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Quantum theory of the one-mode Λ-type micromaser and laser

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We derive the master equations for the fields in the one-mode Λ-type micromaser and laser with injected atoms in a superposition of their states. In terms of effective parameters, these equations are equivalent to those for a two-level micromaser and laser. The interference terms in the master equations may cancel the absorption terms independent of statistical properties of the field. Due to the atomic coherence between the two degenerate lower levels, the generation of field states with sub-Poissonian photon statistics is possible in the micromaser without the need for a population inversion. If an equivalent level system is used in a laser operating without population inversion it can produce field states with Poisson distributions of the photon number.

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I. INTRODUCTION

Recently there has been considerable interest in noninversion lasers [1–20]. Many schemes for laser action without population inversion have been proposed [1–10]. Kocharovskaya and Khanin have predicted amplification of ultrashort pulses in an active medium consisting of three-level atoms with two nearly degenerate lower levels [1]. Harris has considered the difference between emission and absorption spectra due to Fano interference between two lifetime-broadened discrete levels which decay into an identical continuum [2]. Scully *et al.* have studied the crucial role played by atomic coherence in driven atomic systems [3]. Experimental observations of amplification without inversion have recently been reported [18–20]. The study of noninversion lasers operating on three-level atoms may provide us with interesting information on coherent effects resulting from interference between different channels of atomic transitions.

Most of the previous studies [1–10] were, however, limited to the classical treatment of the field. Only a few contributions [11–17] addressed the statistical properties of the radiated field. By using the density-operator method, Zhu [11] has shown that the conditions for lasing without inversion in the degenerate quantum-beat laser are independent of the intensity of the laser field. Analyzing the coefficients of the Fokker-Planck equation, Bergou and Bogar [12] predicted that the inversionless degenerate quantum-beat laser in steady state would be characterized by a standard deviation of the photon number approaching the Poisson value under appropriate conditions. Manka *et al.* [13] found that for selected values of the parameters the radiated field in a three-level laser driven by a coherent external field shows strong sub-Poissonian characteristics. Squeezed lasing has been studied by Gheri and Walls [14]. The reduction of spectral linewidth in noninversion lasers has been predicted by Agarwal [15] and Ritsch, Marte, and Zoller [16]. The noise-free energy transfer in a Λ-type atomic medium has been shown by Agarwal, Scully, and Walther [17].

On the other hand, recent developments in quantum optics have provided a new kind of maser, the micromaser, operating with one atom at a time inside the cavity [21–32]. These devices have allowed new tests of the basic models in quantum optics, displaying a variety of interesting quantum phenomena.

Though preparing three-level atoms in a mixture of states with internal coherence is not easy, using three-level atoms is attractive since such systems may yield quantum noise quenching [33], lasers that emit squeezed light [34], lasing without inversion [1–20], and new optical materials with a substantially enhanced index of refraction [35].

The purpose of this paper is to develop the quantum theory of the one-mode Λ-type micromaser and laser with injected coherence. We show that the interference terms in the master equations may cancel the absorption terms independent of statistical properties of the field. Due to the atomic coherence between the two degenerate lower levels, the generation of field states with sub-Poissonian photon statistics is possible in the micromaser without the need for a population inversion. If an equivalent level system is used in a laser operating without population inversion it can produce field states with Poisson distributions of the photon number.

In Sec. II we derive the master equations, present the steady-state photon distributions and discuss the results. In Sec. III we summarize our conclusions.

II. MASTER EQUATIONS AND STEADY-STATE PHOTON DISTRIBUTIONS

We consider a micromaser or laser cavity in which the atoms interact with the cavity mode for a finite time

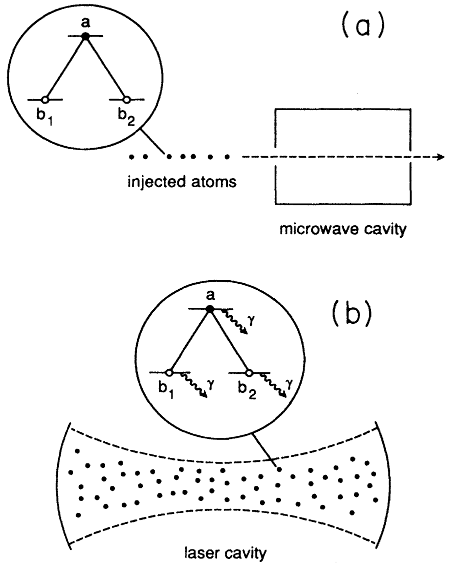

FIG. 1. The schemes of the one-mode Λ-type micromaser (a) and laser (b).

