Pulse matching and correlation of phase fluctuations in Λ systems

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The interaction of a bichromatic quantized field with three-level Λ-type atoms is analyzed on the basis of *c*-number Langevin equations. In the presence of ground-state coherence one particular combination of field modes is selectively absorbed leading to a correlation of the output fields. This correlation manifests itself in a matching of Fourier components of the field modes and in a strong reduction of the diffusion of the difference phase. This correlation phenomenon is studied for the case of externally generated coherence as well as for coherence produced by population trapping in conjunction with a nonadiabatic atomic response.

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I. INTRODUCTION

The optical properties of a three-level Λ- or V-type atom interacting with a bichromatic field can be best understood in terms of so-called normal modes [1]. If perfect coherence is established between the pair of upper or lower levels, one particular combination of field modes or normal mode does not couple to the atom under conditions of two-photon resonance, while the corresponding orthogonal combination does. This separation into coupled and decoupled normal modes due to the presence of coherence leads to interesting correlation phenomena.

For example, in a two-mode laser with three-level V-type atoms in which coherence is established between the upper levels, the emission of photons is possible only into the coupled normal mode and hence correlated fields are generated. Since the same restriction applies to spontaneous transitions, there are no noise contributions due to spontaneous emission to the beat signal which corresponds to the uncoupled normal mode. This is the essence of the so-called correlated spontaneous-emission laser (CEL) [2], which has interesting potential applications for active gravity wave detectors and laser gyros [3].

Similar effects are to be expected for the case of absorption by a coherently prepared Λ system. Here only field components corresponding to the coupled normal mode are absorbed, whereas the medium is transparent for the orthogonal superposition. Eventually correlated fields are generated by the interaction process. This situation is investigated in the first part of the present paper in a perturbative approach using *c*-number Langevin equations and standard linearization techniques [4]. In particular we study the propagation of a bichromatic field through a beam of three-level atoms which are injected into the interaction region in a coherent superposition of the lower levels. In the case of two pulses, a nearly perfect matching of the Fourier components of the semiclassical field amplitudes occurs. Furthermore, a correlation of the phase fluctuations of two cw fields is found. The latter is of particular interest since it could lead to an essential noise reduction in measurements of phase differences.

The selective absorption of the coupled normal mode in a Λ system with ground-state coherence is also the basis of the recently predicted pulse matching in electromagnetically induced transparency [5]. A pair of strong fields in two-photon resonance dumps the atom into a decoupled coherent superposition of the lower levels [6]. Since this trapping state involves the relative amplitude and phase of the fields, fast oscillations of these quantities, which do couple to the trapping state, are absorbed. Only oscillations fast compared to the atomic response time are affected by this process, since slowly changing fields drive the atom quasi-instantaneously into a new coherent superposition which therefore remains decoupled at all times [7]. Associated with the nonadiabatic correlation of Fourier components is a reduction of quantum fluctuations of the relative amplitude [8] and phase [7]. A detailed analysis of the correlation of phases, which was recently reported by one of the authors (M.F.) in Ref. [7] will be the subject of the second part of the paper. In particular the propagation of two quantized cw fields through a vapor cell with three-level atoms is studied.

II. CORRELATION PHENOMENA FOR INITIALLY PREPARED COHERENCE

A. Stochastic differential equations for fields propagating through a beam of coherently prepared atoms

We here consider the one-dimensional situation shown in Fig. 1(a). Two fields propagate along the z axis through a beam of atoms with a level structure indicated in Fig. 1(b). The atoms enter the interaction region at a time t_j and a position z_l in a coherent superposition of

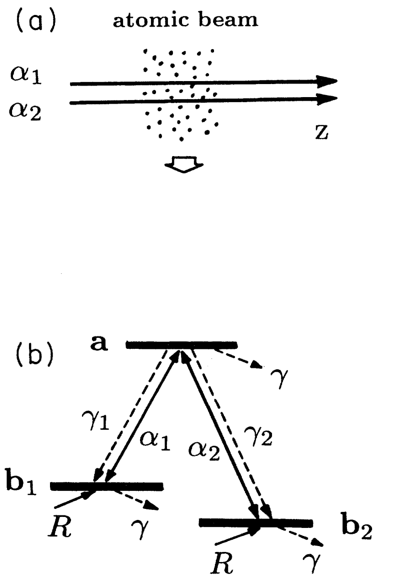

FIG. 1. Two light fields propagate through a beam of coherently prepared three-level atoms (a) with a level structure shown in (b).

levels b_1 and b_2. The interaction of the atoms with the two fields can then be described by a set of Heisenberg-Langevin equations [4].

For the case of a single atom characterized by the atomic operators

$$\begin{aligned}\sigma_a&=|a\rangle\langle a|,\quad \sigma_0=|b_1\rangle\langle b_2|\ ,\\ \sigma_{b_1}&=|b_2\rangle\langle b_1|,\quad \sigma_1=|b_1\rangle\langle a|\ ,\\ \sigma_{b_2}&=|b_2\rangle\langle b_2|,\quad \sigma_2=|b_2\rangle\langle a|\ ,\end{aligned} \tag{1}$$

and two field modes described by the annihilation and creation operators $a_\mu, a_\mu^\dagger$ we find, in a rotating frame,

$$\dot\sigma_a=-\Gamma'\sigma_a-ig_1\Theta(t-t_j)(a_1^\dagger\sigma_1-\text{H.a.})-ig_2\Theta(t-t_j)(a_2^\dagger\sigma_2-\text{H.a.})+F_a\ , \tag{2a}$$

$$\dot\sigma_{b_1}=-\gamma\sigma_{b_1}+\gamma_1\sigma_a+ig_1\Theta(t-t_j)(a_1^\dagger\sigma_1-\text{H.a.})+F_{b_1}\ , \tag{2b}$$

$$\dot\sigma_{b_2}=-\gamma\sigma_{b_2}+\gamma_2\sigma_a+ig_2\Theta(t-t_j)(a_2^\dagger\sigma_2-\text{H.a.})+F_{b_2}\ , \tag{2c}$$

$$\dot\sigma_0=-\Gamma_0\sigma_0-ig_1\Theta(t-t_j)a_1\sigma_2^\dagger+ig_2\Theta(t-t_j)a_2^\dagger\sigma_1+F_{\sigma_0}\ , \tag{2d}$$

$$\dot\sigma_1=-(i\Delta+\tfrac{1}{2}\Gamma)\sigma_1+ig_1\Theta(t-t_j)a_1(\sigma_{b_1}-\sigma_a)+ig_2\Theta(t-t_j)a_2\sigma_0+F_{\sigma_1}\ , \tag{2e}$$

$$\dot\sigma_2=-(i\Delta+\tfrac{1}{2}\Gamma)\sigma_2+ig_2\Theta(t-t_j)a_2(\sigma_{b_2}-\sigma_a)+ig_1\Theta(t-t_j)a_1\sigma_0^\dagger+F_{\sigma_2}\ . \tag{2f}$$

The generalization to many-atom variables and to propagating, i.e., multimode, fields will be done later. The injection into the cavity is modeled by a turn-on of the atom-field coupling at time t_j and the finite interaction time is taken into account by an effective decay out of the system with rate γ [9]. The g's are the coupling strength of the corresponding transitions, which are given by the dipole moments $\wp_{1,2}$, the transition frequencies $\omega_{ab_{1/2}}$, the beam cross section A and interaction length L, $g_\mu=\sqrt{\hbar\omega_\mu/2\epsilon_0 AL}\,\wp_\mu/\hbar$. The dipole moments are assumed to be real, such that $g_\mu=g_\mu^*$. We have assumed in Eq. (2) two-photon resonance conditions, that is, $\omega_{ab_1}-\omega_{ab_2}=\nu_1-\nu_2$. $\Delta=\omega_{ab_1}-\nu_1=\omega_{ab_2}-\nu_2$ is the detuning of the fields from the one-photon resonance and $\Gamma=\gamma_1+\gamma_2+2\gamma$, $\Gamma'=\gamma_1+\gamma_2+\gamma$, and $\Gamma_0=\gamma$. The noise operators in Eqs. (2) have zero mean value and are δ correlated. The corresponding diffusion coefficients can be obtained from the generalized fluctuation-dissipation theorem [4,9].

In order to describe the propagation of the two fields we follow an approach used in Ref. [10] and introduce space- and time-dependent, collective atomic variables. For this the interaction volume is subdivided in $2M+1$ cells of length $L/(2M+1)$, L being the total interaction length, with center points $z_l=lL/(2M+1)$, $l=-M,\dots,M$. We define

$$\sigma_\mu(z,t)\equiv\frac{1}{N}\lim_{M\to\infty}(2M+1)\sum_j\Theta(t-t_j)\sigma_\mu^{jl}\bigg|_{z_l\to z}\ , \tag{3}$$

where N is the mean number of atoms in the sample. The superscripts j and l in Eq. (3) characterize the injection time and position of the particular atom. As shown in Appendix A, the interaction of the atomic beam with two propagating fields can be described by replacing the single-mode operators $a_1(t)$ and $a_2(t)$ in Eqs. (2) by space- and time-dependent operators $a_1(z,t)$ and $a_2(z,t)$, which obey the Maxwell equations in slowly varying amplitude and phase approximation

$$\left[\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right]a_1(z,t)=ig_1N\sigma_1(z,t)\ , \tag{4a}$$

$$\left[\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right]a_2(z,t)=ig_2N\sigma_2(z,t)\ . \tag{4b}$$

Due to the presence of the step function in Eq. (3), the equation of motion for the collective atomic variables obtains an additional term which accounts for the injection of the atoms

$$\frac{d}{dt}\sigma_\mu(z,t)=\frac{1}{N}\lim_{M\to\infty}(2M+1)\times\sum_j\delta(t-t_j)\sigma_\mu^{jl}(t_j)\bigg|_{z_l\to z}+\cdots \tag{5}$$

