Laser cooling a trapped atom in a cavity: Bad-cavity limit

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We analyze theoretically a one-dimensional model of laser cooling of an atom or ion trapped in a cavity. We assume that the cavity loss rate is much larger than the atom-cavity coupling (bad-cavity limit) and that the atomic excited state is weakly occupied (low saturation limit). After elimination of the cavity mode and the atomic excited state, we derive rate equations for the populations of the trap states. We find that in the Lamb-Dicke limit the atom can be cooled to the ground state of the trap even in the strong confinement limit. This result is interpreted in terms of quantum interferences between different cooling and heating processes involving spontaneous emission in the cavity.

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I. INTRODUCTION

The main goal of laser cooling of atoms confined in electromagnetic traps is to leave the atoms in the ground state of the trapping potential. In this state the atoms reach their minimum temperature, which has important implications in both high-resolution spectroscopy and the developing field of quantum statistical mechanics of laser-cooled atoms [1].

In laser cooling of trapped atoms or ions one usually distinguishes two limits, depending on the typical size of the ground state of the trap a_0 and the wavelength of the laser λ [2]. For $a_0 \gg \lambda$ one expects that the trapping potential does not play an important role in the cooling process and therefore one can use the cooling schemes developed for free atoms [3]. In the opposite limit $\lambda \gg a_0$, the so-called Lamb-Dicke limit (LDL), the (quantized) motion of the atom in the trapping potential becomes important, which leads to different cooling mechanisms than those for free atoms. In the LDL and for harmonic traps, laser cooling to the ground state of the trap may be achieved by tuning the laser to the lower motional sideband in such a way that the atom loses energy every time a photon absorption-emission cycle takes place. This cooling mechanism is known as sideband cooling [4] and requires the trap frequency ν to be larger than the spontaneous emission rate Γ_s of the atomic transition excited by the laser $\nu \gg \Gamma_s$ (strong confinement limit). Sideband cooling has been demonstrated experimentally with a single Hg^+ ion confined in a Paul trap [5]. In the weak confinement limit $\Gamma_s \gtrsim \nu$ several schemes have been proposed to cool the atoms to very low temperatures [6,7].

In a recent paper [8], a scheme to cool free atoms using a cavity has been proposed. The cavity is tuned to one of the Mollow's sideband of the atomic emission spectrum in order to enhance the transitions that decelerate the atoms in a Sisyphus cooling scheme. Zaugg *et al.* have proposed the adiabatic cooling of atoms in cavities [9]. Laser cooling of trapped atoms strongly coupled to a cavity mode (i.e., in the good-cavity limit) has been studied [10], showing that the temperature of the atoms reflect the ion–cavity-mode interaction spectrum. However, in this good-cavity limit, the minimum temperature is the same as that achieved with traditional mechanisms for laser cooling of trapped atoms.

In this paper we analyze laser cooling of an atom trapped in a harmonic potential and placed inside a cavity. We concentrate on the bad-cavity limit, where the cavity loss rate is much larger than the atom-cavity interaction. We show that in the LDL and when emission into the cavity mode dominates the spontaneous emission into the background modes, the atom may be cooled down to the ground state of the trap. We interpret this result as a consequence of the existence of a destructive interference effect between two quantum paths for heating transitions between trap levels. This effect disappears outside the LDL and therefore the cooling mechanism does not work for free atoms or weak traps. To derive analytical results, we eliminate both the cavity-mode degrees of freedom and the excited atomic state. We use two formal proce-

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dures to perform these eliminations. The present paper shows how these procedures can be combined to simplify the problem of atomic motion in cavities in some limiting cases.

The paper is organized as follows. In Sec. II we introduce the model in the form of a master equation. In Sec. III we perform several approximations to simplify the problem in the bad-cavity and low-intensity limit and derive a set of rate equations. In Sec. IV we discuss the results. Finally, a summary is given in Sec. V.