τ in the micromaser case or spend their life interacting with the cavity mode in the laser case. The atoms have an upper level a with the energy $\hbar\Omega_a$ and a lower-state doublet b_1 and b_2 with the energy $\hbar\Omega_b$, as shown in Fig. 1. The dipole-allowed transitions between the upper level a and the lower levels b_1 and b_2 are resonant with the field mode. The transition between the two lower levels is dipole forbidden.

The interaction of the cavity mode with an injected atom is described by the Hamiltonian

$$H = H_{\rm A} + H_{\rm F} + H_{\rm AF} \,. \tag{1}$$

Here, $H_{\rm A}$ and $H_{\rm F}$ describe the free atom and free field, respectively, and $H_{\rm AF}$ describes the atom-field interaction in the dipole and rotating-wave approximations:

$$\begin{aligned} H_{\rm A} &= \hbar\Omega_a\,|a\rangle\langle a| + \hbar\Omega_b \sum_{\alpha=1}^{2} |b_\alpha\rangle\langle b_\alpha|\,, \\ H_{\rm F} &= \hbar\omega a^\dagger a\,, \\ H_{\rm AF} &= \sum_{\alpha=1}^{2} \hbar g_\alpha \left(a\,|a\rangle\langle b_\alpha| + |b_\alpha\rangle\langle a|\,a^\dagger\right) . \end{aligned} \tag{2}$$

The operator $|j\rangle\langle j|$ ($j = a, b_1, b_2$) describes the atomic population of level j. The operator $|i\rangle\langle j|$ ($i \neq j$) describes the transition from level j to level i. The photon operators a and $a^\dagger$ describe the annihilation and creation of photons in the field mode with the resonance frequency $\omega = \Omega_a - \Omega_b$. The parameter g_α is the atom-mode coupling constant corresponding to the transition $a \leftrightarrow b_\alpha$.

We can show that the time-evolution operator $U(t)$ in the interaction picture has the form

$$\begin{aligned} U(t) &\equiv \exp\left(-iH_{\rm AF}t/\hbar\right) \\ &= \left(1 + \frac{K}{\lambda^2}\right)\cos(\lambda t) - i\left(H_{\rm AF}/\hbar\right)\frac{1}{\lambda}\sin(\lambda t) - \frac{K}{\lambda^2}\,. \end{aligned} \tag{3}$$

Here, the operators

$$\begin{aligned} \lambda^2 &= \left(g_1^2 + g_2^2\right)\left(a^\dagger a + |a\rangle\langle a|\right) , \\ K &= a^\dagger a[g_1 g_2(|b_1\rangle\langle b_2| + |b_2\rangle\langle b_1|) \\ &\quad - g_1^2\,|b_2\rangle\langle b_2| - g_2^2\,|b_1\rangle\langle b_1|] \end{aligned} \tag{4}$$

are constants of motion.

The nonzero matrix elements of the operator $U(t)$ are found to be

$$\begin{aligned} \langle a; n|U(t)|a; n\rangle &= \cos\left(Gt\sqrt{n+1}\right) , \\ \langle a; n|U(t)|b_\alpha; n+1\rangle &= -i\,\frac{g_\alpha}{G}\,\sin\left(Gt\sqrt{n+1}\right) , \\ \langle b_\alpha; n|U(t)|a; n-1\rangle &= -i\,\frac{g_\alpha}{G}\,\sin\left(Gt\sqrt{n}\right) , \\ \langle b_\alpha; n|U(t)|b_\alpha; n\rangle &= 1 - 2\,\frac{g_\alpha^2}{G^2}\,\sin^2\!\left(\tfrac{1}{2}\,Gt\sqrt{n}\right) , \\ \langle b_2; n|U(t)|b_1; n\rangle &= \langle b_1; n|U(t)|b_2; n\rangle \\ &= -2\,\frac{g_1 g_2}{G^2}\,\sin^2\!\left(\tfrac{1}{2}\,Gt\sqrt{n}\right) , \end{aligned} \tag{5}$$

