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Direct production of electron-positron pairs by 200-GeV/nucleon oxygen and sulfur ions in nuclear emulsion

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Measurements of direct Coulomb electron-positron pair production have been made on the tracks of relativistic heavy ions in nuclear track emulsion. Tracks of O^{16} and S^{32} at 200 GeV/nucleon were studied. The measured total cross sections and energy and emission angle distributions for the pair members are compared to theoretical predictions. The data are consistent with some recent calculations when knock-on electron contamination is accounted for.

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I. INTRODUCTION

Direct electron-positron pair production results from the electromagnetic interaction of relativistic ions with the Coulomb field of target nuclei. This phenomenon has been studied since the 1930s with calculations [1–17], and starting in the 1950s with experimental measurements [18–32], mostly using nuclear track emulsion. Some recent work on this topic has been stimulated by cosmic ray physics where it is considered for use in measuring the energy of extremely relativistic particles [33,34] and from considerations of heavy-ion collider performance, where this phenomenon may result in the reduction of beam luminosity through the bound electron-positron pair production process [35,36].

When predicting the total cross section for the direct pair production by ions in emulsion, the calculations [1–17] have included semiclassical and quantum electrodynamic treatments. The semiclassical approach, visualized with the aid of the Weizsäcker-Williams (WW) method of virtual quanta [4,5], is appropriate since the energy transfer is small compared to the incident energy of the heavy ion. However, one of the shortcomings of the WW method is that it depends on an undetermined quantity corresponding to the minimum impact parameter. Beginning in the 1950s, the calculations [6–9] were based on the Feynman-Dyson formulation of quantum electrodynamics. Wright [37] points out in his critical evaluation of those treatments that differences are found in the low-energy transfer regime (also see Ref. [38]).

The various approaches to calculating pair production from heavy ions differ in a number of respects. The incident particle is treated as a point particle in Refs. [1,3,14–17] and as a plane wave in Refs. [2,6–13]. These two assumptions can be shown to be equivalent after integration over impact parameters. Screening by atomic electrons is included in Refs. [1,6–11,13,15] and it is not included in Refs. [2,3,14,16,17]. Screening becomes increasingly important at higher incident particle energies. Both pair members are assumed to be relativistic (i.e., the Lorentz factor satisfies the condition $\gamma \gg 1$) in Refs. [6–9], but this assumption is not made in Refs. [2,11,13,14,16,17]. Higher-order corrections in target atomic number Z are included in Refs. [10,13,15–17] and are not included in the other papers listed.

To get accurate values for the total cross section and compare with the data presented here at low-energy transfer, the low-energy pairs must be treated accurately. Angular distributions differential only in the emission angle of one of the produced particles are calculated in Refs. [12,14] and not in the other listed references. These distributions are necessary to compare the data presented here with theory. Finally, as a practical matter, the dependence of the cross section on a low-energy transfer cutoff can be obtained from the method used in Ref. [11] and it is difficult to get this information from the other calculations listed even if they are accurate for low-energy transfers. A low-energy transfer cutoff is part of the microscope scanning criteria used in the data presented here and in Refs. [28–31]. To summarize, for purposes of comparison with the data presented here, a treatment is needed that is accurate for low-energy pairs, allows for an adjustable low-energy transfer cutoff, includes atomic screening, estimates

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higher-order corrections, and predicts angular distributions. All these features are only available in the work reported in Refs. [10–12].