II. MODEL

We consider a one-dimensional model of an atom trapped in a harmonic potential of frequency ν. We describe the internal structure of the atom by a two-level system of ground and excited levels $|g\rangle$ and $|e\rangle$, respectively, with transition frequency ω_0. The ion interacts with a laser beam of frequency ω_l and wave vector $k_l = \omega_l/c$. The trap is placed inside a cavity of frequency ω_c and wave vector $k_c = \omega_c/c$. The master equation describing this situation is ($\hbar = 1$)

$$\dot{\rho} = -i[H_a + H_c + H_{tp} + H_{a-c} + H_{a-l}, \rho] + \mathcal{L}^c\rho, \quad (1)$$

where

$$H_a = \tfrac{1}{2}\omega_0\sigma_z, \quad (2a)$$

$$H_c = \omega_c b^\dagger b, \quad (2b)$$

$$H_{tp} = \nu a^\dagger a \quad (2c)$$

are the free Hamiltonians for the internal structure of the ion, for its motion in the trapping potential, and for the cavity mode, respectively, and

$$H_{a\text{-}l} = \frac{\Omega}{2}[\sigma^+ e^{i(k_l r - \omega_l t)} + \sigma^- e^{-i(k_l r - \omega_l t)}], \quad (2d)$$

$$H_{a\text{-}c} = gS(\sigma^+ b + \sigma^- b^\dagger) \quad (2e)$$

describe the interaction of the atom with the laser and cavity mode, respectively. Finally,

$$\mathcal{L}^c\rho = \kappa(2b\rho b^\dagger - b^\dagger b\rho - \rho b^\dagger b) \quad (3)$$

describes the cavity damping.

Here the σ are the standard spin-$\frac{1}{2}$ matrices describing the internal structure of the atom and a (b) and $a^\dagger$ ($b^\dagger$) are creation and annihilation operators for the harmonic oscillator (cavity mode). We can express the position operator r in terms of the harmonic oscillator operators; in particular, $k_c r \simeq k_l r = \eta(a + a^\dagger)$, where $\eta = \sqrt{k_l^2/(2m\nu)}$ is the Lamb-Dicke parameter. Ω is the Rabi frequency for the laser-ion interaction and g is the coupling constant for the cavity-mode–ion interaction. The operator S is defined as

$$S = \sin(k_c r + \phi), \quad (4)$$

where ϕ depends on the relative position between the trap center and the cavity standing wave. We assume that the cavity mode has a simple sinusoidal spatial dependence. Finally, κ is the cavity loss rate.

Note that in our one-dimensional model spontaneous emission takes place only through the cavity mode. The master equation (1) can be easily generalized to include spontaneous emission in three dimensions, i.e., into the modes other than the cavity mode.

III. APPROXIMATIONS

The master equation (1) cannot be solved analytically. Even solving it numerically using a truncated basis of internal atomic, trap, and photon states represents a formidable task. However, since we are interested in the bad-cavity and low saturation limits, we can simplify the problem considerably. In these limits, one can eliminate the cavity mode and the internal excited state of the atom, which leads to simple rate equations for the populations of the trap levels. In this section we derive such rate equations and give analytical expressions for their solutions in the LDL.

A. Elimination of the cavity mode

We assume that the cavity loss rate is much faster than the laser-ion and cavity-mode–ion interactions, i.e.,

$$\kappa \gg g, \Omega. \quad (5)$$

In this limit, one can eliminate the cavity mode and find a master equation for the atomic degrees of freedom alone. In this subsection we follow the procedure given in Ref. [11] to derive such a master equation.

Let us define a density operator

$$\tilde{\rho}(t) = e^{-\mathcal{L}^c t}\left[e^{iH_0 t}\rho(t)e^{-iH_0 t}\right], \quad (6)$$

where $H_0 = \frac{1}{2}\omega_0\sigma_z + \omega_c b^\dagger b$. Using (1), we find that

$$\begin{aligned}\dot{\tilde{\rho}} = &-i[H_a^{(c)} + H_{tp}, \tilde{\rho}] \\ &-ig\left(e^{-\kappa t}b[\sigma^+ S, \tilde{\rho}] + e^{\kappa t}[b, \tilde{\rho}]\sigma^+ S - \text{H.c.}\right), \end{aligned} \quad (7)$$

where $H_a^{(c)} = -\frac{1}{2}\Delta_c\sigma_z$, with $\Delta_c = \omega_c - \omega_0$ being the cavity-mode–ion detuning. In the derivation of (7) we have used

$$e^{-\mathcal{L}^c t}(bX) = e^{-\kappa t}be^{-\mathcal{L}^c t}X,$$

$$e^{-\mathcal{L}^c t}([b, X]) = e^{\kappa t}[b, e^{-\mathcal{L}^c t}X]$$

and ignored the laser-ion interaction. In the limit (5) this interaction can be added independently to the final master equation [see Eq. (11) below].

