrary as yet. The unconditioned infinitesimally evolved state matrix is then

$$\rho(t+dt) = \sum_\alpha \Omega_\alpha(dt)\rho(t)\,\Omega_\alpha^\dagger(dt). \qquad (2.13)$$

This represents the evolution of the system, ignoring the measurement results. If the $\Omega_\alpha(dt)$ are time-independent, this nonselective evolution is obviously Markovian (depending only on the state of the system at the start of the interval).

A Markovian equation of motion for the density operator of a system is known as a master equation. Consider for simplicity the case where there is single loss channel for the system. Then it can be shown [17] that the master equation is of the form

$$\dot{\rho} = -i[H,\rho] + \mathcal{D}[c]\rho, \qquad (2.14)$$

where I have defined the superoperator $\mathcal{D}[a]$ taking an arbitrary operator a as an argument by

$$\mathcal{D}[a]\rho \equiv a\rho a^\dagger - \tfrac{1}{2}\{a^\dagger a, \rho\}, \qquad (2.15)$$

where curly brackets denote an anticommutator. This representation is not unique; the master equation (2.14) is invariant under the transformation

$$c \to c+\beta\,, \quad H \to H + \tfrac{1}{2}(-i\beta^* c + i\beta c^\dagger), \qquad (2.16)$$

where β is an arbitrary complex number. This will be important later.

Once a particular representation of the master equation (2.14) has been chosen, it can be put in one-to-one correspondence with the continuous measurement theory outlined above. By inspection, there are only two possible measurement results (say 0 and 1), with corresponding operators

$$\Omega_1(dt) = \sqrt{dt}\,c, \qquad (2.17)$$

$$\Omega_0(dt) = 1 - \left[iH + \tfrac{1}{2}c^\dagger c\right]dt. \qquad (2.18)$$

The two unnormalized conditioned density operators are

$$\tilde{\rho}_1(t+dt) = dt\,c\rho c^\dagger, \qquad (2.19)$$

$$\tilde{\rho}_0(t+dt) = \rho + dt\left(-i[H,\rho] - \tfrac{1}{2}\{c^\dagger c,\rho\}\right). \qquad (2.20)$$

It is easy to see that

$$\rho(t+dt) = \tilde{\rho}_1(t+dt) + \tilde{\rho}_0(t+dt) = \rho + dt\dot{\rho}, \qquad (2.21)$$

where $\dot{\rho}$ is given by the master equation (2.14). Evidently, almost all infinitesimal intervals yield the measurement result 0. Upon such a result, the system state evolves infinitesimally (but not unitarily in general). Whenever the result 1 is obtained, however, the system state changes by a finite operation. This discontinuous change can be justifiably called a *quantum jump*, and the measurement event a *detection*. Such jumps have been observed in electron shelving experiments [18]. As with all efficient measurements, if the initial state of the system is pure, then the conditioned state of the system will remain pure. The stochastic evolution of such a conditioned state has been called a *quantum trajectory* by Carmichael [19]. The interpretation and use as calculation tools of such trajectories have been of considerable interest recently [19–22].

The operator $\Omega_0(dt)$ defined in Eq. (2.18) is evidently not Hermitian, because of the Hamiltonian term. However, this can be removed by working in the interaction picture. This is consistent with the definition of first-kind measurements given in Sec. II A. Providing the interaction picture operator $c^\dagger c$ is not explicitly time dependent, this leaves the measurement term $\mathcal{D}[c]$ of the master equation unchanged. An example satisfying this condition is a free cavity with annihilation operator proportional to c, and so $\mathcal{D}[c]$ represents damping. If H can be ignored, then $\Omega_0(dt)$ is Hermitian, and hence the classification of the measurement depends on whether c is Hermitian. In the case of damping, c is not Hermitian, and so the measurements permitted by damping are necessarily quantum-demolition measurements. The non-Hermiticity of c in this case can be seen explicitly using the factorization (2.9)

$$c = e^{i\Phi}\sqrt{c^\dagger c}, \qquad (2.22)$$

where Φ is a phase operator for the single-mode field. The difficulty in defining this operator [23,24] is due to the fact that $c^\dagger c$ is not a bounded operator, which violates the assumptions made in writing Eq. (2.9).

III. BACK-ACTION ELIMINATION BY FEEDBACK

Feedback is the use of a measurement result to control the dynamics of the system being measured. The effectiveness of feedback is thus limited by two factors: the quality of the measurement and the degree of control over the system dynamics. Assuming that the dynamics of the system can be arbitrarily well controlled, then it is evident from the preceding section that feedback can eliminate the back-action of any measurement. The back-action of a particular measurement result α is produced by the unitary operator U_α of Eq. (2.9). Hence, if the result α is obtained, then the Hamiltonian of the system should be changed by a large amount for a short time in order to induce the evolution U_α^{-1}. This turns the QD measurement with measurement operators Ω_α into a QND measurement with Hermitian measurement operators V_α. For the case of a Markovian system with a single output and with trivial free evolution, the requirement is for the feedback to act immediately following a detection. The induced evolution undoes the unitary component of the quantum jump caused by the operator c of Eq. (2.17).