Since the injection times are random, this term has a stochastic nature. We therefore add it to the effective noise operator, after subtracting its average value

$$\frac{1}{N}\lim_{M\to\infty}(2M+1)\left\langle\sum_j\delta(t-t_j)\sigma_\mu^{jl}(t)\right\rangle_{z_l\to z}=\gamma\sigma_\mu^0 . \tag{6}$$

Here σ_μ^0 is the expectation value of the single-atom operator at the time of injection. As discussed in Appendix B, the collective noise operators

$$F_\mu(z,t)\equiv\frac{1}{N}\lim_{M\to\infty}(2M+1)\sum_j\Theta(t-t_j)F_\mu^{jl}(t)+\frac{1}{N}\lim_{M\to\infty}(2M+1)\times\sum_j\delta(t-t_j)\sigma_\mu^{jl}(t_j)\bigg|_{z_l\to z}-\gamma\sigma_\mu^0 \tag{7}$$

again have zero mean value and, if we assume that each atom is coupled to an individual reservoir, are δ correlated in time and space. If we furthermore assume a Poissonian injection statistics we find

$$\langle F_\mu(z,t)F_\nu(z',t')\rangle=\frac{1}{N^2}\lim_{M\to\infty}(2M+1)\delta(z-z')\delta(t-t')L\sum_j\Theta(t-t_j)D_{\mu\nu}\bigg|_{z_l\to z}+\frac{\gamma}{N}L\delta(z-z')\delta(t-t')\langle\sigma_\mu(t_j)\sigma_\nu(t_j)\rangle , \tag{8}$$

where the $D_{\mu\nu}$ are the single-atom diffusion coefficients. Note that the last term contains an operator product evaluated at the time of injection. The equations of motion for the collective atomic operators are then

$$\dot\sigma_a(z,t)=-\Gamma'\sigma_a-ig_1[a_1^\dagger(z,t)\sigma_1-\text{H.a.}]-ig_2[a_2^\dagger(z,t)\sigma_2-\text{H.a.}]+F_a(z,t) , \tag{9a}$$

$$\dot\sigma_{b_1}(z,t)=\gamma\sigma_{b_1}^0-\gamma\sigma_{b_1}+\gamma_1\sigma_a+ig_1[a_1^\dagger(z,t)\sigma_1-\text{H.a.}]+F_{b_1}(z,t) , \tag{9b}$$

$$\dot\sigma_{b_2}(z,t)=\gamma\sigma_{b_2}^0-\gamma\sigma_{b_2}+\gamma_2\sigma_a+ig_2[a_2^\dagger(z,t)\sigma_2-\text{H.a.}]+F_{b_2}(z,t) , \tag{9c}$$

$$\dot\sigma_0(z,t)=\gamma\sigma_0^0-\Gamma_0\sigma_0-ig_1a_1(z,t)\sigma_2^\dagger+ig_2a_2^\dagger(z,t)\sigma_1+F_{\sigma_0}(z,t) , \tag{9d}$$

$$\dot\sigma_1(z,t)=-(i\Delta+\tfrac{1}{2}\Gamma)\sigma_1+ig_1a_1(z,t)(\sigma_{b_1}-\sigma_a)+ig_1a_2(z,t)\sigma_0+F_{\sigma_1}(z,t) , \tag{9e}$$

$$\dot\sigma_2(z,t)=-(i\Delta+\tfrac{1}{2}\Gamma)\sigma_2+ig_2a_2(z,t)(\sigma_{b_2}-\sigma_a)+ig_1a_1(z,t)\sigma_0^\dagger+F_{\sigma_2}(z,t) . \tag{9f}$$

An exact analytic solution of the Heisenberg-Langevin equations (4) and (9) that would give all information about the system is not possible. The properties of the system can, however, be derived to a very good approximation from c-number Langevin equations which originate from the Liouville equation for the system density operator ρ. ρ can be expressed in terms of a generalized quasidistribution which obeys a Fokker-Planck equation. The Fokker-Planck equation, on the other hand, is equivalent to a set of stochastic c-number equations which are much easier to handle than the Heisenberg-Langevin equations given above [4]. Correlation functions of ordered operator products can then be obtained from the correlation functions of the corresponding c-number variables. The operator ordering is thereby defined by the relation between the quasidistribution and the density operator. Here we choose the ordering

$$a_2^\dagger,a_1^\dagger,\sigma_2^\dagger,\sigma_1^\dagger,\sigma_0^\dagger,\sigma_a,\sigma_{b_1},\sigma_{b_2},\sigma_0,\sigma_1,\sigma_2,a_1,a_2 .$$

The equations of motion of the c-number variables are formally identical to the Heisenberg-Langevin equations (4) and (9). We will use in the following the same symbols for the c-number counterparts of the atomic operators. The c-number field variables will be denoted by α_1 and α_2. The corresponding c-number Langevin noise terms $F_\mu(z,t)$ are δ correlated as the operators, however, with different diffusion coefficients $\mathcal{D}_{\mu\nu}(z,t)$

$$\langle F_\mu(z,t)F_\nu(z',t')\rangle=\delta(z-z')\delta(t-t')\frac{L}{N}\mathcal{D}_{\nu\mu}(z,t) , \tag{10}$$

which can be obtained from the quantum diffusion coefficients as shown in Appendix B.

We will now discuss the semiclassical evolution of the system assuming weak fields, so that perturbation with respect to the atom-field coupling strength $g_{1,2}$ is possible.

B. Dynamics of mean amplitudes: Pulse matching

In this section we study the evolution of the mean-field amplitudes in lowest-order perturbation theory, in which case the equations of motion are linear. Under conditions of a stationary atomic beam, the nonvanishing zeroth-order solutions of the averaged equations (9) read

$$\langle\sigma_a^{(0)}\rangle=0 , \tag{11a}$$

$$\langle\sigma_{b_1}^{(0)}\rangle=\sigma_{b_1}^0 , \tag{11b}$$

$$\langle\sigma_{b_2}^{(0)}\rangle=\sigma_{b_2}^0 , \tag{11c}$$

$$\langle\sigma_0^{(0)}\rangle=\sigma_0^0=|\sigma_0^0|e^{i\Theta} . \tag{11d}$$

The variables to be evaluated in first order are functionals of the field amplitudes and hence are time dependent. A Fourier transformation of Eqs. (9e) and (9f) yields, in first order of the atom-field coupling,

$$\langle \sigma_1^{(1)}(z,\omega)\rangle = \frac{ig_1\sigma_{b_1}^0}{\Gamma/2+i(\omega+\Delta)}\langle \alpha_1(z,\omega)\rangle + \frac{ig_2\sigma_0^0}{\Gamma/2+i(\omega+\Delta)}\langle \alpha_2(z,\omega)\rangle , \quad (12a)$$

$$\langle \sigma_2^{(1)}(z,\omega)\rangle = \frac{ig_2\sigma_{b_2}^0}{\Gamma/2+i(\omega+\Delta)}\langle \alpha_2(z,\omega)\rangle + \frac{ig_1\sigma_0^{0*}}{\Gamma/2+i(\omega+\Delta)}\langle \alpha_1(z,\omega)\rangle . \quad (12b)$$

Fourier transforming the field equations of motion and plugging expressions (12a) and (12b) into them yields

$$\left[i\omega+c\frac{d}{dz}\right]\langle \alpha_1(z,\omega)\rangle = -\frac{g_1^2N\sigma_{b_1}^0}{\Gamma/2+i(\omega+\Delta)}\langle \alpha_1(z,\omega)\rangle - \frac{g_1g_2N\sigma_0^0}{\Gamma/2+i(\omega+\Delta)}\langle \alpha_2(z,\omega)\rangle , \quad (13a)$$

$$\left[i\omega+c\frac{d}{dz}\right]\langle \alpha_2(z,\omega)\rangle = -\frac{g_2^2N\sigma_{b_2}^0}{\Gamma/2+i(\omega+\Delta)}\langle \alpha_2(z,\omega)\rangle - \frac{g_1g_2N\sigma_0^{0*}}{\Gamma/2+i(\omega+\Delta)}\langle \alpha_1(z,\omega)\rangle . \quad (13b)$$

The solutions of these equations can be found very easily. They read

$$\langle \alpha_1(z,\omega)\rangle = \tfrac{1}{2}[\langle \alpha_1(0,\omega)\rangle + e^{i\Theta}\langle \alpha_2(0,\omega)\rangle] \times \exp\left\{-(1+\eta)\beta(\omega)z - i\omega\frac{z}{c}\right\} + \tfrac{1}{2}[\langle \alpha_1(0,\omega)\rangle - e^{i\Theta}\langle \alpha_2(0,\omega)\rangle] \times \exp\left\{-(1-\eta)\beta(\omega)z - i\omega\frac{z}{c}\right\} , \quad (14a)$$

$$\langle \alpha_2(z,\omega)\rangle = \tfrac{1}{2}[\langle \alpha_2(0,\omega)\rangle + e^{-i\Theta}\langle \alpha_1(0,\omega)\rangle] \times \exp\left\{-(1+\eta)\beta(\omega)z - i\omega\frac{z}{c}\right\} + \tfrac{1}{2}[\langle \alpha_2(0,\omega)\rangle - e^{-i\Theta}\langle \alpha_1(0,\omega)\rangle] \times \exp\left\{-(1-\eta)\beta(\omega)z - i\omega\frac{z}{c}\right\} , \quad (14b)$$

where we have assumed for simplicity

$$\frac{g_1^2N\sigma_{b_1}^0}{\Gamma/2+i(\omega+\Delta)} = \frac{g_2^2N\sigma_{b_2}^0}{\Gamma/2+i(\omega+\Delta)} = c\beta(\omega) , \quad (15a)$$

$$\frac{g_1g_2N|\sigma_0^0|}{\Gamma/2+i(\omega+\Delta)} = c\eta\beta(\omega) . \quad (15b)$$

