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Reliability of Bell-inequality measurements using polarization correlations in parametric-down-conversion photon sources

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In recent years parametric-down-conversion photon sources have been proposed and used to test local realism via Bell-type inequalities. This study shows that this technique fails to discriminate between quantum mechanics and local realism in all those tests in which polarization correlations are measured, since dichotomic observables cannot be associated satisfactorily with the state produced in these experiments. A more adequate description of these photon sources leads to an alternative inequality for Einstein locality, which is always satisfied by quantum-mechanical predictions. The result can be extended to all the experiments in which only one-half of the emitted pairs are detected.

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In recent years a new technique that produces correlated photon couples has been used to perform tests of Einstein's locality via Bell's inequality for polarization correlation functions [1–4]. This technique uses a laser beam to produce a pair of degenerate down-converted photons in a nonlinear crystal of potassium dihydrogen phosphate. When the condition for degenerate phase matching is satisfied, two down-converted photons with the same linear polarization emerge along different paths. One of the two photons passes through a 90° polarization rotator, while the other crosses a compensating glass plate. The two photons are then reflected from two mirrors and impinge onto a beam splitter from opposite sides (Fig. 1).

Thus, the state of the emerging pair is given by

$$|\psi\rangle=\sqrt{T_xT_y}|x_1\rangle|y_2\rangle+\sqrt{R_xR_y}|y_1\rangle|x_2\rangle-i\sqrt{R_yT_x}|x_1\rangle|y_1\rangle +i\sqrt{R_xT_y}|x_2\rangle|y_2\rangle, \tag{1}$$

where R_x,R_y and T_x,T_y are the beam-splitter reflectivities and transmissivities, respectively, with $R_x+T_x=R_y+T_y=1$; $|x_i\rangle$ [$|y_i\rangle$] is the polarization state along the x direction (y direction) for the photon in the ith output channel of the beam splitter.

The quantum state $|\psi\rangle$, expressed by Eq. (1), has been used for testing the well-known Bell-type inequality obtained by local realism and some *ad hoc* assumptions like the "fair sampling" or "no-enhancement" hypotheses [5,6]:

$$B(\theta_1,\theta_1';\theta_2,\theta_2')=P(\theta_1;\theta_2)-P(\theta_1;\theta_2')+P(\theta_1';\theta_2) +P(\theta_1';\theta_2')-P(\theta_1';\infty)-P(\infty;\theta_2)\leqslant 0. \tag{2}$$

In Eqs. (2), $P(\theta_1;\theta_2)$ is the joint detection probability of a photon pair when the beams emerging from the beam splitter pass through linear polarizers, set at angles θ_1 and θ_2, respectively. $P(\theta_1';\infty)$ and $P(\infty;\theta_2)$ are the corresponding probabilities when either one of the linear polarizers is removed.

In this paper we will show that the local realism expressed in inequality (2) cannot be tested by using quantum state (1). This is due to the fact that the inequality (1) can only be obtained if a binary choice between the transmission and absorption in a polarizer is assumed. However, when a down-conversion photon source is used so as to obtain a correlated pair of photons, like the one described by the quantum state expressed in Eq. (1), the choice at each polarizer is not dichotomic. Besides the probabilities analyzed by Clauser *et al.*, there is another one. In fact, both of the two photons can travel along the same channel and reach only one of the two polarizers.

For every choice of polarizer orientation θ_1 and θ_2 Clauser *et al.* [5] have introduced four probabilities $P(\theta_{1\pm},\theta_{2\pm})$. For instance, $P(\theta_{1+},\theta_{2-})$ represents the probability that the photon that travels along the channel 1 is transmitted through polarizer 1 and the photon that travels along channel 2 is absorbed by polarizer 2. Thus the corre-

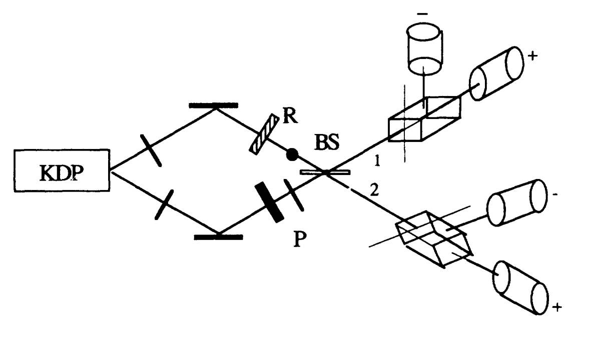

FIG. 1. Setup used to produce correlated pairs of photons. KDP denotes a crystal of potassium dihydrogen phosphate, R is a polarization rotator, and BS is a beam splitter.