The means by which such Markovian feedback could be approximately accomplished in practice is more easily explained if the possible quantum trajectories of the measured system are expressed as a stochastic evolution equation for the conditioned density operator $\rho_c(t)$. Here the subscript c indicates that the density operator is conditioned on the entire measurement record up to time t. As done in Ref. [22], a stochastic increment $dN_c(t)$ of the number of detections in the time interval $(t, t+dt)$ is introduced. This obeys

$$dN_c(t)^2 = dN_c(t), \tag{3.1}$$

$$E[dN_c(t)] = \mathrm{Tr}[W_1(dt)\rho_c(t)] = \langle c^\dagger c\rangle_c(t)dt, \tag{3.2}$$

where E denotes expectation value. Equation (3.1) simply indicates that the number of detections is either zero or 1, while Eq. (3.2) arises from Eq. (2.17). It is easy to verify that $\rho_c(t)$ obeys the following nonlinear stochastic evolution equation

$$d\rho_c(t) = \left\{dN_c(t)\mathcal{G}[c] + dt\mathcal{H}\left[-iH + \tfrac{1}{2}c^\dagger c\right]\right\}\rho_c(t). \tag{3.3}$$

Here, the nonlinear superoperators $\mathcal{G}$ and $\mathcal{H}$ are defined by

$$\mathcal{G}[a]\rho = \frac{a\rho a^\dagger}{\mathrm{Tr}[a\rho a^\dagger]} - \rho, \tag{3.4}$$

$$\mathcal{H}[a]\rho = a\rho + \rho a^\dagger - \mathrm{Tr}[a\rho + \rho a^\dagger]\rho. \tag{3.5}$$

Because of the assumed perfect detection, there exists an equivalent stochastic equation for the state vector [22], but this will not be used. Taking the ensemble average of Eq. (3.3) yields the original master equation (2.14) for the ensemble average state matrix $\rho(t) = E[\rho_c(t)]$.

The measurement results that condition the state matrix in Eq. (3.3) can be expressed as a rate of detections, or current, defined by

$$I_c(t) = dN_c(t)/dt. \tag{3.6}$$

Note that this mathematical quantity is zero almost all of the time, and when it is not zero it is infinite. In reality, the photocurrents generated in experimental quantum optics are smooth functions of time. The current can be used to control the evolution of the system via the time-dependent Hamiltonian

$$H_{\mathrm{fb}}(t) = I_c(t)Z, \tag{3.7}$$

with Z Hermitian. Bearing in mind the smoothness of a real current, and that the feedback must act later than the measurement, the effect of this Hamiltonian can be shown [25] to change Eq. (3.3) into

$$d\rho_c(t) = \left\{dN_c(t)\mathcal{G}[e^{-iZ}c] + dt\mathcal{H}\left[-iH + \tfrac{1}{2}c^\dagger c\right]\right\}\rho_c(t). \tag{3.8}$$

Taking the ensemble average of this equation gives the feedback master equation

$$\dot\rho = -i[H,\rho] + \mathcal{D}[e^{-iZ}c]\rho. \tag{3.9}$$

This equation is a good model for the effect of feedback, providing the time delay in the feedback loop is small compared to the characteristic evolution time of the system [25].

In order to use this feedback to turn a QD into a QND measurement, one simply chooses Z so that $e^{iZ} = U_1$. In practice, this may not be so easy. For the case of direct detection of the photons emitted by a cavity, Eq. (2.22) shows that it would be necessary to have

$$Z = \Phi. \tag{3.10}$$

Obtaining a nonlinear crystal that produces an effective Hamiltonian proportional to the phase of the field, and which in addition can be turned on and off by a current, is exceedingly unlikely. Turning cavity damping into a strict QND measurement therefore seems impractical. However, this is not the case. So far, I have considered only direct detection, with $\Omega_1(dt) = \sqrt{dt}\,c$, where c is proportional to the annihilation operator of the cavity. As noted in Sec. II B, the measurement operator can be altered by an additive constant without changing the master equation. That is, one can consider $\Omega_1(dt) = \sqrt{dt}(c+\beta)$. In quantum optics, this can be achieved by combining the output field of the cavity with a coherent local oscillator field at a beam splitter. This is known as homodyne detection, and is the subject of the following section.

IV. HOMODYNE-BACK-ACTION ELIMINATION BY FEEDBACK

Consider an optical cavity supporting a single mode with annihilation operator c. Let the output of the cavity pass through a low reflectivity beam splitter, where it is combined with a very intense coherent local oscillator. Measuring time in units of the decay rate of the output mirror, the amplitude of the transmitted field is effectively $c + \beta$, where β is a complex number. For simplicity, β will be assumed real, appropriate for measuring the $x = \frac{1}{2}(c + c^\dagger)$ quadrature. To apply the theory of the preceding section, one must first write the measurement operator $\Omega_1(dt) = \sqrt{dt}(c+\beta)$ in a form whereby its factorization (2.9) can be easily accomplished. Keeping terms up to second order in $1/\beta$,

$$\Omega_1(dt) = \sqrt{dt}\,\beta\exp\left(\frac{c}{\beta} - \frac{c^2}{2\beta^2}\right). \tag{4.1}$$