The experimental study of direct electron-positron pair production began with the discovery of electron "tridents" in cosmic ray showers in nuclear track emulsions [18–21]. The tridents were identified with the direct pair process ($e^{\pm} \rightarrow e^{\pm} + e^{-} + e^{+}$) and their frequency of occurrence was in approximate agreement with calculations [21] at Lorentz factors of $\sim 10^5$. Pair production was investigated with muons from cosmic ray showers using calorimeters [33,39], covering primary Lorentz factors to $\sim 10^5$ and direct pair energy transfer down to ~ 100 MeV. These results, corrected for bremsstrahlung, were in general agreement with predictions [38]. Experiments were also performed with monoenergetic, singly charged primary accelerator beams including 13.75 GeV electrons [22], 15.8 GeV/c μ leptons [23], 16.2 GeV/c and 200 GeV π mesons [24,25], and 200 GeV protons [26,27]. These measurements used nuclear emulsion with the exception of the 200 GeV π-meson experiment [25], which used a hydrogen-neon bubble chamber to detect direct pairs. The yield of direct pairs in the accelerator exposures with singly charged ions was typically one per several meters of track length. With microscope scanning rates of ≥ 10 cm/h, limited statistics on pairs were collected from the emulsion exposures and the pair yield was significantly below contemporary predictions. The bubble chamber measurement [25], with good counting statistics above a pair momentum threshold of 10 MeV/c, gave a yield close to the calculations based on the work of Ternovskii [7]. The experimental limitations, the disparity between various measurements, and differences in the calculated cross sections prevented a definitive test of theoretical calculations. Recently, heavy-ion beams up to 200 GeV/nucleon became available at the European Center for Nuclear Research (CERN) and experiments have been performed [28–32] that allow comparisons with the calculations in an energy region where prior calculations of the cross section differ significantly (see, e.g., [2,6]).

II. EXPERIMENTAL PROCEDURE

The objective of this work was to provide a measurement of the direct electron-positron pair yield, energy distributions, and angular distributions down to electron-pair energies of several MeV, encompassing most of the total cross section. A stack of 20 thick emulsion plates oriented parallel to the CERN heavy-ion beam was exposed to beams of both O^{16} and S^{32} of energy 200 GeV/nucleon. Each individual emulsion plate consisted of an 800 μm acrylic substrate coated on both sides with 500 μm of Fuji 7B, an electron sensitive emulsion. A nominal beam intensity of 2000–3000 ions per square centimeter was chosen to reduce the primary track density and background from nuclear interactions. There is on average one primary track in an ~ 100 μm field of view with track dip angles $< 2°$ with respect to the emulsion plane. A reference grid, photographed on one side of each plate, contained a pattern of numbers that located the planar position of each primary track and pair event [40]. The measured depth provided the additional coordinate for identification.

Microscopes, equipped with 100× oil-immersion objectives and possessing an overall magnification of either 1000 or 1250, were used to scan primary ion tracks. The segments of beam tracks in the emulsion plates (≈ 4 cm) were traced at rates ranging from 1 mm/h to 1 cm/h while varying the focus over a depth of field of 40 μm centered on the ion track. Any primary track segment that showed a nuclear interaction or was associated with lightly ionizing detritus from a peripheral nuclear interaction upstream was excluded from the study. Systematic observations of electrons (≥ 3 MeV) that originate from relativistic ion tracks in emulsion provide a starting point for a search of a possible companion track that identifies a "candidate" direct pair event. Most electrons observed are single "recoils" from Rutherford scattering. If the projected origin of a companion track was within 2 μm of the initial electron's origin, the event was identified as a possible pair and the energy of each electron was measured by multiple Coulomb scattering methods. If the second electron was determined to be ≥ 1 MeV and no evidence of nuclear interactions was present, the event was included in the pair data. This procedure defines the total electron-positron pair energy to be $\gtrsim 4$ MeV. This energy is a practical lower limit because the uncertainty in the energy measurement progressively degrades below 1 MeV using the multiple Coulomb scattering method. The double difference technique was applied for electron energies above 30 MeV, obtaining accuracies in the range 25–30%. The tangential method was applied below 30 MeV with an error of $\sim 50\%$ for electrons near 1 MeV. The scattering measurement procedures to determine the momentum of the electron are described in Ref. [41]. The emission angle of the individual pair members was measured with an uncertainty varying from 0.5–5.0% depending on whether the electron track lies near the horizontal plane of the emulsion or outside this plane, where the uncertainty in the emulsion shrinkage introduces error.

