

## Self-phase-modulation and high-power steady-state pulses in a weakly ionized coherent amplifier

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Taking into account the nonlinear loss and nonlinear refraction-index change due to medium ionization, high-power pulse amplification is studied under coherent conditions. An exact self-phase-modulated steady-state pulse solution is found for the coupled Maxwell-Bloch equations with a nonlinear loss and refraction-index change caused by the medium ionization, representing a generalization of the known  $\pi$ -pulse solution. The ultimate parameters of an amplifier as maximum pulse energy or minimum pulse duration are estimated and compared with experimental data.

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### I. INTRODUCTION

In recent years much progress has been made in the generation and amplification of high-power ultrashort laser pulses. Recently many efforts have been invested in solid-state systems [1–4] but also gas-discharge laser systems (e.g., high-pressure CO<sub>2</sub> lasers, excimer lasers) have a large potential for higher intensities and shorter pulses with durations below 500 fs, energy densities larger than 1 J/cm<sup>2</sup>, or peak intensities larger than 1 TW/cm<sup>2</sup> [5–7].

The amplification of pulses with the above-mentioned parameters exhibits two special features, which at least in its combination have not been much investigated up to now. The first one originates from the fact that for a pulse duration in the femtosecond region the polarization relaxation time in many active media is comparable or even larger than the pulse duration and following coherent effects play an essential role. In this situation rate equations fail in the description of pulse amplification and the full Maxwell-Bloch equations has to be solved. Coherent-pulse amplification in an inverted laser medium was first studied analytically by Arecchi and Bonifacio [8], who discovered the steady-state  $\pi$ -pulse solution (SSP) for the case of a homogeneously broadened amplifier with linear loss. Coherent phenomena were recently demonstrated in pico- and subpicosecond amplifier experiments in Refs. [9–11].

In high-power amplification experiments a second effect can play a role at such pulse energies. Due to the heating of the gas-discharge electrons by the laser pulse, ionization of the neutral particles in the amplifier takes place. The ionization-produced plasma contributes to the medium nonlinear polarization, which leads to a considerable change of the properties of the pulse. The nonlinear response due to the field-induced ionization depends mainly on the instantaneous pulse energy and occurs within the time scale of the pulse which is much shorter than the characteristic times of relaxation and transport phenomena in the created plasma. The investi-

gation of the pulse evolution in an amplifying medium under the conditions of medium ionization is of considerable interest since the amplification in such extreme regime determines the ultimate limits of pulse parameters as maximum energy or intensity and minimum pulse duration of an amplified pulse.

In the present paper we study the amplification of ultrashort pulses in the coherent region with pulse durations comparable with the polarization relaxation time and under the influence of avalanche ionization. It will be shown that even under the influence of the time-dependent ionization with a nonlinear loss and a nonlinear refraction index change an exact steady-state pulse solution can be found, determining the main pulse properties and ultimate parameters in long amplifiers. The obtained solution describes a self-phase modulated steady-state pulse with a nearly linear chirp and represents a generalization of the known  $\pi$ -pulse solution. The predicted effects and the calculated parameters will be compared with experimental investigations in high-pressure CO<sub>2</sub> amplifiers.

### II. MODEL OF PLASMA PRODUCTION AND NONLINEAR MEDIUM RESPONSE BY AVALANCHE IONIZATION

In order to investigate the propagation of an ultrashort laser pulse in an amplifying medium under the conditions of medium ionization we first discuss in more detail the mechanism of the formation of a nonstationary medium response due to laser plasma production.

Various mechanisms can contribute to the process of ionization in the field of an intense picosecond pulse. One possible mechanism for the ionization is associated with multiphoton and tunneling ionization of atoms by an intense laser field [12]. For picosecond pulses these processes typically take place at intensities two or three orders of magnitude higher than that of a second possible mechanism for ionization—the avalanche ionization [13,14]. This process starts in a gas or a solid in a high-power laser beam if by any physical reason some initial free electrons  $N_{e0}$  appear at the beginning of the laser pulse and these electrons absorb light quanta by the in-

