Impact-parameter dependence of multiple lepton-pair production from electromagnetic fields

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In relativistic heavy-ion collisions, the strong Lorentz-contracted electromagnetic fields are capable of producing copious numbers of lepton pairs through the two-photon mechanism. Monte Carlo techniques have been developed that allow the exact calculation of production by this mechanism when a semiclassical approximation is made for the motion of the two ions. Here we develop a hybrid Monte Carlo technique that enables us to calculate the impact-parameter dependence of the two-photon mechanism for lepton-pair production, and by using this result we obtain the probability distribution for multiple-pair production as a function of impact parameter. Computations are performed for S+Au and Pb+Pb systems at 200 and 160 A GeV, respectively. We also compare our results with the equivalent-photon approximation and elucidate the differences.

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I. INTRODUCTION

The proposed new colliding-beam accelerators — designed to investigate nuclear matter at high temperatures and densities by accelerating highly charged heavy ions at fixed-target energies per nucleon up to 20 TeV— have motivated great interest during the past decade concerning possible new electromagnetic phenomena. When heavy ions collide at relativistic velocities, the Lorentz-contracted electromagnetic fields in the space-time region near the collision are sufficiently intense to produce large numbers of electron-positron pairs, muon pairs, vector bosons, and possibly the yet-unconfirmed Higgs boson. All these processes occur at nearly atomic distance scales [1–3]. The phenomena involved are pervasive, impinging upon atomic, nuclear, and particle physics. The electromagnetic fields associated with these collisions are intense as they are proportional to the Lorentz factor γ, which is approximately the beam kinetic energy per nucleon, and the charge of the ion Z. These parameters, together with the impact parameter b, determine the fields available in a collision. The feature of electromagnetic particle production by heavy ions, which piqued early studies, is the Z^4 enhancement through the coherent action of all the constituent charges of the colliding partners. The pulsed fields are strongly time dependent with a width of b/γ and thus contain large Fourier components which give rise to relatively large particle production probabilities. Coherent particle production is most clearly distinguished in peripheral collisions.

One of the most interesting of these processes, from the perspective of both fundamental and practical importance, is the sparking of the vacuum to produce electron-positron pairs. Much study has been devoted to the problem of electron-positron pair production during recent years in anticipation of new experimental opportunities at the Relativistic Heavy-Ion Collider (RHIC), currently under construction at Brookhaven National Laboratory, as the immense predicted fluxes of electrons cannot be ignored in detector or accelerator design. This facility will provide extremely relativistic colliding beams of fully stripped ions as heavy as gold, fully exposing the large charge of the atomic nucleus.

The primary physics goal of the RHIC project is the creation and study of a so-called quark-gluon plasma. This unique form of matter is expected to be formed in the central, or near-central, collisions of heavy ions at extreme relativistic energies [4]. The thermodynamic conditions attained in these central collisions are expected to be such that the constituent quarks and gluons of baryons and mesons become deconfined in a new and short-lived plasma state. Electron- and muon-pair production from hadronic interactions have been widely discussed as a possible tool to help probe the formation and the decay of the quark-gluon plasma phase of matter [5,6]. In the conditions of such central collisions, lepton-hadron final-state interactions are usually small, and hence the leptons carry direct information on the space-time region of creation. However, suggestions by several authors indicate that other sources of lepton pairs might possibly mask the leptonic signals originating from the plasma phase [7–9]. Electromagnetic production from the vacuum of single- and multiple-lepton pairs is a major contribution to this physical background [10,1] and therefore must be understood in detail. In addition, two very abundant electromagnetic processes constitute the primary limitation to the lifetime of stored beams at the RHIC. One is a nuclear decay following the electromagnetic excitation of the giant dipole resonance and the second is the creation of an electron-positron pair accompanied by the

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capture of the electron in an atomic bound state of a participant heavy ion. Both processes result in a change in the charge-to-mass ratio of the ion in the storage ring causing it to be deflected out of the beam.

From a fundamental perspective, relativistic heavy-ion collisions provide an opportunity to study nonperturbative quantum electrodynamics (QED) in an entirely new and continuously varying energy regime, using an interaction which is completely known due to the combination of very high collision energies and electric charges. The coherent, electromagnetic production of electron-positron pairs using heavy ions is fundamentally different from other production mechanisms using light particles at high energies, since in the former, the coupling constant is strongly enhanced due to the large charge.

Nonetheless, perturbative methods are useful in studying the problem of lepton-pair production and reliable lowest-order perturbative calculations have been used as input into design models for the RHIC [2,11]. Beginning with quantum-field theory, we derive a classical-field method based on Feynman perturbation theory in the limit that the momentum transfer of the photons is much smaller than the momentum carried by the nuclei [2,12]. This is manifestly true for the coherent production of particles through two-photon processes in relativistic heavy-ion collisions. These lowest-order diagrams (see Fig. 1), coupling lepton fields to classical electromagnetic fields, have been evaluated exactly using Monte Carlo [2,13] and analytical techniques [14,15] and predict cross sections consistent with experiments for free-electron–positron production performed by two independent groups using collisions of S + Au [16] and S + Pt [17], respectively, at fixed-target energies of 200 A GeV. Furthermore, no indication of nonperturbative effects is observed in these experiments using the relatively light sulfur projectile. However, applying lowest-order perturbation theory to the production of electron-positron pairs with heavy ions at high energies and small impact parameters results in probabilities and cross sections which violate various theoretical bounds such as unitarity [18,2]. It is therefore clear that low-order perturbative calculations alone are not adequate for smaller impact parameters at the RHIC energies, as higher-order damping effects must be included (e.g., the creation of multiple pairs [19]). Heavy ions have been used to produce electron-positron pairs in collisions of U + Au at fixed-target energies of 0.96 A GeV and the observed cross sections are not in agreement with the low-order calculations performed at these energies [20]. New experiments using a variety of targets are scheduled using [21] lead beams at fixed-target energies of 160 A GeV and using [22] gold beams at fixed-target energies of 12 A GeV during which electron-pair multiplicities and other nonperturbative features of these collisions will be observed.

