RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 50, NUMBER 2 AUGUST 1994

Amplification of vacuum fluctuations by two-beam coupling in atomic vapors

Martti Kauranen,* Alexander L. Gaeta,† and Robert W. Boyd
The Institute of Optics, University of Rochester, Rochester, New York 14627

Girish S. Agarwal
School of Physics, University of Hyderabad, Hyderabad 500 134, India

(Received 8 April 1994)

We show both theoretically and experimentally that fluctuations of a weak probe beam are increased above the coherent-state shot-noise level as the beam interacts with a strong degenerate pump beam in an atomic vapor. The origin of this noise amplification process is the two-beam-coupling process between the pump beam and the vacuum sidebands of the probe beam. The major contribution to this excess noise arises from the quantum fluctuations of the atomic medium that are induced by the pump beam. We believe that this mechanism can be important in preventing the reduction of noise below the quantum-noise limit in experiments that utilize atomic vapors.

PACS number(s): 42.50.Dv, 42.65.Es

Nonlinear optical techniques make it possible to tailor the quantum fluctuations of an electromagnetic field in such a way that quantum noise in an optical measurement is reduced. One such technique is quadrature squeezing, in which the noise in one of the field quadratures is suppressed below the coherent-state noise level [1]. The noise in the other field quadrature is then necessarily amplified above the quantum-noise limit so that the uncertainty principle is not violated. Another technique of reducing the noise of an optical measurement is amplitude squeezing. One method of producing amplitude-squeezed light is by appropriate control of the injection current that drives a semiconductor laser [2]. Amplitude-squeezed light can also be generated through the use of so-called "twin beams" [3]. Twin beams are two beams with correlated intensity fluctuations such that the fluctuations in the intensity difference of the two beams are below the shot-noise limit. One of the beams can then serve as a reference for the intensity fluctuations of the other beam. This property has recently been utilized to stabilize the intensity fluctuations of one of the beams to below the shot-noise limit to produce amplitude-squeezed light [4]. Twin beams are the high-intensity version of correlated photon pairs [5] that have been used to test fundamental properties of the quantum field [6]. Both quadrature-squeezed states and twin beams have been experimentally realized through the use of second-order or third-order nonlinear materials. In the case of optical parametric amplifiers [7] and oscillators [8] utilizing second-order nonlinear crystals, quantum-noise reduction approaching 90% has been demonstrated [9]. On the other hand, schemes that utilize third- (and higher-) order resonant nonlinearities of atomic vapors [10–12] typically achieve a noise reduction of less than 50%. The inability of the schemes utilizing atomic vapors to achieve large quantum-noise reduction is usually attributed to effects such as atomic fluctuations [13,14] and self-focusing [12,15].

In this Rapid Communication, we present the results of a detailed experimental and theoretical investigation that further elucidates the role of quantum fluctuations in limiting the amount of noise reduction that can be achieved when one utilizes atomic vapors as the nonlinear medium. We focus our attention on the two-beam-coupling process [16,17], which occurs between a single pump wave and a single probe wave as they interact in an atomic medium. In particular, the Rayleigh feature of the two-beam-coupling process is found to increase the number of photons in the probe beam at frequencies that are displayed by less than $\sim 1/T_1$ (where T_1 is the population lifetime of the atoms) from the pump frequency. In the case of a probe beam that is degenerate in frequency with the pump beam, this mechanism therefore gives rise to increased low-frequency (typically less than 100 MHz) intensity fluctuations of an initially shot-noise-limited probe field as the additional photons beat against the carrier mode of the probe beam. This noise mechanism is, in principle, included in several theoretical formulations of wave-mixing processes in atomic media [13,18–22]. However, no detailed experimental investigation of this excess noise mechanism has been performed. We have chosen to adapt the theoretical model of Agarwal [21,22] to explain our experimental results, since this model has the additional advantage that it conveniently distinguishes between the two different physical mechanisms that can contribute to the increase in noise. The first such mechanism is the direct amplification of the vacuum fluctuations of the incoming probe beam as a result of the semiclassical two-beam-coupling gain experienced by the vacuum sidebands of the probe beam. This mechanism occurs only for the case in which the intensity of the pump wave exceeds or is comparable to the saturation intensity of the atomic transition. The other mechanism is the spontaneous scattering of light as a result of quantum fluctuations in the response of the atomic medium, which results in resonance fluorescence radiation being emitted in the spatial mode of the probe beam. In principle, this mechanism can occur in an unsaturated medium. However, in an opti-

---

*Present address: Laboratory of Chemical and Biological Dynamics, University of Leuven, B-3001 Heverlee, Belgium.

