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RAPID COMMUNICATIONS

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Schemes for atomic-state teleportation

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We present two schemes employing cavity QED phenomena to realize the teleportation of quantum states following the principle outlined by Bennett *et al.* [Phys. Rev. Lett. **70**, 1895 (1993)].

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The possibility of entanglement between two separated systems and the projection postulate are two of the most intriguing features of quantum mechanics. Combinations of these features have led to predictions of a variety of novel quantum effects, such as the teleportation of quantum states [1], whereby a sender is able to communicate the unknown state of a given particle to a receiver at another location without sending the particle itself.

Bennet *et al.* [1] have recently described a scheme for quantum-state teleportation involving three particles. The particle whose state is to be teleported (particle 1) is initially in an unknown state $|\phi_1\rangle$. The particle that will receive the teleported state (particle 3) is prepared in a maximally entangled state $|\Psi_{23}\rangle$ with another particle (particle 2). Teleportation is achieved by first performing a joint measurement of the von Neumann (vN) type on particles 1 and 2. The outcome of this measurement is communicated by conventional (classical) means to a receiver, who, with this information, applies an appropriate rotation to particle 3. Following this rotation, the state of particle 3 is identical to the original state of particle 1 and the teleportation is complete. In summary, the teleportation process involves three steps: (i) preparation of two particles in a maximally entangled state, (ii) a joint measurement on one of these particles and the unknown particle, and (iii) communication of the measurement result to the other particle of the initially entangled pair and subsequent rotation of this particle.

Entangled states of polarized photons have been prepared and employed in several experiments testing Bell's inequalities [2]. However, to date no vN measurement of two particles has ever been performed. In this Rapid Communication, we propose a scheme to implement teleportation in which the relevant particles are atoms and the state that we wish to teleport is the state of an atom initially prepared in an unknown linear superposition of two internal states. (We note at this point that the scheme can also be used for the teleportation of a cavity radiation field state in a superposition of the zero- and one-photon Fock states.) In this scheme, both particle entanglement and vN measurement are accomplished via the interaction of the atoms with (microwave or optical) cavity field modes.

Our use of cavity QED effects is encouraged by spectacular recent advances both in microwave and optical experiments [3], in which coherent atom-cavity-field interactions can be made to dominate over dissipative processes due to cavity losses and atomic spontaneous emission. With atoms, it is also straightforward to implement rotations required for teleportation using, e.g., sequences of microwave pulses between hyperfine levels.

Certain important differences exist between the microwave and optical regimes and so we shall present two variations of the teleportation scheme, each suited to one particular regime. An interesting feature of the scheme designed for the optical regime is that, for the entirety of the procedure, only ground atomic states are ever populated, and so, in principle, teleportation could be implemented over large distances (i.e., no time constraints are set by spontaneous emission).

The basic idea of the vN measurement we are proposing is as follows. Consider two two-level atoms (equivalent to spin-$\frac{1}{2}$ particles) with levels $|g\rangle$ and $|e\rangle$. The total Hilbert space of the atoms $\mathcal{H}=\mathcal{H}_2\otimes\mathcal{H}_2$, where $\mathcal{H}_2$ is the Hilbert space of a single atom. A standard vN measurement involves measuring the joint state of these atoms in a maximally entangled basis of $\mathcal{H}$, with, for example, basis vectors

$$|\Psi_{ij}^{(\pm)}\rangle=1/\sqrt{2}(|e\rangle_i|g\rangle_j\pm|g\rangle_i|e\rangle_j), \tag{1a}$$

$$|\Phi_{ij}^{(\pm)}\rangle=1/\sqrt{2}(|e\rangle_i|e\rangle_j\pm|g\rangle_i|g\rangle_j). \tag{1b}$$

Our approach to this measurement is to utilize an appropriate interaction between the atoms and a cavity mode to uniquely map $\mathcal{H}$ onto a different Hilbert space $\mathcal{H}_4$ of *one atom only* (utilizing two additional internal atomic states). Once this mapping has taken place, a single measurement of the state of the four-level atom gives the required vN measurement.