Tracing Eq. (7) over the cavity mode states, we get the following equation for the reduced density operator $\tilde{\mu} = \text{Tr}_c(\tilde{\rho})$:

$$\dot{\tilde{\mu}} = -i[H_a^{(c)} + H_{tp}, \tilde{\mu}] - ige^{-\kappa t}\text{Tr}_c\left(b[\sigma^+ S, \tilde{\rho}] - \text{H.c.}\right). \quad (8)$$

In the limit (5), following arguments identical to those in Ref. [11], it can be shown that

$$e^{-\kappa t}\tilde{\rho} \simeq -i\left([b,\tilde{\rho}]\sigma^+ T - \text{H.c.}\right), \tag{9}$$

where

$$T = g\int_0^\infty d\tau\, e^{-(\kappa+i\Delta_c)\tau} e^{-iH_{tp}\tau} S e^{iH_{tp}\tau}. \tag{10}$$

Substituting (9) into Eq. (8) and coming back to the nonrotating frame, we find that $\mu = \text{Tr}_c(\rho)$ obeys the following master equation:

$$\dot{\mu} = -i[H_a + H_{tp} + H_{a-l}, \mu] + \mathcal{L}^d\mu. \tag{11}$$

Here we have added the atom-laser interaction and defined

$$\mathcal{L}^d\mu = g(\sigma^- T\mu S\sigma^+ - \sigma^+\sigma^- ST\mu + \text{H.c.}). \tag{12}$$

The Liouvillian $\mathcal{L}^d$ describes the effects of the cavity on the behavior of the atom. Apart from the interaction with the laser, the atom emits photons into the cavity mode which leave the cavity rapidly. The corresponding effective spontaneous emission rate depends on the position of the atom through the position-dependent interaction with the cavity mode. Apart from that, there is an energy shift (Lamb shift) which also depends on the position of the atom. To show this, let us consider that the position of the atom remains practically constant during the time $\tau = \kappa^{-1}$ (i.e., $\kappa \gg \nu$). In this case, we have

$$T \simeq \frac{g}{k + i\Delta_c} S \tag{13}$$

and we can write

$$\mathcal{L}^d\mu = -i[\delta(r) - i\gamma(r)]\sigma^+\sigma^-\mu + i\mu\sigma^+\sigma^-[\delta(r) + i\gamma(r)] + 2\gamma_0\sigma^- S\mu S\sigma^+, \tag{14}$$

where

$$\gamma(r) = \gamma\sin(k_c r + \phi)^2, \tag{15a}$$

$$\delta(r) = -\gamma(r)\Delta_c/\kappa, \tag{15b}$$

and $\gamma = g^2\kappa/(\kappa^2 + \Delta_c^2)$. Here $2\gamma(r)$ and $\delta(r)$ are the position-dependent spontaneous emission rate and the Lamb shift, respectively. When $\nu \gtrsim \kappa$, the operator T incorporates the fact that the position of the atom may change appreciably during the time in which the cavity mode is damped.

B. Elimination of the atomic excited state

For low laser intensities the internal excited state $|e\rangle$ will be negligibly occupied. Thus, in this limit, one can eliminate the excited state $|e\rangle$. To do this, we follow standard procedures based on projector techniques. We first rewrite master equation (11) in a frame rotating at the laser frequency ω_l as

$$\dot{\mu} = (\mathcal{L}_0 + \mathcal{L}_1)\mu, \tag{16}$$

where

$$\mathcal{L}_0\mu = -i[H_a^{(l)} + H_{tp}, \mu] + \mathcal{L}^d\mu, \tag{17a}$$

$$\mathcal{L}_1\mu = -i[H_{a-l}^{(l)}, \rho], \tag{17b}$$

and

$$H_a^{(l)} = -\frac{1}{2}\Delta_l\sigma_z, \tag{18a}$$

$$H_{a\text{-}l}^{(l)} = \frac{1}{2}\Omega(\sigma^+ e^{ik_l r} + \sigma^- e^{-ik_l r}), \tag{18b}$$

with $\Delta_l = \omega_l - \omega_0$ being the laser-ion detuning.

