RAPID COMMUNICATIONS

PHYSICAL REVIEW A VOLUME 50, NUMBER 5 NOVEMBER 1994

Quantum conversion between the cavity fields and the center-of-mass motion of ions in a quantized trap

Heping Zeng and Fucheng Lin
Shanghai Institute of Optics and Fine Mechanics, Academia Sinica, P.O. Box 800-211, Shanghai, 201800, People's Republic of China
(Received 4 April 1994)

We propose a technique to generate nonclassical vibrational states of the quantized center-of-mass motion of an ion in a harmonic trap, based on the quantum conversion between the quantum cavity field and the quantized center-of-mass motion. It is shown that when an ion trap system interacts with the eigenmode of a single-mode Fabry-Pérot cavity, where the trap is set in, and with an external classical driving electromagnetic field through Raman transitions, the interchange of the quantum features between the quantum cavity field and the quantized trap occurs. This kind of quantum conversion can be used to prepare some nonclassical trap states for the ion trap as well as to measure the quantum statistics of an initial nonclassical vibrational state.

PACS number(s): 32.80.Pj, 42.50.Vk

Considerable experimental and theoretical work has been devoted recently to the investigation of laser cooling and trapping of ions and neutral atoms. Experimental advances in this field have enabled the achievement of extremely low temperatures of trapped ions and neutral atoms, at which the quantum nature of the center-of-mass (c.m.) motion was dominant. Diedrich *et al.* [1] have shown that a trapped ion can be cooled down to its zero-point energy of motion. Current experiments [2–6] on laser cooling of free atoms have observed transitions between different vibrational states, which belonged to the quantized c.m. motions of atoms constrained to move in optical molasses. Moreover, these experiments presented some evidence that a majority of the population was in the vibrational ground state. With enough population accumulated in the vibrational ground state, an atom trapped in the optical molasses approximates a minimum-uncertainty wave packet. In a Paul or Penning ion trap system, the trap potential approximates to be harmonic and the corresponding c.m. motion behaves as a standard harmonic oscillator. On the other hand, for neutral atoms in optical molasses, a theoretical investigation performed by Castin and Dalibard [7] for the lin⊥lin configuration (two counterpropagating laser beams with orthogonal linear polarizations) has predicted that the periodic potential, which was associated with the light shifts of atomic Zeeman sublevels, gave rise to atom localization on the optical wavelength scale and to energy-band structures dependent upon the atomic c.m. motion. The experimental demonstration of atom localization in three-dimensional optical molasses was first carried out by Westbrook *et al.* by observing Dicke narrowing of fluorescence [8]. At very low temperature, neutral atoms are well localized near the valley of the well, where the periodic potential is approximately harmonic. Therefore, the vibrational ground state of the ionic or atomic trap is a coherent state with zero-point quantum fluctuations for position and momentum operators, respectively.

The previous and ongoing experiments give impetus to the investigation of the possibility of generating some nonclassical vibrational states. To our knowledge, three kinds of schemes have been theoretically suggested to date. Agarwal and Kumar [9] showed that the vibrational states of the c.m. motion of an ion in a Paul trap with time-dependent trap frequency had some remarkable nonclassical properties, such as squeezing and sub-Poissonian statistics. Much current theoretical work has furthered the investigation on this subject [10]. Cirac *et al.* [11] demonstrated that multichromatic excitation of a trapped ion by standing- and traveling-wave light fields with certain selected laser frequencies led to the occurrence of a dark resonance in the fluorescence emitted by the ion and to the generation of coherent squeezed states of the c.m. motion. Furthermore, they proposed to prepare nonclassical states, especially Fock states, of the c.m. motion in an ion trap, by the observation of quantum jumps [12]. More recently, considerable interest has been attracted to laser cooling in the strong-sideband (SSB) limit [13]. It has been predicted that both the population inversion and the average trap number [14,15] of a two-level trapped ion will exhibit collapses and revivals in the SSB limit, which result from the discreteness of the vibrational states. These quantum phenomena actually have proved the existence of quantum interferences between the individual responses of the different vibrational states in the trap quanta creation and annihilation processes, which can be exerted to prepare some vibrational states with nonclassical features, such as quadrature and amplitude-squared squeezing, sub-Poissonian statistics, and even some unrecognized nonclassical characteristics [16].