†Present address: School of Engineering and Applied Physics, Cornell University, Ithaca, NY 14850.

1050-2947/94/50(2)/929(4)/$06.00

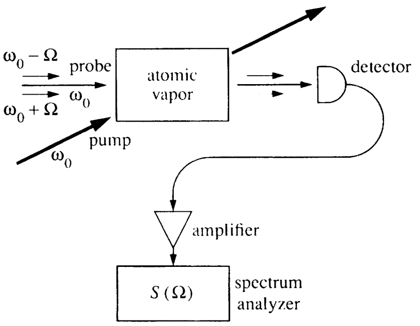

FIG. 1. A pump beam at frequency ω_0 and a degenerate probe beam with vacuum sidebands at frequencies $\omega_0 \pm \Omega$ interacting in an atomic vapor cell. The transmitted probe beam is detected and the resulting photocurrent is amplified and spectrally analyzed.

cally thick medium such as that considered here, the scattered light as well as the probe beam will be absorbed without the presence of a saturating pump beam.

Our theoretical description of the two-beam-coupling noise amplification mechanism is based on the schematic setup shown in Fig. 1, in which a strong pump beam and a weak probe beam at the same frequency ω_0 interact in an atomic vapor cell. The transmitted probe beam is detected and the resulting photocurrent is amplified and spectrally analyzed. To determine the power spectrum $S(\Omega)$ of the fluctuations of the photocurrent, we calculate the absorption or the gain experienced by the probe beam at the frequency ω_0 and the number of noise photons generated by the two-beam-coupling process at the sideband frequencies $\omega_0 \pm \Omega$. The interaction of the probe beam and its sidebands with the two-beam-coupling amplifier, whose gain is frequency dependent, is illustrated in Fig. 2. More specifically, we assume that the incoming probe field is composed of many modes that are all in the vacuum state and a mode at frequency ω_0 that is in the coherent state $|\alpha\rangle$. For the case of a quantum amplifier, the annihilation operators $\hat{A}_\omega$ for the outgoing modes of the probe field can be expressed in terms of the operators $\hat{a}_\omega$ for the incoming modes as [19]

$$\hat{A}_\omega = g(\omega)\hat{a}_\omega + \hat{L}_\omega, \tag{1}$$

where $g(\omega)$ is the amplitude gain of the amplifier at frequency ω and $\hat{L}_\omega$ is a Langevin noise operator. The coupling to the Langevin noise operator is necessary to preserve the commutation relations $[\hat{A}_\omega, \hat{A}^\dagger_{\omega'}] = \delta_{\omega\omega'}$ for the outgoing modes of the probe field.

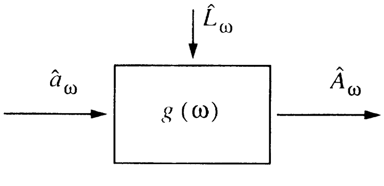

FIG. 2. A quantum-noise-limited amplifier with frequency-dependent gain $g(\omega)$. The noise operator $\hat{L}_\omega$ represents the reservoir mode of the amplifier.

We have employed multimode techniques [23] to calculate the power spectrum of the photocurrent at a nonzero spectrum analyzer frequency Ω. The result of this calculation is given by the equation [24]

$$S(\Omega) = |g(\omega_0)|^2[1 + \langle \hat{n}_{\omega_0-\Omega}\rangle + \langle \hat{n}_{\omega_0+\Omega}\rangle], \tag{2}$$

where $S(\Omega)$ is normalized to the shot-noise level of the incoming probe field and where the number of photons at the frequency sidebands is $\langle \hat{n}_{\omega_0 \pm \Omega}\rangle = \langle \hat{L}^\dagger_{\omega_0\pm\Omega}\hat{L}_{\omega_0\pm\Omega}\rangle$. The first term in square brackets in Eq. (2) represents the shot noise of the output signal. The second and third terms represent the number of photons added spontaneously to the lower and upper frequency sidebands of the probe beam as a consequence of the two-beam-coupling process. We have adapted the full quantum-mechanical treatment of the two-beam-coupling process of Ref. [22] to determine the numerical values of the gain $g(\omega_0)$ and the number of photons at the two sidebands $\langle \hat{n}_{\omega_0\pm\Omega}\rangle$. Our adaptation of this model for an inhomogenously broadened atomic vapor includes the calculation of the Doppler average of the correlation functions $\tilde{C}^{+-}$ and $\tilde{Q}^{+-}$ of Ref. [22] with a phenomenological account of grating-washout effects [25,26] for each velocity group. The correlation function $\tilde{C}^{+-}$ is related to the semiclassical gain of the two-beam-coupling amplifier and hence it accounts for the part of the fluctuations of the transmitted probe beam that corresponds to a quantum-noise-limited amplifier, i.e., this part corresponds to the direct amplification of the vacuum fluctuations of the incoming probe beam at the sideband frequencies. The correlation function $\tilde{Q}^{+-}$, on the other hand, is related to the quantum fluctuations of the atomic medium, i.e., to the amount of resonance fluorescence radiation resulting from the presence of the pump beam that is emitted in the spatial mode of the probe beam at the sideband frequencies.