The Monte Carlo method described in Ref. [2] for evaluating the two-photon Feynman diagrams employs an analytic integration over the impact parameter and thus does not provide information on the impact-parameter dependence of the cross section. In this present work, we generalize these techniques in order to calculate the impact-parameter dependence of the two-photon diagrams which has implications for the description of the electromagnetic background for hadronic interactions, the study of strong-field effects (e.g., pair multiplicities), and the study of the validity of the equivalent-photon approximation, as discussed below. Various possibilities for experimental study of the impact-parameter dependence for free-electron- and/or muon-pair production are discussed in Ref. [23].

Concerning certain classes of detectors designed for nuclear physics experiments at the RHIC, the total integrated cross section may not be the most relevant quantity for studies of the electromagnetic background. If the detector triggers on a limited range of small impact parameters, the pair production probability for these impact parameters would be more relevant [18]. Due to the nature of the electromagnetic interaction, the particle production process occurs over a large range of impact parameters. The larger impact parameters produce weaker, more perturbative, fields, but these impact parameters are favored by the geometrical weight factor in the cross-section integration. An integration over all impact parameters also includes contributions from impact parameters where the two nuclei would collide. The assumption that the nuclei continue on straight-line paths for these nonperipheral impact parameters is not accurate. Thus one would like to isolate the relatively small impact parameters which are still larger than the grazing impact parameter for the study of strong-field effects.

Recent progress has been made in understanding the nonperturbative nature of the production of multiple free-electron-positron pairs [24–27]. Using the usual assumptions of the classical, strong nature of the electromagnetic field generated by the heavy ions and omitting the final-state interactions among the produced leptons, one can express the probability for producing N pairs as a Poisson distribution whose mean value is the probability for producing a single pair in lowest-order perturbation theory. Therefore, the central ingredient needed for describing the multiplicity distribution of electron-positron pairs within this approximation is the impact-parameter dependence of the two-photon mechanism.

Historically, the two-photon process has been modeled through the equivalent-photon approximation, which takes advantage of the close relation between the interaction produced by a relativistic charged particle and those due to incident electromagnetic waves [28–36]. In the equivalent-photon approximation, the equivalent-photon flux associated with a relativistic charged particle is obtained via a Fourier decomposition of the electromagnetic interaction [37,38]. Cross sections are obtained by folding the elementary, real two-photon cross section for the pair-production process with the equivalent-photon flux produced by each ion. The pair-production cross section is easier to compute using the equivalent-photon approximation than by calculating the two-photon diagram, and the results for the total cross section are reasonably accurate provided the incident-particle Lorentz factor γ is much greater than one and the energy transferred via the photon is much less than γ. However, details of the differential cross sections, spectra, and impact-parameter dependence differ, especially when complicated numerical cuts in the coordinates are applied in order to compare with experiments. Among the shortcomings of this approximation is an undetermined parameter which corresponds to the minimum impact parameter or the maximum momentum transfer, which makes it difficult to get specific results. As such, the method loses applicability at impact parameters less than the Compton wavelength of the lepton [15], which is the region of greatest interest for the study of nonperturbative effects. Reference [39] reviews the application of the equivalent-photon method in relativistic heavy-ion collisions.

Section II introduces the basic two-photon approach for computing the single-pair production and the generalization of these results for calculating multipair production cross sections. In this section we also discuss the impact-parameter dependence of these cross sections. In Sec. III we discuss the details of the numerical techniques used for obtaining impact-parameter-dependent cross sections. Section IV outlines the results for S+Au and Pb+Pb heavy-ion reactions. The paper concludes with the discussion of the results in Sec. V.

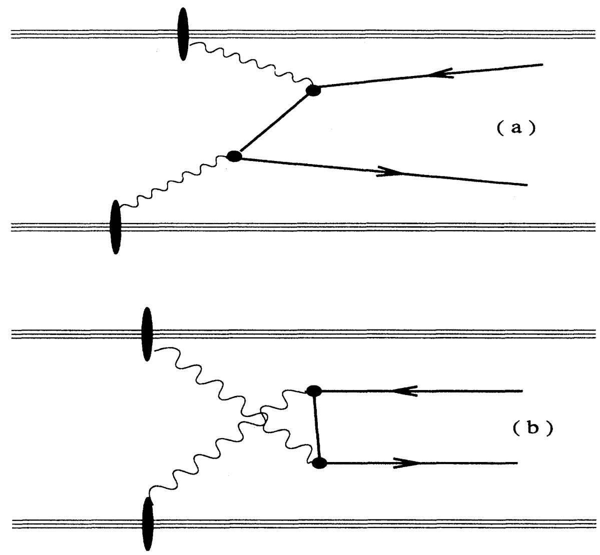

FIG. 1. (a) Direct and (b) crossed Feynman diagrams for pair production in a heavy-ion collision.

II. FORMALISM

A. Single-pair production

The cross section for lepton-pair production in relativistic heavy-ion collisions via the two-photon process has been derived in [2] using a semiclassical least-action principle. The result is equivalent to utilizing the leading-order Feynman diagram and taking the classical limit [12,40,41] of the relative motion of the two ions. In this formalism, the source currents appear as arising from Lorentz-boosted charge distributions. We follow the formalism of [2], only here we develop a technique for calculating cross sections as a function of the impact parameter rather than the integral over all impact parameters, as was done previously. The lowest-order two-photon process is pictured in Fig. 1.