lation function can be written as

$$E(\theta_1;\theta_2)=P(\theta_{1+},\theta_{2+})-P(\theta_{1+},\theta_{2-})-P(\theta_{1-},\theta_{2+})+P(\theta_{1-},\theta_{2-}). \quad (3)$$

When a dichotomous choice between transmission and absorption in a polarizer can be made, the following relations hold:

$$P(\theta_{1+},\theta_{2+})+P(\theta_{1+},\theta_{2-})+P(\theta_{1-},\theta_{2+})+P(\theta_{1-},\theta_{2-})=1, \quad (4a)$$

$$P(\theta_{1+},\theta_{2+})+P(\theta_{1+},\theta_{2-})=P(\theta_{1+},\infty_+), \quad (4b)$$

$$P(\theta_{1+},\theta_{2+})+P(\theta_{1-},\theta_{2+})=P(\infty_+,\theta_{2+}), \quad (4c)$$

$$P(\infty_+,\infty_+)=1, \quad (4d)$$

where $P(\theta_{1+},\infty_+)$, $P(\infty_+,\theta_{2+})$, and $P(\infty_+,\infty_+)$ are the corresponding probabilities with one, the other, or both linear polarizers removed, respectively. Using Eqs. (4), Eq. (3) becomes

$$E(\theta_1;\theta_2)=4P(\theta_{1+},\theta_{2+})-2P(\theta_{1+},\infty_+)-2P(\infty_+,\theta_{2+})+1. \quad (5)$$

In the above equation only cases of double transmission, which can be experimentally detected, appear. This allows us to transform Bell's inequality

$$|E(\theta_1;\theta_2)-E(\theta_1;\theta_2')|+|E(\theta_1';\theta_2)+E(\theta_1';\theta_2')|\le 2 \quad (6)$$

into inequality (2).

However, from Eq. (1) it follows that some photon pairs can travel along the same channel, reaching only one of the two polarizers. Thus this quantum state does not satisfy all of Eqs. (4). In fact, Eqs. (4a) and (4d) must be replaced by

$$P(\theta_{1+},\theta_{2+})+P(\theta_{1+},\theta_{2-})+P(\theta_{1-},\theta_{2+})+P(\theta_{1-},\theta_{2-})=\tfrac{1}{2},$$

$$P(\infty_+,\infty_+)=\tfrac{1}{2}, \quad (7)$$

which causes Eq. (5) to be replaced by

$$E(\theta_1;\theta_2)=4P(\theta_{1+},\theta_{2+})-2P(\theta_{1+},\infty_+)-2P(\infty_+,\theta_{2+})+\tfrac{1}{2}, \quad (8)$$

and, consequently inequality (2) to be written

$$-\tfrac{3}{4}\le B(\theta_1,\theta_1';\theta_2,\theta_2')\le\tfrac{1}{4}. \quad (9)$$

The inequality given by Eq. (9) cannot be violated by the quantum-mechanic joint transmission probabilities for the correlated photon pairs described by Eq. (1), even for ideal polarizers and detector. In fact, the latter are given by

$$P(\theta_1;\theta_2)=\tfrac{1}{2}[\cos\theta_1\sin\theta_2+\sin\theta_1\cos\theta_2]^2=\tfrac{1}{4}\sin^2(\theta_1+\theta_2),$$
$$P(\theta_1;\infty)=\tfrac{1}{4}\sin^2\theta_1+\tfrac{1}{4}\cos^2\theta_1=\tfrac{1}{4}, \quad (10)$$

if we assume, as done in the performed experiments, $R_x/T_x=R_y/T_y\cong 1$. So, for example, the maximum value of observable B according to the quantum-mechanical predictions expressed by Eqs. (10) is

$$B_{\rm QM}=\tfrac{1}{4}(\sqrt{2}-1)<\tfrac{1}{4},$$

and, therefore, the parametric-down-conversion photon source cannot give in this case quantum states with which to test quantum mechanics versus local realism, *not even in the case of ideal behavior of polarizers and detectors.*

This result is in contradiction to the claimed locality violation in this class of experiments but is in complete agreement with theoretical results on the subject. In fact, state (1) can be written in the factorized form

$$|\psi\rangle=(\sqrt{T_x}|x_1\rangle+i\sqrt{R_x}|x_2\rangle)(\sqrt{T_y}|y_2\rangle-i\sqrt{R_y}|y_1\rangle, \quad (11)$$

and it has been proved that factorable states always satisfy Einstein locality or, equivalently, Bell-type inequalities [7]. This theoretical consideration seems to be decisive in proving that the discussed results are reliable.