To study the noise properties of the transmitted probe beam experimentally, we used potassium vapor as the nonlinear medium. The pump and probe beams were derived from a single-mode continuous-wave dye laser that was tuned close to the $4^2S_{1/2} \rightarrow 4^2P_{3/2}$ transition of potassium at a wavelength of 767 nm. The two beams intersected inside a 0.5-cm-long potassium vapor cell. The temperature of the cell was ~175 °C, which corresponds to a potassium number density of $\sim 4 \times 10^{13}$ cm^{-3}. The transmitted probe beam was detected with a fast photodiode, and the resulting photocurrent was amplified and spectrally analyzed. The reflection of the transmitted probe beam off the front window of the fast photodiode was used to monitor the average power of the probe beam. Our detection system was shot-noise-limited for a range of detected light powers from ~0.5 to ~10 mW and for a range of frequencies of 2–3 MHz up to the detection bandwidth of ~100 MHz except for some isolated frequencies where the detection system displayed excess noise. The sources of excess noise were traced to local radio stations or fluctuations of the dye laser. The frequencies with excess noise have been excluded from the data presented below. We note that in some respect our experiment is similar to that of Ref. [14], in which resonance fluorescence from an optically thin sample was beaten against a local oscillator to study the limits placed on squeezing by resonance fluorescence. The results of this experiment were ex-

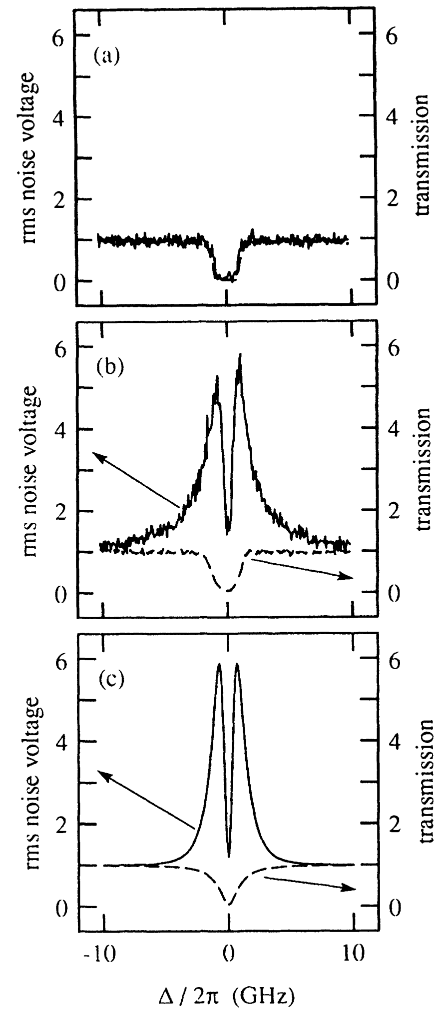

FIG. 3. The average power (dashed line) and the rms noise voltage (solid line) of the transmitted probe beam as a function of laser detuning Δ from resonance. The noise voltage is measured in a narrow frequency band centered at 10 MHz and is normalized to unity for the off-resonance shot-noise level. When the pump beam is off (a), the noise behaves as expected of an attenuated coherent state. When the pump beam is on (b), the absolute noise level is increased although a significant portion of the probe beam is absorbed. Theoretical prediction (c) corresponds to case (b).

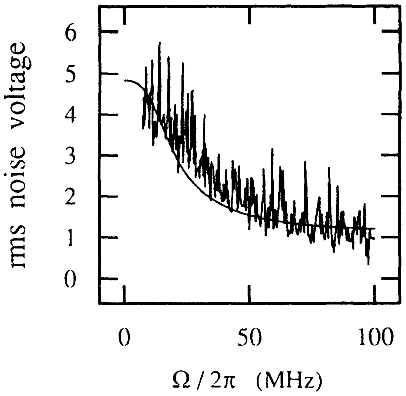

FIG. 4. The rms noise voltage of the transmitted probe beam as a function of the spectrum analyzer frequency Ω for the case where the laser was detuned by ~1.5 GHz to the blue side from the potassium line center. The voltage is normalized to unity for the off-resonance shot-noise level. The smooth line is the theoretical prediction and the noisy line is the experimental result.

plained using a phenomenological model that included an account of the modification of spectral distribution of resonance fluorescence due to atomic motion. In contrast to that, the atomic medium in the experiment of the present paper was optically thick and our theoretical description of the experiment is based on a detailed quantum-mechanical theory of the two-beam-coupling process including propagation effects. Also, in our experimental arrangement, the probe beam transmitted through the atomic vapor is used as the local oscillator for its own sidebands.