Here $\eta=|\sigma_0^0|/\sqrt{\sigma_{b_1}^0\sigma_{b_2}^0}$ is the degree of coherence between the two lower levels. It is quite instructive to express the results in terms of the normal modes

$$\langle \alpha_1(z,\omega)\rangle + \langle \alpha_2(z,\omega)\rangle e^{i\Theta} = [\langle \alpha_1(0,\omega)\rangle + \langle \alpha_2(0,\omega)\rangle e^{i\Theta}] \times \exp\left\{-(1+\eta)\beta(\omega)z - i\omega\frac{z}{c}\right\} , \quad (16a)$$

$$\langle \alpha_1(z,\omega)\rangle - \langle \alpha_2(z,\omega)\rangle e^{i\Theta} = [\langle \alpha_1(0,\omega)\rangle - \langle \alpha_2(0,\omega)\rangle e^{i\Theta}] \times \exp\left\{-(1-\eta)\beta(\omega)z - i\omega\frac{z}{c}\right\} . \quad (16b)$$

Equations (16) show that the "sum" mode (16a) is damped out very rapidly, whereas the "difference" mode (16b) can propagate essentially undamped if the coherence degree η is close to unity. This means the interaction leads to pulse matching after a sufficiently long propagation distance and the field amplitudes at the output obey the relation

$$\langle \alpha_1(L,\omega)\rangle \approx -\langle \alpha_2(L,\omega)\rangle e^{i\Theta} . \quad (17)$$

As can be seen from Eqs. (15), the pulse matching is perfect for Fourier frequencies small compared to $\Gamma/2$, that is, for not too fast varying pulse amplitudes. For $\omega^2 \gg \Gamma^2/4$ we have $\beta(\omega)\to 0$ and there is no matching of Fourier components.

If the relevant range of Fourier frequencies is such that $\beta(\omega)\approx\beta(0)$, i.e., in the adiabatic limit, Eqs. (14) can immediately be transformed back into the time domain

$$\langle \alpha_1(z,t)\rangle = \tfrac{1}{2}[\langle \alpha_1(0,t-\tau)\rangle + e^{i\Theta}\langle \alpha_2(0,t-\tau)\rangle] \times \exp\{-(1+\eta)\beta(0)z\} + \tfrac{1}{2}[\langle \alpha_1(0,t-\tau')\rangle - e^{i\Theta}\langle \alpha_2(0,t-\tau')\rangle] \times \exp\{-(1-\eta)\beta(0)z\} , \quad (18a)$$

$$\langle \alpha_2(z,t)\rangle = \tfrac{1}{2}[\langle \alpha_2(0,t-\tau)\rangle + e^{-i\Theta}\langle \alpha_1(0,t-\tau)\rangle] \times \exp\{-(1+\eta)\beta(0)z\} + \tfrac{1}{2}[\langle \alpha_2(0,t-\tau')\rangle - e^{-i\Theta}\langle \alpha_1(0,t-\tau')\rangle] \times \exp\{-(1-\eta)\beta(0)z\} , \quad (18b)$$

where τ and τ' are the retardation times

$$\tau = \frac{z}{c} + \frac{2(1+\eta)\mathrm{Re}[\beta(0)]z}{\Gamma} , \quad \tau' = \frac{z}{c} + \frac{2(1-\eta)\mathrm{Re}[\beta(0)]z}{\Gamma} . \quad (19)$$

One can see that the normal modes propagate with different group velocities. The strongly damped mode is considerably slowed down, whereas the weakly damped mode propagates in the case of ideal coherence with the speed of light c. Figure 2 shows the propagation of two resonant pulses with envelopes $\tilde{\alpha}_1(0,t)=\exp\{-(\Gamma t/10-0.3)^2\}$ and $\tilde{\alpha}_2(0,t)$

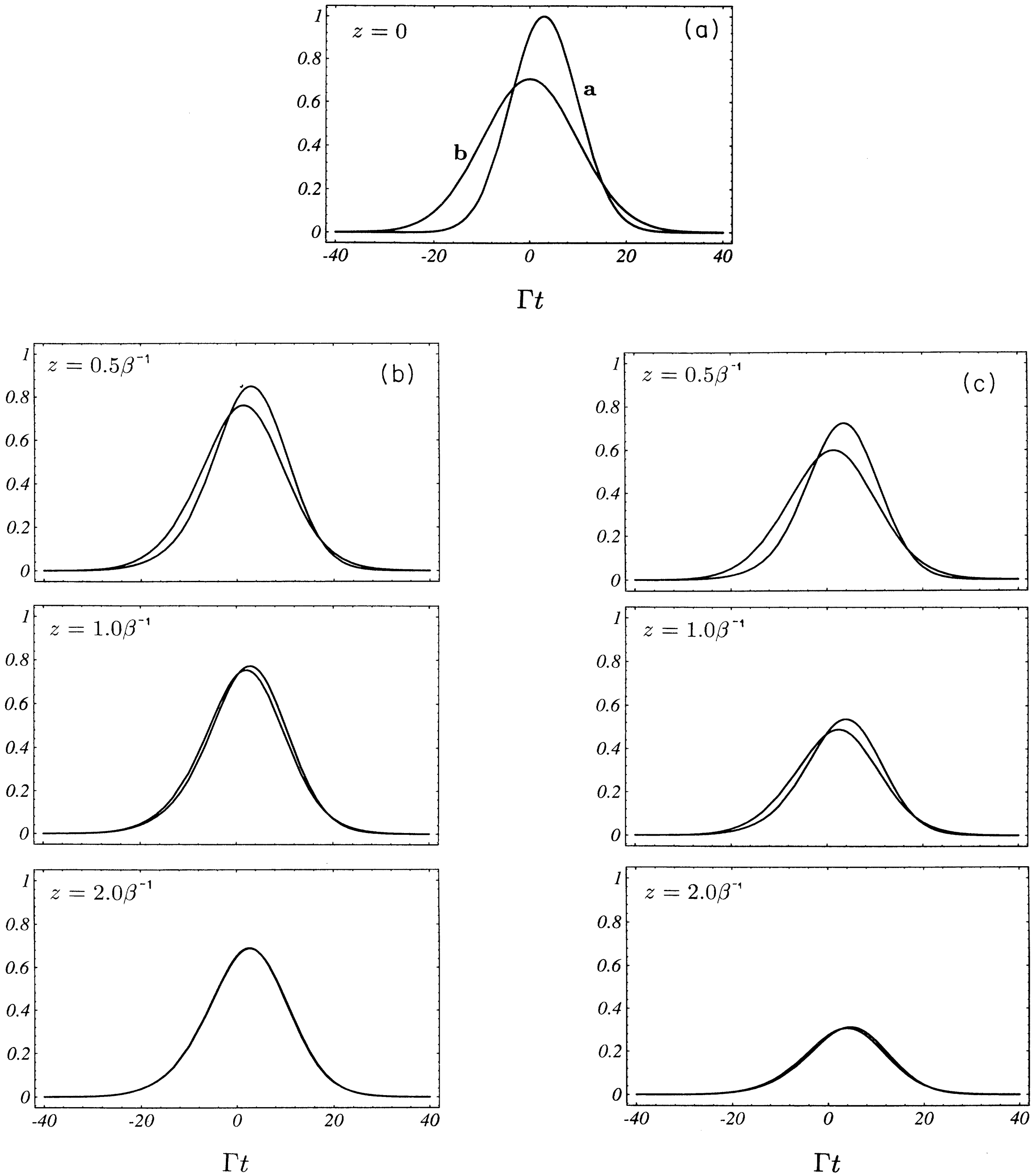

FIG. 2. Evolution of coherent amplitudes of two pulses with envelopes $\tilde{\alpha}_1(0,t)=\exp\{-(\Gamma t/10-0.3)^2\}$ (curve a) and $\tilde{\alpha}_2(0,t)=\sqrt{0.5}\exp\{-0.005\Gamma^2t^2\}$ (curve b). The initial pulse envelopes are shown in (a). The pulse envelopes for different propagation length through the medium are shown in (b) for $\gamma_1=\gamma_2=\gamma=\Gamma/4$ and $\eta=0.9$, and for $\eta=0.5$ in (c).

$=\sqrt{0.5}\exp\{-0.005\Gamma^2t^2\}$ in the adiabatic limit for two different degrees of coherence and $\Theta=\pi$. For smaller degrees of coherence the field amplitudes converge slower and are much stronger absorbed.

Since the originally weak pulse (b) gets amplified for a not too large interaction length at the expense of the stronger pulse, one might think of using the present scheme as an amplifier with one pulse replaced for instance by a strong cw field. One should note, however, that the signal modulation depth is not amplified by this process, but rather a background is added depending on the shape of the "pump" field. This can be seen from Eqs. (18). Let α_1 be a strong pump field and α_2 a weak signal field. One finds after a sufficiently long propagation distance for $\Theta=\pi$:

$$\langle \alpha_2(t)\rangle_{\text{out}} \sim \tfrac{1}{2}\langle \alpha_2(t-\tau')\rangle_{\text{in}} e^{-(1-\eta)\beta z} + \tfrac{1}{2}\langle \alpha_1(t-\tau')\rangle_{\text{in}} e^{-1(1-\eta)\beta z} .$$

Since $\eta \le 1$ the input signal is diminished by at least a factor $\frac{1}{2}$ and amplification is due to the *addition* of the pump field amplitude. One can see that the system can be used for an energy transfer from one field to the other, which, as shown by Agarwal, Scully, and Walther [11] can be noiseless, but not as an amplifier.