In examining recoil electrons along the primary ion track, the chance association of two knock-on electrons (KO-KO) that fit the criteria for a candidate electron pair event is a background that varies with the energy of the two individual electrons and the charge and energy of the primary ion species. The chance association of knock-on electrons within 2 μm was treated by a simulation that is described in the Appendix.

III. EXPERIMENTAL RESULTS

In Table I we list the theoretical direct pair interaction path length from recent calculations [10,11] for the oxygen and sulfur tracks in emulsion, both with and without the chance KO-KO electron background for two pair energy thresholds 4 and 10 MeV. Direct pair production from the incident ion interacting with the target atomic electrons has not been included in the calculations, but is thought to be small ($\leq 2\%$). As one can see from Table I, the direct electron-positron pair yield for both ions is contaminated by the chance KO-KO electron back-

TABLE I. Theoretical calculations [11] of the interaction path length for producing direct pairs with pair energies greater than 4 and 10 MeV. The entry denoted by the dagger includes a contribution from the change association of KO-KO electrons within a 2 μm interval, one of which exceeds 3 MeV and the other 1 MeV (see the Appendix). The composite atomic number for the emulsion used in the calculation was 23.

Projectile	Energy (GeV/nucleon)	Theoretical interaction path length (cm) ($E_{\text{pair}} \geq 4$ MeV)	($E_{\text{pair}} \geq 10$ MeV)
S^{32}	200	1.4	1.8
$S^{32\dagger}$	200	0.83	1.1
O^{16}	200	5.8	7.2
$O^{16\dagger}$	200	5.0	6.2

ground, but significantly less so for the oxygen beam.

The measurements in the emulsions corresponding to Table I are presented in Table II. We see that the agreement between the measurement and the theory (Table I, including the KO-KO probability) for the direct pair yield of relativistic heavy ions in emulsion is within one standard deviation. The theoretical predictions from Ref. [11] in Table I give an equivalent total cross section of 17.7 b for sulfur ions and 3.0 b for oxygen ions (including the KO-KO contribution). The total theoretical cross section was reduced by 9% to account for the 4 MeV pair energy threshold of the measurement. The measurements are 19.1 and 3.0 b, respectively, with counting statistics errors of about 10%. The corresponding theoretical direct pair production cross section without KO-KO contamination or pair energy cutoff is 11.5 b for 200 GeV/nucleon sulfur ions and 2.9 b for 200 GeV/nucleon oxygen ions interacting with an emulsion target. However, predictions using the methods of [1,6,7] are at least twice these values.

The predictions in Table I include the effect of atomic screening. Reference [11] predicts that atomic screening reduces the cross section by about 7% and this is in close agreement with Ref. [13] for intermediate absorber Z. Note that calculations using the methods of Refs. [2,14] give results within a few percent for the unscreened case evaluated in Ref. [11]. Higher-order corrections are estimated to give a 2–3 % reduction in the total cross section for intermediate absorber Z [11] and this is in agreement with the recent calculations in Ref. [17]. With our statistical errors, we cannot test these differences in the calculations.

TABLE II. Measurements of the interaction mean free path for "candidate" direct pairs from sulfur and oxygen ions at 200 GeV/nucleon. The length of the ion tracks scanned were 120 and 565 cm and the number of candidate direct pairs above pair energy 4 MeV are 156 and 113, respectively.