In the context of teleportation, given that an entangled state of atoms 2 and 3 has been prepared (see [4] and below for schemes), atoms 1 and 2 pass through a cavity initially in the vacuum state. The total state of the atoms plus the cavity mode before the interaction is

$$|\Psi_{23}\rangle|\phi_1\rangle|0\rangle_c, \tag{2}$$

where $|\phi_1\rangle=a|g\rangle_1+b|e\rangle_1$, and $|0\rangle_c$ denotes the vacuum state of the cavity mode (i.e., the zero-photon Fock state). Through the interaction, the state (2) becomes

$$|\tilde{\Psi}_{23}\rangle|g\rangle_1|0\rangle_c, \tag{3}$$

where $|\tilde{\Psi}_{23}\rangle$ is now an entangled state of a four-level atom (atom 2) and a two-level atom (atom 3). A proper measurement of the state of atom 2 will have the same effect as a vN joint measurement on atoms 1 and 2.

(a) Microwave regime: Rydberg atoms. We consider three atoms with atomic Rydberg states $|g\rangle_i,|g'\rangle_i,|e\rangle_i,|e'\rangle_i$ $(i=1,2,3)$. Here, g and e represent states with quantum numbers (n,l) and $(n+1,l+1)$, respectively $(n\gg 1)$, and the primes denote different magnetic quantum numbers. As we will show, teleportation can be accomplished by utilizing the interaction of these atoms with two *identical* microwave cavities. The cavity modes are σ^+ polarized and are on resonance with the $(n,l)\rightarrow(n+1,l+1)$ transition. In the following we will assume that the velocities of these atoms have been selected so that the interaction times between the atoms and the cavity modes are $t_1=t_2=2t_3=\pi/(2g)$, where g is the atom-cavity-mode coupling constant and t_i refers to atom i. We will also assume that the matrix element of the dipole moment for the transition $|g\rangle\rightarrow|e\rangle$ is the same as that for the transition $|g'\rangle\rightarrow|e'\rangle$ [5].

We want to prepare atom 3 in the same state as that of atom 1, given by

$$|\phi_1\rangle=a|g\rangle_1+b|e\rangle_1. \tag{4}$$

In the following, we outline the three fundamental steps involved in the teleportation process. A schematic representation of the process is given in Figs. 1 and 2.

Step 1: Preparation of the entangled state $|\Psi_{23}\rangle$. Atom 3, initially prepared in the state $|e\rangle_3$, is sent through a cavity initially in the vacuum state $|0\rangle$. Subsequently, atom 2, initially prepared in the state $|g\rangle_2$, is sent through the same cavity. Given the interaction times specified above, the state of the system composed of atoms 2 and 3 is, after the interaction [4],

$$|\Psi_{23}\rangle=1/\sqrt{2}(|g\rangle_2|e\rangle_3-|e\rangle_2|g\rangle_3), \tag{5}$$

with the cavity mode again in the vacuum state. After this, two consecutive $\pi/2$ microwave pulses are applied to atom 2: the first with π (or σ^-) polarization and the second with σ^+ polarization. Population in atom 2 is thus transferred coherently from states $|e\rangle$ and $|g\rangle$ to states $|g\rangle$ and $|g'\rangle$, respectively (dashed lines in Fig. 2), and the state $|\Psi_{23}\rangle$ becomes

$$|\Psi_{23}\rangle=1/\sqrt{2}(|g'\rangle_2|e\rangle_3-|g\rangle_2|g\rangle_3). \tag{6}$$