We define the projector operators $\mathcal{P}$ and $\mathcal{Q}$, fulfilling

$$\mathcal{P}X = \sum_{n=0}^{\infty} |n,g\rangle\langle n,g|\langle n,g|X|n,g\rangle \tag{19}$$

and $\mathcal{Q} = 1 - \mathcal{P}$. Here $|n,g\rangle$ denotes a state with the atom in its internal ground state and in the nth trap level. Projecting master equation (11), we obtain

$$\mathcal{P}\dot{\mu} = \mathcal{P}(\mathcal{L}_0 + \mathcal{L}_1)\mathcal{Q}\mu, \tag{20a}$$

$$\mathcal{Q}\dot{\mu} = \mathcal{L}_1\mathcal{P}\mu + \mathcal{Q}(\mathcal{L}_0 + \mathcal{L}_1)\mathcal{Q}\mu, \tag{20b}$$

where we have used that $\mathcal{P}\mathcal{L}_1\mathcal{P} = \mathcal{L}_0\mathcal{P} = 0$. We then solve formally Eq. (20b) for $\mathcal{Q}\mu$ and substitute the result into (20a). Expanding the result in powers of Ω and keeping the lowest nonvanishing orders, we find the following equation for the population of the nth trap level $P_n = \langle n,g|\mu|n,g\rangle$:

$$\dot{P}_n = -\left[\sum_{n'=0}^{\infty}\Gamma_{n'\leftarrow n}\right]P_n + \sum_{n'=0}^{\infty}\Gamma_{n\leftarrow n'}P_{n'}. \tag{21}$$

Here

$$\Gamma_{n\leftarrow n'} = \frac{\Omega^2}{2}\text{Re}\left[\left\langle n\left|T\frac{1}{\Delta_l + \nu(n' - a^\dagger a) + igST}e^{ik_l r}\right|n'\right\rangle \times\left\langle n'\left|e^{-ik_l r}\frac{1}{\Delta_l + \nu(n' - a^\dagger a) - igT^\dagger S)}S\right|n\right\rangle\right] \tag{22}$$

are the Raman-type transition rates from state $|n'\rangle$ to $|n\rangle$.

The rate equation (21) can be easily solved numerically using a truncated set of harmonic oscillator states. According to the above discussion, it is valid in the limits $\kappa \gg g$ and $\Omega^2/[\gamma(r)^2 + \delta(r)^2] \ll 1$. In the following, we will further assume that the trap frequency is much smaller than the cavity loss rate ($\kappa \gg \nu$). In this limit, the coefficients $\Gamma_{n\leftarrow n'}$ can be written as

$$\Gamma_{n\leftarrow n'} = \frac{\Omega^2}{2}\gamma\left|\left\langle n,g\left|\sigma^- S\frac{1}{E_{n',g} - H_{\text{eff}}(r)}\sigma^+ e^{ik_l r}\right|n',g\right\rangle\right|^2, \tag{23}$$

where $H_{\text{eff}}(r) = H_a^{(l)} + H_{tp} + [\delta(r) - i\gamma(r)]\sigma^+\sigma^-$.

The interpretation of (23) is simple since it has the typical form of a Raman transition rate. The process $|n'\rangle \to |n\rangle$ takes place through the absorption of a laser photon (which is accompanied by a recoil kick $e^{ik_l r}$), followed by a spontaneous emission into the cavity mode. The spontaneous emission rate is $\gamma(r)$, i.e., depends on the coupling constant between the atom and the cavity mode at the position r of the atom. There is also a position-dependent Lamb shift $\delta(r)$ induced by the cavity.

Finally, it is worth mentioning that the expression (23) for the rates can be easily generalized to the case where there is spontaneous emission in modes other than the cavity mode. In such a case, one can write $\Gamma_{n\leftarrow n'} = \Gamma^c_{n\leftarrow n'} + \Gamma^s_{n\leftarrow n'}$, where $\Gamma^c_{n\leftarrow n'}$ is given in (23), and

$$\Gamma^s_{n\leftarrow n'} = \frac{\Omega^2}{2}\gamma^s \int_{-1}^{1} du N(u) \left| \left\langle n, g \left| \sigma^- e^{-ikru} \frac{1}{E_{n',g} - H_{\rm eff}(r)} \sigma^+ e^{ik_l r} \right| n', g \right\rangle \right|^2, \tag{24}$$

where $2\gamma^s$ is the spontaneous decay rate into the background modes, $N(u)$ is the usual dipole emission rate, and the replacement $H_{\rm eff}(r) \to H_{\rm eff}(r) + i\gamma^s$ is understood.