In this Rapid Communication, we present a proposal to generate nonclassical vibrational states of the c.m. motion of an ion confined to move in a quantized trap, based on the conversion of nonclassical properties from the quantized electromagnetic field to the c.m. motion. This kind of quantum conversion between two light waves with different frequencies was theoretically predicted and experimentally realized previously [17,18]. In our scheme here, we consider a situation in which a harmonically trapped ion of mass M with well-resolved absorption sidebands (known as the resolved-sideband limit) is set in a Fabry-Pérot cavity with the cavity-mode frequency ω_c. For simplicity, we assume that the ion oscillates near one node of the standing cavity field. An external laser field with frequency ω_l is applied to the cavity. We suppose that both the cavity mode and the external laser field are tuned far from any ionic internal resonance, but on two-photon Raman resonance with c.m. vibrational states, i.e., their frequencies fulfill a condition: $\omega_c-\omega_l=\nu$, where ν is the harmonic trap frequency. Thereby, Raman transitions between the c.m. vibrational states can be driven. In a high-Q cavity, the ion counteracts significantly the cavity field. An appropriate description of the dynamical behaviors of the cavity field and the c.m. motion requires that they both be treated quantum mechanically. What should be emphasized is the fact that the Raman transitions result in a dramatic alteration for the ionic c.m. motion and for the quantum cavity field as well. Our results here confirm that complete quantum conversion takes place under the circumstance of a properly selected external laser pulse. If the quantum cavity field is initially in a nonclassical state, the c.m. motion will obtain the same nonclassical properties by complete conversion. The conversion can also take place in the reverse way. The output of the complete conversion from the c.m. motion to the cavity-mode field characterizes the features of an initial nonclassical vibrational state, which can be exerted to measure the associated nonclassical properties; particularly, a dark squeezed state in Ref. [11].

To simplify matters, we assume that the cavity is an ideal one-dimensional one bounded by two perfect mirrors. This corresponds to an approximation in which the cavity loss is omitted. The ion placed within the cavity is confined to move in a three-dimensional trap potential. A triplet of quantum numbers (n_1,n_2,n_3) is needed to label the vibrational states. If one of the principal trap axes is taken to coincide with the axis of the Fabry-Pérot cavity, taken to be the x axis, the problem can be reduced to one dimension. One quantum number now suffices, for the other two are traced out by summing over the corresponding degrees of freedom. In one dimension, the external potential can be expressed in terms of the creation and annihilation operators $b^\dagger$ and b as

$$H_{\mathrm{TP}}=\hbar\nu(b^\dagger b+\tfrac{1}{2}). \tag{1}$$

The discrete nature of the external energies and the Raman transitions between the trap states are schematically shown in Fig. 1. We make a further simplification by assuming that the spatial variation of the external laser pulse along the direction of the x axis can be ignored in the Lamb-Dicke limit. This can be arranged by setting the external field to propagate along a direction perpendicular to the x axis. The external classical light excites the ion from the ground state $|g\rangle$ with the c.m. motion being in different trap-number states $|n\rangle$, i.e., $|g\rangle\otimes|n\rangle$ $(n=0,1,2,\ldots)$, to intermediate states $|j,n\rangle$. The succeeding emission stimulated by the privileged cavity mode drives the ion back to the ground state with decreasing trap numbers $|g,n-1\rangle$ $(n=1,2,\ldots)$. We omit spontaneous emission to modes other than the privileged cavity mode. Those processes can also take place in reverse. Since the intermediate states $|j,n\rangle$ $(n=0,1,2,\ldots)$ are far off resonance, population pumped in those states is negligible and can be adiabatically eliminated. Effectively speaking, two processes are driven: i.e., absorbing a cavity-field photon accompanied by increasing one trap quantum number, and emitting a cavity photon accompanied by decreasing one trap quantum number.