where

$$G = \sqrt{g_1^2 + g_2^2}\,. \tag{6}$$

We suppose that, before entering the cavity, each atom is prepared either in the upper state or in a superposition of the two lower states. The density matrix of the initial state of each atom is given in the interaction picture by

$$\varrho_{\rm A} = \varrho_{aa}\,|a\rangle\langle a| + \sum_{\alpha,\beta=1}^{2} \varrho_{b_\alpha b_\beta}\,|b_\alpha\rangle\langle b_\beta|\,, \tag{7}$$

where

$$\begin{aligned} &\varrho_{jj} \geq 0, \quad \sum_{j=a,b_1,b_2} \varrho_{jj} = 1, \quad \varrho_{b_1 b_2} = \varrho^*_{b_2 b_1}, \\ &|\varrho_{b_1 b_2}| = |\varrho_{b_2 b_1}| \leq \sqrt{\varrho_{b_1 b_1}\varrho_{b_2 b_2}}\,. \end{aligned} \tag{8}$$

Let the atoms be injected into the cavity according to a Poissonian process with an average rate r. Below, we will study separately the micromaser and laser cases.

A. Micromaser

In the micromaser, the injected atoms pass through the cavity and the atomic flux is so low that only one atom is in the cavity at a time. The atomic decay is negligible. The time of interaction of each atom with the cavity field is much shorter than the cavity damping time so that the relaxation of the cavity field can be ignored while an atom is inside the cavity. For simplicity, we suppose that the injected atoms have the same velocity and, therefore, interact with the cavity field for the same time. We denote this interaction time by τ. As discussed in Ref. [36], the time evolution of the density matrix ϱ of the cavity field in the interaction picture is governed by the equation

$$\dot{\varrho} = r\delta_\tau \varrho + L\varrho \, , \tag{9}$$

where $\delta_\tau \varrho$ is the change in ϱ due to an atom interacting with the field for the time τ, and $L\varrho$ is the Liouvillian operator which describes losses due to the coupling of the cavity mode to a thermal bath.

The expression of $\delta_\tau \varrho$ is

$$\begin{aligned}\delta_\tau \varrho &= \mathrm{Tr}_{(\mathrm{A})} \left\{ U(\tau) \varrho_{\mathrm{A}} \otimes \varrho \, U^\dagger(\tau) \right\} - \varrho \\ &= \varrho_{aa} \, \delta_\tau^{(aa)} \varrho + \sum_{\alpha,\beta=1}^{2} \varrho_{b_\alpha b_\beta} \, \delta_\tau^{(b_\alpha b_\beta)} \varrho \, ,\end{aligned} \tag{10}$$

where

$$\begin{aligned}\delta_\tau^{(aa)} \varrho &= \mathrm{Tr}_{(\mathrm{A})} \left\{ U(\tau) \, |a\rangle\langle a| \otimes \varrho \, U^\dagger(\tau) \right\} - \varrho \, , \\ \delta_\tau^{(b_\alpha b_\beta)} \varrho &= \mathrm{Tr}_{(\mathrm{A})} \left\{ U(\tau) \, |b_\alpha\rangle\langle b_\beta| \otimes \varrho \, U^\dagger(\tau) \right\} - \varrho \, \delta_{\alpha\beta} \, .\end{aligned} \tag{11}$$

The expression of the Liouvillian operator $L\varrho$ is given by [36]

$$\begin{aligned}L\varrho = {} & \tfrac{1}{2} C(n_{\mathrm{b}} + 1)(2a\varrho a^\dagger - a^\dagger a \varrho - \varrho a^\dagger a) \\ & + \tfrac{1}{2} C n_{\mathrm{b}} (2a^\dagger \varrho a - a a^\dagger \varrho - \varrho a a^\dagger) \, .\end{aligned} \tag{12}$$

Here, n_{b} is the number of photons in thermal equilibrium, and C is the cavity damping rate.