C. Lamb-Dicke limit

In the LDL, the motion of the atom is restricted to a region small compared with the laser wavelength. In this limit we have $\eta \ll 1$. Thus we expand the rates $\Gamma_{n\leftarrow n'}$ in powers of η and keep only the lowest nonvanishing order. The rates $\Gamma_{n\leftarrow n}$ are of zero order in η, whereas the rates $\Gamma_{n\pm1\leftarrow n}$ are of second order. The rest of the rates are, at least, of fourth order in η. Since the rates $\Gamma_{n\leftarrow n}$ do not appear in the rate equations (21), we only derive the expression for the rates that change the quantum oscillator number by unity. We first expand $e^{ik_l r}$ and S, obtaining

$$e^{ik_l r} = 1 + i\eta(a + a^\dagger) + o(\eta^2), \tag{25a}$$

$$S = \sin(\phi) + \eta\cos(\phi)(a + a^\dagger) + o(\eta^2). \tag{25b}$$

On the other hand,

$$\frac{1}{E_{n',g} - H_{\rm eff}(r)} = \frac{1}{E_{n',g} - H_{\rm eff}(0)} \left[1 + 2\eta(\delta - i\gamma)\cot(\phi)(a + a^\dagger) \times \frac{1}{E_{n',g} - H_{\rm eff}(0)} + o(\eta^2) \right], \tag{26}$$

where we have used the shorthand notation $\delta = \delta(0)$ and $\gamma = \gamma(0)$. Substituting these expressions in (23), we obtain

$$\Gamma_{n\leftarrow n+1} = F(\nu), \tag{27a}$$

$$\Gamma_{n+1\leftarrow n} = F(-\nu), \tag{27b}$$

with

$$F(\nu) = (n+1)\eta^2\gamma \frac{\Omega^2/2}{(\Delta_l - \delta)^2 + \gamma^2} \times \left| \cos(\phi) \left[1 + \frac{2(\delta - i\gamma)}{\Delta_l - \delta + \nu + i\gamma} \right] + i\sin(\phi) \frac{\Delta_l - \delta + i\gamma}{\Delta_l - \delta + \nu + i\gamma} \right|^2. \tag{28}$$

In the LDL limit, the rate equations (21) can be solved analytically. In steady state, transitions from $|n\rangle \to |n+1\rangle$ are balanced with those from $|n+1\rangle \to |n\rangle$ (detailed balance), i.e., $\Gamma_{n\leftarrow n+1} P_{n+1} = \Gamma_{n+1\leftarrow n} P_n$, which results in an occupation probability of level n

$$P_n = (1 - \langle n \rangle_{SS}) \left[\frac{\langle n \rangle_{SS}}{\langle n \rangle_{SS} + 1} \right]^n, \tag{29}$$

where $\langle n \rangle_{SS}$ is the mean oscillator quantum number and is given by

$$\langle n \rangle_{SS} = \frac{\Gamma_{n+1\leftarrow n}}{\Gamma_{n\leftarrow n+1} - \Gamma_{n+1\leftarrow n}}. \tag{30}$$

IV. DISCUSSION

The most interesting feature of laser cooling in a cavity in the limits studied here is the possibility of cooling to the ground state of the harmonic oscillator. This can be achieved when $\Gamma_{n+1\leftarrow n}$ equals zero, since in this case no transition increasing the oscillator quantum number takes place (in the Lamb-Dicke limit). This effect is the consequence of a destructive quantum interference effect between two processes: The first is $|n, g\rangle \to |n+1, e\rangle \to |n+1, g\rangle$ [see Fig. 1(a)] and corresponds to the first term in the formula (28); the second is $|n, g\rangle \to |n, e\rangle \to |n+1, g\rangle$ [Fig. 1(b)] and corresponds to the second term in the formula (28). Note that the tran-

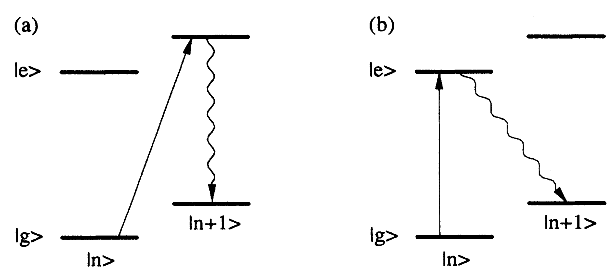

FIG. 1. Heating transitions between trap levels in the Lamb-Dicke limit. In (a), the atom absorbs a laser photon, increasing the quantum number n. This photon is emitted into the cavity without changing n. In (b), the atom absorbs a laser photon without changing n and then emits a photon into the cavity, increasing n by one.