The total system can be described by the Hamiltonian

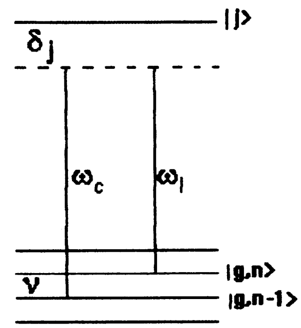

FIG. 1. Vibrational level scheme for the ion-trap system. The trap states are well resolved, each being $\hbar\nu$ apart in the energy spectrum. Raman transitions are driven by an external laser pulse and a single-mode cavity field, with frequencies of ω_l and ω_c, respectively. Both fields are tuned far from any ionic internal resonance but on two-photon Raman resonance, i.e., $\omega_c-\omega_l=\nu$. $|j\rangle$ is an intermediate state, $|g\rangle$ is the ionic internal ground state, and $|n\rangle$ is the number trap state.

$$H=H_a+H_{\mathrm{TP}}+H_f+H_{\mathrm{int1}}+H_{\mathrm{int2}}, \tag{2}$$

with H_a being the Hamiltonian for the ionic internal states, H_f that for the cavity field, and H_{int1} and H_{int2} the Hamiltonians for couplings of the trapped ion with the external and cavity laser fields, respectively. Those terms can be written out in detail as

$$H_a=\sum_j \hbar\omega_j|j\rangle\langle j|, \tag{3}$$

$$H_f=\hbar\omega_c a^\dagger a, \tag{4}$$

$$H_{\mathrm{int1}}=\hbar\sum_{j,n} (\Omega_j|g,n\rangle\langle j,n|e^{i\omega_l t+i\phi}+\mathrm{H.c.}), \tag{5}$$

$$H_{\mathrm{int2}}=\hbar\sin(kx)\sum_{j,n} (g_j a|j,n\rangle\langle g,n|+\mathrm{H.c.}), \tag{6}$$

where $a^\dagger$ and a are creation and annihilation operators of the cavity field, respectively, $\hbar\omega_j$ is the energy separation between internal levels $|j\rangle$ and $|g\rangle$, ϕ is the phase of the external laser field, and $\Omega_j=\mu_j E_0/2\hbar$, $g_j=\sqrt{2\pi\omega_c/V}\mu_j/\hbar$, in which μ_j represents the transition moment between states $|g\rangle$ and $|j\rangle$, and V is the quantization volume of the cavity field. In the Lamb-Dicke limit, the operator function $\sin(kx)$ can be expanded approximately as $\sin(kx)=\eta(b^\dagger+b)$, where η is the Lamb-Dicke parameter defined by $\eta=\sqrt{E_r/E_\nu}$, with $E_r=\hbar^2k^2/2M$ and $E_\nu=\hbar\nu$. The Hamiltonian can be expressed in a more convenient form in a rotating frame: $a=\tilde{a}e^{-i\omega_c t}$, $b=\tilde{b}e^{-i\nu t}$. Hereafter we use tildes to indicate operators in the rotating frame.

We set the system wave function as

$$|\psi\rangle=\sum_{j,n,m} c_{j,n,m}|j,n,m-1\rangle+\sum_{n,m} c_{g,n,m}|g,n,m\rangle, \tag{7}$$

where $|g,n,m\rangle=|g\rangle\otimes|n\rangle\otimes|m\rangle$ and $|j,n,m-1\rangle=|j\rangle\otimes|n\rangle\otimes|m-1\rangle$, in which $|n\rangle$ and $|m\rangle$ represent the Fock states of the trap quantum and the cavity field, respectively. From the Schrödinger equation $i\hbar(\partial/\partial t)|\psi\rangle=\tilde{H}|\psi\rangle$, we derive a set of differential equations to first order of the Lamb-Dicke parameter η for $c_{g,n,m}$,

$$\begin{aligned}\dot{c}_{g,n,m}=&-iSc_{g,n,m}-ig^{*}e^{i\phi}\sqrt{n(m+1)}c_{g,n-1,m+1}\\&-ige^{-i\phi}\sqrt{(n+1)m}c_{g,n+1,m-1}\\&-ige^{-i\phi+2i\nu t}\sqrt{nm}c_{g,n-1,m-1}\\&-ig_{--}e^{i\phi-2i\nu t}\sqrt{(n+1)(m+1)}c_{g,n+1,m+1},\end{aligned}\tag{8}$$

where, $S\equiv\Sigma_j\Omega_j\Omega_j^{*}/\delta_j$, $g_{--}\equiv\Sigma_j\Omega_j g_j\eta/(\delta_j+2\nu)$, $g\equiv\Sigma_j\Omega_j^{*}g_j^{*}\eta/\delta_j$, in which, $\delta_j=\omega_l-\omega_j$. The Stark shifts (the first term) have no observable effects on the problem we are concerned with here, and hence can be ignored. Finally, we obtain an effective Hamiltonian

