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Preparation of macroscopic superpositions in many-atom systems

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We propose a technique to prepare two or more atoms in certain entangled states, based on the interaction of the atoms with a cavity mode. After the atomic state is prepared, the cavity mode is left in the vacuum state, so dissipation does not affect the generated entangled states. These states could be used for improved tests that challenge local realistic theories, and to examine related phenomena, which, thus far, have been realized only with photons.

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Entangled states of two or more particles which are spatially separated give rise to quantum phenomena that cannot be explained in classical terms. One important example of such states is a pair of spin-$\frac{1}{2}$ particles, in the singlet state

$$|\Psi\rangle_S=\frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\downarrow_2\rangle-|\uparrow_2\rangle|\downarrow_1\rangle), \qquad (1)$$

where the subscripts 1 and 2 label the particles in this state. This particular state is in the realm of the original derivation of Bell's theorem [1], which states that local hidden-variable theories of quantum mechanics are not possible, and is the basis of some quantum effects introduced recently, such as the "teleportation" of quantum states [2]. Another particularly interesting entangled state of three particles is the state

$$|\Psi\rangle_{\rm GHZ}=\frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle-|\downarrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle), \qquad (2)$$

proposed by Greenberger, Horne, and Zeilinger [3]. With this state, a single set of observations is required in order to confront quantum mechanics with local realistic theories.

During the past decade, a number of interesting experiments related to the quantum nature of *two*-particle entangled states have been proposed and carried out. In all these experiments, the particles are photons, and entanglement results from different paths taken by the photons [4] or from different polarizations [5]. Several experiments have shown violations of Bell's inequalities [1,6], providing overwhelming support for quantum mechanics versus local realistic theories. However, as noted by Clauser and Horne [7], the statement that these experiments with photons violate Bell's inequality is strictly true only if some supplementary assumptions are invoked. In order to test Bell's inequality without any of these assumptions, it would thus be desirable to prepare in a controlled way two different particles in an entangled state, each particle going to a different region of space, and to obtain the correlation between appropriate observables by means of two spatially separated measurements. Furthermore, the detection has to be carried out with high efficiency [7].

In this paper we propose a simple scheme to prepare superpositions of atoms [8] with *macroscopic* separation in the forms (1) and (2) which facilitate tests challenging local realistic theories, as well as of other related phenomena that have been only observed with photons so far. The spin-$\frac{1}{2}$ particles are, in our case, two-level atoms, and the entangled states (1) and (2) are produced by the interaction with the electromagnetic field contained in a cavity. Note that with atoms in the state (1), one would be able to test Bell's in-

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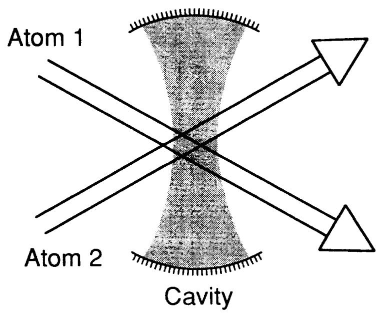

FIG. 1. Proposed configuration for the preparation of the entangled state (singlet state) of two two-level atoms. Initially the cavity is in the vacuum state. Atom 1 enters the cavity in its excited state and atom 2 in its ground state. After crossing the cavity, the atoms become entangled and the cavity is left in its original state: the vacuum.

equality [1] without resorting to any additional assumptions. Recent advances in cavity QED, both in the microwave [9,10] and in the optical regimes [11], imply that these states should be realizable with existing or planned technology. This fact, together with the near-unity detection efficiency for Rydberg atomic states [12], makes this technique a promising path for a variety of experiments related to the quantum nature of entangled states.

Our scheme to prepare the state (1) is based on the interaction of two atoms, initially in their excited and ground states, respectively, with a resonant cavity mode in a vacuum state. An important feature of this scheme lies in the fact that after the preparation of the entangled state, the cavity mode is left in its original state, the vacuum, which is not entangled with the two atoms. As a consequence, cavity damping mechanisms before and after the interaction will not affect the feasibility of the singlet state, and therefore cavity dissipation must be negligible only during the time the entangled state is prepared. We will also show that with a slight modification, the proposed scheme would serve to prepare the state (2). This technique can be easily generalized for the preparation of n-particle entangled states.

The preparation of the state (1) takes place in two steps. First, one atom (atom 1) initially prepared in its excited state $|\uparrow_1\rangle$ is sent through a cavity in the vacuum state $|0\rangle$. After the atom has left the cavity, a second atom (atom 2) prepared in its ground state $|\downarrow_2\rangle$ is sent through the cavity in a different direction (see Fig. 1). The atom-cavity mode interaction is described by the Jaynes-Cummings model. The interaction Hamiltonian in a rotating frame at the cavity mode frequency and in the rotating-wave approximation is $H=ig(t)(\sigma_+a-a^\dagger\sigma_-)$, where $\sigma_+=(\sigma_-)^\dagger=|\uparrow\rangle\langle\downarrow|$ is the usual spin-$\frac{1}{2}$ excitation operator for the two-level system, and $a^\dagger$ and a are the creation and annihilation operators for the cavity mode. For simplicity, we have taken the two-level transition on resonance with the cavity mode. The coupling parameter between the atom and the cavity mode is $g(t)$, which is time dependent as the atom crosses the cavity, due to the spatial structure of the cavity mode.