It is interesting to note, furthermore, that in contrast to the case of electromagnetically induced transparency (EIT), no front-end losses [1,12] of the pulses occur in the present scheme and in principle very weak fields can be used. This is because the atoms are prepared here in a trapping state, whereas in EIT the fields have to dump the atoms into a decoupled coherent superposition. The necessary energy for this has to be provided by the pulses, which leads to absorption at the front edges.

C. Phase correlation of quasimonochromatic fields

We now study the evolution of the phase fluctuations of two quasimonochromatic, resonant ($\Delta=0$) laser beams propagating through the medium. In particular we calculate the spectrum of the phase-difference fluctuations as a function of the propagation distance. We again concentrate on a lowest-order perturbation analysis and assume small fluctuations of the dynamical variables around their semiclassical steady-state values.

We introduce an intensity-phase representation according to

$$\alpha_{1,2}=|\alpha_{1,2}|e^{i\phi_{1,2}} , \tag{20a}$$

$$\sigma_{1,2}=|\sigma_{1,2}|e^{i\Theta_{1,2}} , \tag{20b}$$

$$\sigma_0=|\sigma_0|e^{i\Theta_0} . \tag{20c}$$

With these definitions we find, from Eqs. (4) and (9), the phase equations

$$\left[\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right]\phi_1(z,t)=g_1N\frac{|\sigma_1|}{|\alpha_1|}\cos(\Theta_1-\phi_1) , \tag{21a}$$

$$\left[\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right]\phi_2(z,t)=g_2N\frac{|\sigma_2|}{|\alpha_2|}\cos(\Theta_2-\phi_2) , \tag{21b}$$

and

$$\dot{\Theta}_1=g_1N\frac{|\alpha_1|}{|\sigma_1|}(\sigma_{b_1}^{(0)}-\sigma_a^{(0)})\cos(\Theta_1-\phi_1) +g_2N\frac{|\alpha_2|}{|\sigma_1|}|\sigma_0|\cos(\Theta_1-\phi_2-\Theta_0)+\text{Im}\left[\frac{F_{\sigma_1}}{\sigma_1}\right] , \tag{22a}$$

$$\dot{\Theta}_2=g_2N\frac{|\alpha_2|}{|\sigma_2|}(\sigma_{b_2}^{(0)}-\sigma_a^{(0)})\cos(\Theta_2-\phi_2) +g_1N\frac{|\alpha_1|}{|\sigma_2|}|\sigma_0|\cos(\Theta_2-\phi_1+\Theta_0)+\text{Im}\left[\frac{F_{\sigma_2}}{\sigma_2}\right] , \tag{22b}$$

$$\dot{\Theta}_0=-\gamma\sin(\Theta_0-\Theta)+\text{Im}\left[\frac{F_{\sigma_0}}{\sigma_0}\right] . \tag{22c}$$

We now consider small fluctuations of the dynamical variables around their semiclassical steady-state values. The semiclassical steady-state values are obtained by neglecting the noise terms in Eqs. (22) and by setting all time derivatives equal to zero. We assume the following relation between the semiclassical steady-state field amplitudes:

$$\alpha_1(z)=-\alpha_2(z)e^{i\Theta} . \tag{23}$$

Note that the interaction process generates fields which will anyway obey this relation after a sufficiently large interaction distance. Under this condition we find the semiclassical stationary phase values

$$\Theta_0=\Theta,\ \ \Theta_1-\phi_2=-\frac{\pi}{2}+\Theta,\ \ \Theta_2-\phi_1=-\frac{\pi}{2}-\Theta ,\quad \Theta_1-\phi_1=\frac{\pi}{2},\ \ \Theta_2-\phi_2=\frac{\pi}{2} . \tag{24}$$

We now linearize Eqs. (21) and (22) around these steady-state values, Fourier transform the results, and plug the expressions for the atomic phases back into the field equations. We arrive at

$$\left[i\omega+c\frac{d}{dz}\right]\delta\phi_1=-\frac{g_1^2N\sigma_{b_1}}{\Gamma/2+i\omega}\delta\phi_1+g_1N\frac{|\sigma_1|}{|\alpha_1|}\delta\phi_1+\frac{g_1g_2N|\sigma_0|}{\Gamma/2+i\omega}\frac{|\alpha_2|}{|\alpha_1|}\delta\phi_2 +\frac{g_1g_2N|\sigma_0|}{(\Gamma/2+i\omega)(\gamma+i\omega)}\frac{|\alpha_2|}{|\alpha_1|}\text{Im}\left[\frac{F_{\sigma_0}}{\sigma_0}\right]-\frac{g_1N|\sigma_1|}{|\alpha_1|}\frac{1}{\Gamma/2+i\omega}\text{Im}\left[\frac{F_{\sigma_1}}{\sigma_1}\right] , \tag{25a}$$

$$\left[i\omega+c\frac{d}{dz}\right]\delta\phi_2=-\frac{g_2^2N\sigma_{b_2}}{\Gamma/2+i\omega}\delta\phi_2+g_2N\frac{|\sigma_2|}{|\alpha_2|}\frac{|\alpha_1|}{|\alpha_2|}\delta\phi_2+\frac{g_1g_2N|\sigma_0|}{\Gamma/2+i\omega}\frac{|\alpha_1|}{|\alpha_2|}\delta\phi_1 -\frac{g_1g_2N|\sigma_0|}{(\Gamma/2+i\omega)(\gamma+i\omega)}\frac{|\alpha_1|}{|\alpha_2|}\text{Im}\left[\frac{F\sigma_0}{\sigma_0}\right]-\frac{g_2N|\sigma_2|}{|\alpha_2|}\frac{1}{\Gamma/2+i\omega}\text{Im}\left[\frac{F\sigma_2}{\sigma_2}\right] . \tag{25b}$$

The fluctuation of the phase difference between the two modes $\delta\psi \equiv \delta\phi_1 - \delta\phi_2$ obeys the equation

$$c\frac{d}{dz}\delta\psi(z,\omega) = -\left[i\omega\left[1-(1+\eta)c\beta\frac{\Gamma/2}{\Gamma^2/4+\omega^2}\right] + c\eta\beta\left[1+\frac{\Gamma^2/4}{\Gamma^2/4+\omega^2}\right] - c\beta\frac{\omega^2}{\Gamma^2/4+\omega^2}\right]\delta\psi(z,\omega) + \frac{c\eta\beta\Gamma}{(\Gamma/2+i\omega)(\gamma+i\omega)}\mathrm{Im}\left[\frac{F_{\sigma_0}}{\sigma_0}\right] - \frac{c\beta(1-\eta)}{\Gamma/2+i\omega}\mathrm{Im}\left[\frac{F\sigma_1}{\sigma_1} - \frac{F_{\sigma_2}}{\sigma_2}\right], \quad (26)$$

where $\beta = \beta(\omega = 0)$. Equation (26) can easily be solved and we can calculate the spectrum of phase-difference fluctuations, which we define as

$$S_\psi(z,\omega) \equiv \omega^2 \int_{-\infty}^{\infty} d\tau \langle \delta\psi(z,t)\delta\psi(z,t-\tau)\rangle e^{-i\omega\tau} = \frac{\omega^2}{2\pi}\int_{-\infty}^{\infty} d\omega' \langle \delta\psi(z,\omega)\delta\psi(z,\omega')\rangle . \quad (27)$$

The factor ω^2 is introduced in this definition of the spectrum, so that for a single field with freely diffusing phase, S is equal to the linewidth. Since, as mentioned in Sec. II A, c-number correlation functions correspond to normally ordered correlation functions of operators, vacuum contributions are not included in S_ψ. Therefore $S = 0$ corresponds to the vacuum limit. We find

$$S_\psi(z,\omega) = S_\psi(0,\omega)e^{-2\kappa(\omega)z} + \frac{3\lambda_1^2}{\pi A}\frac{\gamma_1}{\Gamma}(1-\eta)\frac{\gamma\omega^2}{(\gamma^2+\omega^2)(\Gamma^2+4\omega^2)} \times \frac{\omega^2+\Gamma^2/4}{(\omega^2(\eta-1)+\eta\Gamma^2/2)}[2(\gamma^2+\omega^2)+3\Gamma^2] \times [1-e^{-2\kappa(\omega)z}] , \quad (28)$$

where

$$\kappa(\omega) = \beta\frac{\omega^2(\eta-1)+\eta\Gamma^2/2}{\Gamma^2/4+\omega^2} . \quad (29)$$

In Eq. (28) we have substituted the coupling strength g_1 in terms of the radiative decay rate γ_1 and the wavelength λ_1 [9]:

$$g_1^2 = \frac{\wp_1^2\nu_1}{2\hbar\epsilon_0 AL} = \frac{3}{4\pi}\frac{\lambda_1^2}{\alpha}c\gamma_1 . \quad (30)$$

One can see from Eq. (29) that for large coherence degrees ($\eta \approx 1$), initial phase-difference fluctuations above the shot-noise limit are absorbed due to the locking to the phase of the injected coherence. It is interesting to note that, in the absence of coherence ($\eta = 0$), there is an amplification of high-frequency phase fluctuations. This can be understood in the following way: The incoming field induces an oscillating atomic dipole, whose phase is correlated with the phase of the field. Due to the finite response time of the atom, there is, however, a small retardation. For Fourier frequencies larger than the inverse response time of the atom Γ, the phase fluctuations of the oscillating dipole become independent of the field fluctuations and the total phase noise is amplified. For intermediate values of η both mechanisms compete such that phase fluctuations are quenched for $\omega^2 < [\eta/(1-\eta)]\Gamma^2/2$, but are amplified outside this frequency region. Figure 3 shows the exponential damping factor $\kappa(\omega)$ as function of ω/Γ and βz for 80% coherence.