$$\begin{aligned}\tilde{H}_{\rm eff}=&\hbar g\tilde{b}\tilde{a}^{\dagger}e^{-i\phi}+\hbar g^{*}\tilde{b}^{\dagger}\tilde{a}\,e^{i\phi}+\hbar g_{--}\tilde{b}\tilde{a}\,e^{i\phi-2i\nu t}\\&+\hbar g\tilde{b}^{\dagger}\tilde{a}^{\dagger}e^{-i\phi+2i\nu t}.\end{aligned}\tag{9}$$

If the trapped ion is in the resolved-sideband limit ($\nu\gg0$), the last two counterrotating terms can be dropped, in the spirit of the rotating-wave approximation. From the above derivation, we can see that, if the external laser field is a pulsed field, the effective Hamiltonian is the same, except that Ω is replaced by $\Omega(t)=\Omega f(t)$,

$$H_{\rm eff}=(\hbar g\tilde{a}^{\dagger}\tilde{b}\,e^{-i\phi}+\text{ H.c})f(t),\tag{10}$$

where $f(t)$ is dependent upon the time waveform of the external laser pulse. Then, the corresponding master equations for cavity field and c.m. motion become

$$\frac{d\tilde{a}}{d\xi}=-ig\tilde{b}\,e^{-i\phi},\tag{11}$$

$$\frac{d\tilde{b}}{d\xi}=-ig^{*}\tilde{a}\,e^{i\phi},\tag{12}$$

where ξ is a new variable defined as $\xi\equiv\int_{-\infty}^{t}f(\tau)d\tau$. Those equations can be analytically solved in terms of initial values a_0 and b_0 at the time when the external laser pulse is absent ($t=-\infty$). After the external laser pulse, i.e., at $t=+\infty$, the cavity field and c.m. motion become

$$\tilde{a}(t)|_{t=+\infty}=a_0\cos(|g|\xi_0)-i\,e^{-i(\phi+\phi_0)}b_0\sin(|g|\xi_0),\tag{13}$$

$$\tilde{b}(t)|_{t=+\infty}=-i\,e^{i(\phi+\phi_0)}a_0\sin(|g|\xi_0)+b_0\cos(|g|\xi_0),\tag{14}$$

where the phase ϕ_0 is defined by $e^{-i\phi_0}\equiv g/|g|$, and ξ_0 by $\xi_0\equiv\int_{-\infty}^{+\infty}f(\tau)d\tau$. ξ_0 corresponds to the normalized external laser pulse area. If it is selected to satisfy

$$|g|\xi_0=(N+\tfrac{1}{2})\pi,\tag{15}$$

a complete conversion takes place, i.e., $\tilde{a}=e^{i\chi_1}b_0$ and $\tilde{b}=e^{i\chi_2}a_0$, in which χ_1 and χ_2 are defined as $\chi_1=(N-\frac{1}{2})\pi-\phi-\phi_0$ and $\chi_2=(N-\frac{1}{2})\pi+\phi+\phi_0$, respectively. If the system is in a state $|\phi\rangle_a\otimes|\phi\rangle_b$, then initially (at $t=-\infty$), the quantum properties of the quantum cavity field and the c.m. motion are determined by the individual states $|\phi\rangle_a$ and $|\phi\rangle_b$, respectively. This means that the corresponding operator functions $\hat{F}_a(\mathbf{a})$ and $\hat{F}_b(\mathbf{b})$ have expectation values of

$$\langle\hat{F}_a\rangle_{t=-\infty}={}_a\langle\phi|\hat{F}_a(a_0)|\phi\rangle_a,\tag{16}$$

$$\langle\hat{F}_b\rangle_{t=-\infty}={}_b\langle\phi|\hat{F}_b(b_0)|\phi\rangle_b.\tag{17}$$