Using the Baker-Hausdorff theorem [28] for the first-order terms, this can be factorized (to second order in $1/\beta$) as

$$\Omega_1(dt) = U_1 V_1(dt), \tag{4.2}$$

where

$$U_1 = \exp\left(\frac{c - c^\dagger}{2\beta} - \frac{c^2 - c^{\dagger 2}}{4\beta^2}\right) = \exp(iZ) \tag{4.3}$$

is unitary, and

$$V_1(dt) = \sqrt{dt}\,\beta\exp\left(\frac{c + c^\dagger}{2\beta} - \frac{c^2 + c^{\dagger 2} + [c^\dagger, c]}{4\beta^2}\right) = \sqrt{dt\left[\beta^2 + \beta(c + c^\dagger) + c^\dagger c\right]} \tag{4.4}$$

is Hermitian. This $V_1(dt)$ correctly gives the probability operator $W_1(dt)$ (3.2), which generates the expected count rate for homodyne detection.

From the expression for U_1 (4.3), it is evident that the

Hermitian operator Z is

$$Z = \frac{-ic + ic^\dagger}{2\beta} + \frac{ic^2 - ic^{\dagger 2}}{4\beta^2} = \frac{y}{\beta} - \frac{xy + yx}{2\beta^2}, \quad (4.5)$$

where $c = x + iy$. Thus, according to Sec. III, the feedback Hamiltonian needed to turn the homodyne measurement into a QND measurement of x is

$$H_{\rm fb}(t) = [dN_{\rm c}(t)/dt] \left(\frac{y}{\beta} - \frac{xy + yx}{2\beta^2} \right). \quad (4.6)$$

Now, keeping terms of two orders in $1/\beta$ gives

$$H_{\rm fb}(t) = \beta y - \tfrac{1}{2}(xy + yx) + \frac{dN_{\rm c}(t) - \beta^2 dt}{\beta dt} y. \quad (4.7)$$

The first term in this expression is yielded automatically by the transformation (2.16). Thus the Hamiltonian that must be added to produce a QND measurement is

$$H(t) = -\tfrac{1}{2}(xy + yx) + I_{\rm c}^x(t) y, \quad (4.8)$$

where the signal $I_{\rm c}^x(t)$ in the homodyne photocurrent is [22]

$$I_{\rm c}^x(t) = \lim_{\beta \to \infty} \frac{dN_{\rm c}(t) - \beta^2 dt}{\beta dt}. \quad (4.9)$$

With this Hamiltonian, the evolution of the system is exactly that required of a Markovian QND measurement of the x quadrature. In the nonselective case,

$$\dot\rho = \mathcal{D}[x]\rho \equiv \mathcal{L}\rho. \quad (4.10)$$

This equation is also obtainable directly from the theory of homodyne-mediated feedback [26,27] with Hamiltonian (4.8). The expression for the current (4.9) is unchanged by the feedback; it still measures the x quadrature. However, the two-time correlation function is

$$E[I_{\rm c}^x(t+\tau) I_{\rm c}^x(t)] = {\rm Tr}\left\{ 2x e^{\mathcal{L}\tau}[x\rho(t) + \rho(t)x] \right\} + \delta(\tau). \quad (4.11)$$

This will always give a super-shot-noise spectrum because it measures symmetrically ordered moments for x, rather than normally ordered moments as from homodyne detection without feedback:

$$E[I_{\rm c}^x(t+\tau) I_{\rm c}^x(t)] = {\rm Tr}\left\{ 2x e^{\mathcal{L}\tau}[c\rho(t) + \rho(t)c^\dagger] \right\} + \delta(\tau), \quad (4.12)$$

where here $\mathcal{L}$ would be $\mathcal{D}[c]$, not $\mathcal{D}[x]$.

Unlike the direct detection case analyzed in Sec. III, the scheme proposed here is quite practical. The feedback Hamiltonian y simply corresponds to controlling the driving onto the cavity. This could be achieved by controlling the intensity of a strong coherent beam (the same source as the local oscillator could be used) incident on another mirror of the cavity. If the transmittivity of this mirror is sufficiently low, the extra damping it causes can be ignored. Alternatively, the driving could take place at the same mirror as the damping, but the modulated amplitude is removed from the output beam before it is detected. This will be explained in detail in Sec. VI. The intensity modulator could be effected by using an electro-optic polarization modulator combined with a polarization-dependent beam splitter. The auxiliary Hamiltonian $H = -(xy+yx)/2$ is well approximated by the action of a degenerate parametric oscillator below threshold [29]. In fact, the magnitude of this DPO nonlinearity puts it at threshold in the cavity. However, this difficulty can be avoided by assuming that there are other linear losses apart from that allowing the measurement to be made. This will be the case in practice, and is necessary if the device is to be used to monitor a traveling wave, as will be investigated in the following sections.