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verse bremsstrahlung if they collide with neutral atoms. The electrons gain enough energy from the laser pulse to allow electron impact ionization to occur. Cascade ionization or electron avalanche follows with the resultant formation of a plasma. The rate of energy gain of a free electron under the influence of the light wave is given by [13]

$$\frac{dW_e}{dt} = \frac{e^2 |E|^2 \tau_c}{2m(1 + \omega^2 \tau_c^2)}, \quad (1)$$

where  $W_e$  is the energy of the electrons,  $E$  is the optical field at frequency  $\omega$ ,  $\tau_c = (N_0 v \sigma_{tr})^{-1}$  is the momentum transfer collision time,  $v$  is the average electron velocity,  $\sigma_{tr}$  the transport cross section,  $N_0$  the number density of atoms, and  $e$  and  $m$  the electron charge and mass, respectively. If the ionization energy of the atoms is  $I_i$ , then using (1) the ionization rate of the atoms by the free electrons is given by

$$r = \frac{qe^2 |E|^2 \tau_c}{2m(1 + \omega^2 \tau_c^2) I_i}, \quad (2)$$

where the factor  $q$  accounts for possible reduction of the ionization rate due to excitation of atomic levels [17]. With the knowledge of the ionization rate the growth of the free plasma electrons is described by

$$\frac{dN_e}{dt} = r(t) N_e, \quad (3)$$

with the solution

$$N_e(t) = N_{e0} \exp \left\{ \int_{-\infty}^t r(|E(t')|) dt' \right\}. \quad (4)$$

The initial free electron density  $N_{e0}$  here is determined by the gas discharge. Since  $r$  mainly depends on the inverse of the collision time,  $\tau_c^{-1} \sim N_0$ , avalanche ionization occurs in not too rarefied media (high-pressure gas, condensed media). For a high laser intensity the exponential growth (4) of plasma electrons leads to the optical breakdown of the gas. However a noticeable change in pulse amplitude and phase can take place even for much lower intensities as the breakdown threshold intensity. In the following we study this region, where the ionization rate takes only small or moderate values ( $r\tau_p < 1$ ) and the exponential function in (4) can be expanded. The response of the electrons to the optical pulse is mainly determined by the contribution of the plasma to the medium dielectric constant described by

$$\epsilon_{pl} = -(1 + i\delta) \frac{N_e(t)}{N_{cr}}, \quad N_{cr} = \frac{m\omega^2(1 + \delta^2)}{4\pi e^2}, \quad (5)$$

where  $N_{cr}$  is the critical density of the plasma and  $\delta = (\omega\tau_c)^{-1}$  is connected with the transfer collision time of the electrons. With the relation (4) the dielectric constant can approximately be rewritten in the form

$$\epsilon_{pl} = \epsilon_1 + \epsilon_2 \int_{-\infty}^t |E(t')|^2 dt' \quad (6)$$

with

$$\epsilon_1 = -(1 + i\delta) N_{e0} / N_{cr}, \quad \epsilon_2 = \epsilon_1 \frac{q\delta e^2}{2m\omega(1 + \delta^2) I_i}.$$

### III. FORMATION OF A STEADY-STATE SELF-MODULATED PULSE IN A WEAKLY IONIZED COHERENT AMPLIFIER

Let us now consider a laser pulse with a complex amplitude  $E(z, t)$  and a carrier frequency  $\omega$  propagating along the  $OZ$  axis in an extended weakly ionized coherent amplifying medium. In describing the pulse amplification we will use the model of a generalized two-level medium with the concentration  $N_a$  of active two-level atoms (molecules) much less than the total medium concentration  $N$ . During the interaction with the high-power pulse the medium is assumed to remain a low-ionized one with the plasma contribution to the dielectric constant  $\epsilon_{pl} = \epsilon_{pl}(E)$  small compared with the nonresonant linear index  $\epsilon_0(\omega) = \eta_0^2(\omega)$ .