However, the situation is reversed after the Raman coupling (at $t=+\infty$), because the associated quantum properties are determined by the expectation values

$$\langle\hat{F}_a\rangle_{t=+\infty}={}_b\langle\phi|\hat{F}_a(e^{i\chi_1}b_0)|\phi\rangle_b,\tag{18}$$

$$\langle\hat{F}_b\rangle_{t=+\infty}={}_a\langle\phi|\hat{F}_b(e^{i\chi_2}a_0)|\phi\rangle_a.\tag{19}$$

Especially, if the cavity field is initially in a vacuum state, i.e., $a_0|0\rangle=0$, then at $t=+\infty$, the c.m. motion of the trapped ion will be in a vibrational ground state $|0\rangle$ for $b|0\rangle=e^{i\chi_2}a_0|0\rangle=0$. Moreover, if $|\phi\rangle_a$ is a squeezed state, i.e.,

$$|\phi\rangle_a=\hat{D}(\alpha,t=-\infty)\hat{S}(\theta,t=-\infty)|0\rangle$$

with $\hat{D}(\alpha,t=-\infty)=\exp(\alpha a_0^{\dagger}-\alpha^{*}a_0)$ and $\hat{S}(\theta,t=-\infty)=\exp[\frac{1}{2}\theta^{*}a_0^{2}-\frac{1}{2}\theta a_0^{\dagger2}]$, where θ is the squeezing parameter, then squeezing occurs at $T=+\infty$ for the trap states, because the state generated by the operator

$$\begin{aligned}&\hat{D}_b(\alpha,t=+\infty)\hat{S}_b(\theta,t=+\infty)\\&\quad=\exp[\alpha b^{\dagger}(t)-\alpha^{*}b(t)]\\&\qquad\times\exp[\tfrac{1}{2}\theta^{*}b^{2}(t)-\tfrac{1}{2}\theta b^{+2}(t)]|_{t=+\infty}\end{aligned}\tag{20}$$

is $\hat{D}(\alpha_1,t=-\infty)\hat{S}(\theta_1,t=-\infty)|0\rangle$, which is a coherent squeezed trap state with squeezing parameter θ_1, where $\alpha_1=\alpha\,e^{-i\chi_2}$ and $\theta_1=\theta e^{-2i\chi_2}$. Particularly, if the phase of the external laser field is selected to satisfy $\phi+\phi_0=\pi/2$, the trap states will achieve the same squeezing characteristics as the cavity field. On the other hand, any initially squeezing characteristics of the trap states will be converted to the cavity field after the external laser pulse. This kind of nonclassical property interchange is independent of the initial quantum statistics of the cavity field. Therefore, it is expected that the quantum conversion can be used to measure the quantum features of an initial nonclassical trap state [9,11,12,16].

We next come to a brief estimate of the experimental feasibility. The above-predicted quantum conversion can take place only when two conditions are fulfilled. First, there should exist strong Raman coupling between the vibrational states. Second, spontaneous emission to modes other than the privileged cavity mode should be negligible. Experimentally, the first condition can be fulfilled by employing a strong external laser pulse and a strong cavity field, and the second one by the use of inhibited spontaneous emission in cavity QED. Very recently, very-high-Q cavities have been constructed for optical frequencies [19–22]. In a high-Q cavity, spontaneous emission is suppressed, and only stimulated emission to the cavity mode is dominant. Therefore, it is possible to create an experimental arrangement in a high-Q cavity to test the prediction here. The position of the trapped ion relative to the nodes of the cavity standing-wave field can be set by adjusting the positions of the cavity mirrors with the aid of a piezoelectric crystal. The condition for complete conversion $|g|\xi_0=(N+1/2)\pi$ can be obtained by a careful selection of the external laser pulse. Another important parameter that should be taken into account is the linewidth $\Delta\omega_l$ of the external laser field. It is clear that the above analyses hold in the limit of $\Delta\omega_l \ll \nu$. Let us consider a typical ion-trap system, such as that described in Ref. [1]: $^{198}Hg^{+}$ in a Paul trap. The relevant trap frequency is $\nu=2.96$ MHz. The external laser pulse with linewidth $\Delta\omega_l \ll \nu=2.96$ MHz can be achieved by chopping the cw laser field. On the other hand, as mentioned in the above text, the results of complete conversion from the c.m. motion to the cavity field characterize the trap states and can be used as a novel measurement technique. However, the cavity field is well confined in the high-Q cavity. A little tedious technique is needed to probe the quantum statistics of this confined field. One efficient way to accomplish this task is to employ the atomic homodyne scheme proposed by Wilkens and Meystre [23] and developed by Dutra, Knight, and coworker [24].