sition amplitudes for these processes are proportional to $\sin(\phi)$ and $\cos(\phi)$, respectively. On the other hand, their phases depend (in a different manner) on the atom-laser detuning Δ_l. It is then clear that by varying these two parameters the destructive interference can occur. A simple analysis shows that the rate $\Gamma_{n+1\leftarrow n}$ is zero when

$$\sin(2\phi) = \Delta_l/(2\gamma), \quad (31a)$$

$$\Delta_l = [\nu \pm \sqrt{\nu^2 + 4\gamma^2 \sin(\phi)^4}]/2. \quad (31b)$$

Figure 2 shows a contour plot of the mean quantum oscillator number $\langle n\rangle_{SS}$ as a function of the detuning Δ_l and the phase ϕ for $\eta = 0.03$, $\kappa = 200\nu$, $g = 20\nu$, and $\Delta_c = 0$. This figure has been plotted after solving numerically Eq. (21) with the rates given in (22). For $\Delta_l \simeq -0.9\nu$ and $\phi \simeq 2.18$, the mean quantum oscillator number is practically zero. Note that the parameters for this plot have been chosen so that the weak confinement applies ($2\gamma = 4\nu$).

From the formulas given in Sec. III we can derive the cooling rate Γ. In the LDL, it is defined from the evolution equation of $\langle n\rangle_{SS}$,

$$\langle \dot{n}\rangle = -\Gamma\langle n\rangle + D, \quad (32)$$

where D is the diffusion coefficient. In our case, the cooling rate is given by

$$\Gamma = \Gamma_{n\leftarrow n+1} - \Gamma_{n+1\leftarrow n}. \quad (33)$$

Note that, as expected, it is proportional to $\eta^2\Omega^2$ and also depends on the particular values taken by the detunings, etc.

We stress the fact that due to this interference effect, the ground state of the harmonic potential $|0\rangle$ is a *dark state*, since the atom, once it has reached this state, cannot leave it. We wish to emphasize that the cooling mechanism based on this interference effect is completely different from the sideband cooling [4,2], whereby the state $|0\rangle$ is a dark state as well (in the strong confinement limit $\nu \ll \gamma$). In that case, the atom cannot leave the state $|0\rangle$ since the laser photons are very far from resonance for all possible transitions. In fact, the interference effect described here works in the weak confinement limit as well.

This cooling mechanism based on the existence of a destructive interference only operates in the LDL. This is clear since outside this limit, although transitions $n \to n+1$ are suppressed, other transitions increasing the quantum number n can occur. To show this, we have plotted in Fig. 3 $\langle n\rangle_{SS}$ [Fig. 3(a)] and Γ as given by Eq. (33) [Fig. 3(b)] as a function of the Lamb-Dicke parameter η for the same parameters as in Fig. 2 and $\Delta_c = -0.9$ and $\phi = 2.18$ [which correspond to the values for which $\langle n\rangle_{SS} \simeq 0$ in the LDL]. Figure 3(a) shows that $\langle n\rangle_{SS}$ is an increasing function of η. In fact, for $\eta > 0.2$ the population of the ground level is already significant. On the other hand, in Fig. 3(b) we have computed the cooling rate Γ as the maximum nonzero eigenvalue of the rate equations (21). In order to compute Γ we have solved numerically these rate equations taking a truncated basis of harmonic oscillator states, namely, 25, 50, and 100 states (solid, dashed, and dash-dotted lines, respectively). For small values of η, the dependence is quadratic, in agreement with the results derived above for the LDL. For increasing values of η, this dependence changes. As one takes more states for the calculation, the cooling rate changes its quadratic dependence for smaller values of η. This is due to the fact that the cooling starting from higher trap levels is slower, because the transitions increasing and decreasing the quantum oscillator number tend to the same values. We remind the reader that for most of the ions trapped in Paul traps, $\eta \lesssim 0.1$.