Step 2: Joint measurement of atoms 1 and 2. Atom 1, initially in the "unknown" state (4), is sent through a second cavity initially in the vacuum state $|0\rangle_c$. Before this interaction, the state of the whole system is given by (2). After the interaction, the state of the cavity mode is

$$|\phi\rangle_c=a|0\rangle_c+b|1\rangle_c, \tag{7}$$

and atom 1 is in the state $|g\rangle_1$. Atom 2 then crosses the same cavity. In the interaction between this atom and the cavity mode, the states $|g\rangle_2|0\rangle_c$ and $|g'\rangle_2|0\rangle_c$ remain unaltered, whereas the states $|g\rangle_2|1\rangle_c$ and $|g'\rangle_2|1\rangle_c$ are transformed into $|e\rangle_2|0\rangle_c$ and $|e'\rangle_2|0\rangle_c$, respectively. Thus the state of the system after the interaction is (3), with

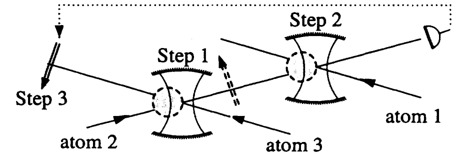

FIG. 1. Schematic of cavity configurations and atomic trajectories for atomic state teleportation from atom 1 to atom 3. The dashed circles apply to the adiabatic passage scheme and represent laser fields which partially overlap the cavity fields. The dashed arrow between the cavities applies to the Rydberg atom scheme and represents a pair of microwave pulses applied to atom 2.

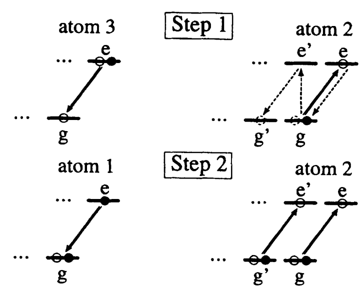

FIG. 2. Atomic level configurations for Rydberg-atom scheme showing required transitions for steps 1 and 2. The solid (open) circles show the initial (final) populations. Double-lined arrows represent coupling to the cavity mode. Upward (downward) arrows denote that a photon is absorbed from (emitted into) the cavity mode respectively. The dashed arrows denote application of microwave pulses (after the cavity interaction) and dashed circles the atomic populations following these pulses (see text).

$$|\tilde{\Psi}_{23}\rangle=\tfrac{1}{2}[|\Psi_2^{(+)}\rangle(-a|g\rangle_3+b|e\rangle_3)+|\Psi_2^{(-)}\rangle(a|g\rangle_3+b|e\rangle_3)$$
$$+|\Phi_2^{(+)}\rangle(a|e\rangle_3-b|g\rangle_3)+|\Phi_2^{(-)}\rangle(-a|e\rangle_3-b|g\rangle_3)], \tag{8}$$

where

$$|\Psi_2^{(\pm)}\rangle=1/\sqrt{2}(|e'\rangle_2\pm|g\rangle_2), \tag{9a}$$

$$|\Phi_2^{(\pm)}\rangle=1/\sqrt{2}(|e\rangle_2\pm|g'\rangle_2). \tag{9b}$$

A single measurement on the state of atom 2 is performed in the basis $\{|\Psi_2^{(\pm)}\rangle,|\Phi_2^{(\pm)}\rangle\}$, projecting atom 3 into one of four possible states.

Step 3: Rotation of atom 3. With the outcome of the joint measurement of step 2, an appropriate rotation is applied to atom 3, leaving this particle in the same state as (4). This rotation is performed by a suitable σ^+-polarized microwave pulse.