Nucleus 1 is the nucleus that (in the center of velocity frame) moves with a velocity $-\beta$ and nucleus 2 (in Fig. 2) moves with velocity $+\beta$, both parallel to the z axis. Their trajectories are taken to be straight lines separated by an impact parameter b. Throughout this paper, we use a system of units with $\hbar = c = m = 1$. The semiclassical coupling of electrons to the electromagnetic field is given by the Lagrangian density

$$\mathcal{L}_{\rm int}(x) = -\overline{\Psi}(x)\gamma_\mu \Psi(x) A^\mu(x) \ , \tag{2.1}$$

which only depends on the field variables via the classical four-potential A^μ, where

$$A^\mu = A^\mu(1) + A^\mu(2) \ , \tag{2.2}$$

and in the momentum space, the nonzero components of the potential from nucleus 1 are

$$\begin{aligned} A^0(1) &= -8\pi^2 Z\gamma^2 \frac{\delta(q_0 - \beta q_z)}{q_z^2 + \gamma^2(q_x^2 + q_y^2)} \exp\left[i\mathbf{q}_\perp \cdot \frac{\mathbf{b}}{\mathbf{2}}\right] \ , \\ A^z(1) &= \beta A^0(1). \end{aligned} \tag{2.3}$$

The potentials from nucleus 2 can be obtained from (3) by the substitutions $\mathbf{b} \to -\mathbf{b}, \beta \to -\beta$. If we assume that the heavy-ion motion can be localized along definite impact parameters, we can write the total inclusive singles σ_s and pair σ_p cross sections

$$\begin{aligned} \sigma_s &= \int d^2b \mathcal{N}_s(\mathbf{b}) \ , \\ \sigma_p &= \int d^2b \mathcal{N}_p(\mathbf{b}) \ , \end{aligned} \tag{2.4}$$

where $\mathcal{N}_s(\mathbf{b})$ is the singles multiplicity and $\mathcal{N}_p(\mathbf{b})$ is the pair multiplicity of produced electrons. In the strong field limit the multiplicity of electrons represents the mean number of electrons produced out of the vacuum. It has been shown that, in the perturbative limit, the pair multiplicity is the same as the singles multiplicity and the inclusive pair cross section is $\sigma_p \simeq \sigma$,

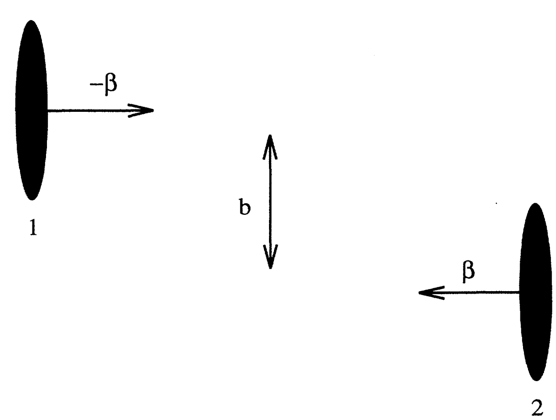

FIG. 2. Schematic diagram depicting a relativistic heavy-ion collision.

$$\sigma = \int d^2b \sum_{k>0} \sum_{q<0} |\langle \chi_k^{(+)} | S | \chi_q^{(-)} \rangle|^2 , \tag{2.5}$$

where the summation over the states k is restricted to those above the Dirac sea and the summation over the states q is restricted to those occupied in the Dirac sea. The summation of these states is over spin and momentum,

$$\sum_p = \sum_{\sigma_p} \sum_{\mathbf{p}} \to \sum_{\sigma_p} \int \frac{d^3p}{(2\pi)^3} .$$

The momentum in intermediate states is composed of parts transverse and parallel to the motion of the heavy ions, $\mathbf{p} = \mathbf{p}_\perp + \mathbf{p}_z$. The transition matrix element for direct Feynman diagrams in Eq. (2.5) has been derived in [2] as

$$\begin{aligned}\langle \chi_k^{(+)} | S | \chi_q^{(-)} \rangle &= \frac{i}{2\beta} \int \frac{d^2p_\perp}{(2\pi)^2} \exp\left\{ i \left[\mathbf{p}_\perp - \left(\frac{\mathbf{k}_\perp + \mathbf{q}_\perp}{2} \right) \right] \cdot \mathbf{b} \right\} \\ &\quad \times \mathcal{A}^{(+)}(k, q : \mathbf{p}_\perp) ,\end{aligned} \tag{2.6}$$

where $\mathbf{p}_\perp$ is the transverse momentum.

Including both the direct and crossed Feynman diagrams, the results for the cross section, as a function of impact parameter, can be obtained as

$$\begin{aligned}\frac{d\sigma}{db} = \frac{1}{4\beta^2} \sum_{\sigma_k, \sigma_q} \int &\frac{d^3kd^3qd^2p_\perp d^2p'_\perp}{(2\pi)^9} bJ_0(b|\mathbf{p}_\perp - \mathbf{p}'_\perp|) \\ &\times [\mathcal{A}^{(+)}(k, q : \mathbf{p}_\perp) + \mathcal{A}^{(-)}(k, q : \mathbf{k}_\perp + \mathbf{q}_\perp - \mathbf{p}_\perp)] \\ &\times l[\mathcal{A}^{(+)}(k, q : \mathbf{p}'_\perp) + \mathcal{A}^{(-)}(k, q : \mathbf{k}_\perp + \mathbf{q}_\perp - \mathbf{p}'_\perp)]^* ,\end{aligned} \tag{2.7}$$

where $\mathbf{k}$ ($\mathbf{q}$) is the momentum of the produced lepton (antilepton) and $\mathcal{A}^{(\pm)}(k, q; \mathbf{p}_\perp)$ are given by

$$\begin{aligned}\mathcal{A}^{(+)}(k, q : \mathbf{p}_\perp) &= F(\mathbf{k}_\perp - \mathbf{p}_\perp : \omega_1) F(\mathbf{p}_\perp - \mathbf{q}_\perp : \omega_2) \\ &\quad \times \mathcal{T}_{kq}(\mathbf{p}_\perp : +\beta) , \\ \mathcal{A}^{(-)}(k, q : \mathbf{p}_\perp) &= F(\mathbf{k}_\perp - \mathbf{p}_\perp : \omega_2) F(\mathbf{p}_\perp - \mathbf{q}_\perp : \omega_1) \\ &\quad \times \mathcal{T}_{kq}(\mathbf{p}_\perp : -\beta) .\end{aligned} \tag{2.8}$$