V. QND EVALUATION CRITERIA

It was shown in the preceding section that feedback of the homodyne photocurrent, combined with a $\chi^{(2)}$ nonlinearity, can turn damping into a perfect measurement of the first kind of the x quadrature of the intracavity field. In reality, imperfections would arise due to inefficient photodetectors, time delay in the feedback loop, and other losses. Also, it is usually more desirable to be able to measure the quadrature of a traveling wave, rather than that of a single-mode cavity. The device I have proposed can be used to this effect. The traveling wave to be measured (called the signal) would reflect off a second mirror to the cavity, which is controlled as before. However, in order to evaluate the effectiveness of such a measurement, it is necessary to introduce a means of discrimination other than simply inspecting master equations for ideal cases. The criteria I will use are those defined in Ref. [7]. This section summarizes that work.

The type of measurements to which the criteria apply can be modeled as a "black box" with two inputs and two outputs. One input is the signal or system to be measured, labeled $S_{\rm in}$, and the other is the probe, labeled $B_{\rm in}$. After interacting with the probe, the system leaves as the output $S_{\rm out}$, while the probe output that contains the information of the measurement is $B_{\rm out}$. For a good measurement, $B_{\rm out}$ will be highly correlated with $S_{\rm in}$. This can be quantified by the accuracy A, which varies between 0 and 1, defined by

$$A = \frac{|\langle S_{\rm in}, B_{\rm out}\rangle|^2}{V(S_{\rm in})V(B_{\rm out})}. \quad (5.1)$$

Here, angle brackets denote quantum expectation values, and

$$\langle a, b\rangle \equiv \tfrac{1}{2}\langle ab + ba\rangle - \langle a\rangle\langle b\rangle \quad (5.2)$$

for arbitrary operators a and b, and $V(a) = \langle a, a\rangle$. A good QND measurement must have the additional property that $S_{\rm out}$ is little changed from $S_{\rm in}$. This is quantified by the conservativity C defined by

$$C = \frac{|\langle S_{\rm in}, S_{\rm out}\rangle|^2}{V(S_{\rm in})V(S_{\rm out})}. \quad (5.3)$$

In order to compare different quadrature QND schemes, it is useful to define the standard inputs for the probe and signal beams to be coherent states (possibly the vacuum). It is also useful to define an obviously non-QND standard device: a beam splitter. For such a classical device, it is easy to show that the following inequality holds:

$$A + C \leq 1. \tag{5.4}$$

The extent to which $A + C > 1$ and approaches its maximum value of 2 thus measures how non-classical the QND device is. If $A + C = 2$, it is a perfect QND device, and knowing the probe output implies complete knowledge of the signal output. In general, however, there is no simple relation between A, C and the predictive ability of the device. This predictive ability is best quantified by the conditional variance of the signal output. For Gaussian statistics (which is all that is of interest here), it is given by [7]

$$V(S_{\rm out}|B_{\rm out}) = V(S_{\rm out}) - \frac{|\langle B_{\rm out}, S_{\rm out}\rangle|^2}{V(B_{\rm out})}. \tag{5.5}$$

An output conditional variance less than unity (the input value) is a sure measure of the nonclassicality of the device. For the beam splitter, the output variance equals unity. The generalization of these formulas for the frequency components of traveling waves (used in Sec. VI) are found in Ref. [7].

VI. EVALUATING THE FEEDBACK QND SCHEME

A. Inputs and outputs for the device

As explained in the preceding section, the scheme for eliminating back-action by feedback described in Sec. V can be used as a QND scheme with input and output beams. The QND variable is the x quadrature of the signal, reflected off one end of the cavity. The probe output is the beam that is detected by the homodyne apparatus, whose current controls the feedback driving. This modulated driving can act at the probe mirror. An experimental configuration that removes the modulated reflections from the probe beam before detection is shown in Fig. 1. The modulated beam (call it m) is added to the input probe beam (usually a vacuum) by a low reflectance ($\eta \ll 1$) beam splitter (LRBS). Call the transmitted part (almost all) of the modulated beam $t \simeq m$, and the reflected part $r \simeq \sqrt{\eta}m$. The beam t is reflected by a transverse mirror, and put through the LRBS again, where almost all of it is again transmitted, but a small part (call it $r' \simeq \sqrt{\eta}m$) is reflected outwards. Meanwhile the original reflected beam r from the LRBS, plus the probe input $B_{\rm in}$ that entered from the other port of the LRBS, drives the cavity. The modulation r is reflected off the cavity mirror along with the probe output $B_{\rm out}$. This will be almost entirely transmitted at the LRBS, and will travel outwards along with r'. If the optical path lengths are set correctly, destructive interference will cause the cancellation of the two modulated components in the output beam r and r'. Thus the light

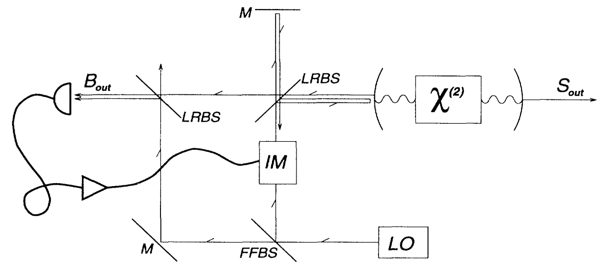

FIG. 1. Schematic diagram of traveling wave QND device using feedback. The current-carrying wire is indicated by a thick curve; narrow lines represent light beams. $S_{\rm out}$ and $B_{\rm out}$ are the signal and probe output beams, as explained in the text. LO denotes the local oscillator source, IM a current controlled intensity modulator, M a mirror, LRBS a low reflectivity beam splitter, and FFBS a 50-50 beam splitter (although any finite ratio will do).