The scheme presented here for Rydberg atoms requires (a) controlled atomic velocities, (b) detection of atoms in given states, (c) negligible cavity loss during the atom-cavity interactions, and (d) no spontaneous decay during the whole teleportation process. In cavity QED experiments with Rydberg atoms, control over atomic velocities is achieved utilizing velocity-selective chopping of an atomic beam. The detection of atoms in the states $|\Psi^{(\pm)}\rangle$ and $|\Phi^{(\pm)}\rangle$ could be performed by applying a sequence of microwave pulses to the atom such that these four superposition states are transformed in a one-to-one fashion to four distinct pure states, which can then be detected and distinguished using selective ionization techniques [6]. Cavity lifetimes for high-Q superconducting cavities can be as long as 0.01 s, which is three orders of magnitude longer than typical atom-cavity interaction times in present experiments. Finally, atomic excited-state lifetimes for Rydberg atoms in circular states $l=n-1$ are of the order of 0.01 s. Given atomic velocities of approximately 1000 m/s, this would impose a maximum distance between cavities of the order of 1 m.

(b) Optical regime: Adiabatic passage. Scheme (a) employs coherent Rabi oscillations between atoms and cavity-mode fields to produce the initial entangled state and then to facilitate the vN measurement. An alternative scheme for producing the required transformations of atomic and cavity field states is adiabatic passage, whereby a suitable time-dependent interaction of atoms and fields yields a controlled evolution from some initial system eigenstate to a desired final eigenstate. Such a scheme has been proposed recently as a means of "mapping" states between a cavity field and the ground-state Zeeman sublevels of an atom [7]. A significant feature of this particular scheme is that the adiabatic transformation is performed on a so-called "dark" eigenstate of the system, i.e., a state containing no contribution from excited atomic states. In this way, only ground atomic states are ever populated and atomic spontaneous emission is avoided. This has particular appeal for investigations in the optical regime, and experiments testing dark-state adiabatic passage have been carried out (with laser fields rather than few-photon cavity fields), demonstrating coherent population transfer between atomic ground states [8] and coherent atomic beam deflection [9].

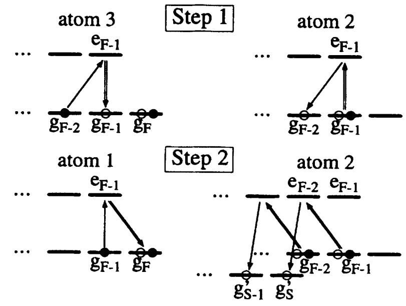

FIG. 3. Atomic level configurations for optical scheme showing required transitions for steps 1 and 2. The description is as in Fig. 2.

In practice, the adiabatic passage scheme for state mapping between a cavity field and an atom requires the passage of the atom through partially overlapping laser and cavity fields, such that the atom effectively "sees" a pair of time-delayed pulsed fields (the order of which is important). Figure 1 again gives a schematic representation of the scheme, with the relevant internal atomic states now shown in Fig. 3. The atomic level scheme could, e.g., apply to the $D2$ line of Cs or Rb.

Step 1: Preparation of the entangled state $|\Psi_{23}\rangle$. Atoms 2 and 3 pass through overlapping cavity and laser fields in the manner shown in Fig. 1. The cavity mode is π polarized and initially in the vacuum state $|0\rangle$. The laser field is σ^+ polarized. Atom 3 is initially prepared in the state $\sqrt{1/2}(|g_{F-2}\rangle_3-|g_F\rangle_3)$ (F is the total angular momentum quantum number of the ground-state level and g_F, g_{F-2} denote substates within this level with magnetic quantum numbers $m_F=F$ and $m_F=F-2$, respectively) and passes through the fields first, producing the adiabatic transformation

$$1/\sqrt{2}(|g_{F-2}\rangle_3-|g_F\rangle_3)|0\rangle\rightarrow 1/\sqrt{2}(|g_{F-1}\rangle_3|1\rangle-|g_F\rangle_3|0\rangle).$$

Atom 2, prepared in the state $|g_{F-1}\rangle_2$, then passes through the fields, and an entangled state is prepared through the adiabatic transformation

$$1/\sqrt{2}|g_{F-1}\rangle_2(|g_{F-1}\rangle_3|1\rangle-|g_F\rangle_3|0\rangle)$$
$$\rightarrow 1/\sqrt{2}(|g_{F-2}\rangle_2|g_{F-1}\rangle_3-|g_{F-1}\rangle_2|g_F\rangle_3)|0\rangle\equiv|\Psi_{23}\rangle|0\rangle.$$