By using Eqs. (5), the matrix elements of the operators $\delta_\tau^{(aa)} \varrho$ and $\delta_\tau^{(b_\alpha b_\beta)} \varrho$ are easily found to be

$$\begin{aligned}\delta_\tau^{(aa)} \varrho(n,n') &= \delta_\tau^{(a)} \varrho(n,n') \, , \\ \delta_\tau^{(b_\alpha b_\beta)} \varrho(n,n') &= \left(g_\alpha g_\beta / G^2 \right) \delta_\tau^{(b)} \varrho(n,n') \, ,\end{aligned} \tag{13}$$

where

$$\begin{aligned}\delta_\tau^{(a)} \varrho(n,n') &= \left[\cos\left(G\tau\sqrt{n+1}\right) \cos\left(G\tau\sqrt{n'+1}\right) - 1 \right] \varrho(n,n') + \sin\left(G\tau\sqrt{n}\right) \sin\left(G\tau\sqrt{n'}\right) \varrho(n-1,n'-1) \, , \\ \delta_\tau^{(b)} \varrho(n,n') &= \left[\cos\left(G\tau\sqrt{n}\right) \cos\left(G\tau\sqrt{n'}\right) - 1 \right] \varrho(n,n') + \sin\left(G\tau\sqrt{n+1}\right) \sin\left(G\tau\sqrt{n'+1}\right) \varrho(n+1,n'+1) \, .\end{aligned} \tag{14}$$

According to Eq. (12), the matrix elements of the loss operator $L\varrho$ are

$$\begin{aligned}L\varrho(n,n') = {} & C(n_{\mathrm{b}}+1)[\sqrt{(n+1)(n'+1)} \; \varrho(n+1,n'+1) - \tfrac{1}{2}(n+n')\varrho(n,n')] \\ & + C n_{\mathrm{b}} [\sqrt{nn'} \; \varrho(n-1,n'-1) - \tfrac{1}{2}(n+n'+2)\varrho(n,n')].\end{aligned} \tag{15}$$

From Eqs. (9), (10), and (13), we find

$$\begin{aligned}\dot{\varrho}(n,n') = {} & r \varrho_{aa} \delta_\tau^{(a)} \varrho(n,n') \\ & + r \sum_{\alpha,\beta=1}^{2} \left(\varrho_{b_\alpha b_\beta} g_\alpha g_\beta / G^2 \right) \delta_\tau^{(b)} \varrho(n,n') \\ & + L\varrho(n,n') \, .\end{aligned} \tag{16}$$

In the above equation, the first term corresponds to the emission from the upper level a. The terms in the sum correspond to the absorption from the lower levels b_1 and b_2 and to the interference between the channels of transitions. It is interesting to note that in the micromaser model considered above the interference terms, proportional to $\varrho_{b_1 b_2}$ or $\varrho_{b_2 b_1}$, may be different from zero and, therefore, may cancel the absorption terms, proportional to $\varrho_{b_1 b_1}$ or $\varrho_{b_2 b_2}$, independent of statistical properties of the field.

As it stands, the master equation (16), together with the expressions (14) and (15) of the matrix elements of the operators $\delta_\tau^{(a)} \varrho$, $\delta_\tau^{(b)} \varrho$, and $L\varrho$, is the basic equation for the micromaser considered. Note that this equation coincides with the master equation for a micromaser that operates on two-level atoms with the atom-field coupling parameter G, the injection rate $\bar{r}$, and the initial populations $\bar{\varrho}_a$ and $\bar{\varrho}_b$ of the upper and lower levels [31, 32], where

$$\begin{aligned}\bar{r} &= r \left[\varrho_{aa} + \sum_{\alpha,\beta=1}^{2} \left(\varrho_{b_\alpha b_\beta} g_\alpha g_\beta / G^2 \right) \right] \, , \\ \bar{\varrho}_a &= \frac{\varrho_{aa}}{\varrho_{aa} + \sum_{\alpha,\beta=1}^{2} \left(\varrho_{b_\alpha b_\beta} g_\alpha g_\beta / G^2 \right)} \, , \\ \bar{\varrho}_b &= 1 - \bar{\varrho}_a \, .\end{aligned} \tag{17}$$

In terms of the above effective parameters, the dynamics of the field in the degenerate three-level micromaser is identical to that of the field in a two-level micromaser. This result is not surprising and is due to the existence of an atomic trapping state (dark state), $|\Psi_0\rangle = (g_2|b_1\rangle - g_1|b_2\rangle)G^{-1}$. The state $|\Psi_0\rangle$ is not coupled to the upper state $|a\rangle$. Therefore, the Hamiltonian H, given by Eqs. (1) and (2), can be reduced to the Hamiltonian for a two-level system with the upper state $|a\rangle$ and the lower state $|\Psi\rangle = (g_1|b_1\rangle + g_2|b_2\rangle)G^{-1}$.