Projectile	Energy (GeV/nucleon)	Mean interaction path length (cm) ($E_{\text{pair}} \geq 4$ MeV)	($E_{\text{pair}} \geq 10$ MeV)
S^{32}	200	0.77±0.06	0.94±0.08
O^{16}	200	5.0±0.5	5.8±0.6

In order to further compare the data with theory we show the energy transfer and angle distributions. Figures 1 and 2 show the energy-transfer distributions for oxygen and sulfur compared with theory. The pluses show the results of the simulation described in the Appendix with chance knock-on electrons that appear within 2 μm of each other included. The crosses show the results of the simulation without the knock-on electrons in the energy regions where the KO's make a significant contribution. The measured energy spectra appear to be significantly flatter than the calculated spectra. A major contribution to the apparent discrepancy is the large energy measurement error by the multiple-scattering method, which at higher energies is 25% when sufficient electron track length is available for measurement, but degrades to ~50% for energies near 1 MeV and for electrons with short paths for measurements [41]. This energy-dependent measurement error flattens the observed spectrum. Since event by event differences in error are involved and the observed statistics are low, a simulation including the measurement errors has not been performed. However, the total cross-section measurement is not significantly affected by these momentum errors. The KO-KO chance coincidences explain the overall difference in the sulfur and oxygen energy spectrum in the lower-energy (≤ 35 MeV) pair region.

For the electron and positron emission angles which are measured with an accuracy of 1–5 %, the comparison between calculation and measurement is more revealing. The distribution of emission angles of the pair members is the histogram in Fig. 3 for oxygen and Fig. 4 for sulfur. The pluses and crosses are the results of the simulation as in Figs. 1 and 2, where the calculated emission-angle distribution for direct pair production is from Ref. [12] based on earlier work [10,11]. The KO-KO events do not affect the oxygen data much, but they do explain the

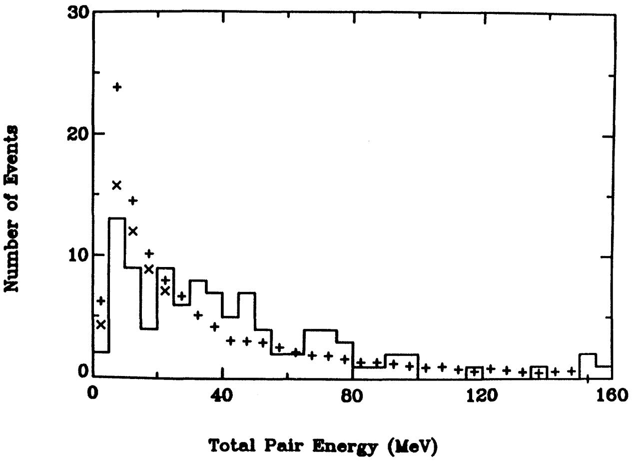

FIG. 1. Energy-transfer distribution for 200 GeV/nucleon oxygen. The histogram represents the data and the pluses are the results of the simulation described in the Appendix which includes double knock-on "false pairs." The crosses are the results of the simulation without false pairs in energy regions where it differs significantly from the results that include the false pairs. Six candidate pairs were in the energy range (160–250) MeV.

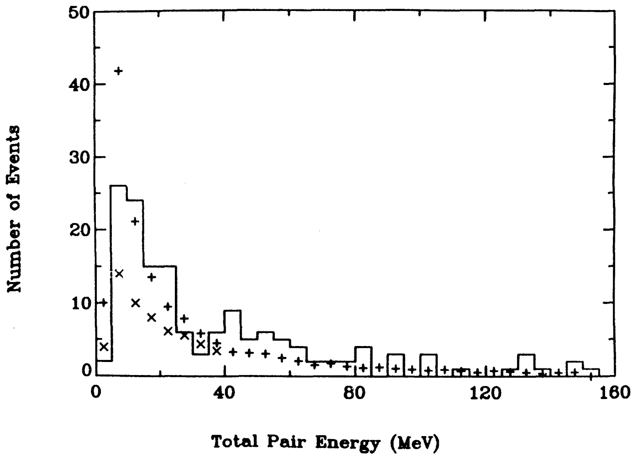

FIG. 2. Same as Fig. 1, except for 200 GeV/nucleon sulfur ions. Three candidate pairs were in the (160–250)-MeV energy region.