The second term in Eq. (28) accounts for atomic noise contribution due to absorption and spontaneous reemission of photons. It vanishes if the degree of coherence η approaches unity. Since the atomic noise also vanishes as the Fourier frequency goes to zero, the long-time diffusion coefficient of the difference phase is not affected by it. The atomic noise term is plotted in Fig. 4 in units of $(1-\eta)(3\lambda_1^2\gamma_1\gamma/\pi A\Gamma)$.

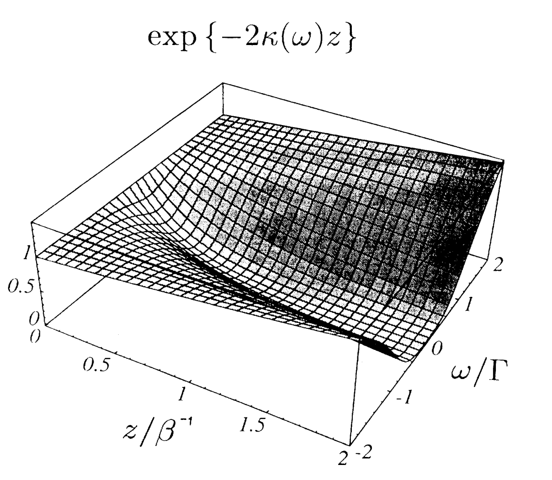

FIG. 3. Exponential decay factor of initial phase-difference fluctuations for $\eta = 0.8$ and $\gamma = \gamma_1 = \gamma_2 = \Gamma/4$.

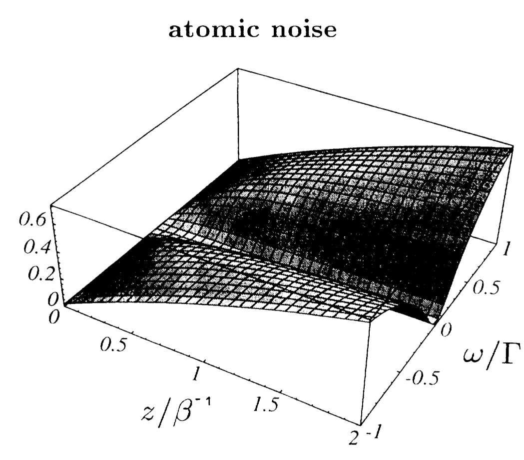

FIG. 4. Atomic noise contribution in units of $(1-\eta)(3\lambda_1^2\gamma\gamma_1/\pi A\Gamma)$ for the same parameters as in Fig. 3.

III. CORRELATIONS DUE TO ELECTROMAGNETICALLY INDUCED TRANSPARENCY

A. Stochastic differential equations and semiclassical dynamics in the adiabatic limit

We here analyze the propagation of two strong fields through a vapor cell with three-level Λ-type atoms such as those shown in Fig. 1, however, in a closed system, i.e., without the injection and decay channels. Since collisions cannot be neglected in a cell as opposed to the situation in a beam, we take into account a collisional dephasing of the lower level coherence with a rate γ_0. We assume that this contribution to the decay rate of the lower-level coherence is the dominant one and laser linewidth contributions [13] are small compared to it. In this scheme there is no externally generated coherence. However, the two strong fields will dump the atomic population into a coherent superposition of the two lower levels, which is essentially hidden from the fields. This phenomenon is well known as coherent population trapping [6]. Since the trapping state involves the relative phase and amplitudes of the two fields, small perturbations with different relative phases and amplitudes do couple to the trapped state and are absorbed if they happen on a time scale short compared to the atomic response time. Slow fluctuations on this time scale will drive the atom adiabatically into a new coherent superposition and the atom will remain in a dark state.

The interaction of the two fields with the atomic system is again described with c-number Langevin equations for space- and time-dependent variables, similar to those of Sec. II A. The equations of motion for the atomic and field variables are formally identical to Eqs. (9) and (4) with $R=\gamma=0$ and $\Gamma_0=\gamma_0$. The collective atomic operators are defined as in Eq. (3) without the Θ functions. The noise operators are δ correlated according to Eq. (10), with diffusion coefficients listed in Appendix C. Since coherent population trapping is a nonperturbative effect, we cannot apply perturbation theory as in the first part of the paper. For simplicity we assume, however, resonance of the carrier frequencies of the bichromatic field with the corresponding atomic transitions.

In a first step we discuss the semiclassical properties of the system in the adiabatic limit. For this we neglect the time derivatives on the left-hand side of Eq. (9) and disregard all noise operators. Solving the set of algebraic equations yields

$$\sigma_a=\frac{4\gamma_0\Omega_1^2\Omega_2^2}{D}\ , \tag{31a}$$

$$\sigma_{b_1}=\frac{4\gamma_0\Omega_1^2\Omega_2^2+2\Omega_2^2(\gamma_1\Omega_2^2+\gamma_2\Omega_1^2)+\gamma_0\gamma_1\Gamma\Omega_2^2}{D}\ , \tag{31b}$$

$$\sigma_{b_2}=\frac{4\gamma_0\Omega_1^2\Omega_2^2+2\Omega_1^2(\gamma_1\Omega_2^2+\gamma_2\Omega_1^2)+\gamma_0\gamma_2\Gamma\Omega_1^2}{D}\ , \tag{31c}$$

$$\sigma_1=\frac{2i\gamma_0\gamma_1\Omega_1\Omega_2^2e^{i\phi_1}}{D}\ , \tag{31d}$$

$$\sigma_2=\frac{2i\gamma_0\gamma_2\Omega_2\Omega_1^2e^{i\phi_2}}{D}\ , \tag{31e}$$

$$\sigma_0=\frac{2(\gamma_1\Omega_2^2+\gamma_2\Omega_1^2)e^{i(\phi_1-\phi_2)}\Omega_1\Omega_2}{D}\ . \tag{31f}$$

In Eqs. (31) we have introduced the Rabi frequencies of the fields according to $g_\mu\alpha_\mu(z,t)=\Omega_\mu(z,t)\exp\{i\phi_\mu(z,t)\}$ and

$$D=12\gamma_0\Omega_1^2\Omega_2^2+(\gamma_2\Omega_1^2+\gamma_1\Omega_2^2)[\gamma_0\Gamma+2(\Omega_1^2+\Omega_2^2)]\ . \tag{32}$$

Plugging the adiabatic values for the coherence $\sigma_{1,2}$ in the field equations (4), we find

$$\left[\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right]\alpha_{1,2}(z,t)=-\frac{2g_{1,2}^2N\Omega_{2,1}^2\gamma_0\gamma_{1,2}}{D}\alpha_{1,2}(z,t)\ . \tag{33}$$

One can immediately recognize from Eq. (33) that there is no mutual coupling of the phases of the two fields. Furthermore, the normalized differences of the intensities $|\alpha_1(z)|^2/\gamma_1-|\alpha_2(z)|^2/\gamma_2$ is a constant of motion. This implies that any initial difference of the phases or normalized amplitudes is unaffected by the interaction process in the adiabatic limit, which means that there is no matching of Fourier components. One can also see from Eqs. (33) that, due to the collisional dephasing of the lower-level coherence, which corresponds to a decay of the population trapped state, the absorption of the two fields is nonzero. The corresponding intensity absorption rates are

$$\kappa_{1,2}=\frac{4g_{1,2}^2N\Omega_{2,1}^2\gamma_0\gamma_{1,2}}{Dc}\ . \tag{34}$$

Figure 5 shows the dependence of $\kappa=\kappa_1=\kappa_2$ for the symmetric situation of equal Rabi frequencies ($\Omega_1=\Omega_2\equiv\Omega$), decay rates ($\gamma_1=\gamma_2\equiv\gamma$), and coupling strength as a function of the Rabi frequency Ω. One recognizes electromagnetically induced transparency for $\Omega^2\geq\gamma_0\gamma$ [12].

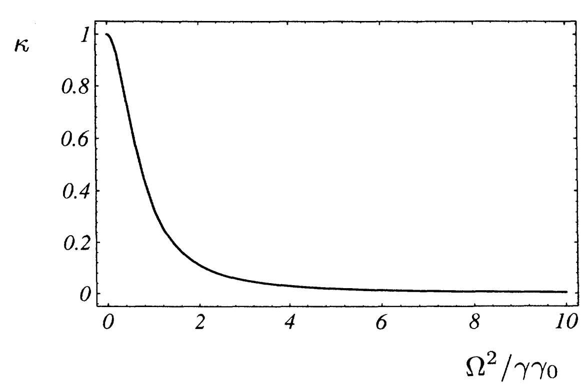

FIG. 5. Absorption rate κ in arbitrary units as a function of $\Omega^2/\gamma_0\gamma$. For $\Omega^2\geq\gamma_0\gamma$ one recognizes electromagnetically induced transparency.

