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Realization of trapping in a two-level system with frequency-modulated fields

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We demonstrate the existence of population-trapping states in a two-level system driven by a frequency-modulated field. We present detailed numerical results on trapping and also on jumps in the system which occur when the energy levels cross.

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The trapping states of a system occupy a very special place in quantum optics. A large number of papers have discussed the importance and applications of coherent population-trapping states [1] which occur in a Λ system driven by two semiclassical fields. The trapping states of a Λ system driven by quantized fields have been very recently discovered [2]. Most of the existing investigations discuss the trapping states in three-level systems. The two-level systems so far are not known to exhibit trapping states, except when the two-level atom is interacting with a quantized field [3].

In this Rapid Communication we show that we can realize trapping states in two-level systems by driving the system by frequency modulated semiclassical fields. We present both analytical and numerical results.

In order to keep the analysis as simple as possible, we consider the frequency-modulated field on resonance with the frequency of the atomic transition. The interaction Hamiltonian in the interaction picture can be written as

$$H=\hbar\left(\frac{g}{2}S^{+}e^{-i\Phi(t)}+\text{H.c.}\right), \tag{1}$$

$$\Phi(t)=M\sin(\Omega t), \tag{2}$$

where M and Ω are the index of modulation and the frequency of modulation, respectively. The $S^{\pm},S^{z}$ are the spin-1/2 angular momentum operators for the two-level system and g is the Rabi frequency. On using the generating function for the Bessel functions

$$\exp(iz\sin\theta)=\sum_{k=-\infty}^{+\infty}e^{ik\theta}J_k(z), \tag{3}$$

the interaction can also be written as

$$H=\frac{\hbar g}{2}\sum_{k=-\infty}^{+\infty}e^{-ik\Omega t}J_k(M)S^{+}+\ \text{H.c.} \tag{4}$$

Assuming that Ω is large, we can make a second rotating-wave approximation, leading to

$$H\simeq\frac{\hbar g}{2}J_0(M)S^{+}+\text{H.c.} \tag{5}$$

For weak coupling g and large Ω one expects this rotating-wave approximation to be a good one. Note further that if M is chosen such that

$$J_0(M)=0, \tag{6}$$

then the interaction Hamiltonian (5) vanishes and in such a case one is left only with the fast oscillating terms in (4). Clearly, under these conditions one would expect that no dynamical evolutions will take place on a time scale that is slower than the scale of periodic exponentials in (4) and that the populations will thus be trapped on this slow scale. For time $t\approx\pi/2\Omega$ (say), the other exponentials in (4) become important and they would lead to a transition between the two states of the system. Again one would expect trapping in the time interval $(\pi/2\Omega,\pi/\Omega)$. We will verify these qualitative results by integrating numerically the time-dependent Schrödinger equation. The jump at time $\pi/2\Omega$ can also be understood in terms of the crossing of the energy levels. We can transform (1) into a frame that is rotating with the instantaneous frequency of the field; then the effective Hamiltonian becomes

$$H_{\text{eff}}=\hbar M\Omega\cos(\Omega t)S^{z}+\left(\frac{\hbar g}{2}S^{+}+\text{H.c.}\right). \tag{7}$$

The two bare levels cross whenever $\cos(\Omega t)=0$, i.e., when $t=n\pi/2\Omega$ ($n=$integer). The Landau-Zener theory [4] predicts that a transition from the ground to the excited state will occur with a probability

$$p=1-e^{-2\pi\kappa},$$

$$\kappa=\frac{\left(\frac{\hbar g}{2}\right)^2}{\hbar\left|\frac{d}{dt}[\hbar M\Omega\cos(\Omega t)]\right|}\approx\frac{g^2}{4M\Omega^2},\quad \sin(\Omega t)\approx 1. \tag{8}$$

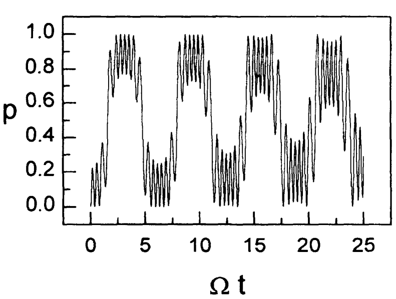

FIG. 1. Probability p of excitation as a function of Ωt, where we choose $M=14.930\,917\,708\,6$ the fifth zero of J_0 and $g/\Omega=8.0$; $\gamma/\Omega=0.0$ (i.e., in the absence of spontaneous emission).

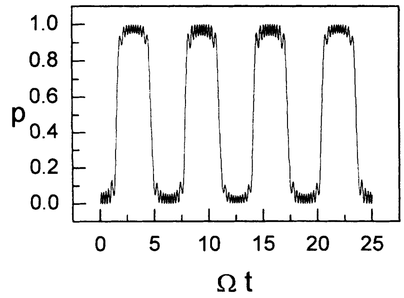

FIG. 2. Probability p of excitation as a function of Ωt, where we choose the tenth zero of J_0, i.e., $M=30.634\,606\,468\,4$; $g/\Omega=8.0$; $\gamma/\Omega=0.0$ (i.e., in the absence of spontaneous emission).