To sum up, we consider in this paper the system consisting of a single ion well localized in a Fabry-Pérot cavity and interacting with an external laser field as well as with the cavity mode, and demonstrate that quantum features of the cavity field can be exchanged with those of the quantized c.m. motion under certain situations. This kind of quantum conversion can be used to prepare nonclassical trap states, as well as to probe the quantum statistics of a previously prepared nonclassical trap state.

[1] F. Diedrich, J. C. Bergquist, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **62**, 403 (1989).
[2] P. Verkerk, B. Lounis, C. Salomon, C. Cohen-Tannoudji, J.-Y. Courtois, and G. Grynberg, Phys. Rev. Lett. **68**, 3861 (1992).
[3] P. S. Jessen, C. Gerz, P. D. Lett, W. D. Phillips, S. L. Rolston, R. J. C. Spreeuw, and C. I. Westbrook, Phys. Rev. Lett. **69**, 49 (1992).
[4] G. Grynberg, B. Lounis, P. Verkerk, J.-Y. Courtois, and C. Salomon, Phys. Rev. Lett. **70**, 2249 (1993).
[5] A. Hemmerich, C. Zimmermann, and T. W. Hänsch, Europhys. Lett. **22**, 89 (1993).
[6] A. Hemmerich, C. Zimmermann, and T. W. Hänsch, Phys. Rev. Lett. **72**, 625 (1994).
[7] Y. Castin and J. Dalibard, Europhys. Lett. **14**, 761 (1991).
[8] C. I. Westbrook, R. N. Watts, C. E. Tanner, S. L. Rolston, W. D. Phillips, P. D. Lett, and P. L. Gould, Phys. Rev. Lett. **65**, 33 (1990); see also N. P. Bigelow and M. G. Prentiss, *ibid.* **65**, 29 (1990).
[9] G. S. Agarwal and S. A. Kumar, Phys. Rev. Lett. **67**, 3665 (1991).
[10] C. F. Lo, Phys. Rev. A **43**, 404 (1991); F. L. Li, Phys. Lett. A **168**, 400 (1992); C. M. Cheng and P. C. W. Fung, J. Phys. A **21**, 4115 (1988).
[11] J. I. Cirac, A. S. Parkins, R. Blatt, and P. Zoller, Phys. Rev. Lett. **70**, 656 (1993).
[12] J. I. Cirac, R. Blatt, A. S. Parkins, and P. Zoller, Phys. Rev. Lett. **70**, 762 (1993).
[13] C. A. Blockley and D. F. Walls, Phys. Rev. A **47**, 2115 (1993).
[14] C. A. Blockley, D. F. Walls, and H. Risken, Europhys. Lett. **17**, 509 (1992).
[15] J. I. Cirac, R. Blatt, A. S. Parkins, and P. Zoller, Phys. Rev. A **49**, 1202 (1994).
[16] H. P. Zeng and F. C. Lin, Phys. Rev. A **48**, 2393 (1993).
[17] P. Kumar, Opt. Lett. **15**, 1476 (1990).
[18] Jianming Huang and P. Kumar, Phys. Rev. Lett. **68**, 2153 (1992).
[19] M. G. Raizen, R. J. Thompson, R. J. Brecha, H. J. Kimble, and H. J. Carmichael, Phys. Rev. Lett. **63**, 240 (1989).
[20] R. J. Thompson, G. Rempe, and H. J. Kimble, Phys. Rev. Lett. **68**, 1132 (1992).
[21] G. Rempe, R. J. Thompson, R. J. Brecha, W. D. Lee, and H. J. Kimble, Phys. Rev. Lett. **67**, 1727 (1991).
[22] G. Rempe, R. J. Thompson, H. J. Kimble, and R. Lalerari, Opt. Lett. **17**, 363 (1992).
[23] M. Wilkens and P. Meystre, Phys. Rev. A **43**, 3832 (1991).
[24] S. M. Dutra, P. L. Knight, and H. Moya-Cessa, Phys. Rev. A **48**, 3168 (1993); S. M. Dutra and P. L. Knight, *ibid.* **49**, 1506 (1994).