B. Nonadiabatic correlation of phase fluctuations

In this section we investigate the phase fluctuations of two quasimonochromatic fields interacting with the atomic sample. As discussed in the Introduction and shown in the preceding subsection no correlation effects arise in the adiabatic limit. We therefore have to take into account the full dynamics of the atomic evolution. In order to simplify the analysis, however, we eliminate the fast decaying variables, which are the upper level population σ_a, the sum of the population of the lower levels $\sigma_{b_1}+\sigma_{b_2}=1-\sigma_a$, and the two optical polarizations σ_1 and σ_2. Plugging the solutions of the corresponding algebraic equations into the equations of motion for the phases $\phi_{1,2}$ of the fields and the phase Θ_0 of the lower-level coherence σ_0, we find, after some algebra,

$$\left[\frac{\partial}{\partial t}+c\frac{\partial}{\partial z}\right]\phi_{1,2}=g_{1,2}^2 N\,\mathrm{Re}\left[\frac{\sigma_{1,2}}{\Omega_{1,2}}e^{-i\phi_{1,2}}\right]$$
$$=\pm\frac{2g_{1,2}^2 N}{\Gamma}\frac{\Omega_{2,1}}{\Omega_{1,2}}|\sigma_0|\sin(\phi_1-\phi_2-\Theta_0)$$
$$+2g_{1,2}^2 N\,\mathrm{Re}\left[\frac{F\sigma_{1,2}}{\Omega_{1,2}\Gamma}e^{-i\phi_{1,2}}\right] \tag{35}$$

and

$$\frac{d}{dt}\Theta_0=-\frac{2\Omega_1\Omega_2}{|\sigma_0|}\frac{(\gamma_1+\gamma_2)}{D'}\sin(\phi_1-\phi_2-\Theta_0)+\frac{24\Omega_1\Omega_2}{|\sigma_0|}\frac{(\Omega_1^2-\Omega_2^2)\Sigma}{\Gamma D'}\sin(\phi_1-\phi_2-\Theta_0)$$
$$+\frac{48\Omega_1^2\Omega_2^2}{\Gamma D'}\cos(\phi_1-\phi_2-\Theta_0)\sin(\phi_1-\phi_2-\Theta_0)+\frac{6\Omega_1\Omega_2}{|\sigma_0|}\frac{F}{\Gamma D'}\sin(\phi_1-\phi_2-\Theta_0)$$
$$+\mathrm{Im}\left[\frac{F\sigma_0}{\sigma_0}\right]-2\,\mathrm{Re}\left[\frac{\Omega_1 F_{\sigma_2}^*}{\Gamma\sigma_0}e^{i\phi_1}\right]-2\,\mathrm{Re}\left[\frac{\Omega_2^* F_{\sigma_1}}{\Gamma\sigma_0}e^{i\phi_2}\right]. \tag{36}$$

Here $D'=(\gamma_1+\gamma_2)\Gamma+6(\Omega_1^2+\Omega_2^2)$, $F=(2i\Omega_1 e^{i\phi_1}F_{\sigma_1}^* +2i\Omega_2 e^{i\phi_2}F_{\sigma_2}^*+\text{c.c.})+\Gamma F_a$, and we have introduced the difference of the lower-level populations $\Sigma=(\sigma_{b_1}-\sigma_{b_2})/2$. In order to make the above given equations tractable we apply standard linearization techniques and assume small fluctuations of the variables of interest around the semiclassical steady-state values $x(z,t)=x(z)+\delta x(z,t)$. Higher-order terms in the fluctuations are disregarded and all dynamical variables appearing in the diffusion coefficients are replaced by their semiclassical steady-state values. Since we have, in the steady state,

$$\phi_1-\phi_2-\Theta_0=\pi\ , \tag{37}$$

we can disregard the fourth term on the right-hand side of the (linearized) equation (36). A Fourier transformation turns the linearized Eq. (36) into an algebraic equation. Extracting $\delta\Theta_0(z,\omega)$ from this equation and substituting the result into Eq. (35) yields an ordinary linear stochastic differential equation for the phase-difference fluctuation $\delta\psi(z,\omega)\equiv\delta\phi_1(z,\omega)-\delta\phi_2(z,\omega)$:

$$\frac{d}{dz}\delta\psi(z,\omega)=-\tfrac{1}{2}[\kappa_\psi(z,\omega)-i\tilde{\kappa}_\psi(z,\omega)]\delta\psi(z,\omega)$$
$$+F_\psi(z,\omega)\ , \tag{38}$$

with κ_ψ and $\tilde{\kappa}_\psi$ being real and positive. The damping rate of the phase-difference fluctuations reads

$$\kappa_\psi(z,\omega)=\frac{8g_1^2g_2^2N(\Omega_1^2/g_1^2+\Omega_2^2/g_2^2)(\gamma_1\Omega_2^2+\gamma_2\Omega_1^2)}{\Gamma Dc}$$
$$\times\frac{\omega^2}{\Gamma_g^2+\omega^2}$$
$$\approx\frac{8g_1^2g_2^2N(\Omega_1^2/g_1^2+\Omega_2^2/g_2^2)}{\Gamma^2\Gamma_g c}\frac{\omega^2}{\Gamma_g^2+\omega^2}\ , \tag{39}$$

where in the second line we have assumed that γ_0 is small compared to the radiative decay rates. The effective noise term F_ψ has the form

$$F_\psi(z,\omega)=\frac{8g_1^2g_2^2(\Omega_1^2/g_1^2+\Omega_2^2/g_2^2)(\gamma_1\Omega_2^2+\gamma_2\Omega_1^2)}{\Gamma Dc}\frac{\Gamma_g+i\omega}{\Gamma_g^2+\omega^2}\left\{2\,\mathrm{Re}\left[\frac{\Omega_2 F_{\sigma_1}}{\Gamma\sigma_0}e^{-i\phi_2}\right]-2\,\mathrm{Re}\left[\frac{\Omega_1 F_{\sigma_2}^*}{\Gamma\sigma_0}e^{i\phi_1}\right]+\mathrm{Im}\left[\frac{F_{\sigma_0}}{\sigma_0}\right]\right\}$$
$$+2N\left\{g_1^2\,\mathrm{Re}\left[\frac{F_{\sigma_1}}{\Omega_1\Gamma}e^{-i\phi_1}\right]-g_2^2\,\mathrm{Re}\left[\frac{F_{\sigma_2}}{\Omega_2\Gamma}e^{-i\phi_2}\right]\right\}. \tag{40}$$

The damping rate of the phase-difference fluctuations displays a Lorentzian dip at $\omega=0$, which means—in agreement with the adiabatic result—that the damping vanishes as $\omega\to 0$. The width of the Lorentzian dip

$$\Gamma_g=\gamma_0+2(\Omega_1^2+\Omega_2^2)/\Gamma \tag{41}$$

decreases when the Rabi frequencies of the fields become smaller. That is, in order to correlate low-frequency phase fluctuations, one has to either increase the propagation length or reduce the rabi frequencies of the fields. On the other hand, as can be seen from Eq. (34) and Fig. 5, the transmittivity of the vapor cell decreases as well.

The extent to which phase-difference fluctuations can be suppressed by the interaction process therefore depends on the ratio of the phase damping length $\Lambda_\psi \equiv 1/\kappa_\psi$ to the absorption length $\Lambda \equiv 1/\kappa$ (where symmetric conditions are assumed for simplicity, such that $\kappa_1=\kappa_2=\kappa$). This ratio is plotted in Fig. 6 as a function of the Fourier frequency ω for different values of the Rabi frequency $\Omega=\Omega_1=\Omega_2$. An optimum quenching of phase-difference fluctuations is achieved if Ω is of the order of a few $\gamma\gamma_0$.

We now consider the spectrum of phase-difference fluctuations $S_\psi(z,\omega)$, as defined in Eq. (27). From the equation of motion (38) of the phase-difference fluctuations $\delta\psi$ we obtain

$$\frac{d}{dz}S_\psi(z,\omega) = -\kappa_\psi(z,\omega)S_\psi(z,\omega) + \frac{\omega^2}{2\pi c}\int_{-\infty}^{+\infty} d\omega' \langle F_\psi(z,\omega)\delta\psi(z,\omega')\rangle + \frac{\omega^2}{2\pi c}\int_{-\infty}^{\infty} d\omega' \langle \delta\psi(z,\omega)F_\psi(z,\omega')\rangle \ . \quad (42)$$

Formally integrating Eq. (38) and substituting the result into the second line of Eq. (42) yields

$$\frac{d}{dz}S_\psi(z,\omega) = -\kappa_\psi(z,\omega)S_\psi(z,\omega) + \frac{\omega^2}{2\pi c}\int d\omega' \int_0^z dz' \langle F_\psi(z,\omega)F_\psi(z',\omega')\rangle e^{-(1/2)(\kappa_\psi + i\tilde{\kappa}_\psi)(z-z')} + \frac{\omega^2}{2\pi c}\int d\omega' \int_0^z dz' \langle F_\psi(z',\omega)F_\psi(z,\omega')\rangle e^{-(1/2)(\kappa_\psi + i\tilde{\kappa}_\psi)(z-z')} \ , \quad (43)$$

where we made use of the fact that phase fluctuations at the input of the cell $z=0$ are statistically independent from the noise operators corresponding to some location inside the medium. Making use of the correlation property (10) of the noise operators

$$\langle F_\psi(z,\omega)F_\psi(z',\omega')\rangle = 2\pi \frac{L}{N} D_{\psi\psi}(z,\omega)\delta(z-z')\delta(\omega+\omega')$$

we find

$$\frac{d}{dz}S_\psi(z,\omega) = -\kappa_\psi(z,\omega)S_\psi(z,\omega) + \kappa_\psi(z,\omega)N_\psi(z,\omega) \ , \quad (44)$$

with

$$\kappa_\psi(z,\omega)N_\psi(z,\omega) = \frac{\omega^2 L}{c^2 N} D_{\psi\psi}(z,\omega) \ .$$

Equation (44) can be solved analytically if the propagation distance is small compared to the absorption length. In this case the z dependence of κ_ψ and N_ψ can be neglected and we find the solution

$$S_\psi(z,\omega) = S_\psi(0,\omega)e^{-\kappa_\psi(0,\omega)z} + N_\psi(0,\omega)[1 - e^{-\kappa_\psi(0,\omega)z}] \ . \quad (45)$$

Figure 7 shows $e^{-\kappa_\psi(z,\omega)z}$ for $\gamma \equiv \gamma_1 = \gamma_2 \gg \gamma_0$ and $\Omega_1=\Omega_2=5\gamma\gamma_0$. Outside a certain frequency region initial fluctuations above the shot-noise level are damped out very rapidly.