The quantity $F(\mathbf{q}, \omega)$ is the scalar part of the electromagnetic field of the moving heavy ions in momentum space

$$F(\mathbf{q} : \omega) = \frac{4\pi Z \gamma^2 \beta^2}{\omega^2 + \beta^2 \gamma^2 |\mathbf{q}|^2} G_E(q^2) f_Z(q^2) , \tag{2.9}$$

where $G_E(q^2)$ is the form factor of the nucleon and $f_Z(q^2)$ is the form factor of the nucleus. The frequencies ω_1 and ω_2 of the virtual photons are fixed by energy conservation at the vertex where the photon is absorbed

$$\begin{aligned}\omega_1 &= \frac{E_q^{(-)} - E_k^{(+)} + \beta(q_z - k_z)}{2} , \\ \omega_2 &= \frac{E_q^{(-)} - E_k^{(+)} - \beta(q_z - k_z)}{2} .\end{aligned} \tag{2.10}$$

The quantity $\mathcal{T}$ contains the propagator of the intermediate lepton and the matrix elements for the coupling of the photon to the leptons

$$\begin{aligned}\mathcal{T}_{kq}(\mathbf{p}_\perp : \beta) = \sum_s \sum_{\sigma_p} &\left[E_p^{(s)} - \left(\frac{E_k^{(+)} + E_q^{(-)}}{2} \right) \right. \\ &\left. + \beta \left(\frac{k_z - q_z}{2} \right) \right]^{-1} \\ &\times \langle u_{\sigma_k}^{(+)} | (1 - \beta\alpha_z) | u_{\sigma_p}^{(s)} \rangle \\ &\times \langle u_{\sigma_p}^{(s)} | (1 + \beta\alpha_z) | u_{\sigma_q}^{(-)} \rangle .\end{aligned} \tag{2.11}$$

Here $|u_{\sigma_q}^{(\pm)}\rangle$ is the usual Dirac spinor and α_z is the z component of the Dirac matrices.

B. Multiple-pair production

In this section we will provide the basic formulas used in our calculation of multipair cross sections. Recently, a number of differing approaches have been developed to compute the multipair cross sections [24–27], all resulting in a Poisson distribution for the multipair impact-parameter-dependent probability function. The prevailing assumption is that the heavy ions do not suffer any recoil or energy loss while electron-positron pairs are produced from the vacuum, which seems to be a reasonable assumption when the pair energy is compared with that of the heavy ions. One implication of the above assumption is the approximation of straight-line, heavy-ion trajectories for all impact parameters. Although this may not be correct for central impact parameters, its reliability for cross-section predictions depends on the physical region for producing pairs. Provided that most of the pair-production cross section stems from peripheral heavy-ion trajectories, the use of this assumption seems to be safe.

It has also been recognized that in most lowest-order perturbative calculations, pair production probability violates unitarity for small impact parameters and extreme energies. It has been suggested that the summation of the classes of diagrams resulting from the independent pair approximation can be used to restore unitarity to the lowest-order perturbation theory. These have been discussed in Ref. [24] using the sudden and quasiboson approximations, in Ref. [25] where a similar result is obtained by a straightforward summation of the diagrammatically defined series in the perturbational expansion, and in Ref. [26] using a nonperturbative treatment and neglecting the interference terms. The resulting multipair production probability is described by a Poisson form

$$P_N(b) = \frac{\mathcal{P}(b)^N \exp[-\mathcal{P}(b)]}{N!} , \tag{2.12}$$

where N denotes N-pair production and

$$\mathcal{P}(b) = \sum_{k>0} \sum_{q<0} |\langle \chi_k^{(+)} | S | \chi_q^{(-)} \rangle|^2 \tag{2.13}$$

is the lowest-order perturbation result for the pair-production probability. $\mathcal{P}(b)$ can also be interpreted as the average number of electrons produced out of the vacuum. The N-pair cross section $\sigma_{N\text{pair}}$ is obtained by integrating the N-pair probability over the impact parameter b

$$\sigma_{N\text{pair}} = \int d^2 b P_N(b), \quad N = 1, 2, \ldots \ . \tag{2.14}$$

In the two-photon formalism, as discussed above, the total one-pair cross section is given by

$$\sigma = \int d^2 b \mathcal{P}(b) \ . \tag{2.15}$$

Therefore, from the above equation, we can write $\mathcal{P}(b)$

$$\mathcal{P}(b) = \frac{1}{2\pi b}\frac{d\sigma}{db}, \tag{2.16}$$

which can be used in Eq. (2.12) to calculate N-pair cross sections. Consequently, our task in this paper is to determine a well-behaved impact-parameter-dependent cross section. In terms of the multipair cross section, the total cross section for producing any number of pairs is given by

$$\sigma_{\text{pair}} = \sum_{N=1} \sigma_{N\,\text{pair}} \ . \tag{2.17}$$

III. NUMERICAL TECHNIQUES

For the calculation of the total cross section, the differential cross section given by Eq. (2.7) can be integrated over all impact parameters analytically (it produces a simple δ function) and yields the result for the total cross section as given in Ref. [2]. However, the impact-parameter-dependent cross section is numerically much more difficult and evaded any computations until the present. This is due to the fact that the function $J_0(b\,|\mathbf{p}_\perp - \mathbf{p}'_\perp|)$ is a rapidly oscillating function, especially for large b. One can add an additional $d^2p'_\perp$ integration to the Monte Carlo technique given in Ref. [2] and attempt to do the integration in Eq. (2.7) directly. We tried up to 100 million Monte Carlo points and, although accuracy for small impact parameters was acceptable, we have not been able to obtain convergence for large impact parameters.