coming from the cavity through the beam splitter will be $B_{\rm out}$, as if it it had a vacuum input (although the input modulation r will of course change the cavity dynamics; this is the point of the feedback). This light is then detected by homodyne detection (using the same local oscillator, which was modulated to give the feedback, and a second low reflectance beam splitter) and the resultant photocurrent used in the feedback loop.

The QND correlation functions defined in the preceding section were defined using quantum averages for the probe and signal beams. For QND schemes based around an intracavity interaction, these functions are usually evaluated using the input-output theory of Gardiner and Collett [30]. The system evolution is expressed as a quantum Langevin equation for the cavity mode operator, in which the noise term arises from the input field operator. The output field operator is simply related to the input field and the intracavity mode. The treatment of the feedback QND scheme in Sec. III was based on a measurement theory approach, which necessarily uses the density operator. It is thus not obvious that there is any corresponding quantum Langevin treatment of feedback. The early treatments of continuous feedback did in fact use the Heisenberg picture [31–33] rather than a measurement theory approach, but were not developed rigorously. Recently, it has been shown that the Langevin equation approach to feedback can be formulated rigorously for all cases, and is equivalent to the measurement theory approach [25]. The proof is essentially related to the recently published theories of cascaded open quantum systems [34,35]. Thus, in this section, the Langevin approach to feedback will be used for convenience.

Let the damping rates at the signal and probe ends of the cavity be γ_1 and γ_2, respectively. Assume that the field inputs at both ends are in vacuum states, and so represented by the vacuum annihilation operators $\nu_1(t)$ and $\nu_2(t)$, respectively. These bath operators have zero mean, but obey the commutation relations

$$[\nu_i(t), \nu_j^\dagger(t')] = \delta_{ij}\delta(t - t'), \tag{6.1}$$

with all other commutators vanishing. The Langevin equation for an arbitrary cavity operator a is

$$\dot{a} = i[H,a] - [a,c^\dagger]\left(\frac{\gamma_1+\gamma_2}{2}c + \sqrt{\gamma_1}\nu_1 + \sqrt{\gamma_2}\nu_2\right) + \left(\frac{\gamma_1+\gamma_2}{2}c^\dagger + \sqrt{\gamma_1}\nu_1^\dagger + \sqrt{\gamma_2}\nu_2^\dagger\right)[a,c]. \quad (6.2)$$

Here, $c = x+iy$ is the annihilation operator for the cavity as before, and H is the Hamiltonian (4.8)

$$H(t) = -\frac{\gamma_2}{2}(xy+yx) + y\int_0^\infty I_c^x(t-s)h(s)ds, \quad (6.3)$$

where I have generalized the feedback by including a response function $h(s)$. The instantaneous feedback of Sec. IV corresponds to $h(s) = \delta(s)$. The output fields, denoted by $b_1(t)$ and $b_2(t)$ for the signal and probe, respectively, are given by [30]

$$b_i(t) = \sqrt{\gamma_i}\nu_i(t) + \gamma_i\, c(t). \quad (6.4)$$

In order to treat Eq. (6.2) consistently, the current $I_c^x(t)$ must be an operator. The correct operator is that which is measured by homodyne detection, namely, the x quadrature of the output probe field. Define new bath operators $\xi_i(t) = \nu_i(t) + \nu_i^\dagger(t)$. These commute, and act as real, normalized, Gaussian white-noise terms satisfying

$$\langle \xi_i(t)\xi_j(t')\rangle = \delta_{ij}\delta(t-t'). \quad (6.5)$$

Then the homodyne current can be defined as

$$I_c^x(t) = b_2(t) + b_2^\dagger(t) = 2\gamma_2 x(t) + \sqrt{\gamma_2}\,\xi_2(t). \quad (6.6)$$

In order to consider detectors of efficiency η_i for the two output beams, it is necessary to modify this expression. The effect of nonunit efficiency can be modeled as placing a beam splitter of transmittance η_i in front of a perfectly efficient detector. Normalizing the transmitted field so that the deterministic part (proportional to x) remains the same, the result is

$$I_c^x(t) = 2\gamma_2 x(t) + \sqrt{\gamma_2}\,\xi_2(t) + \sqrt{\gamma_2\epsilon_2}\,\xi_4(t), \quad (6.7)$$

where

$$\epsilon_i = (1-\eta_i)/\eta_i, \quad (6.8)$$

and $\xi_4(t)$ is an independent Gaussian white-noise term.