From Eq. (16), the steady-state solution for the photon distribution $P(n) = \varrho(n,n)$ is found to be [31, 32]

$$P(n) = P(0) \prod_{k=1}^{n} \frac{C n_b + r \varrho_{aa} \sin^2(G\tau\sqrt{k})/k}{C(n_b+1) + r \sum_{\alpha,\beta=1}^{2} (\varrho_{b_\alpha b_\beta} g_\alpha g_\beta / G^2) \sin^2(G\tau\sqrt{k})/k}, \tag{18}$$

where $P(0)$ is determined by the normalization condition $\sum_{n=0}^{\infty} P(n) = 1$. It can be shown that the steady-state mean photon number $\langle n \rangle$ will be larger than the thermal photon number n_b when

$$\frac{\varrho_{aa}}{\sum_{\alpha,\beta=1}^{2} \left(\varrho_{b_\alpha b_\beta} g_\alpha g_\beta / G^2\right)} > \frac{n_b}{n_b+1}. \tag{19}$$

Moreover, according to Ref. [32], the distribution (18) may be sub-Poissonian with one or more than one peak when

$$r\left[\varrho_{aa} - \sum_{\alpha,\beta=1}^{2} (\varrho_{b_\alpha b_\beta} g_\alpha g_\beta / G^2)\right] > \frac{C}{(G\tau)^2}. \tag{20}$$

In general, the condition (20) does not require $\varrho_{aa} > \varrho_{b_1 b_1}$ or $\varrho_{aa} > \varrho_{b_2 b_2}$. For example, if we choose $\varrho_{b_1 b_2} = \varrho_{b_2 b_1} = -(\varrho_{b_1 b_1}\varrho_{b_2 b_2})^{1/2}$ and $\varrho_{b_1 b_1} g_1^2 = \varrho_{b_2 b_2} g_2^2$, then $\sum_{\alpha,\beta=1}^{2}(\varrho_{b_\alpha b_\beta} g_\alpha g_\beta/G^2) = 0$; therefore, the condition (20) becomes $\varrho_{aa} > C/r(G\tau)^2$, that is, there is no need for population inversion between the upper level a and the lower level b_1 or b_2.

In Fig. 2, we show the steady-state photon distribution, calculated from Eq. (18) for $\varrho_{aa} = 0.2$, $\varrho_{b_1 b_1} = \varrho_{b_2 b_2} = 0.4$, $\varrho_{b_1 b_2} = \varrho_{b_2 b_1} = -0.4$, $r/C = 1000$, $n_b = 0.1$, and $g_1 = g_2 = 0.1\tau^{-1}$. The value of the normalized standard deviation $\sigma = (\langle n^2\rangle - \langle n\rangle^2)^{1/2}/\langle n\rangle^{1/2}$ for this distribution is $\sigma = 0.82 < 1$, indicating sub-Poissonian statistics.

In Fig. 3, we plot the normalized standard deviation σ of the distribution (18) as a function of $G\tau$ for the parameters $\varrho_{aa} = 0.2$, $\varrho_{b_1 b_1} = \varrho_{b_2 b_2} = 0.4$, $\varrho_{b_1 b_2} = \varrho_{b_2 b_1} = -0.4$, $r/C = 1000$, $n_b = 0.1$, and $g_1 = g_2 = G/\sqrt{2}$. The result of this figure shows sub-Poissonian statistics of the field ($\sigma < 1$) for some intervals of the interaction time $G\tau$. Note that the minimum value of σ is 0.29 and is reached for $G\tau = 1.44$.

Thus, due to the injected atomic coherence the generation of field states with sub-Poissonian peaked photon distributions is possible in the micromaser operating on three-level atoms without population inversion.