In the framework of this model, the propagation of the pulse can be described by the wave equation [15]

$$\frac{\partial E}{\partial z} + \frac{1}{V_0} \frac{\partial E}{\partial t} = -ig\mathcal{P} - \frac{i}{2} \left[ \frac{K_0}{\epsilon_0} \right] \epsilon_{pl}(E)E - \alpha_0 E, \quad (7)$$

where  $V_0$  is the group velocity in the linear medium,  $K_0 = (\omega/c)\eta_0$ ,  $\alpha_0$  is the index of linear nonresonant loss,  $g = 2\pi\omega_{21}N_a\mu/c\eta_0$ ,  $\mu$  is the dipole moment of the resonant transition with the characteristic frequency  $\omega_{21}$ .

The resonant polarization  $\mathcal{P}$  is determined by the solution of the Bloch equations of the homogeneously broadened amplifier [16]

$$\frac{\partial \mathcal{P}}{\partial t} = -i\Delta\omega\mathcal{P} + i\frac{\mu}{\hbar}En - \gamma_2\mathcal{P}, \quad (8a)$$

$$\frac{\partial n}{\partial t} = \frac{\mu}{\hbar} \text{Im}\{E\mathcal{P}^*\}, \quad (8b)$$

where  $n = n_2 - n_1$  is the population difference between the upper (2) and the lower (1) level of the transition  $\Delta\omega = \omega - \omega_{21}$ . In a gas-discharge amplifier the spectral gain profile is much more complex as described by the Lorentzian line in (8) caused by inhomogeneous broadening and vibrational-rotation transitions. Nevertheless for the effects considered here the detailed structure of the gain profile does not play an essential role and we can model such an amplifier by Eqs. (8), where the effective relaxation rate  $\gamma_2$  is related with the full width at half maximum (FWHM)  $\Delta\omega_g$  of the gain profile by  $\gamma_2 = \frac{1}{2}\Delta\omega_g$ . In Eq. (8b) the relaxation of the population of the excited level is neglected. Since the transverse relaxation time in gaseous media is in the range of nanoseconds this approximation is well justified for pulses in the picosecond and femtosecond region ( $\tau_p \ll T_1$ ). The initial conditions of Eqs. (8) are given by

$$n(z, t = -\infty) = +n_0, \quad \mathcal{P}(z, t = -\infty) = 0, \quad (9)$$

where  $n_0$  is the initial inversion on the resonant transition.

In general, the coupled Eqs. (6)–(8) admit only numerical solutions. However, it is well known that without the

influence of the medium ionization after a certain amplification length the amplifier develops a steady-state pulse denoted as  $\pi$  pulse [8], which propagates unchanged through an amplifier. By the  $\pi$  pulses some important features and ultimate parameters as the largest possible pulse energy or the shortest pulse duration in coherent amplification can be studied. On the first view one could assume that the medium ionization with nonlinear loss and refractive index change destroys steady-state pulse propagation. But as we will show, under certain conditions a self-consistent exact steady-state solution of Eqs. (6)–(8) can be found, which can be considered as a generalization of the known  $\pi$ -pulse solution with some analogous properties but also with certain distinct features as a self-phase modulation. We try to find a steady-state solution of Eqs. (6)–(8) using the solution ansatz

$$E(z, t) = A_0 \operatorname{sech}(\xi) \exp\{i\Phi\}, \quad (10)$$

$$\Phi = \kappa z + \beta \ln[\cosh(\xi)],$$

with  $\xi = (t - z/v)\tau_p^{-1}$ . The maximum amplitude  $A_0$ , the pulse duration  $\tau_p$ , the chirp parameter  $\beta$ , the group velocity  $V$ , the phase parameter  $\kappa$ , and the carrier frequency  $\omega = \omega_p$  will be considered as free parameters, self-consistently determined by the Eqs. (6)–(8).

From Eq. (7) it follows that the polarization  $\mathcal{P}$  of the steady-state pulse should have the same phase exponential as the field (10), i.e.,  $\mathcal{P} = P e^{i\phi}$ , and in the limit  $\xi \rightarrow \infty$   $P(\xi)$  should vanish as  $\exp(-\xi)$ . One can find that the solution of Eqs. (8) satisfying the initial conditions (9) and the above requirements is given by

$$P = \frac{n_0[(\Delta\omega/\gamma_2) + i]}{[1 + \tau_p \gamma_2][1 + (\Delta\omega/\gamma_2)^2]^{1/2}} \operatorname{sech}(\xi), \quad (11)$$