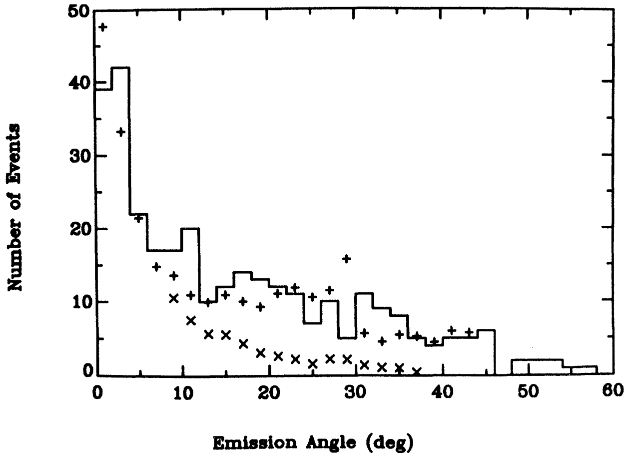

FIG. 4. Same as Fig. 3, except for 200 GeV/nucleon sulfur ions.

large number of events with large angles in the sulfur data. The satisfactory fit to the simulations that include the KO contamination confirms that we have adequately accounted for the KO-KO contribution in the total yields listed in Table I. The peak at 30° in the simulation is due to the 3 MeV cutoff in the scanning criteria, which corresponds to a production angle of 30° derived from the KO energy-angle kinematic relationship (see the Appendix). Candidate pairs with both members having kinetic energy below this value are not included in the simulation. The angle distribution used to generate the simulation from Ref. [12] differs from that of Ref. [14], Fig. 5. However, Fig. 5 in Ref. [14] may actually be for the center-of-mass collider frame so that the same calculation in the laboratory frame would be close to that of Ref. [12].

In the course of this study, a number of other possible background sources have been considered: (a) a single π^0 emission decaying directly to $\gamma + e^- + e^+$ (branching ratio 1.2%) requiring at least ~135 MeV excitation energy; (b) internal pair conversion between nuclear transition states; (c) multiple ionization of atomic electrons with their individual energies all exceeding 1 MeV (several three- and four-prong electron events have been observed whose individual energies were all ≥1 MeV; (d) pair conversion within 1 μm of the primary track from ($d1$) π^0 decay into two γ's and from ($d2$) bremsstrahlung emitted when a relativistic heavy ion interacts with an atomic electron; (e) "white star" events (i.e., multiplicity equal to 2, nuclear peripheral interactions with no target evaporation particles). All these background sources (a)–(e) are judged to be second-order effects when compared to direct electron-positron pair production. In general, our data do not contain off track pair contamination, nor is it influenced by accompanying KO electrons that are not associated with the interaction vertex.

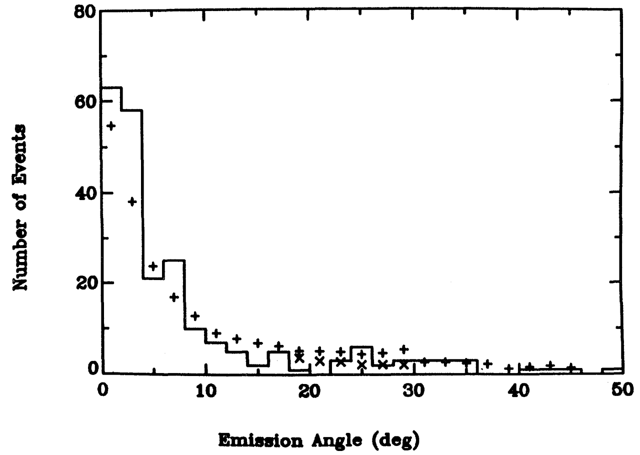

FIG. 3. Distribution of angles that pair members make relative to the incident primary ion for 200 GeV/nucleon oxygen. The pluses and crosses have the same meaning as in Fig. 1.