B. Laser

In the laser, the injected atoms stay and spend their life in the cavity. In addition to participating in the laser action, levels a, b_1, and b_2 can decay to other levels. We assume that the Wigner-Weisskopf theory of atomic decay prevails and the decay rates are the same for all three lasing levels: $\gamma_a = \gamma_{b1} = \gamma_{b2} \equiv \gamma$. The distribution $\mathcal{P}(\tau)$ of the atomic lifetime τ is given by

$$\mathcal{P}(\tau) = \gamma \exp(-\gamma\tau). \tag{21}$$

Following the procedure of Scully and Lamb [36], we can write the coarse-grained time derivative of the field density matrix ϱ in the form

$$\dot{\varrho} = r \int_0^\infty \delta_\tau \varrho \mathcal{P}(\tau)\, d\tau + L\varrho. \tag{22}$$

By averaging Eqs. (16) and (14) over the atomic lifetime τ, the master equation for the laser field is found to be

$$\begin{aligned}\dot{\varrho}(n,n') = {} & r\varrho_{aa}\delta^{(a)}\varrho(n,n') \\ & + r\sum_{\alpha,\beta=1}^{2}\left(\varrho_{b_\alpha b_\beta} g_\alpha g_\beta/G^2\right)\delta^{(b)}\varrho(n,n') \\ & + L\varrho(n,n'),\end{aligned} \tag{23}$$

where

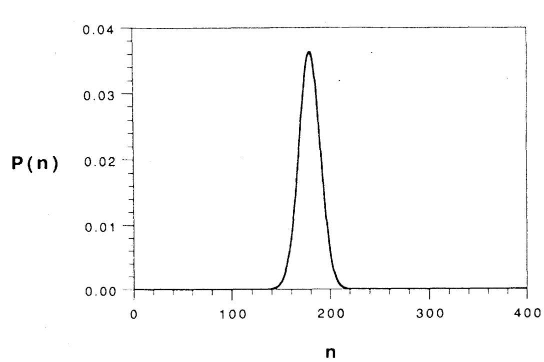

FIG. 2. The steady-state photon distribution of the micromaser, calculated from Eq. (18) for $\varrho_{aa} = 0.2$, $\varrho_{b_1 b_1} = \varrho_{b_2 b_2} = 0.4$, $\varrho_{b_1 b_2} = \varrho_{b_2 b_1} = -0.4$, $r/C = 1000$, $n_b = 0.1$, and $g_1 = g_2 = 0.1\tau^{-1}$.

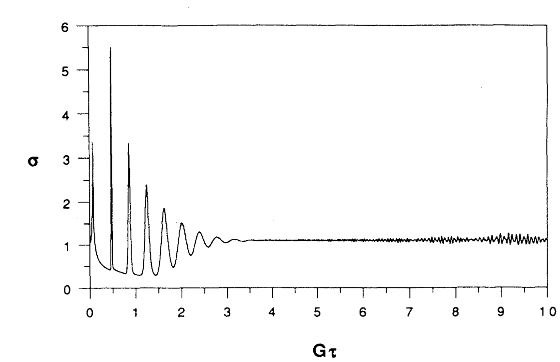

FIG. 3. The normalized standard deviation σ of the distribution (18) as a function of $G\tau$ for the parameters $\varrho_{aa} = 0.2$, $\varrho_{b_1 b_1} = \varrho_{b_2 b_2} = 0.4$, $\varrho_{b_1 b_2} = \varrho_{b_2 b_1} = -0.4$, $r/C = 1000$, $n_b = 0.1$, and $g_1 = g_2 = G/\sqrt{2}$.

$$\delta^{(a)}\varrho(n,n') = -W_c(n+1,n'+1)\,\varrho(n,n') + W_s(n,n')\,\varrho(n-1,n'-1)\,,$$
$$\delta^{(b)}\varrho(n,n') = -W_c(n,n')\,\varrho(n,n') + W_s(n+1,n'+1)\,\varrho(n+1,n'+1)\,, \quad (24)$$

and

$$W_c(n,n') = 1 - \frac{1}{2}\left[1+\left(\sqrt{n}-\sqrt{n'}\right)^2 \frac{G^2}{\gamma^2}\right]^{-1} - \frac{1}{2}\left[1+\left(\sqrt{n}+\sqrt{n'}\right)^2 \frac{G^2}{\gamma^2}\right]^{-1},$$
$$W_s(n,n') = 2\,\frac{G^2}{\gamma^2}\,\sqrt{nn'}\left[1+\left(\sqrt{n}-\sqrt{n'}\right)^2 \frac{G^2}{\gamma^2}\right]^{-1} \times \left[1+\left(\sqrt{n}+\sqrt{n'}\right)^2 \frac{G^2}{\gamma^2}\right]^{-1}. \quad (25)$$