Since the probe output beam has to be measured in order to carry out the feedback, it is only sensible to define the probe output to be that measured photocurrent. Normalized to unit shot noise,

$$B_{\rm out}(t) = \sqrt{\eta_2}[2\sqrt{\gamma_2}x(t) + \xi_2(t) + \sqrt{\epsilon_2}\,\xi_4(t)]. \quad (6.9)$$

Since this expression includes the possibility of inefficient detectors, it would seem consistent to define the signal output in the same way:

$$S_{\rm out}(t) = \sqrt{\eta_1}[2\sqrt{\gamma_1}\,x(t) + \xi_1(t) + \sqrt{\epsilon_1}\,\xi_3(t)], \quad (6.10)$$

where $\xi_3(t)$ is another noise term. It was assumed above that both probe and signal inputs were in the vacuum state. This is a convenient choice, as explained in the preceding section. Thus, with the same normalization,

$$B_{\rm in}(t) = \xi_2(t), \quad S_{\rm in}(t) = \xi_1(t). \quad (6.11)$$

This completes the definitions.

The probe and signal outputs depend only on the noise operators and the intracavity quadrature operator $x(t)$. From Eq. (6.2), this obeys

$$\dot{x}(t) = -\frac{\gamma_1+\gamma_2}{2}x(t) - \frac{\gamma_2}{2}x(t) - \frac{\sqrt{\gamma_1}}{2}\xi_1(t) - \frac{\sqrt{\gamma_2}}{2}\xi_2(t) + \int_0^\infty \left[\gamma_2 x(t-s) + \frac{\sqrt{\gamma_2}}{2}\xi_2(t-s) + \frac{\sqrt{\gamma_2\epsilon_2}}{2}\xi_4(t-s)\right]h(s)ds, \quad (6.12)$$

where the definition (6.7) has been used. For the ideal case $[\epsilon_i = 0,\ h(s) = \delta(s)]$, it is simple to see from this equation how the device works. The feedback plus nonlinearity completely removes the effect of the damping at the first (probe) mirror, for the x quadrature. The x quadrature evolves as if the probe mirror were perfect, and is damped only through the signal mirror. Thus the signal is reflected unchanged (apart from a phase shift), but there is nevertheless a probe output that is measured, and which directly gives information about the signal input. This is the essence of a good QND apparatus.

Transforming Eq. (6.12) to the frequency domain yields

$$\tilde{x}(\omega) = \frac{-\sqrt{\gamma_1}\,\tilde{\xi}_1(\omega) - \sqrt{\gamma_2}[1-\tilde{h}(\omega)]\tilde{\xi}_2(\omega) + \sqrt{\gamma_2\epsilon_2}\,\tilde{h}(\omega)\tilde{\xi}_4(\omega)}{2\{\gamma_1/2 + \gamma_2[1-\tilde{h}(\omega)] - i\omega\}}, \quad (6.13)$$

where the noise terms in the frequency domain are complex and obey

$$\xi_i(\omega)^* = \xi_i(-\omega), \quad (6.14a)$$

$$\langle \xi_i(\omega)\xi_j(\omega')\rangle = 2\pi\delta_{ij}\delta(\omega+\omega'). \quad (6.14b)$$

The Fourier transformed probe and signal outputs are

$$\frac{\tilde{B}_{\text{out}}}{\sqrt{\eta_2}} = \frac{-\sqrt{\gamma_1\gamma_2}\tilde{\xi}_1 + (\gamma_1/2 - i\omega)\tilde{\xi}_2 + \sqrt{\epsilon_2}(\gamma_1/2 + \gamma_2 - i\omega)\tilde{\xi}_4}{\gamma_1/2 + \gamma_2(1-\tilde{h}) - i\omega}, \tag{6.15}$$

$$\frac{\tilde{S}_{\text{out}}}{\sqrt{\eta_1}} = (\tilde{\xi}_1 + \sqrt{\epsilon_1}\,\tilde{\xi}_3) + \frac{-\gamma_1\tilde{\xi}_1 - \sqrt{\gamma_1\gamma_2}(1-\tilde{h})\tilde{\xi}_2 + \sqrt{\epsilon_2\gamma_1\gamma_2}\,\tilde{h}\tilde{\xi}_4}{\gamma_1/2 + \gamma_2(1-\tilde{h}) - i\omega}, \tag{6.16}$$

where the argument ω has been suppressed.

B. QND correlation coefficients

Using these results and the noise statistics (6.14), the correlation coefficients A, C, and $V(S_{\text{out}}|B_{\text{out}})$ may be found. First, however, it is useful to define the dimensionless quantities

$$\Omega \equiv \omega/\gamma_1, \quad G \equiv \gamma_2/\gamma_1. \tag{6.17}$$

The symbol G is used for the ratio of the damping rates of the end mirrors, because it is effectively the gain of the QND measurement. In terms of these parameters,

$$C(\Omega) = \left(1 + \frac{G|1-\tilde{h}|^2 + G\epsilon_2|\tilde{h}|^2 + \epsilon_1\left|\frac{1}{2} + G(1-\tilde{h}) - i\Omega\right|^2}{\left|-\frac{1}{2} + G(1-\tilde{h}) - i\Omega\right|^2}\right)^{-1}, \tag{6.18}$$

$$A(\Omega) = \left(1 + \frac{\frac{1}{4} + \Omega^2}{G} + \epsilon_2\frac{\left(G + \frac{1}{2}\right)^2 + \Omega^2}{G}\right)^{-1}, \tag{6.19}$$

$$V(S_{\text{out}}|B_{\text{out}})(\Omega) = 1 - \frac{1+\epsilon_2}{1+\epsilon_1}A(\Omega). \tag{6.20}$$

It is interesting to note that the feedback gain $\tilde{h}$ only enters into the expression for the conservativity C.