In Fig. 3 we show the experimental and theoretical results for the average power and the rms noise voltage in a narrow frequency band centered at 10 MHz of the transmitted probe beam as a function of the laser detuning from potassium line center. The transmitted power and the noise voltage are normalized to the off-resonance signal levels. For these particular measurements, the crossing angle between the pump and probe beams was 2.6°, and the pump intensity was ~20 W/cm^2. Figure 3(a) shows the experimental results when the pump beam is blocked. Close to resonance the probe beam is completely absorbed and the noise level of the transmitted probe beam behaves as expected of a field in a coherent state. In the presence of the pump beam [Fig. 3(b)], however, the absolute noise level of the transmitted probe beam is amplified when the laser is tuned close to resonance, even though a significant portion of the probe beam is absorbed. Figure 3(c) shows the theoretical prediction based on Eq. (2) for the noise of the transmitted probe beam in the presence of the pump field. For a best fit between the experimental and theoretical results, we assumed that $\alpha_0 L = 2000$ (where α_0 is the line-center absorption coefficient and L is the length of the cell) and that the ratio of the dephasing time and population lifetime was $T_2/T_1 = 0.3$. These values are in good order-of-magnitude agreement with the best estimates of the values of these parameters under our experimental conditions. We further assumed that the intensity of the pump beam entering the medium was 17 W/cm^2. The general agreement between the experimental data and the theoretical prediction is seen to be good, although the width of the noise feature in the theoretical plot is somewhat narrower than that observed experimentally. We attribute the slight asymmetry of the experimental noise plot on the blue and red sides of the line center to the effects of self-focusing and self-defocusing, respectively. We also note that, according to our theoretical model, the dominant contribution to the excess noise under our experimental conditions arises from the fluctuations of the atomic medium.

For further evidence of the two-beam-coupling mechanism, we have also studied the dependence of the intensity fluctuations of the transmitted probe beam on the spectrum analyzer frequency. In Fig. 4 we show the experimental and theoretical results for the rms noise voltage of the transmitted probe beam in a narrow frequency band normalized to the off-resonance shot-noise level as a function of the spectrum analyzer center frequency for the case where the laser was detuned by ~1.5 GHz to the blue side from exact resonance.

The experimental conditions and the parameters used in the theoretical model are the same as in Fig. 3 and for the best fit with the experimental result we assumed a detuning of 1.2 GHz in the theory. The noise level is seen to decrease and approach the shot-noise limit as a function of the frequency but to exceed the shot-noise limit for all frequencies studied. The noise level was also found to be sensitive to the crossing angle between the pump and probe beams, in agreement with the expected results for the two-beam-coupling process in a Doppler-broadened medium [27].

In conclusion, we have investigated a mechanism that we believe can be important in limiting the amount of quantum-noise reduction that can be obtained by schemes utilizing four-wave mixing in atomic vapors. In such noise-reduction experiments, the four-wave-mixing process is expected to give rise to correlated fluctuations in the intensities of the signal and idler beams. However, the two-beam-coupling process that occurs between the vacuum sidebands of the signal or idler beam and a single pump beam at a time gives rise to uncorrelated fluctuations in the intensities of the signal and idler beams. We believe that these uncorrelated fluctuations are sufficient to prevent large amounts of noise reduction due to the four-wave-mixing process from occurring. Our full quantum-mechanical description of this noise amplification process is in very good agreement with the experimental results. We expect that the amount of excess noise due to this mechanism can be reduced by looking at the noise at frequencies higher than ~100 MHz [10], which is the typical width of the Rayleigh feature of the two-beam-coupling process. On the other hand, the frequency should be smaller than the Rabi frequency corresponding to the intensity of the pump beam, since a similar noise amplification mechanism is expected to take place at the Rabi sidebands.

We gratefully acknowledge useful discussions of this work with W. V. Davis. This work has been supported by the U.S. Army Research Office. G.S.A. also thanks NSF, Grant No. INT 9100685, for supporting this collaborative effort.