We propose the following technique for circumventing this difficulty. We divide the integration according to

$$\frac{d\sigma}{db} = \int_0^\infty dq\, q\, b\, J_0(qb)\mathcal{F}(q) \ , \tag{3.1}$$

where $\mathcal{F}(q)$ is given by a nine-dimensional integral

$$\begin{aligned}\mathcal{F}(q) = \frac{\pi}{8\beta^2}\sum_{\sigma_k}\sum_{\sigma_q}\int_0^{2\pi} d\phi_q \int \frac{dk_z dq_z d^2k_\perp d^2K d^2Q}{(2\pi)^{10}} \Bigg\{ & F\left[\frac{1}{2}(\mathbf{Q}-\mathbf{q});\omega_1\right] F[-\mathbf{K};\omega_2]\,\mathcal{T}_{kq}\left[\mathbf{k}_\perp - \frac{1}{2}(\mathbf{Q}-\mathbf{q});+\beta\right] \\ & + F\left[\frac{1}{2}(\mathbf{Q}-\mathbf{q});\omega_1\right] F[-\mathbf{K};\omega_2]\,\mathcal{T}_{kq}[\mathbf{k}_\perp - \mathbf{K};-\beta]\Bigg\} \\ \times \Bigg\{ & F\left[\frac{1}{2}(\mathbf{Q}+\mathbf{q});\omega_1\right] F[-\mathbf{q}-\mathbf{K};\omega_2]\,\mathcal{T}_{kq}\left[\mathbf{k}_\perp - \frac{1}{2}(\mathbf{Q}+\mathbf{q});+\beta\right] \\ & + F\left[\frac{1}{2}(\mathbf{Q}+\mathbf{q});\omega_1\right] F[-\mathbf{q}-\mathbf{K};\omega_2]\,\mathcal{T}_{kq}[\mathbf{k}_\perp + \mathbf{q} - \mathbf{K};-\beta]\Bigg\} \ ,\end{aligned} \tag{3.2}$$

where we have changed variables to

$$\begin{aligned}\mathbf{p}_\perp &= \mathbf{k}_\perp - \frac{1}{2}(\mathbf{Q}-\mathbf{q}) \ , \\ \mathbf{q}_\perp &= \mathbf{k}_\perp - \frac{1}{2}(\mathbf{Q}-\mathbf{q}) + \mathbf{K} \ , \\ \mathbf{p}'_\perp &= \mathbf{k}_\perp - \frac{1}{2}(\mathbf{Q}+\mathbf{q}) \ .\end{aligned} \tag{3.3}$$

For a fixed value of q, the Monte Carlo technique of Ref. [2] can be generalized to calculate the nine-dimensional integral of Eq. (3.2). The details are provided in the Appendix.

This procedure results in a function $\mathcal{F}(q)$, which is a relatively smooth function of q. The number of Monte Carlo points used is chosen such that there is approximately a 5% error in each point we generate for $\mathcal{F}(q)$. We fit a smooth function to the $\mathcal{F}(q)$ generated by the Monte Carlo integration and then perform the q integral analytically. We check the procedure by integrating over the impact parameter and find that we are always within the 5% error we set for ourselves in comparison with the total cross section calculated using the methods of Ref. [2].

IV. RESULTS

To demonstrate the ability of the technique presented here to produce the impact parameter dependence of the two-photon electron-positron pair-production cross sections, we present results for two typical cases. These are for electron-positron pairs produced in the S + Au collision at 200 A GeV per nucleon and in the Pb + Pb collision at 160 A GeV per nucleon in the fixed-target frame. These two systems are studied because they correspond to experiments either completed or planned for the immediate future at CERN [16,21]. In all cases we set the form factor of the nucleon equal to 1 and use a Woods-Saxon form factor for the nucleus. The use of regular form factors for the nucleon or uniform or Gaussian form factors for the nucleus does not alter the results at these energies.

In Fig. 3 we present the function $\mathcal{F}(q)$ both from the Monte Carlo calculation and from our smooth fit. The smooth functions fit to $\mathcal{F}(q)$ are given by (in barns), for the Pb + Pb and S + Au cases, respectively,

$$\mathcal{F}(q) = 3835.7\, e^{-1.340\,29q},$$
$$\mathcal{F}(q) = 163.5\, e^{-1.3519q}\,. \tag{4.1}$$