Observation of the effects predicted above in a *real* three-dimensional trap would require

$$\gamma \gg \gamma^s, \quad (34)$$

where $2\gamma_s$ is the spontaneous emission decay rate in

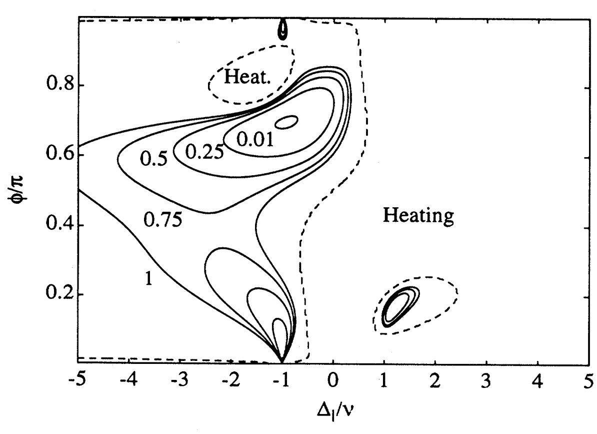

FIG. 2. Contour plot of the mean quantum number of the harmonic oscillator $\langle n\rangle_{SS}$ in steady state as a function of the laser detuning Δ_l and the angle ϕ. The parameters are $g = 20\nu$, $\kappa = 200\nu$, $\Delta_c = 0$, and $\eta = 0.03$. Dashed lines delimit regions where cooling or heating occurs.

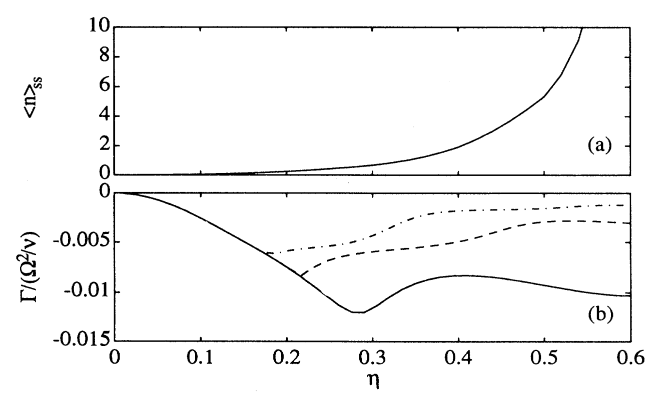

FIG. 3. (a) Mean quantum oscillator number $\langle n\rangle_{SS}$ in steady state and (b) cooling rate as a function of the Lamb-Dicke parameter η for the same parameters as in Fig. 2 and $\Delta_l = -0.9\nu$ and $\phi = 2.18$. These parameters correspond to those which give the minimum value of $\langle n\rangle_{SS}$ in the Lamb-Dicke limit. In (b), the solid, dashed, and dash-dotted curves have been calculated using a truncated basis of 25, 50, and 100 states.

modes other than the cavity mode. In this case there are few photons emitted into those other modes (compared to those emitted into the cavity mode), which is the required condition to neglect spontaneous emission into the background modes. For this, one should be able to enhance the atomic spontaneous emission with the cavity [12]. At present, there are two regimes for experiments in cavity QED, namely, the optical and the microwave ones. In the optical regime, the condition $\gamma \gtrsim \gamma_s$ has been approached and there are attempts to improve it [13]. In the microwave regime, the condition (34) has been succesfully achieved, for instance, in the recent experiments of Lange and Walther [14].

Finally, we would like to point out that there is an alternative possibility to use a cavity to cool a trapped particle. It consists in tuning the cavity to the upper motional sideband $\Delta_c = \nu$. In this case and for $\kappa \gg \nu$ only transitions $n \to n-1$ would take place since the other would be very far from resonance. However, this cooling mechanism operates in the same regime as the sideband cooling (note $\nu \gg \kappa \gg g^2/\kappa = \gamma$) and requires very good quality cavities ($\nu \sim 3$ MHz $\gg \kappa$).

V. CONCLUSIONS

We have studied laser cooling of a trapped atom in a cavity in one dimension. We have derived a rate equation valid in the bad-cavity and low saturation limits. We have obtained analytical expressions for the transition rates between neighboring levels in the LDL. These expressions predict that the ground state of the harmonic oscillator is a dark state for certain parameters. This effect is the consequence of a destructive quantum interference and is not restricted to the strong confinement limit.

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