Step 2: Joint measurement of atoms 1 and 2. We wish to teleport the following state of atom 1:

$$|\phi_1\rangle=a|g_F\rangle_1+b|g_{F-1}\rangle_1. \tag{10}$$

Adiabatic passage in a second cavity is now used to implement transformations such that a single measurement on atom 2 (following the transformations) implements the required vN measurement. The state of the system formed by the atoms and the second cavity is, following step 1, given again by (2). Atom 1 now passes through overlapping cavity and laser fields, which are resonant with the $F\rightarrow F-1$ transition and σ^- and π polarized, respectively. The resulting adiabatic passage leaves the cavity in the state (7) and atom 1 in the state $|g_F\rangle_1$.

For the passage of atom 2, the frequency and (if necessary) the polarization of the laser field are changed such that it is resonant with the transitions $|e_{F-2}\rangle_2\rightarrow|g_s'\rangle_2$ and

$|e_{F-3}\rangle_2 \rightarrow |g'_{s-1}\rangle_2$, where g'_s, g'_{s-1} denote magnetic substates in, e.g., another ground-state hyperfine level of the atom. Given these conditions, as a result of adiabatic passage, the states $|g_{F-1}\rangle_2|0\rangle_c$ and $|g_{F-2}\rangle_2|0\rangle_c$ remain unaltered, while the states $|g_{F-1}\rangle_2|1\rangle_c$ and $|g_{F-2}\rangle_2|1\rangle_c$ are transformed into $|g'_s\rangle_2|0\rangle_c$ and $|g'_{s-1}\rangle_2|0\rangle_c$, respectively. The state of the entire system thus becomes $|\tilde{\Psi}_{23}\rangle|g_F\rangle_1|0\rangle_c$, where

$$|\tilde{\Psi}_{23}\rangle = \tfrac{1}{2}[|\Psi_2^{(+)}\rangle(-a|g_F\rangle_3 + b|g_{F-1}\rangle_3) + |\Psi_2^{(-)}\rangle(a|g_F\rangle_3 + b|g_{F-1}\rangle_3) + |\Phi_2^{(+)}\rangle(a|g_{F-1}\rangle_3 - b|g_F\rangle_3) + |\Phi_2^{(-)}\rangle \times(-a|g_{F-1}\rangle_3 - b|g_F\rangle_3)], \tag{11}$$

with

$$|\Psi_2^{(\pm)}\rangle = 1/\sqrt{2}(|g'_{s-1}\rangle_2 \pm |g_{F-1}\rangle_2), \tag{12a}$$

$$|\Phi_2^{(\pm)}\rangle = 1/\sqrt{2}(|g'_s\rangle_2 \pm |g_{F-2}\rangle_2). \tag{12b}$$

That is, atom 3 is projected into one of four states by a single measurement of the state of atom 2.

Step 3: Rotation of atom 3. With an appropriate rotation of the state of atom 3, depending on the result of the measurement, the teleportation of the state of atom 1 to atom 3 is completed. Such a rotation could again be achieved using, e.g., microwave pulses. We note that this scheme could also be applied to the teleportation of light fields, since suitable adiabatic passage with atom 3 and a third cavity would amount to teleportation of the field state $a|0\rangle_c + b|1\rangle_c$ to this third cavity.

