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Electron field-emission data, quantum mechanics, and the classical stochastic theories

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Electron field-emission experiments conclusively eliminate a family of classical stochastic theories that have been proposed as alternatives to quantum mechanics. Modified stochastic theories with certain nonlocal interactions may be postulated and may agree with the experimental data, but these revised theories should not be described as *classical* stochastic theories.

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In a recent paper [1] it was shown that electron-tunneling spectroscopy provides a straightforward experimental test between quantum mechanics and a class of alternative theories, the classical stochastic theories. The classical stochastic theories assume that the Maxwell equations, Lorentz force law, and classical mechanics are rigorously true even for subatomic distances, but each particle undergoes random fluctuations in position and momentum due to a postulated fluctuating submicroscopic medium. Within these theories the Schrödinger equation is viewed essentially as a diffusion equation and probability enters the theories only through classical ideas. Some examples of classical stochastic theories can be found in the literature [2–7]. The conclusions of this paper do not apply to theories which modify classical physics through the addition of nonlocal interactions; for example, Bohm's theory is "distinctly non-Newtonian" in the words of Durr, Goldstein, and Zanghi [8] since it introduces a quantum potential. Thus Bohm's theory is not a classical stochastic theory and it is not covered by the arguments in this paper.

A particle cannot tunnel through a potential barrier according to classical physics, but in the classical stochastic theories a particle may acquire sufficient kinetic energy from the fluctuating background medium and escape over the barrier. It must be emphasized that this escape of particles over a barrier is a selection process which favors the particles with high kinetic energies.

On the other hand, according to quantum mechanics, particles tunneling through a potential barrier at very low temperatures typically lose energy due to inelastic collisions within the barrier region. Bruinsma and Bak [9] have derived a rather general quantum theory of inelastic tunneling and have obtained expressions for $\langle \Delta E \rangle$, the mean change in energy per tunneling electron. If E is the initial energy of an electron, E' is its energy after tunneling, and $\langle \Delta E \rangle = \langle E' \rangle - \langle E \rangle$, then according to quantum mechanics [9] inequality (1) is always valid at $T=0$ K and is generally valid for low but nonzero temperatures

$$\langle \Delta E \rangle \leq 0 \; . \tag{1}$$

The equation $\langle \Delta E \rangle = 0$ describes elastic tunneling.

It is also possible to make quantitative predictions for electron transmission across a potential barrier for the classical stochastic theories. Let V_0 be the maximum value of the barrier and choose coordinates as in Fig. 1 so that the potential barrier is located at positions $x \geq 0$. Consider an ensemble ε composed of all free electrons initially on the left-hand side of the barrier $x \leq 0$. The average energy of an electron in this ensemble will be denoted by $\langle E \rangle_\varepsilon$. Now, let us consider the subensemble S of those electrons which have crossed over the barrier potential by a later time t_2. The average energy of an electron in this subensemble is represented by $\langle E' \rangle_s$. Each electron in this subensemble must have attained an energy greater than V_0, at least for a brief time, in order to surmount the potential barrier [1,3]. Unless the electrons experienced a subsequent correlated decrease in energy immediately after crossing the barrier, then

$$\langle E' \rangle_s \geq V_0 \; . \tag{2}$$

The difference in mean energies, according to the classical stochastic theories, can be represented by $\langle \Delta E \rangle_{\rm CS} = \langle E' \rangle_s - \langle E \rangle_\varepsilon$ and from (2) we find

$$\langle \Delta E \rangle_{\rm CS} \geq V_0 - \langle E \rangle_\varepsilon \; . \tag{3}$$

Inequalities (1) and (3) yield very different predictions since quantum mechanics permits electrons to tunnel elastically through a barrier with no change in energy, but according to the classical stochastic theories electrons may pass over a potential barrier only if they have sufficient kinetic energy. An experimental test between

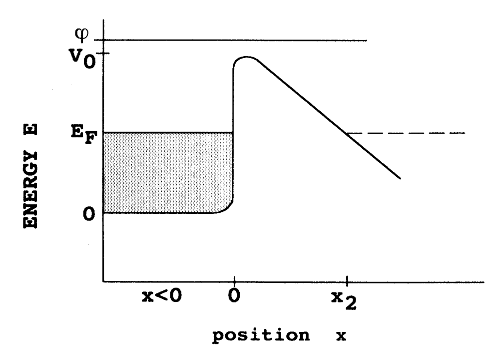

FIG. 1. Electrons in the metal at $x<0$ may tunnel through the vacuum in the region $0<x<x_2$.

these alternatives requires only that the change in energy of electrons be measured after traversing a barrier of potential energy $\langle V_0 \rangle \gg \langle E \rangle$. It was suggested in Ref. [1] that the vacuum tunneling of electrons at low temperatures might provide such an experimental situation and it was noted that photoemission measurements would permit an independent measurement of the potential barrier V_0.