We next discuss the numerical results, which support the above findings. It should be noted that the response of the two-level atom subject to a bichromatic field has been very extensively studied [5–8]. However, most of the literature concerns the steady state, though some papers deal explicitly with the transient response [9,10]. In light of our earlier discussion we concentrate on the trapping situation; i.e., when the condition (6) is satisfied. We assume that the system has a very long lifetime and that the atom is in the ground state at time $t=0$. Some typical results are shown in Figs. 1 and 2, where the probability of excitation is plotted as a function of Ωt. In the interval $0<\Omega t<\pi/2$ the atom remains trapped in the ground state except for the small oscillations at fast time scales. At $\Omega t=\pi/2$ the atom can make a transition to the excited state. However, here in the region $\pi/2<\Omega t<\pi$ we find that the atomic polarization (Fig. 3) is also significant. Thus the atomic state in this region is a coherent state [11]. In Fig. 3 we show different contour plots. Other initial conditions, like the atom prepared in a dressed state, lead to similar behavior. Using Figs. 1 and 2 we have made an estimate of the jump probability at $t=\pi/2\Omega$. This result agrees closely with the approximate result (8). Finally in Fig. 4 we show the effect of the atomic spontaneous emission, which is expected to change the characteristics of the trapping state, particularly if $\gamma\sim\Omega$. Thus the conditions under which the trapping states can be observed are

$$\Omega\gg\gamma;\quad J_0(M)=0,\quad t\sim\frac{\pi}{\Omega}. \tag{9}$$

In earlier transient experiments Golub and Mossberg [10] used the Yb atom transition, which has $\tau\sim 875$ nsec. This atom seems suited to observing the trapping states discussed here, as we can use $\Omega\sim 1$ MHz and observation time in the range of nanoseconds to microseconds.

Thus in conclusion we have demonstrated the possibility of producing trapping in two-level systems, the existence of which depends on the conditions (9). We further show that the system jumps whenever the energy levels in the frame rotating with the instantaneous frequency of the field cross each other. The issues discussed in this paper would be even more interesting in the context of higher spins or multilevel systems where several levels can cross at the same time, and we hope to discuss these in the future.

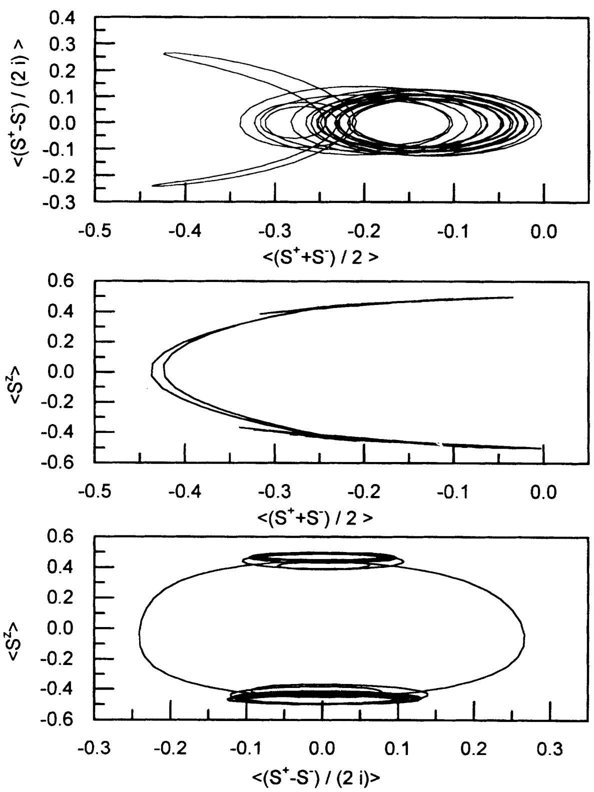

FIG. 3. Contour plots for the atomic polarizations and inversion when $M=30.634\,606\,468\,4$; $g/\Omega=8.0$; $\gamma/\Omega=0.0$ (i.e., in the absence of spontaneous emission).

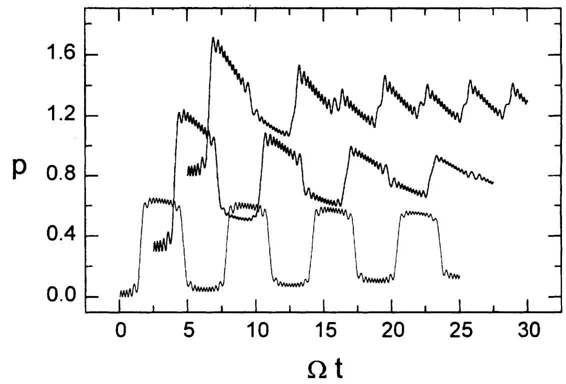

FIG. 4. Effect of weak spontaneous emission on the probability p of excitation for different values of γ ($=1/2$ Einstein A coefficient), $\gamma/\Omega=0.01$ ($x=x+0.0$, $y=2/3y$), $\gamma/\Omega=0.05$ ($x=x+2.5$, $y=y+0.3$), $\gamma/\Omega=0.1$ ($x=x+5.0$, $y=y+0.8$); $M=30.634\,606\,468\,4$; $g/\Omega=8.0$. For clarity the different curves have been displaced as indicated by the transformations in brackets. For example, $y=y+0.8$ means: the value on the y axis equals the actual value plus 0.8.

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