$$n = n_0 \left[ 1 - \frac{(1 + \tanh \xi)}{(1 + \tau_p \gamma_2)} \right], \quad (12)$$

where the pulse duration  $\tau_p$  is connected with the pulse amplitude  $A_0$  and the chirp parameter  $\beta$  by the relation

$$\tau_p^2 A_0^2 = \left[ \frac{\hbar}{\mu} \right]^2 [1 + \beta^2], \quad (13)$$

and the frequency detuning  $\Delta\omega = \omega_p - \omega_{21}$  is proportional to the chirp parameter

$$\Delta\omega = -\gamma_2 \beta. \quad (14)$$

Substitution of Eq. (11) into Eq. (7) yields the set of algebraic relations between the remaining parameters of the problem

$$i\kappa A_0 = \frac{in_0 g(\beta - i)}{(1 + \tau_p \gamma_2)(1 + \beta^2)^2} - \alpha_0 A_0 - \frac{i}{2} \left[ \frac{K_0}{\varepsilon_0} \right] A_0 (\varepsilon_1 + \varepsilon_2 A_0^2 \tau_p^2), \quad (15)$$

$$\left[ \frac{1}{V_0} - \frac{1}{V} \right] (i\beta - 1) = -\frac{i}{2} \left[ \frac{K_0}{\varepsilon_0} \right] \varepsilon_2 A_0^2 \tau_p^2. \quad (16)$$

By introducing the notation  $\varepsilon_{1,2} = \varepsilon'_{1,2} + i\varepsilon''_{1,2}$  ( $\varepsilon_i$  and  $\varepsilon''_i$  stand for the real and imaginary part of  $\varepsilon_i$ , respectively), from Eq. (16) we get for the chirp parameter  $\beta$  the expression

$$\beta = \frac{\varepsilon'_2}{\varepsilon''_2} = \frac{1}{\delta}. \quad (17)$$

The pulse duration then can be determined from Eq. (15) and is given by

$$\tau_p = \frac{1}{2\gamma_2} \frac{(1+a)}{(b-1)} \left\{ 1 + \left[ 1 + \frac{4a(b-1)}{(1+a)^2} \right]^{1/2} \right\}. \quad (18)$$

Here

$$a = \gamma_2 \left[ \frac{\hbar}{\mu} \right]^2 \left[ \frac{1 + \delta^2}{\delta^2} \right] \left[ \frac{\varepsilon''_2}{\varepsilon'_1} \right],$$

$$b = \left[ \frac{\delta^2}{1 + \delta^2} \right] G_L / \alpha_L,$$

$G_L = (gn_0\mu/\gamma_2\hbar)$  is the gain in the line center, and  $\alpha_L = \alpha_0 + \alpha_{pl}$  is the effective absorption index, where

$$\alpha_{pl} = \frac{1}{2} \left[ \frac{K_0}{\varepsilon_0} \right] (-\varepsilon''_1)$$

is the linear absorption index of plasma.

First, from expression (18) it follows that the steady-state pulse is formed only if  $b > 1$  or under the condition  $(1 + \delta^{-2})^{-1} G_L > \alpha_L$ . Consequently the linear gain must exceed the linear loss and the formation of the steady-state pulse with the chirp parameter  $\beta = \delta^{-1}$  requires a higher value of gain  $G_L$ .

According to Eq. (16), the pulse velocity obeys the relation

$$\frac{1}{V} = \frac{1}{V_0} + \frac{1}{2} \varepsilon''_2 \left[ \frac{K_0}{\varepsilon_0} \right] \left[ \frac{\hbar}{\mu} \right]^2 \left[ \frac{1 + \delta^2}{\delta^2} \right]. \quad (19)$$

From (19) we can see that in the presence of the nonlinear ionization loss ( $\varepsilon''_2 < 0$ ) the velocity is higher than the pulse velocity  $V_0$  in the linear medium. This is explained by the fact that the nonlinear absorption index  $\varepsilon''_2$  increases with the instantaneous pulse energy, whereas the amplifying component of the medium provides a constant gain during the pulse [see Eq. (11)]. Therefore the pulse front experiences a positive net gain, while the trailing edge has a negative one, which results in the propagation with the effective velocity  $V > V_0$ .