Equation (23) together with Eqs. (24) and (25) is equivalent to the master equation for the field in a two-level laser with the atom-field coupling parameter G, the injection rate $\bar{r}$, and the initial populations $\bar{\varrho}_a$ and $\bar{\varrho}_b$ of the upper and lower levels. The reason is that the atomic state $|\Psi_0\rangle = (g_2|b_1\rangle - g_1|b_2\rangle)G^{-1}$ is trapped and, therefore, the Hamiltonian H, given by Eqs. (1) and (2), can be reduced to the Hamiltonian for a two-level system with the upper state $|a\rangle$ and the lower state $|\Psi\rangle = (g_1|b_1\rangle + g_2|b_2\rangle)G^{-1}$. Analogous to the micromaser case, the interference terms, proportional to $\varrho_{b_1b_2}$ or $\varrho_{b_2b_1}$, may cancel the absorption terms, proportional to $\varrho_{b_1b_1}$ or $\varrho_{b_2b_2}$, independent of statistical properties of the field.

The steady-state solution for the photon distribution in the laser is found from Eq. (23) to be

$$P(n) = P(0)\prod_{k=1}^{n} \frac{n_b + \varrho_{aa}(A/C)\,[1+(B/A)k]^{-1}}{n_b + 1 + \sum_{\alpha,\beta=1}^{2}\left(\varrho_{b_\alpha b_\beta} g_\alpha g_\beta/G^2\right)(A/C)\,[1+(B/A)k]^{-1}}\,, \quad (26)$$

where

$$A = 2r\,(G/\gamma)^2, \qquad B = 4\,(G/\gamma)^2 A\,. \quad (27)$$

Analogous to the micromaser case, the steady-state mean photon number $\langle n\rangle$ in the laser case will be larger than the thermal photon number n_b when the condition (19) is fulfilled. It can be shown that the distribution (26) has a peak when

$$\left[\varrho_{aa} - \sum_{\alpha,\beta=1}^{2}\left(\varrho_{b_\alpha b_\beta} g_\alpha g_\beta/G^2\right)\right]\frac{A}{C} > 1 + \frac{B}{A}. \quad (28)$$

The position of the peak is approximately determined by

$$n_0 = \frac{A}{B}\left\{\left[\varrho_{aa} - \sum_{\alpha,\beta=1}^{2}\left(\varrho_{b_\alpha b_\beta} g_\alpha g_\beta/G^2\right)\right]\frac{A}{C} - 1\right\} - 1\,. \quad (29)$$

In ordinary laser systems, A/B is much larger than unity. Therefore, if

$$n_b = 0\,, \quad \sum_{\alpha,\beta=1}^{2}\left(\varrho_{b_\alpha b_\beta} g_\alpha g_\beta/G^2\right) = 0\,, \quad \varrho_{aa}\frac{A}{C} \gg 1\,, \quad (30)$$

the distribution (26) will approach a Poisson distribution

$$P(n) \approx e^{-\langle n\rangle}\frac{\langle n\rangle^n}{n!}\,, \quad (31)$$

where

$$\langle n\rangle = \varrho_{aa}\frac{A^2}{BC}\,. \quad (32)$$

The conditions (30) do not require inversion of level populations. Thus the generation of field states with Poisson photon distributions is possible in the noninversion one-mode Λ-type laser.

III. CONCLUSIONS

By using the Scully-Lamb quantum theory of the laser [36], we have derived the master equations for the fields in the one-mode Λ-type micromaser and laser with injected coherence. These equations, in terms of effective parameters, are equivalent to those for a two-level micromaser and laser. Furthermore, the interference terms may cancel the absorption terms independent of statistical properties of the field. Such behavior is not surprising because of the existence of an atomic trapping state which is not coupled to the upper level by the field. The steady-state photon distributions have been obtained. Due to the atomic coherence between the two degenerate lower levels, the generation of field states with sub-Poissonian photon statistics is possible in the micromaser without the need for a population inversion. If an equivalent level system is used in a laser operating without population inversion it can produce field states with Poisson distributions of the photon number.

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