To understand these formulas, first consider the ideal case $\epsilon_1 = \epsilon_2 = 0$, $\tilde{h} = 1$. Then it is easy to see that

$$C(\Omega) = 1, \tag{6.21}$$

$$A(\Omega) = \frac{G}{G + \frac{1}{4} + \Omega^2}, \tag{6.22}$$

$$V(S_{\text{out}}|B_{\text{out}})(\Omega) = 1 - \frac{G}{G + \frac{1}{4} + \Omega^2}. \tag{6.23}$$

That is to say, the system is unaffected by the interaction, as required for a true QND measurement. The accuracy and predictive ability of the measurement depend on the gain G. For large gain, A approaches 1 and the conditioned variance approaches zero, indicating that the device is a perfect QND detector in this limit. The bandwidth for these correlations is (for large G) approximately equal to $\sqrt{G}$, which is the geometric mean of the two decay rates in original units. This behavior is shown in Fig. 2, which plots $A + C$ and $V(S_{\text{out}}|B_{\text{out}})$ versus Ω for $G = 4$. Evidently this value of gain can be considered quite large.

This apparently excellent role for back-action elimination by feedback is somewhat diminished when one considers the same case, but with the feedback turned off ($\tilde{h} = 0$). As noted above, the values for A and $V(S_{\text{out}}|B_{\text{out}})$ are identical to those with feedback. The conservativity is no longer unity, but rather

$$C(\Omega) = 1 - \frac{G}{G^2 + \frac{1}{4} + \Omega^2}. \tag{6.24}$$

Evidently this is still a QND measurement in the sense

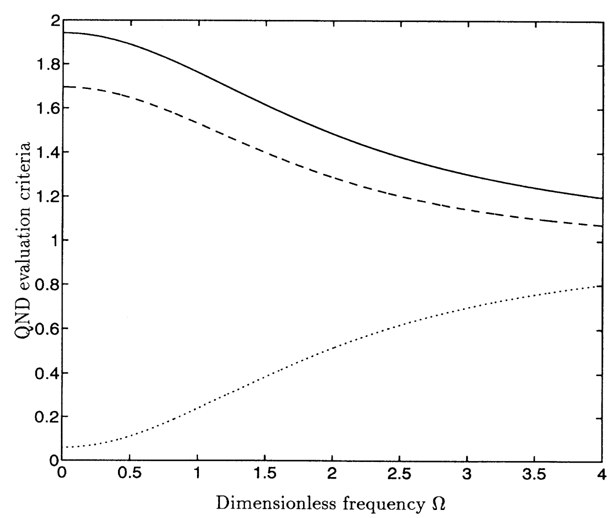

FIG. 2. Plot of the QND evaluation criteria $A + C$ (solid and dashed lines) and $V(S_{\text{out}}|B_{\text{out}})$ (dotted line) versus dimensionless frequency Ω. The solid line is with the feedback on, and a response function $\tilde{h} = 1$. The dashed line is with the feedback off. For both cases, the efficiency of the photodetectors is 100%, and the QND gain $G = 4$.

that $A + C > 1$, as shown in Fig. 2. Furthermore, in the limit $G \to \infty$, C as given in Eq. (6.24) also approaches unity. It thus appears that the central role in the QND device is being played by the $\chi^{(2)}$ medium, rather than by the feedback. This is particularly emphasized by the fact that the conditional output variance, which is the only directly measurable correlation, is unchanged by the feedback. Nevertheless, the QND property itself, the conservativity, is unquestionably enhanced by the feedback. It has been brought to my attention that the scheme to which I am referring here, with the nonlinearity but no feedback, has been analyzed recently in Ref. [9]. (In that work, the nonlinearity was not assumed to equal the loss rate at the probe mirror, but this turns out to be the optimal condition in the limit $G \to \infty$.) The effectiveness of such a simple QND apparatus also raises doubts about the usefulness of the more involved schemes proposed in the past [3–8].