For the symmetric situation of equal oscillator strength, decay rates, and Rabi frequencies, the atomic noise term $N_\psi(0,\omega)$ takes on the simple form

$$N_\psi(z,\omega) = \frac{g^2 L \gamma_0}{\gamma c}\left[1 + \frac{2\omega^2\gamma_0}{\Omega^2\Gamma}\right] = \frac{3}{4\pi}\frac{\lambda^2}{A}\gamma_0\left[1 + \frac{2\omega^2\gamma_0}{\Omega^2\Gamma}\right] \ , \quad (46)$$

where we have substituted in the second equation the coupling strength in terms of the radiative decay rate γ, the wavelength λ, and the beam cross section A. Since λ is small compared to the beam diameter and γ_0 can be of

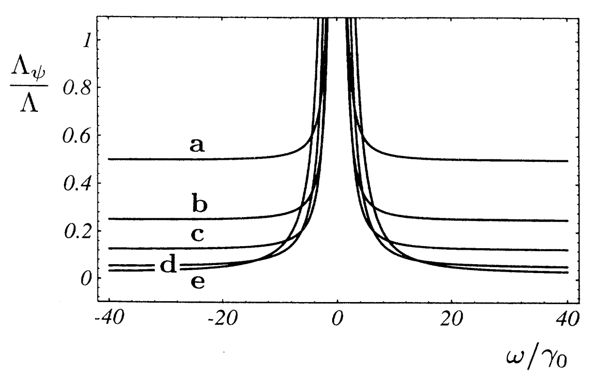

FIG. 6. Ratio of fluctuation damping length Λ_ψ to absorption length Λ for a symmetric situation. The parameters are $\Omega^2/\gamma\gamma_0=0.5$ (curve a), 1 (curve b), 2 (curve c), 5 (curve d), and 10 (curve e).

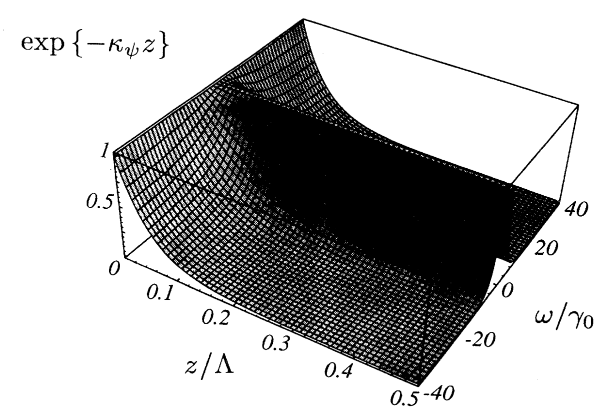

FIG. 7. Damping of initial above-shot-noise fluctuations for symmetric situation and $\Omega^2=5\gamma\gamma_0$ as a function of propagation distance z and Fourier frequency ω.

the order of kilohertz, the atomic noise contribution can be neglected for Fourier frequencies $\omega \ll \gamma$. For example, taking $A \sim 0.1\ \mathrm{cm}^2$, $\gamma_0 \sim 1$ kHz, and $\lambda \sim 0.5\ \mu$m, it is of order 10^{-4} Hz.

In summary, we have seen that in the absence of any externally generated coherence, bichromatic fields interacting with three-level Λ-type atoms can become correlated. At first glance this may be counterintuitive, since there is no phase to which the beat note of the two fields could lock. In fact, we have shown that in the adiabatic regime, where the atom follows any change of the fields instantaneously, no correlation occurs. Only when the finite memory of the atomic response is taken into account may correlation phenomena be observed. Dressed-state optical pumping established a ground-state coherence with a phase equal to that of the beat note of the fields. Due to the finite response time of the atom, the fields always experience an atom being in a trapped state, which involves the difference phase and difference amplitude of the fields at a slightly earlier time. Therefore fast fluctuations of the difference phase and difference amplitude couple to the trapped state and are absorbed.

IV. SUMMARY

In the present paper we have studied the interaction of two fields with three-level Λ-type atoms, in which lower-level coherence is generated either by external means or by coherent population trapping. Under conditions of two-photon resonance with the Λ system, a certain superposition of the fields, called an antisymmetric normal mode, does not couple to the coherent superposition state. Since the orthogonal superposition does couple, any components in this symmetric normal mode will be absorbed by the atoms. The interaction thus generates fields with a certain correlation between the coherent amplitudes.

In the case of externally generated coherence, such as, for example, in a beam where the atoms are injected in a coherent superposition, the antisymmetric normal mode is determined by the phase of the initial coherence. The beat note of the bichromatic field is locked to this phase, giving rise to nearly perfect pulse matching without front-end losses. Associated with the matching of Fourier components is a reduction of noise contributions above the shot-noise level in the beat signal of the two fields. We have shown that, due to this locking mechanism, the diffusion of the phase difference of two independent input lasers can be completely suppressed for frequencies smaller than the natural linewidth of the transition. Atomic noise contributions that result from a residual absorption and reemission of photons are small and do not affect the long-time diffusion coefficient. This scheme represents therefore a powerful tool for strong noise reduction of the difference phase of two input lasers and, as the CEL may have interesting applications in high-precision measurements, of phase differences. As opposed to the CEL, it is passive, i.e., it does not amplify the input fields. It is, however, interesting to note that the locking mechanism persists even if a few percent of the atoms are injected in the upper level leading to noninversion amplification of the fields [14]. The precision of the locking of course depends on the ability to prepare the atoms in a coherent superposition with a well-defined phase, since any fluctutions of the latter will be imposed on the fields.

Also, in the absence of any externally generated coherence, bichromatic fields interacting with three-level Λ-type atoms can become correlated due to the finite response time of the atom. The most interesting consequence of this correlation phenomenon is the reduction of the short-time diffusion of the difference phase between the fields. We have shown that the characteristic time over which diffusion is suppressed is given by the width of $\exp\{-\kappa_\psi(0,\omega)z\}$. Since the medium has a finite transmittivity, the propagation distance z has to be smaller than the damping length Λ. In this case the width of the exponential is of the order of $\Gamma_g = \gamma_0 + 2\Omega^2/\gamma$. If γ_0 is of the order of a few kilohertz, a suppression of phase diffusion is possible for fractions of a millisecond. Noise contributions from the absorption and reemission of photons are small and may be disregarded.

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APPENDIX A: PROPAGATING FIELDS

To describe the noise properties of a field propagating through a sample of atoms we apply a technique described in Ref. [10]. To illustrate the method we consider here the case of a *single*, quasimonochromatic field with mean frequency ν and a corresponding wave number k. Since a description of a propagating field requires a multimode approach, we include a finite number of modes with creation and annihilation operators $c_n^\dagger$ and c_n. The corresponding wave numbers are

$$k_n = k + \frac{2n\pi}{L}, \quad n = -M, \ldots, M \,. \tag{A1}$$

The interaction operator of the probe field with the atoms reads

$$H^{\mathrm{int}} = -\hbar g \sum_{j,l,n} \Theta(t-t_j)(c_n^\dagger e^{-ik_n z_l}\sigma^{jl} e^{ikz_l} + \mathrm{H.c.}) \,, \tag{A2}$$

where we have separated the fast oscillating spatial phase from the atomic polarization. Note that $\sum_i = N/(2M+1)$ is the number of atoms in one cell.

It is convenient to introduce the field variables

$$b_l = \frac{1}{(2M+1)^{1/2}} \sum_{n=-M}^{M} c_n \exp\left\{\frac{2\pi i n l}{2M+1}\right\}, \quad l = -M, \ldots, M \,, \tag{A3}$$

which fulfill Bose-commutation relations. In terms of these field variables the deterministic part of the total Hamiltonian can be written as

$$\begin{aligned} H &= H^0_{\text{atom}} + \sum_n \hbar \nu_n c_n^\dagger c_n + H^{\text{int}} \\ &= H^0_{\text{atom}} + \hbar\nu \sum_l b_l^\dagger b_l + \hbar \sum_{l\neq l'} \nu_{ll'} b_l^\dagger b_{l'} \\ &\quad - \hbar g \sum_{j,k} \Theta(t-t_j)\{(2M+1)^{1/2} b_l^\dagger \sigma^{jl} + \text{H.c.}\}\ , \end{aligned} \tag{A4}$$

where ν is the carrier frequency of the field and

$$\nu_{ll'} = \sum_{n=-M}^{M} \frac{2\pi n c}{(2M+1)L} \exp\left\{\frac{2\pi i n(l-l')}{2M+1}\right\} . \tag{A5}$$

From Eq. (A4) we find the equation of motion

$$\dot{b}_l = -i\sum_{l'} \nu_{ll'} b_{l'} + ig\sum_j (2M+1)^{1/2}\Theta(t-t_j)\sigma^{jl}\ , \tag{A6}$$

where we have introduced slowly varying amplitudes. We proceed by applying a continuous approximation in the limit $M\to\infty$. In this limit we have the following correspondences:

$$\begin{aligned} &\frac{lL}{2m+1} \to z\ , \\ &(2M+1)^{1/2} b_l \to a(z,t)\ , \\ &-i\sum_{l'} \nu_{ll'} b_{l'} (2M+1)^{1/2} \to -c\frac{\partial}{\partial z} a(z,t)\ . \end{aligned} \tag{A7}$$

Thus the equation of motion for the space- and time-dependent dimensionless field amplitude reads

$$\left[\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right] a(z,t) = ig \lim_{M\to\infty} (2M+1)\sum_j \Theta(t-t_j)\sigma^{jl}\Big|_{z_l\to z} . \tag{A8}$$

With the definition for the collective atomic variables given in Eq. (3), this expression goes over into Eqs. (4).