The differential pair production $d\sigma/db$ is then calculated by integrating the smooth fit to $\mathcal{F}(q)$ according to Eq. (3.1). The results are presented in Fig. 4 for the S + Au and Pb + Pb cases, respectively. We see a smooth exponential fall for large b, as expected. We may further integrate these results over impact parameter. In Fig. 5 we show the integrated cross section when integrated to a maximum impact parameter $b_{\max}$, given by

$$\begin{aligned}\sigma &= \int_0^\infty dq\, q \int_0^{b_{\max}} db\, b\, J_0(qb)\mathcal{F}(q) \\ &= \int_0^\infty dq\, b_{\max}\, J_1(qb_{\max})\mathcal{F}(q)\ ,\end{aligned} \tag{4.2}$$

which, as $b \longrightarrow \infty$, reproduces the total cross section obtained in Ref. [2] using the exact integration over b. Analytically one could also show that $\mathcal{F}(q = 0)$ converges for large $b_{\max}$ to the total cross section of [2]. This is done by taking the $b_{\max} \longrightarrow \infty$ limit of Eq. (4.2) as

$$\begin{aligned}\sigma &= \int_0^\infty dx\, J_1(x)\, \mathcal{F}\left(\frac{x}{b_{\max}}\right),\ x = q\, b_{\max} \\ &\approx \mathcal{F}(0) \int_0^\infty dx\, J_1(x) \\ &= \mathcal{F}(0)\ ,\end{aligned} \tag{4.3}$$

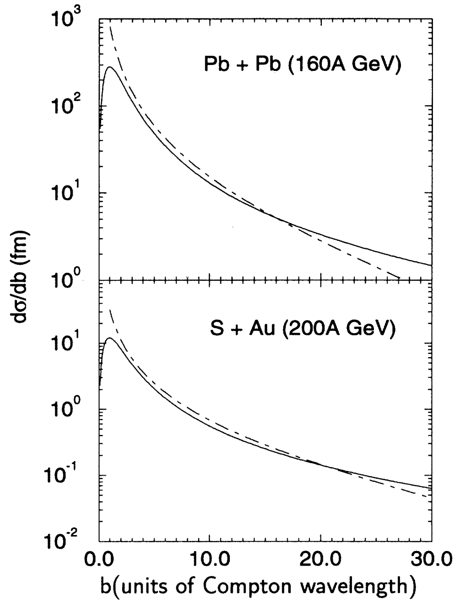

FIG. 4. Differential cross section $d\sigma/db$ versus impact parameter b for electron-positron pair production. Solid lines, exact numerical result; dot-dashed lines, equivalent-photon approximation (Ref. [25]).

where we have taken $\mathcal{F}$ out of the integral since, in the range of x for which the integral converges, $\mathcal{F}$ is essentially a constant with argument very close to zero. The same result can also be obtained by actually setting $q = 0$ in the definition of $\mathcal{F}(q)$ given by Eq. (3.2) and realizing that the resulting expression is the same as the total

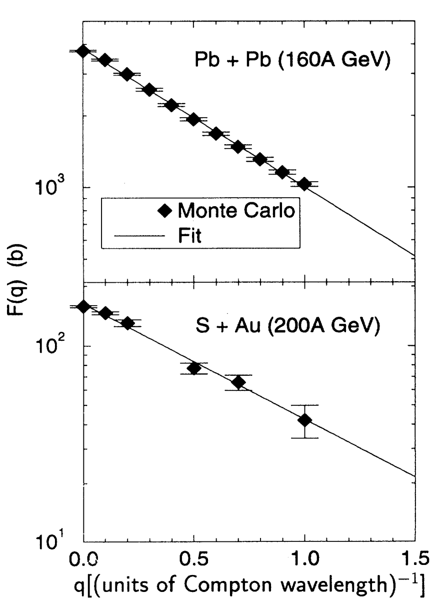

FIG. 3. Function $\mathcal{F}(q)$ versus q for the production of electron-positron pairs. The points are the results of the Monte Carlo calculation and the smooth curve is our fit to these points.

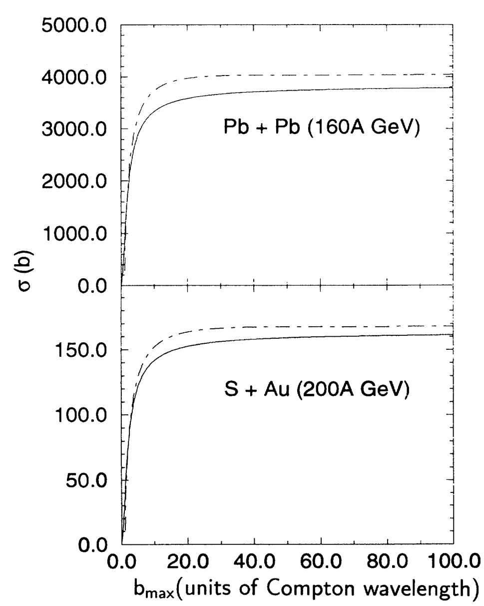

FIG. 5. Integrated cross section as a function of $b_{\max}$ for electron-positron pair production. Solid lines, exact numerical result; dot-dashed lines, equivalent-photon approximation (Ref. [25]).

cross section expression given in Ref. [2]. Numerically, we have obtained this result in Eq. (4.1) with a less than 2% error.

We compare our two-photon results in Figs. 4 and 5 with results obtained using the equivalent-photon approximation as expressed in [39,25]. Using this equivalent photon method, a simple expression for $\mathcal{P}(b)$ is given by the lowest-order perturbative result as

$$\mathcal{P}(b) \simeq \frac{14}{9\pi^2}(Z^2\alpha^2)^2 \left[\frac{\lambda_C}{b}\right]^2 \ln^2\left[\frac{\gamma\delta\lambda_C}{2b}\right] + \Delta(Z), \quad (4.4)$$

where γ is approximately the energy of one ion measured in the frame of the other ion ($\gamma = 2\gamma_{\rm col}^2 - 1$) and δ is a constant that has a value of 0.681. $\Delta(Z)$ is a Coulomb modification which is ignored here. In this equation, for impact parameters smaller than λ_C, $\mathcal{P}(b)$ goes to infinity very rapidly; i.e., the equivalent-photon formula diverges for very small impact parameters [15]. In addition, impact parameters larger than $\gamma\delta\lambda_C/2$ exceed the region of validity for this formula. However, for valid impact parameters, i.e., $\gamma\delta\lambda_C/2 \geq b \geq \lambda_C$, the simple equivalent-photon method gives quite reasonable results, although it overestimates the contribution to pair production from the two-photon process by approximately 5% in both cases considered, with the largest disagreement coming at small impact parameters. In Fig. 5, the equivalent-photon probabilities have been integrated in their region of validity, whereas the integration limits for the Monte Carlo evaluation of the two-photon diagrams are from zero to infinity. We note that other formulations of the equivalent-photon approximation may implement slightly different approximations from the ones used to obtain Eq. (4.4) [42] and, as a result, may produce results somewhat different from the simple formula used here.