---

[1] See the feature issue (October) of J. Opt. Soc. Am. B. **4** (1987); D. F. Walls, Nature **306**, 141 (1983); H. P. Yuen, Phys. Rev. A **13**, 2226 (1976).
[2] Y. Yamamoto, N. Imoto, and S. Machida, Phys. Rev. A **33**, 3243 (1986).
[3] H. P. Yuen, Phys. Rev. Lett. **56**, 2176 (1986); A. Heidmann, R. J. Horowicz, S. Reynaud, E. Giacobino, C. Fabre, and G. Camy, *ibid.* **59**, 2555 (1987).
[4] J. Mertz, A. Heidmann, C. Fabre, E. Giacobino, and S. Reynaud, Phys. Rev. Lett. **64**, 2897 (1990).
[5] D. C. Burnham and D. L. Weinberg, Phys. Rev. Lett. **25**, 84 (1970); S. Friberg, C. K. Hong, and L. Mandel, *ibid.* **54**, 2011 (1985).
[6] For reviews, see L. Mandel, Phys. Scr. **T12**, 34 (1986); Z. Y. Ou and L. Mandel, Quantum Opt. **2**, 71 (1990).
[7] O. Aytür and P. Kumar, Phys. Rev. Lett. **65**, 1551 (1990).
[8] L.-A. Wu, H. J. Kimble, J. L. Hall, and H. Wu, Phys. Rev. Lett. **57**, 2520 (1986).
[9] J. Mertz, T. Debuisschert, A. Heidmann, C. Fabre, and E. Giacobino, Opt. Lett. **16**, 1234 (1991).
[10] R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. Lett. **55**, 2409 (1985).
[11] M. Vallet, M. Pinard, and G. Grynberg, Europhys. Lett. **11**, 739 (1989).
[12] M. W. Maeda, P. Kumar, and J. H. Shapiro, J. Opt. Soc. Am. B **4**, 1501 (1987).
[13] M. D. Reid and D. F. Walls, Phys. Rev. A **31**, 1622 (1985); **34**, 4929 (1986).
[14] R. E. Slusher, L. W. Hollberg, B. Yurke, J. C. Mertz, and J. F. Valley, Phys. Rev. A **31**, 3512 (1985).
[15] S. T. Ho, N. C. Wong, and J. H. Shapiro, Opt. Lett. **16**, 840 (1991).
[16] See, for example, R. W. Boyd, *Nonlinear Optics* (Academic, San Diego, 1992).
[17] B. R. Mollow, Phys. Rev. A **5**, 2217 (1972); F. Y. Wu, S. Ezekiel, M. Ducloy, and B. R. Mollow, Phys. Rev. Lett. **38**, 1077 (1977); D. J. Harter and R. W. Boyd, IEEE J. Quantum Electron. **QE-16**, 1126 (1980); R. W. Boyd, M. G. Raymer, P. Narum, and D. J. Harter, Phys. Rev. A **24**, 411 (1981); M. T. Gruneisen, K. R. MacDonald, and R. W. Boyd, J. Opt. Soc. Am. B **5**, 123 (1988); M. T. Gruneisen, K. R. MacDonald, A. L. Gaeta, and R. W. Boyd, Phys. Rev. A **40**, 3464 (1989).
[18] C. M. Caves, Phys. Rev. D **26**, 1817 (1982); H. P. Yuen and J. H. Shapiro, IEEE Trans. Inf. Theory **IT-24**, 657 (1978).
[19] S. Stenholm, Phys. Scr. **T12**, 56 (1986); Y. Yamamoto and H. A. Haus, Rev. Mod. Phys. **58**, 1001 (1986).
[20] M. Sargent III, D. A. Holm, and M. S. Zubairy, Phys. Rev. A **31**, 3112 (1985); S. Stenholm, D. A. Holm, and M. Sargent III, *ibid.* **31**, 3124 (1985).
[21] G. S. Agarwal, Phys. Rev. A **34**, 4055 (1986); G. S. Agarwal and R. W. Boyd, *ibid.* **38**, 4019 (1988).
[22] A. L. Gaeta, R. W. Boyd, and G. S. Agarwal, Phys. Rev. A **46**, 4271 (1992).
[23] B. Yurke, Phys. Rev. A **32**, 300 (1985); **32**, 311 (1985); R. Horrowicz, M. Pinard, and S. Reynaud, Opt. Commun. **61**, 142 (1987).
[24] M. Kauranen, A. L. Gaeta, and R. W. Boyd, Opt. Commun. **103**, 211 (1993).
[25] M. Kauranen and R. W. Boyd, Phys. Rev. A **44**, 584 (1991).
[26] G. S. Agarwal (unpublished).
[27] M. T. Gruneisen, Ph.D. thesis, University of Rochester (1989).