An important property of the Jaynes-Cummings Hamiltonian is that it conserves the excitation number $a^\dagger a+|\uparrow\rangle\langle\uparrow|$. In particular, the state $|\downarrow\rangle|0\rangle$ (which is the ground state of the Jaynes-Cummings model) does not change during the interaction, and the states $|\uparrow\rangle|0\rangle$ and $|\downarrow\rangle|1\rangle$ will experience *vacuum Rabi oscillations* [10,11], since they have the same excitation number. Here, $|0\rangle$ and $|1\rangle$ are Fock states of the cavity mode with zero and one photons, respectively. These Rabi oscillations will depend on the interaction time t_i of the atom i with the cavity mode, and therefore on the atomic velocity $v_i=L/t_i$ (L is the length of the cavity).

We assume that the velocity of the first atom v_1 has been selected in such a way that it undergoes 1/4 of a Rabi oscillation due to the interaction with the cavity mode. Hence we have for the state of the system

$$|\Psi\rangle=\frac{1}{\sqrt{2}}(|\uparrow_1\rangle|0\rangle-|\downarrow_1\rangle|1\rangle)|\downarrow_2\rangle. \quad (3)$$

The state of the system after the second atom crosses the cavity can be readily calculated if one takes into account that the state $|0\rangle|\downarrow_2\rangle$ remains unchanged during the interaction. By selecting the velocity of the second atom v_2 in such a way that the state $|1\rangle|\downarrow_2\rangle$ performs half a Rabi cycle, the final state of the system becomes

$$|\Psi\rangle=\frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\downarrow_2\rangle|0\rangle-|\downarrow_1\rangle|\uparrow_2\rangle|0\rangle)$$
$$=\frac{1}{\sqrt{2}}(|\uparrow_1\rangle|\downarrow_2\rangle-|\downarrow_1\rangle|\uparrow_2\rangle)|0\rangle=|\Psi\rangle_S|0\rangle, \quad (4)$$

and therefore the state (1) has been prepared. Note that the state of the cavity mode (the vacuum) factorizes, so that there is no "projection noise" when one traces over the unobserved cavity field. Furthermore, the cavity mode is left in its initial vacuum state, so that it is ready to "prepare" another singlet pair. Hence the cavity serves as a quantum mechanical apparatus to produce entangled states of two two-level atoms.

So far we have assumed that the velocity of the atom is fixed. However, in a real experiment, there will be dispersion in the velocities of both atoms, Δv_1 and Δv_2. To evaluate their effect we take the coupling parameter to be constant during the interaction time $[g(t)=g]$. Then, the final state of the system *for any velocities* can be readily calculated, resulting in

$$|\Psi\rangle=[\cos(gt_1)|\uparrow_1\rangle|\downarrow_2\rangle-\sin(gt_1)\sin(gt_2)|\downarrow_1\rangle|\uparrow_2\rangle]|0\rangle$$
$$-\sin(gt_1)\cos(gt_2)|\downarrow_1\rangle|\downarrow_2\rangle|1\rangle. \quad (5)$$

The singlet state will be produced for $gt_1^0=n\pi/4$ and $gt_2^0=m\pi/2$ (n,m integers), or, equivalently, for velocities $v_1^0=4gL/\pi$ and $v_2^0=2gL/\pi$ (we have taken $n=m=1$ for the sake of simplicity). Let us assume that the atoms have a velocity $v_i=v_i^0+\Delta v_i$. Then, in order for the state $|\Psi\rangle$ not to depart appreciably from the desired state $|\Psi\rangle_S$, we require $g\Delta t_i\simeq gt_i\Delta v_i/v_i\ll 1$. This condition can be reexpressed as $\Delta v_i/v_i\ll 2/\pi$. Taking as a cavity a superconducting cylinder of length L of the order of 1 cm, and the states $|\downarrow\rangle$ and $|\uparrow\rangle$ circular Rydberg levels with principal quantum numbers 50