IV. DISCUSSION

The results of this experiment for the total cross section for the production of direct pairs by oxygen and sulfur ions at 200 GeV/nucleon in track emulsion are consistent with calculations of Ref. [11] when a chance association of KO electrons is included in the calculations. The directly measured angular distributions are consistent with predictions [12] in the angular region where chance KO association makes a negligible contribution (<10°). Above ~10°, where the KO contribution is major or dominant, the measured distributions are consistent with simulations containing the KO's which support the conclusion that their effect has been properly accounted for. The measured energy distributions suffer from large and variable (event by event) experimental errors spreading the peak in the total pair energy, which occurs near 5 MeV, and preventing definitive tests of the pair energy distribution calculations.

Some of the calculations cited in the Introduction give values for the total pair production cross section that differ from Ref. [11]. For example, Ref. [1] Eq. (47); Ref. [6], Eq. (39); and Ref. [7], Sec. 4 all give results that are more than twice as large as those of Refs. [10,11] for intermediate absorber Z and at a primary energy of 200 GeV/nucleon (see Ref. [11] for a detailed discussion of these differences). A previous cross-section measurement [25], which measured pair production (from 200 GeV/nucleon π mesons) above a pair momentum of 10 MeV/c, was stated to be in agreement with the prediction of Ref. [7]. We note that the various calculations tend to converge at higher Lorentz factors. Also shown in Ref. [10] (Fig. 2) is that above 10 MeV the theoretical distributions used for comparison here do not differ much from Ref. [6], Eqs. (23)–(25) and Ref. [7], Eqs. (19)–(21), which have adjustable parameters. The differences are principally in the lower-energy pairs not measured by Ref. [25]. We have included in Table I a prediction [11] for direct pair production for pair energies above 10 MeV both with and without the KO-KO contribution.

Direct comparison with the recent results of Vane *et al.* [32] are hampered by the disparate techniques in experimental measurement and data analysis methods. That experiment used a magnetic field to identify positrons and measure their momentum spectra. A simulation of the experimental arrangement with a Monte Carlo calculation, using a recent theoretical calculation of pair production [14], was used to derive the comparisons with energy spectra and angular distributions. The simulation and data on the positrons were used to derive the total cross section.

To compare the results of [32] with the total cross-section measurement of this experiment we must adjust for the projectiles and targets used in Ref. [32] and also the fraction of the cross section sampled by each measurement. Adjusting to a gold target and sulfur projectile (assuming Z^2 dependence [32]), correcting for the pair energy threshold, and subtracting the predicted KO-KO contamination from our data, for our measurements we find a scaled total cross section of 136 b for oxygen and 146 b for sulfur projectiles with $\sim 10\%$ statistical errors only. The corresponding measurement of Vane *et al.*, adjusted for the fraction of the cross section measured ($\sim 70\%$), was 121 b with an estimated error of 25%. These measurements are all within the bounds of the stated errors. A Monte Carlo evaluation of the terms in a lowest order QED calculation (MCQED calculation) [14,32] predicts a total cross section of 140 b (without screening) while Eby's [11] theoretical value is 136 b (with screening).

The angular distributions directly measured by this emulsion experiment are consistent with the recent calculations of Eby [12] within statistical errors. The method used by Vane *et al.* [32] gives $1/e$ widths for the positron angular distribution in various momentum bins, which is consistent with their MCQED calculations at low momenta. The calculations of Eby [12] give a result very close to their calculation shown in Fig. 2 of Ref. [32]. Direct comparison of energy spectra with Ref. [32] is prevented by our measurement errors which tend to flatten the spectra, and also our poor statistics.

The present measurements of direct pairs using the heavy-ion beams from CERN have greatly improved the experimental situation and generally confirm the recent calculations of the total cross section and angular distribution and the shortcomings of earlier calculations [1,6,7]. Tests of the energy spectra calculations are less clear, although Ref. [32] shows satisfactory agreement of the positron spectra up to ~ 8 MeV/c with the calculations of Ref. [14], but an experimental deficit above this momentum. To clarify this issue as well as probe the finer details in the theoretical development (screening, higher-order corrections [10,11,13,14,17]) will require further experiments using magnetic fields.