#### IV. DISCUSSION

Before the steady-state pulse (10) with the parameters (13)–(19) are formed the pulse evolution can be roughly described by two characteristic amplifier lengths. The first one is described by the linear loss in the system

$$L_1 = (\alpha_0 + \alpha_{pl})^{-1} = \left[ \alpha_0 + \frac{1}{2} \left[ \frac{K_0}{\varepsilon_0} \right] (-\varepsilon''_1) \right]^{-1}. \quad (20)$$

The second characteristic length is connected with the

nonlinear loss due to the avalanche ionization

$$L_2 = \left[ \frac{K_0}{2\varepsilon_0} |\varepsilon_2''| \int_{-\infty}^{\infty} |E|^2 dt \right]^{-1} \\ = \left[ \frac{K_0}{2\varepsilon_0} |\varepsilon_2''| \frac{\hbar^2 (1+\delta^2)}{\mu^2 \delta^2} \frac{1}{\tau_p} \right]^{-1}. \quad (21)$$

Let us first assume that the condition

$$L_1 \ll L_2 \quad (22)$$

is fulfilled. Then the pulse evolution can be divided into two stages. In the first stage ( $z < L_1$ ) the pulse will be amplified until it reaches its maximum energy. In the limit  $\varepsilon_2 \rightarrow 0$  or  $L_2 \rightarrow \infty$ ,  $\delta \rightarrow \infty$  and at  $z \sim L_1$  this maximum corresponds to a steady-state  $\pi$  pulse [8], with a pulse duration obtained from (18) with  $a \rightarrow 0$  and  $\delta \rightarrow \infty$

$$\tau_{p_1} = \frac{1}{\gamma_2} \frac{1}{\left[ \frac{G_L}{\alpha_L} - 1 \right]}, \quad (23)$$

a pulse energy

$$W_{p_1} = 2A_0^2 \tau_{p_1} = 2 \left[ \frac{\hbar}{\mu} \right]^2 \gamma_2 \left[ \frac{G_L}{\alpha_L} - 1 \right], \quad (24)$$

and

$$\beta = 0, \quad \Delta\omega = 0, \quad V = V_0. \quad (25)$$

If avalanche ionization plays a role but the condition (22) is fulfilled the maximum pulse at  $z \sim L_1$  approximately can be described by the relations (23) and (24), but now the evolution of the pulse for  $z > L_1$  has not reached the steady-state regime. In the intermediate region  $L_1 < z < L_2$  due to the nonlinear loss by the ionization the pulse energy decreases and the pulse duration increases. At  $z \sim L_2$  a self-phase modulated steady-state pulse is formed with parameters given in (13)–(19). Note that in the case  $\alpha_{p1} > \alpha_0$  the parameter  $a$  in (18) can be expressed by  $a \approx (L_1/L_2)\gamma_2\tau_p$  which under the condition (22) can be estimated as small compared with unity. With the condition  $a \ll 1$  from (18), (13) we get

$$\tau_{p_2} = \frac{1}{\gamma_2} \frac{(1+\delta^{-2})}{\left[ \frac{G_L}{\alpha_L} - \left[ 1 + \frac{1}{\delta^2} \right] \right]}, \quad (26)$$

$$W_{p_2} = 2 \left[ \frac{\hbar}{\mu} \right]^2 \gamma_2 \left[ \frac{G_L}{\alpha_L} - (1+\delta^{-2}) \right], \quad (27)$$

$$\beta = \frac{1}{\delta}, \quad \Delta\omega = -\frac{\gamma_2}{\delta}, \quad V = \frac{V_0}{1 - \tau_{p_2} V_0 / L_2}. \quad (28)$$

Note that pulse duration (26) and pulse energy (27) depend only on the plasma parameter  $\delta$  but do not depend on all other plasma parameters as  $\varepsilon_2$ . By the nonlinear loss coefficient  $\varepsilon_2$  the length  $z \sim L_2$  is determined, where the steady-state pulse is formed. Comparing (23) with (26), (24) with (27), and (25) with (28) one can see that the steady-state chirped pulse formed under the influence of

avalanche ionization has a longer pulse duration, a smaller pulse energy, and a narrower pulse spectrum than the  $\pi$  pulse. These different parameters can be explained mainly by the rapid increase in the electron density of the plasma during the optical pulse and the shift of the carrier frequency  $\Delta\omega = -\gamma_2/\delta$ .