APPENDIX B: DIFFUSION COEFFICIENTS FOR INJECTED COHERENCE

The noise operators for the single-atom variables are δ correlated in time

$$\langle F_x^{jl}(t) F_y^{j'l'}(t')\rangle = D_{xy}\delta_{jj'}\delta_{ll'}\delta(t-t')\ . \tag{B1}$$

Since the atoms are coupled to individual reservoirs, only correlations between noise operators corresponding to the same atom are nonzero. The Heisenberg-Langevin equations (2) have the structure

$$\dot{x}(t) = A_x(t) + F_x(t)\ , \tag{B2}$$

where A_x is the deterministic part of the equation and F_x is the quantum noise operator. The associated diffusion coefficients can be calculated using the generalized dissipation-fluctuation theorem [4,9]

$$D_{xy} = -\langle x A_y\rangle - \langle A_x y\rangle + \frac{d}{dt}\langle xy\rangle\ . \tag{B3}$$

We find the following nonvanishing terms:

$$\begin{aligned} &D_{\sigma_0^\dagger\sigma_0} = \gamma\langle\sigma_{b_2}\rangle + \gamma_2\langle\sigma_a\rangle,\quad D_{\sigma_1^\dagger\sigma_1} = \gamma\langle\sigma_a\rangle\ , \\ &D_{\sigma_2^\dagger\sigma_2} = \gamma\langle\sigma_a\rangle,\quad D_{\sigma_0^\dagger\sigma_1} = \gamma\langle\sigma_2\rangle,\quad D_{\sigma_0\sigma_2} = \gamma\langle\sigma_1\rangle\ . \end{aligned} \tag{B4}$$

We now derive the correlation function of the collective noise operator $F_\mu(z,t)$ defined in Eq. (7). Since the correlation between an atomic noise operator at any time $t>t_j$ and an atomic operator at the injection time t_j vanishes we have

$$\begin{aligned} \langle F_x(z,t)F_y(z',t')\rangle = {} & \frac{1}{N^2}\lim_{M\to\infty}(2M+1)^2\left\langle \sum_{j,j'}\Theta(t-t_j)\Theta(t'-t_{j'})\langle F_x^{jl}(t)F_y^{j'l'}(t')\rangle\right\rangle_s \\ & + \frac{1}{N^2}\lim_{M\to\infty}(2M+1)^2\left\langle\sum_{j,j'}\delta(t-t_j)\delta(t'-t_{j'})\langle\sigma_x^{jl}(t)\sigma_y^{j'l'}(t')\rangle\right\rangle_s - \gamma^2\sigma_x^0\sigma_y^0\ . \end{aligned} \tag{B5}$$

Here $\langle\ \rangle_s$ denotes an average with respect to the injection statistics. Since operators corresponding to different atoms are uncorrelated at the time of injection, we can rewrite Eq. (B5) in the form

$$\begin{aligned} \langle F_x(z,t)F_y(z',t')\rangle = {} & \frac{1}{N^2}\lim_{M\to\infty}(2M+1)^2\delta_{ll'}\delta(t-t')\left\langle\sum_j\Theta(t-t_j)D_{xy}\right\rangle_s \\ & + \left[\frac{1}{N^2}\lim_{M\to\infty}(2M+1)^2\left\langle\sum_{j\neq j'}\delta(t-t_j)\delta(t'-t_{j'})\right\rangle_s - \gamma^2\right]\sigma_x^0\sigma_y^0 \\ & + \frac{1}{N^2}\lim_{M\to\infty}(2M+1)^2\delta_{ll'}\left\langle\sum_j\delta(t-t_j)\delta(t'-t_j)\right\rangle_s\langle\sigma_x(t_j)\sigma_y(t_j)\rangle\ . \end{aligned} \tag{B6}$$

For a Poissonian injection statistics the second term vanishes and we have [15]

$$\left\langle\sum_j\delta(t-t_j)\delta(t'-t_j)\right\rangle_s = \frac{N}{2M+1}\gamma\delta(t-t')\ . \tag{B7}$$

Noting furthermore that $\lim_{M\to\infty}(2M+1)\delta_{ll'} = \delta(z-z')L$, we find

$$\langle F_x(z,t)F_y(z',t')\rangle = \delta(z-z')\delta(t-t')\frac{L}{N} \times \left[\frac{1}{N}\lim_{M\to\infty}(2M+1)\sum_j \Theta(t-t_j)D_{xy}\Big|_{z_l\to z} + \gamma\langle\sigma_x(t_j)\sigma_y(t_j)\rangle \right] . \tag{B8}$$

The c-number diffusion coefficients can be obtained from the quantum diffusion coefficients (B4) by transforming the expressions in the fluctuation-dissipation theorem (B3) into normally ordered operator products. If the operator product $\hat{x}\hat{y}$ is normally ordered, its expectation value is equal to the expectation value of the corresponding c-number product. Hence we have

$$\frac{d}{dt}\langle\hat{x}\hat{y}\rangle = \frac{d}{dt}\langle xy\rangle . \tag{B9}$$

Using again the generalized dissipation-fluctuation theorem, Eq. (B3) and its classical counterpart, we find, from Eq. (B9),

$$D_{xy} + \langle\hat{x}\hat{A}_y\rangle = \langle\hat{A}_x\hat{y}\rangle = \mathcal{D}_{xy} + \langle xA_y\rangle + \langle A_x y\rangle , \tag{B10}$$

where A_x and $\mathcal{D}_{xy}$ are the drift terms and the diffusion coefficients of the c-number Langevin equations. Thus we obtain

$$\mathcal{D}_{xy} = D_{xy} + \langle\hat{x}\hat{A}_y\rangle + \langle\hat{A}_x\hat{y}\rangle - \langle xA_y\rangle - \langle A_x y\rangle . \tag{B11}$$

The right-hand side of Eq. (B11) can be expressed in terms of c-number variables by normal ordering of the operator products. Hence we finally find

$$\begin{aligned}
&\mathcal{D}_{\sigma_0^*\sigma_0} = \gamma\langle\sigma_{b_2}\rangle + \gamma_2\langle\sigma_a\rangle + (ig_1\langle\alpha_1\sigma_1^*\rangle + \text{c.c.}) + \gamma\sigma_{b_2}^0 ,\\
&\mathcal{D}_{\sigma_1\sigma_1} = -2ig_1\langle\alpha_1\sigma_1\rangle ,\\
&\mathcal{D}_{\sigma_1^*\sigma_1} = \gamma\langle\sigma_a\rangle ,\\
&\mathcal{D}_{\sigma_2\sigma_2} = -2ig_2\langle\alpha_2\sigma_2\rangle ,\\
&\mathcal{D}_{\sigma_2^*\sigma_2} = \gamma\langle\sigma_a\rangle ,\\
&\mathcal{D}_{\sigma_0\sigma_1} = -ig_1\langle\alpha_1\sigma_0\rangle ,\\
&\mathcal{D}_{\sigma_0^*\sigma_1} = \gamma\langle\sigma_2\rangle ,\\
&\mathcal{D}_{\sigma_0\sigma_2} = \gamma\langle\sigma_1\rangle + ig_2\langle\alpha_2\sigma_0\rangle + ig_1(\langle\alpha_1\sigma_{b_1}\rangle - \langle\alpha_1\sigma_{b_2}\rangle) ,\\
&\mathcal{D}_{\sigma_1\sigma_2} = -ig_2\langle\alpha_2\sigma_1\rangle - ig_1\langle\alpha_1\sigma_2\rangle .
\end{aligned} \tag{B12}$$

APPENDIX C: DIFFUSION COEFFICIENTS FOR EIT

The c-number diffusion coefficients of the closed system including the phase decay of the lower-level coherence can be obtained in a similar way as outlined in Appendix B. We find

$$\begin{aligned}
&\mathcal{D}_{\sigma_0^*\sigma_0} = 2\gamma_0\langle\sigma_{b_2}\rangle + \gamma_2\langle\sigma_a\rangle + (ig_1\langle\alpha_1\sigma_1^*\rangle + \text{c.c.}) ,\\
&\mathcal{D}_{\sigma_1\sigma_1} = -2ig_1\langle\alpha_1\sigma_1\rangle ,\\
&\mathcal{D}_{\sigma_1^*\sigma_1} = \gamma_0\langle\sigma_a\rangle ,\\
&\mathcal{D}_{\sigma_2\sigma_2} = -2ig_2\langle\alpha_2\sigma_2\rangle ,\\
&\mathcal{D}_{\sigma_2^*\sigma_2} = \gamma_0\langle\sigma_a\rangle ,\\
&\mathcal{D}_{\sigma_0\sigma_1} = -ig_1\langle\alpha_1\sigma_0\rangle ,\\
&\mathcal{D}_{\sigma_0^*\sigma_1} = \gamma_0\langle\sigma_2\rangle ,\\
&\mathcal{D}_{\sigma_0\sigma_2} = \gamma_0\langle\sigma_1\rangle + ig_2\langle\alpha_2\sigma_0\rangle + ig_1(\langle\alpha_1\sigma_{b_1}\rangle - \langle\alpha_1\sigma_{b_2}\rangle) ,\\
&\mathcal{D}_{\sigma_1\sigma_2} = -ig_2\langle\alpha_2\sigma_1\rangle - ig_1\langle\alpha_1\sigma_2\rangle .
\end{aligned} \tag{C1}$$

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