To assess the usefulness of the feedback in practice, it is necessary to consider nonideal conditions, in particular, imperfect photodetectors. Even with better than 95% efficient detectors ($\epsilon_1 = \epsilon_2 = 0.05$), the effect on the quality of the measurement is dramatic, as shown in Fig. 3. The feedback still gives an improvement, but it is less than in the ideal case of Fig. 2. In Fig. 3, the feedback loop response function $\tilde{h}(\omega)$ is no longer unity. By analysis of the formulas for C, one finds that the result is better at $\Omega = 0$ if $\tilde{h}(0) > 1$. However, the feedback loop gain is limited by the stability requirement that $\frac{1}{2}\kappa_1 + \kappa_2[1 - \tilde{h}(0)] > 0$ [see Eq. (6.12)]. Working at the limit of stability suggests $\tilde{h}(0) = 1 + \kappa_1/(2\kappa_2)$. In Fig. 3, I have also included a time delay and some exponential smoothing in the feedback loop by taking the total response function to be

$$\tilde{h} = \left(1 + \frac{\kappa_1}{2\kappa_2}\right) \frac{\exp(\mathrm{i}\omega/\kappa_2)}{1 + \mathrm{i}\omega/\kappa_2} = \left(1 + \frac{1}{2G}\right) \frac{\exp(\mathrm{i}\Omega/G)}{1 + \mathrm{i}\Omega/G}. \tag{6.25}$$

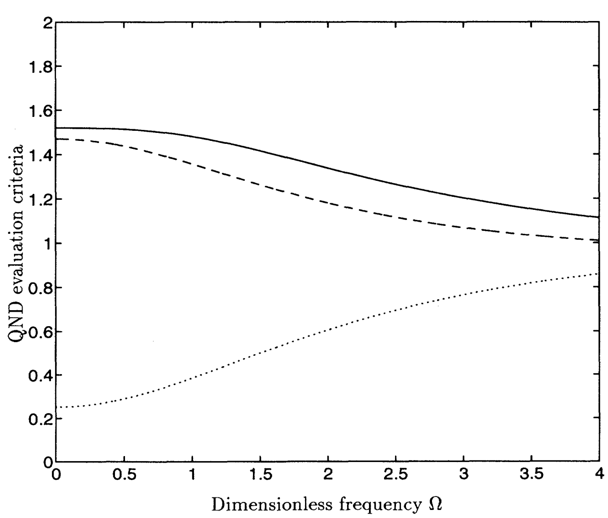

FIG. 3. As in Fig. 2, but with 95% efficient detectors, and with $\tilde{h}$ given by Eq. (6.25)

The nonflat response has little effect at low frequencies, where the QND correlations are best.

A complete analysis of this QND apparatus would require one to relax the assumption that the strength of the nonlinearity χ equals the damping rate κ_2 of the second mirror. Then, for a fixed χ, κ_1, and ϵ_i, one would have to maximize some suitable combination of $A + C$ and $V(S_{\rm out}|B_{\rm out})$, as a function of ω, κ_2, and $\tilde{h}$. This would be necessary to find the optimal operating region, and to find out how much improvement the feedback can offer over the nonlinearity alone. In any case, for making a QND measurement of the intracavity x quadrature, as explored in Sec. IV, the feedback is essential. Without it, a homodyne detection would simply be measuring the squeezing produced by the DPO. If a detector of efficiency η is included in that model, the QND master equation (4.10) becomes

$$\dot{\rho} = \mathcal{D}[x]\rho + \frac{1-\eta}{\eta}\mathcal{D}[y]\rho \equiv \mathcal{L}\rho. \tag{6.26}$$

The extra term causes the variance in x to increase linearly with time, which obviously violates the QND definition. However, for η close to 1, an approximate first-kind measurement could be carried out for a short time. As well as modifying $\mathcal{L}$, the loss increases the noise to signal ration in the two-time correlation function (4.11)

$$E[I_{\rm c}^x(t+\tau)I_{\rm c}^x(t)] = {\rm Tr}\left\{2xe^{\mathcal{L}\tau}[x\rho(t) + \rho(t)x]\right\} + \frac{1}{\eta}\delta(\tau). \tag{6.27}$$

VII. CONCLUSION

Any quantum-demolition measurement can in principle be turned into a quantum-nondemolition measurement simply by using the measurement result to alter the dynamics of the system. In particular, the back-action of continuous observations can be eliminated by a feedback Hamiltonian proportional to the measured current. This was shown to be impractical for the case of direct detection of the light from a single-mode cavity, as the required Hamiltonian is equal to the phase operator for the field (but see [36]). However, it is possible to turn a homodyne measurement of the x quadrature into a QND measurement, because the only nonlinearity required is that of a degenerate parametric oscillator. The homodyne photocurrent is used to control the amplitude of driving on the cavity, while the DPO remains constantly operational. For perfect detectors, and the correct strength of nonlinearity and driving, the device effects a perfect QND measurement of the intracavity x quadrature.

The analysis of the proposed device was extended to allow for measurements on traveling waves. The signal to be measured is reflected off a second mirror of the cavity. Using the appropriate QND evaluation criteria for such measurements, the device was shown to act as a perfect broadband QND measurement apparatus in the limit of large gain and perfect detectors. Here, the gain is equal to the ratio of the $\chi^{(2)}$ value of the DPO (which must be equal to the damping rate of the first mirror) to the damping rate of the second mirror. However, even slightly imperfect detectors degrade the measurement correlations markedly. Also, the perfect results in the limit of infinite gain are reproduced with the feedback turned off. The crucial element in the device appears to be the nonlinearity, rather than the feedback. Nevertheless, there may be a role for feedback, because it produces significantly better results for the case of good detectors but finite gain. The possibilities for back-action elimination by feedback in other contexts are still to be explored.

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