A very interesting feature of our obtained exact solution (10) is the self-phase-modulation of the steady-state pulse

$$\dot{\varphi}(t) = \frac{1}{\delta\tau_p} \tanh \left[ \frac{t}{\tau_p} \right], \quad (29)$$

which linearly increases with time [ $\dot{\varphi} \approx t/(\delta\tau_p^2)$ ] in the pulse centrum. This linear chirp can only be explained by the combined action of the coherent amplifier and the variation in the plasma density. Note that without the amplifier and taking into account only the plasma contribution, from (6) and (7) one can derive a self-phase modulation

$$\dot{\varphi}(t) = -\frac{1}{2} \frac{K_0}{\varepsilon_0} \varepsilon_2' |E(t)|^2 z, \quad (30)$$

which is nonlinear time-dependent at the pulse wings but nearly constant in the pulse centrum  $|t| \lesssim \tau_p/2$ . In the presence of an amplifier the relation (30) is approximately valid for small amplifier length  $z \ll L_2$  before the steady-state pulse (10) is formed. The self-phase modulation is also the reason that the area of the steady-state pulse (10)  $\theta = (\mu/\hbar) \int_{-\infty}^{\infty} |E(t)| dt = \pi(1+\beta^2)^{1/2}$  differs from the area of a  $\pi$  pulse.

Let us give some numerical estimates and compare these estimations with some experimental observations in high-pressure regenerative CO<sub>2</sub> amplifiers [5]. The most important parameter is the electron-atom momentum-transfer time  $\tau_c = (N_0\sigma_{tr}v)^{-1}$ , where  $N_0$  is the helium atom concentration. For  $p = 10$  atm the transfer time  $\tau_c \approx 0.6 \times 10^{-13}$  s and  $\delta \approx 0.1$  can be estimated. We assume an initial gas-discharge density  $N_{e0} = 3 \times 10^{12}$  cm<sup>-3</sup>, the ionization energy of the He atom  $I_i = 24.6$  eV,  $q = 0.05$  [18], the dipole moment of the CO<sub>2</sub> amplifying transition  $\mu = 9.2 \times 10^{-30}$  C cm. With the effective polarization relaxation rate  $\gamma_2 = T_2^{-1} \approx 10^{11}$  s<sup>-1</sup> [19] and with the outcoupling of 1% over the resonator length of 300 cm, the linear loss coefficient can be estimated by  $\alpha_L \approx 1.4 \times 10^{-4}$  cm<sup>-1</sup>, while the gain coefficient  $G_1 \approx 0.04$  cm<sup>-1</sup>. From (21)–(25) we find for the  $\pi$ -pulse parameters:  $\tau_{p_1} \approx 35$  fs,  $W_{p_1} \approx 6$  J/cm<sup>2</sup>. On the other hand, the steady-state pulse parameters (13), (17), and (18) yield  $\tau_p \approx 5$  ps,  $W_p \approx 4$  J/cm<sup>2</sup>,  $\beta \approx 10$ ,  $(V/V_0 - 1) \approx 1.5 \times 10^{-5}$ . For the critical lengths for these parameters we find  $L_1 \approx 70$  m,  $L_2 \approx 100$  m.