Taking advantage of our demonstrated ability to compute the impact-parameter dependence of the two-photon mechanism and the simple Poissonian form for the impact-parameter dependence of the multiple-pair cross section [Eq. (2.12)], we compute the probability distributions for N-pair production as a function of impact parameter b for S + Au (200 A GeV) and Pb + Pb (160 A GeV) collisions. Our results show that the N-pair production probability has a finite value at impact parameter $b = 0$ and is continuous everywhere. In Figs. 6 and 7 we see that one-pair production completely dominates both the S + Au and Pb + Pb collisions. The two- and three-pair probability distributions are small and, as expected, decrease rapidly for impact-parameter values larger than the Compton wavelength, although for the Pb + Pb case the two-pair cross section has an appreciable 95-b value. The other values for two- and three-pair cross sections are shown in Figs. 6 and 7.

For comparison, we also plot in Figs. 6 and 7 the multiple-pair, impact-parameter-dependent probability distributions calculated using the equivalent-photon approximation [Eq. (4.4)]. However, the inability of the equivalent-photon approximation to accurately approximate the two-photon mechanism at impact parameters below one Compton wavelength, along with the general

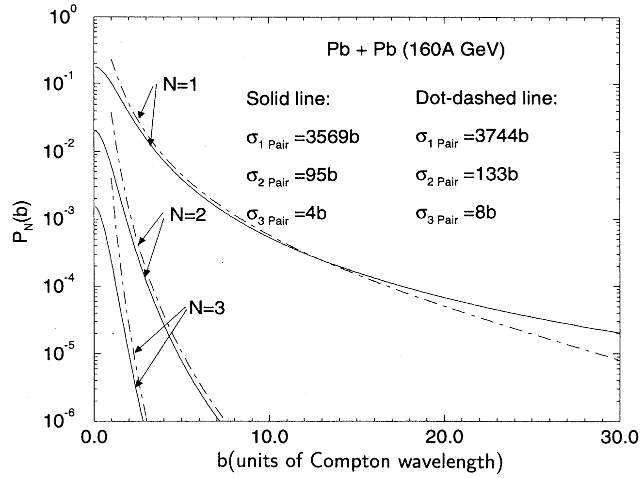

FIG. 6. Probability distribution for N-pair production as a function of impact parameter b for the Pb + Pb collision. Solid lines, exact numerical result; dot-dashed lines, equivalent-photon approximation (Ref. [25]). Total N-pair cross sections are given in units of barns.

overestimation of the total pair-production cross section by Eq. (4.4), results in the equivalent-photon approximation overestimating the N-pair multiplicities. For the lead collision considered, the equivalent-photon approximation overestimates the two-pair cross section by 40% and the three-pair cross section by a factor of 2. Once again, the equivalent-photon approximation has been integrated here only over the range of valid impact parameters.

In summary, we have found the total one-pair cross section for S + Au to be 160 b and Pb + Pb to be 3569 b by integrating the two-photon diagram and using the

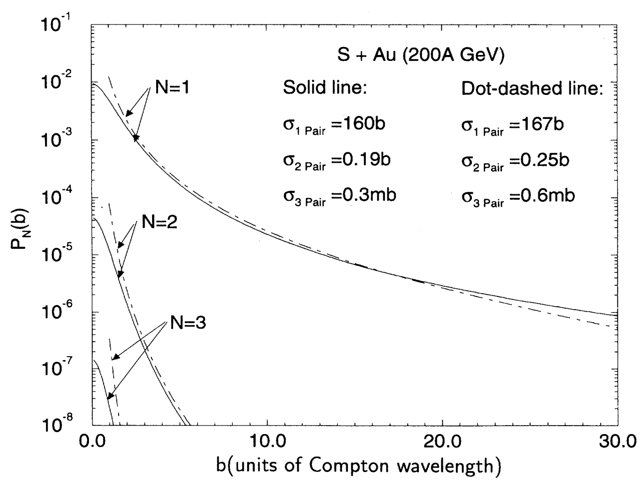

FIG. 7. Probability distribution for N-pair production as a function of impact parameter b for the S + Au collision. Solid lines, exact numerical result; dot-dashed lines, equivalent-photon approximation (Ref. [25]). Total N-pair cross sections are given in units of barns.

Poissonian distribution to extract the cross section for one pair to be produced. The cross section for any number of pairs can be found by summing the contributions from all multipair cross sections as given by Eq. (2.17). Another interesting quantity is the total cross section for multipairs, i.e., excluding the single-pair cross section. For the lead collision considered, this cross section has an appreciable value of approximately 100 b, which is about 3% of the total cross section.

V. CONCLUSIONS

We have found a technique that is capable of calculating the semiclassical two-photon mechanism for lepton-pair production as a function of the impact parameter. This required a generalization of the previously developed Monte Carlo technique in order to be able to handle the oscillating Bessel function which occurs in Eq. (2.7). We find that utilizing a Monte Carlo technique to generate the remainder of the integral $\mathcal{F}(q)$ as a function of q, fitting a smooth function to this result, and then performing the final integral utilizing the smooth fit function produces a workable hybrid Monte Carlo technique.