Recently it has been pointed out [10] that previously published electron field-emission measurements have provided data [11] for such a test. The field emission of electrons from a cold metal surface under the influence of a strong electric field involves electron tunneling through the vacuum as shown in Fig. 1. The electrons in the metal are filled to the Fermi energy E_F. The vacuum state normally would be drawn as a horizontal line a few electron volts above the Fermi energy, but in the presence of a strong electric field the potential outside the metal will be deformed so that an approximately triangular potential results. Electrons that tunnel through this barrier are attracted to the anode and may pass through a tiny probe hole in a screen. A third electrode may be placed to the right of the anode and a retarding potential electron energy analyzer can be operated [12]. The collected current can be measured as a function of the voltage between the emitter and analyzer, and if this collected current is differentiated with respect to the bias potential then a "total energy distribution" of the field emitted electrons can be obtained [12].

A wealth of field emission energy distribution data has been collected with this technique, but the present discussion will concentrate on measurements from the (100) surface of tungsten since it has been studied in great detail. Plummer and Gadzuk [11,13] found that the mean energy of electrons which had tunneled from the W(100) surface through the vacuum at $T = 78$ K was $\langle E' \rangle = E_F - 0.2 \pm 0.1$ eV. Since the tunneling electrons originate with energies near the Fermi level [12], this experimental value obviously agrees with the quantum-mechanical prediction of inequality (1). The data of Plummer and Gadzuk also showed that more than 99% of the field-emitted electrons had final energies $E' \le E_F$. The few electrons with relatively high energies after tunneling, $E_F < E' < E_F + 1.5$ eV, correspond to the "tail" of the energy distribution and may have experienced electron-electron interactions either during the tunneling process or in the electron beam outside the metal [11]. It is also relevant to note that Modinos and Nicolaou [14] have calculated the energy distribution of electrons emitted from the W(100) surface, assuming an applied electric field of 5×10^7 V/cm and a work function of $\phi = 4.5$ eV. Their quantum-mechanical calculation is in good agreement with the experimental results of Plummer and Gadzuk [11].

The height of the potential barrier V_0 is nearly equal [15] to the work function ϕ, but there are some complications when field-emission data are used to determine ϕ. The image potential correction [15] must be included and more seriously it is difficult to measure independently the applied electric field during field-emission experiments. Vorburger, Penn, and Plummer [16] have taken a series of field-emission measurements as a function of the applied electric field and have extrapolated to zero field to determine the work function of W(100). Their result $\phi = 4.57 \pm 0.14$ eV is consistent with the measurement $\phi = 4.63 \pm 0.02$ eV, which was obtained for W(100) by the field-emission retarding potential technique [17], and a quoted value [18] of $\phi = 4.6$ eV based on photoemission data from polycrystalline W. It must be emphasized that the consistency of these data and the measured energy distribution of the field-emitted electrons from W(100) provide strong confirmation of the quantum-mechanical predictions.

Although the empirical results agree with quantum mechanics, they are in conspicuous disagreement with inequality (2), which has been derived for a family of classical stochastic theories. Expression (2) for these classical stochastic theories indicates $\langle E' \rangle_s \ge V_0$ and a plausible value [15,16] for V_0 must be at least 4 eV above the Fermi energy, or, in other words, about 0.9ϕ. The previously cited experimental value [11,13] for the mean electron energy after tunneling $\langle E' \rangle = E_F - 0.2 \pm 0.1$ eV violates this inequality $\langle E' \rangle_s \ge E_F + 4$ eV obtained for a family of classical stochastic theories.

In conclusion, electron-field emission data are in excellent agreement with quantum-mechanical predictions. A proponent of classical stochastic theories now must argue either that the effective barrier potential V_0 in field-emission measurements of W(100) is about 4 eV less than the work function or that each electron after it escapes over the barrier must experience a correlated decrease in energy. A stochastic theory proponent might hypothesize the existence of new forces or nonlocal interactions to account for the decrease in energy after an electron escapes over the barrier, but such forces are, in themselves, significant departures from classical physics. In effect, the classical stochastic theory will have been modified to resemble Bohmian mechanics [8] and it must be emphasized that Bohm's mechanics is not a classical stochastic theory and is not covered by the present analysis. If a classical stochastic theorist admits that electrons passed over the barrier and does not introduce the quantum potential or a similar nonlocal interaction, then one must argue that it is more probable for an electron to have negative fluctuations than positive fluctuations in energy after it has crossed the potential barrier. This is equivalent to the implausible idea that a person flipping coins, who repeatedly has obtained heads, is more likely to obtain tails on the next flip.

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