As an example we compare our predicted parameters with some experimental observations [5] in a high-pressure regenerative CO<sub>2</sub> amplifier. After a certain round trip the output pulse train of the amplifier reached a maximum energy 15 mJ decreasing in the following round trips. With an input pulse duration of 2 ps a minimum output pulse duration of about 600 fs was reached ten pulses after the peak energy pulse. After an

intermediate blue shifting the output pulses experienced a red shifting in the subsequent round trips. In [5] the pulse shortening of a high-power 2 ps pulse to a pulse duration of 600 fs was explained by “plasma pulse compression.” It was assumed that the self-phase modulation by plasma production in combination with negative group velocity dispersion by the NaCl windows and the beam splitter for  $\lambda=9.3 \mu\text{m}$  plays an analog role for pulse shortening as the Kerr effect in fiber-grating compressors. This qualitative explanation was analyzed more quantitatively in Ref. [20]. But note that in distinction to fibers the frequency change caused by the electron density of the plasma depends on the instantaneous pulse energy and, therefore, the frequency chirp depends on the pulse intensity [see (30)]. With  $\varepsilon'_2 < 0$  by this mechanism a blue shift of the carrier frequency  $\Delta\omega = \dot{\varphi}(0)$  can be explained, but the chirp parameter  $\ddot{\varphi}(t=0)$  vanishes. Due to a possible time shift of the maximum amplitude a non-vanishing chirp parameter  $\ddot{\varphi}(t=t_{\text{max}}) \neq 0$  may arise, nevertheless the chirp remains highly nonlinear in time and therefore self-phase modulation by plasma production alone does not present a suitable mechanism for pulse shortening.

Note that for a comparison of our distributed amplifier model with a regenerative amplifier the most essential requirement is that by corresponding pumping conditions after every round trip of the pulse the population inversion is recovered and the plasma is recombined and consequently at every round trip the pulses find the same initial conditions. The most observations of Ref. [5] described above are not directly related to the steady-state region, nevertheless on the basis of our obtained results we can make a qualitative comparison. Because the characteristic length  $L_1$  and  $L_2$  in the discussed experiment can be estimated to be of the same order ( $L_1 \lesssim L_2$ ), the scenario of an intermediate  $\pi$ -pulse formation is practically not realized. Nevertheless one can assume that at the first stage of pulse amplification  $z < L_1$  a coherent shortening mechanism in the kind of  $\pi$ -pulse formation is dominating, which explains the observed shortening up to 600 fs in an intermediate region in Ref. [5]. At this round trip number the self-phase modulation by plasma production can still be considered independently from the amplifier, which explains the observed blue shift at intermediate amplification length. The last part of the pulse train in [5] can be compared with the steady-state parameters (13)–(18). For the amplifier and plasma parameters

as given above we predict for the steady-state a red shift of the carrier frequency of about  $\Delta\omega \approx -10^{12}$  Hz, a pulse energy of  $W_p \approx 4 \text{ J/cm}^2$ , and a pulse duration of  $\tau_p \approx 5$  ps.

By the combined action of coherent amplifier and plasma production the chirp  $\dot{\varphi}(t)$  now is nearly linear time depending. With a corresponding compensation by negative group velocity dispersion one could compress this steady-state pulse up to  $\tau'_p = \tau_p (1 + \beta^2)^{-1/2} \approx 500$  fs. Note that this pulse duration is longer than the pulse duration of a  $\pi$  pulse and refers to the pulses in the final steady-state pulse train region. The pulse shortening in the intermediate pulse train region can only be explained by the coherent interaction mechanism in the kind of  $\pi$ -pulse formation.

## V. CONCLUSION

High-power amplification of ultrashort pulses was studied under the conditions of a weak medium ionization and a coherent interaction of the pulses with the amplifier medium. The nonlinear medium response due to laser plasma production caused by avalanche ionization leads to a nonlinear loss and a nonlinear refraction index change depending on the instantaneous pulse energy. An exact steady-state solution of the coupled Maxwell-Bloch equations with a nonlinear loss and nonlinear refraction index change due to weak plasma production was found. The obtained solution describes the amplified pulses after a long amplification length and can be used for the prediction of the ultimate pulse parameters in high-power amplifiers in the steady-state region. Under such conditions an almost linear chirp with a chirp parameter  $\beta = \delta^{-1}$ , a red shift of the carrier frequency  $\Delta\omega = -\gamma_2/\delta$ , a higher group velocity, an increased pulse duration, and a decreased pulse energy compared with the  $\pi$ -pulse solution were predicted. The calculated parameters were compared with some experimental measurements in  $\text{CO}_2$  high-pressure amplifiers and some observed phenomena as pulse shortening and carrier frequency shifts were explained.

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