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APPENDIX

The problem considered first is the chance association of two knock-on electrons produced by heavy ions in emulsion that occur within 2 μm of each other with both energies above 1 MeV. The first step in estimating the KO-KO electron background is to simulate the point of production of KO electrons along the heavy ion track. If λ is the mean distance between electrons with energy $E > 1$ MeV, then the probability of finding such an electron within the distance dx is dx/λ. Letting $P_0(x/\lambda)$ be the probability of finding no such events in the path length x, then by the laws of probability

$$P_0[(x+dx)/\lambda] = P_0(x/\lambda)[1-(dx/\lambda)] ,$$

where the right-hand side is the probability of finding no events in path length x times the probability of finding no events in path length dx. Expanding the left-hand side yields

$$\frac{dP_0(x/\lambda)}{dx} = -\frac{P_0(x/\lambda)}{\lambda} ,$$

which has the solution

$$P_0(x/\lambda) = e^{-x/\lambda} .$$

If $f(x)dx$ is the probability of finding the first event between x and $x+dx$, then

$$f(x)dx = e^{-x/\lambda}(dx/\lambda) ,$$

where the right-hand side is the probability of finding no events in the distance x times the probability of finding an event in the length dx. That is, the intervals between events are distributed exponentially.

In order to simulate the position of KO electron events ($E > 1$ MeV) simply sample from the exponential distribution $f(x)$. The procedure is to accumulate a number of samples x_i from this distribution until a thickness t of the target is traversed. Then the number of events is determined by the condition

$$\sum_{i=1}^{n} x_i < t < \sum_{i=1}^{n+1} x_i$$

and for differing sets of samples x_i, n will be Poisson distributed. For a given set of x_i $(i=1,\ldots,n)$ the number of x_i's that satisfy $x_i < 2$ μm is determined, which gives the number of chance associated KO-KO electrons with $E_i > 1$ MeV. However, the observation criteria require that one of the energies of each false pair be greater than 3 MeV. So for each false pair the energies are found by sampling the distribution

$$F(E) = \frac{2\pi N Z_1^2 mc^2 r_0^2}{\beta^2 E^2} ,$$

where N is the number of target electrons per cubic centimeter, mc^2 is the rest mass energy of the electron, r_0 is the classical electron radius, and βc is the velocity and Z_1 is the charge of the incident ion. $F(E)$ is the number of electrons per unit path length per unit energy. It follows from the Born approximation to the Mott scattering cross section transformed to the initial rest frame of the electron. The condition $E \ll E_{\text{max}} = 2mc^2\beta^2\gamma^2$ is used, where E_{max} is the maximum energy transferred to the electron. From this distribution a pair of energy samples $E_1, E_2 > 1$ MeV is obtained, where the ones for which both energy samples are less than 3 MeV are eliminated. The number of remaining false pairs gives the number that is compared with the data, along with the energy samples for each false pair. The emission angles are determined from the equation

$$\cos(\theta_i) = \left[\frac{E_i}{E_i + 2mc^2} \right]^{1/2} ,$$

which follows from the conservation of energy and momentum. Atomic binding effects have been neglected in this treatment. This should be adequate for electrons with $E > 1$ MeV since the maximum binding energies for the target atoms in emulsion are at least a factor of 40 smaller than this.

The average of approximately 100 histories of the incident ion traversing the actual path length scanned in the emulsion constituted the simulations in Figs. 1–4. This suppresses the fluctuations by a factor of about 10, facilitating comparisons with the data. The quantity λ is determined by the equation

$$\frac{1}{\lambda} = \int_{1\ \text{MeV}}^{\infty} F(E)\,dE .$$

At higher projectile energies, the KO-KO contamination becomes smaller relative to the direct pair yield and should be less of a problem. Restricting emission angles to smaller values or using the energy-angle correlations could be used to eliminate this contamination in the future, provided there is a sufficient number of observed direct pair events.

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