Even the approach of Clauser and Horne [6] fails in describing the limit of the locality in this class of experiments. In fact, in deducing their inequality it is essential to consider only the dichotomous case of measurement results ± 1; every attempt of the authors to prove that, in the case of a no-detection result, the upper limit of (2) remains zero, was unsuccessful. Moreover, it is possible to prove that the upper limit is equal to zero only in few particular cases and that, in general, it is greater than zero [8].

It must be noted that our criticism of the use of this source, for testing the locality, is different from Santos's criticism of atomic cascade sources connected with the selection of a subensemble of emerging pairs [9]. Specifically, the joint detection probabilities are computed not only by us, but also by all the authors of Refs. [1–4], using state vector (1); i.e., referring to the total set of produced photon pairs, without any selection of any subensemble. Our approach differs from the others insomuch as we deduce a new Bell-type inequality that is explicitly applicable in this case, whereas the authors of [1–4] use an inequality that does not correctly represent the locality in these experiments.

Furthermore, it has been observed that in the Bell-type inequality (2) only joint probabilities are present. Hence to overcome the problems related to the use of state (1), it may seem "reasonable" to cut out the last two terms in quantum state (1). If this procedure is applied to this case, the result will be the entangled (but not normalized) state

$$|\psi\rangle=\tfrac{1}{2}(|x_1\rangle|y_2\rangle+|y_1\rangle|x_2\rangle), \quad (12)$$

and no violation of locality will occur.

Only if the normalization of the quantum state to *the subset of photon pairs traveling in both channels* is imposed, will the wave function

$$|\psi\rangle=\frac{1}{\sqrt{2}}(|x_1\rangle|y_2\rangle+|y_1\rangle|x_2\rangle), \quad (13)$$

which leads to the violation of Bell-type inequality, be obtained.

It is worth noting that, in this case, this operation is not correct in principle. In fact, implicitly it considers the ensemble as consisting of two subensembles: one in which one photon is in channel 1 and the other in channel 2, and a second in which both photons are in same channel. In quantum mechanics each pair described by state (1) has the photons traveling at the same time along the two different channels and in the same channel. A set consisting of the two previous subsets is described by a state that is a 50%-50% mixture of

$$\frac{1}{\sqrt{2}}(|x\rangle_1|y\rangle_2+|y\rangle_1|x\rangle_2),$$

and (14)

$$\frac{1}{\sqrt{2}}(|x\rangle_1|y\rangle_1-|y\rangle_2|x\rangle_2).$$

States (1) and (14) are physically distinguishable, for example, overlapping the two beams onto another beam splitter.

Moreover, the selection of the subensemble of the coincidence pairs is neither a measuring process nor a state preparation. It is not a measuring process because the measurements are made behind the polarizers. Thus they give a factorized state of two photons in a well-defined polarization state. It is not a state preparation since other pairs of photons are traveling along the two channels, and these contribute to the single-photon detection rates.

At least, this procedure can lead to error when quantum mechanics is compared to local realism. In fact, in a previous paper [10] we argued that state (12) can be reproduced in a hidden-variable local realistic model for physical correlated systems. If the quantum operation of normalization of the probability amplitudes is imposed on state (12), state (13) can be obtained, but any possibility of physical and local interpretation of the model will be lost. This result shows the critical importance of the probability interpretation of the wave function. In fact, quantum mechanics can be obtained starting from local realism, but only after the normalization step, which allows the raising of correlation-function values and, consequently, the violation of Bell's inequality.

Our criticism can be easily extended to all those tests in which state (1) or similar states are obtained starting from a factorized state and the measurement of coincidence selects only a subensemble of photon pairs less than or equal to one-half of the initial generated pair number, as, for example, the experiment of Kiess *et al.* [11] with a type-II parametric-down-conversion source, the experiment of Kwiat *et al.* [12] on energy-time correlation, and the experiment with Franson's setup [13].

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