Using the simple results obtained recently in understanding the nonperturbative nature of the production of multiple, free-electron–positron pairs [24–27], we use the calculated impact-parameter dependence of the two-photon diagrams as the central ingredient needed to calculate the multiple-pair-production cross sections. Our calculations show that, for the systems and energies considered, single-pair production is the dominant part of the total production cross section as expected, since, at these energies, the two-photon mechanism does not violate unitarity, indicating that nonperturbative effects remain relatively small. For the Pb + Pb case we find a substantial two-pair cross section of 95 b, which may yield itself to measurement. However, the impact-parameter dependence of the two- and three-pair cross sections are limited to a few Compton wavelengths ($b < 10\lambda\!\!\!^{-}_C$). For S + Au and Pb + Pb collisions, we have found that a large portion of the cross section of e^-e^+ pair production resides in the region $b > 2R$ ($R \simeq 6.8$ fm for Au, with a Compton wavelength for the electron of $\lambda\!\!\!^{-}_C$=386 fm); thus, for low invariant masses we can eliminate much of the concern over the hadronic debris. The simple equivalent-photon approximation does a fairly good job of predicting the total pair-production cross section, but its utility decreases when multiple-pair cross sections are desired due to its inability to accurately describe the small impact-parameter collisions.

It appears that for the lepton-pair production problem, classical field treatment and quantum field theory have complementary roles. In the classical field treatment, we assumed that the momentum transfer of the photons is much smaller than the momentum possessed by the nuclei; therefore, this is in good agreement with the quantum field theory where the energy and momentum conservation is exactly satisfied at all vertices of Feynman diagrams. In relativistic heavy-ion collisions, coherent particle production via two-photon processes does allow us to make the same approximation because the nuclear form factor constrains the virtual photon momenta where such an approximation is particularly good. Quantum field theory provides guidelines for the classical field treatment and tells us what the classical counterpart is, especially for nuclear systems with spin, and when such a treatment is valid. Classical field treatment also produces a clear physical picture for the classical trajectories of heavy ions which can be tagged experimentally and enables us to properly simulate the dependence of particle production in impact-parameter space.

With the advent of higher-energy heavy-ion colliders, the study of the physics of the two-photon process emerges as an exciting new field. The process may be used as a means of production of exotic particles — perhaps the study of nonperturbative effects in QED. In all cases, an understanding of the impact-parameter dependence of the production is necessary and it is useful to be able to work in regions that are beyond the validity of the equivalent-photon approximation.

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APPENDIX: MONTE CARLO INTEGRATION

The $\mathcal{F}$(q) function in Eq. (3.2) is of the form

$$\mathcal{F}(q) = F_0 \int f(x_1, x_2, ..., x_8, q) dx_1 dx_2 \cdots dx_8 \ , \tag{A1}$$

where we denote these coordinates by the eight-dimensional vector

$$\mathbf{x} = \{\xi, \eta, k, \phi_q, \theta_Q, \phi_Q, \theta_K, \phi_K\} \ . \tag{A2}$$

These variables are related to the variables defined in Eq. (3.2) by

$$\begin{pmatrix} k_z \\ q_z \end{pmatrix} = \gamma\, e^{\xi} \begin{pmatrix} \cos\eta \\ \sin\eta \end{pmatrix} , \tag{A3}$$

$$\begin{pmatrix} Q_x \\ Q_y \end{pmatrix} = a_Q \tan\theta_Q \begin{pmatrix} \cos\phi_Q \\ \sin\phi_Q \end{pmatrix} , \tag{A4}$$

$$\begin{pmatrix} K_x \\ K_y \end{pmatrix} = a_K \tan\theta_K \begin{pmatrix} \cos\phi_K \\ \sin\phi_K \end{pmatrix} , \tag{A5}$$

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = q \begin{pmatrix} \cos\phi_q \\ \sin\phi_q \end{pmatrix}, \tag{A6}$$

$$\begin{pmatrix} k_x \\ k_y \end{pmatrix} = \begin{pmatrix} k \\ 0 \end{pmatrix}. \tag{A7}$$

There are only eight integrals because we find that upon changing to the above variables, the remaining integration can be done by symmetry. Here η, ϕ_Q, ϕ_K, and ϕ_q lie between 0 and π, while θ_Q and θ_K lie between 0 and $\pi/2$. The variable ξ is used to set the upper limit for k_z and q_z. The scale factors a_Q and a_K are taken to be

$$a_Q = \omega/\gamma \, , a_K = \omega/\gamma \, . \tag{A8}$$

Monte Carlo methods reduce the Eq. (3.2) to a summation

$$\mathcal{F}(q) = \sum f(x_{i1}, x_{i2}, ..., x_{i8}, q)\Delta^8 x \, , \tag{A9}$$

where $\Delta^n x$ is the volume element of the subregion. For a finite volume V, we divide the integral region to equal volume elements and obtain

$$\mathcal{F}(q) = \frac{V}{N} \sum f(x_{i1}, x_{i2}, ..., x_{i8}, q) \, , \tag{A10}$$

where N is the number of subvolume elements. We can also monitor the fluctuation of the results and, according to the central limit theorem for large values of N, we can write the associated error

$$\delta^2 = (\langle f^2 \rangle - \langle f \rangle^2)/N \, , \tag{A11}$$

where

$$\langle f^2 \rangle = \frac{1}{N} \sum f^2(x_{i1}, x_{i2}, ..., x_{i8}, q) \, ,$$
$$\langle f \rangle = \frac{1}{N} \sum f(x_{i1}, x_{i2}, ..., x_{i8}, q) \, . \tag{A12}$$

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