The conditions required for successful implementation of the adiabatic passage transformations have been detailed elsewhere [7–10]. A fundamental requirement is that $g\tau \gg 1$, where g is the atom-cavity coupling strength and τ is the flight time of each atom through the interaction region. This condition, being in the form of an inequality, does not require precise interaction times as in the case of the Rydberg atom scheme. Spontaneous emission is, in principle, irrelevant, whereas cavity losses must be negligible for the time during which the cavity mode is excited (i.e., in between the pairs of atoms). Such conditions of strong coupling g and small cavity losses should be possible using dielectric microspheres [11,12], in which the "cavity" mode is a whispering-gallery mode circulating just inside the surface of the sphere, and this mode is coupled through its evanescent field to atoms grazing the surface of the sphere. Using parameters relevant to such spheres [12], adiabatic passage is found to operate almost ideally [10]. Finally, detection of the states $|\Psi_2^{(\pm)}\rangle$ and $|\Phi_2^{(\pm)}\rangle$ could be achieved in a similar fashion to that described for scheme (a). Using sequences of $\pi/2$ or π optical-Raman or microwave pulses, possibly in the presence of magnetic fields, these four states could be coherently mapped to four pure Zeeman substates, which could then be detected using standard state-sensitive detection techniques (e.g., fluorescence detection or beam deflection by a magnetic field gradient).

In summary, we have presented two schemes for the teleportation of an atomic state, one suited to experiments with Rydberg atoms and the other to experiments in the optical regime. The schemes employ cavity QED phenomena, which enable the required joint measurement on two atoms to be implemented by a single measurement of the state of one atom. Beyond teleportation, variations of the schemes presented here offer interesting possibilities in, e.g., quantum computing.

Note added: After completion of this work, we learned of alternative proposals for teleportation by Sleator and Weinfurter [13] and by Davidovich *et al.* [14] which employ Rydberg atoms and microwave cavities. In contrast to the present scheme, they require both resonant and off-resonant interactions of two-level atoms with the cavity fields, and the vN measurement is implemented by measurements of the states of *two* two-level atoms.

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[1] C.H. Bennett *et al.*, Phys. Rev. Lett. **70**, 1895 (1993).
[2] See, e.g., A. Aspect, J. Dalibard, and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982); Y.H. Shih and C.O. Alley, *ibid.* **61**, 2921 (1988).
[3] See, e.g., *Cavity Quantum Electrodynamics, Advances in Atomic, Molecular, and Optical Physics, Supplement 2*, edited by P. Berman (Academic Press, San Diego, 1994).
[4] S.J.D. Phoenix and S.M. Barnett, J. Mod. Opt. **40**, 979 (1993); J.I. Cirac and P. Zoller, Phys. Rev. A **50**, R2799 (1994).
[5] For an $n=50 \rightarrow n=51$ transition, the respective Clebsch-Gordan coefficients are within 1% of each other for $l=n-1$, and the approximation makes little difference in numerical calculations.
[6] See, e.g., the article by S. Haroche in *Fundamental Systems in Quantum Optics*, edited by J. Dalibard, J.M. Raimond, and J. Zinn–Justin (Elsevier Science Publishers, New York, 1992).
[7] A.S. Parkins *et al.*, Phys. Rev. Lett. **71**, 3095 (1993).
[8] U. Gaubatz *et al.*, J. Chem. Phys. **92**, 5363 (1990).
[9] J. Lawall and M. Prentiss, Phys. Rev. Lett. **72**, 993 (1994); L.S. Goldner *et al.*, *ibid.* **72**, 997 (1994).
[10] A.S. Parkins, P. Marte, P. Zoller, O. Carnal, and H.J. Kimble (unpublished).
[11] V.B. Braginsky, M.L. Gorodetsky, and V.S. Ilchenko, Phys. Lett. A **137**, 393 (1989); L. Collot *et al.*, Europhys. Lett. **23**, 327 (1993).
[12] H. Mabuchi and H.J. Kimble, Opt. Lett. **19**, 749 (1994).
[13] T. Sleator and H. Weinfurter, in *IQEC Technical Digest 1994*, Vol. 9 of 1994 OSA Technical Digest Series (OSA, Washington, DC, 1994), p. 140.
[14] L. Davidovich *et al.*, Phys. Rev. A **50**, R895 (1994).