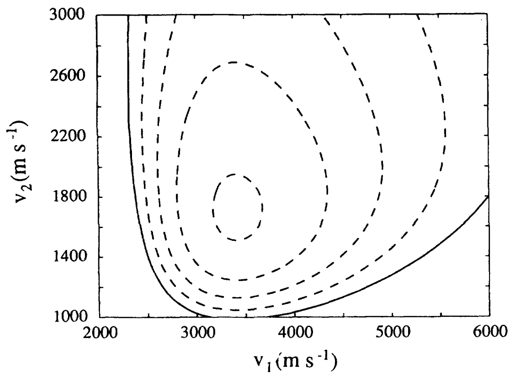

FIG. 2. Proposed test of Bell's inequality: Contour plot of Bell's S parameter as a function of the atomic velocities v_1 and v_2. The cavity length $L=1$ cm, and the coupling parameter varies as a sine arch. Cavity damping as well as spontaneous decays are negligible during the time the measurement takes place.

and 51, we see that g is $\approx 2\times 10^5$ s^{-1} [9]. Hence the velocity must be $v_1^0=2v_2^0\simeq 2000$ m/s, and $\Delta v_i \ll v_i$. These parameters are within reach of current experiments in the microwave regime. On the other hand, it is clear from (5) that general entangled states can be prepared for $v_2=2gL/\pi$, and varying v_1, which, as has been recently reported, can be used to demonstrate nonlocality without using inequalities [13].

In the following, we discuss the possibility of testing Bell's inequality with the scheme we have proposed. In order to do that, one should be able to measure the quantity [6]

$$S=\langle \sigma_a^1\sigma_b^2\rangle - \langle \sigma_a^1\sigma_{b'}^2\rangle + \langle \sigma_{a'}^1\sigma_b^2\rangle + \langle \sigma_{a'}^1\sigma_{b'}^2\rangle. \tag{6}$$

Here, $\sigma_{\vec{\alpha}}^i=\vec{\sigma}^i\vec{\alpha}$ ($\alpha=a,b,a',b'$ and $i=1,2$), where $\vec{\sigma}^i=(\sigma_x^i,\sigma_y^i,\sigma_z^i)$ is the familiar Pauli operator for the two-level atom i. Local hidden-variable theories predict $|S|\leq 2$ while quantum mechanics predicts $|S|>2$ for certain values of the vectors $\vec{a},\vec{b},\vec{a}',\vec{b}'$. For instance, for angles $\angle(\vec{a},\vec{b})=\angle(\vec{b},\vec{a}')=\angle(\vec{a}',\vec{b}')=\pi/4$, the quantum mechanical prediction yields $|S|=2\sqrt{2}$. For two-level atoms such as analyzed in this paper, the measurement of the Pauli operators along any direction $\sigma_{\vec{\alpha}}$ can be carried out by applying a controlled pulse to the atom which transforms the states $|\downarrow\rangle$ and $|\uparrow\rangle$ into the eigenstates of $\sigma_{\vec{\alpha}}$; a subsequent measurement of the state of the atom (ground or excited) would give the desired measurement of the observable corresponding to $\sigma_{\vec{\alpha}}$. For Rydberg atoms, these processes can be very efficient (see, for example, Refs. [9,14,15] and references therein).

Figure 2 shows a contour plot of S for different velocities v_1 and v_2 and for typical parameters found in experiments on cavity QED in the microwave regime: $L=1$ cm, and $g(t)$ a sine arch (which corresponds to the TE_{121} mode of a cylinder cavity) with $\max[g(t)]=4.2\times 10^5$ s^{-1}. The dashed lines in the plot represent $S=-2.2,-2.4,-2.6$, and -2.8, whereas the solid line corresponds to $|S|=2$, i.e., the maximum value allowed by local realistic theories. Note that Bell's inequality $|S|\leq 2$ is violated for an extremely wide range of atomic velocities which would make it possible to perform appropriate tests without having to select the velocities very accurately.

The preparation scheme proposed here can be easily generalized to produce other entangled states of more than two particles. In the following we show how the state $|\Psi\rangle_{\rm GHZ}$ (2) can be prepared. Let us assume that the cavity mode is initially in a coherent superposition of Fock states $|0\rangle$ and $|3\rangle$,

$$|\Psi\rangle_c=\frac{1}{\sqrt{2}}(|0\rangle-|3\rangle). \tag{7}$$

This state could be prepared using some recently proposed schemes which involve either the adiabatic passage of two-level atoms with Zeeman structure through a cavity [16], or, alternatively, the measurement of atoms leaving the cavity [17] which project the state of the cavity mode onto (7). Next we assume that we send three atoms in the ground state, one after another, with given velocities v_1, v_2, and v_3, respectively. The interaction of the first atom with the cavity mode can be easily calculated. As the atom is in its ground state, the state $|0\rangle|\downarrow_1\rangle$ is not modified due to the interaction. If the velocity v_1 is selected in such a way that the state $|3\rangle|\downarrow_1\rangle$ performs half a Rabi cycle after the interaction, the state of the system atom-1–plus–cavity mode will be

$$|\Psi\rangle_1=\frac{1}{\sqrt{2}}(|0\rangle|\downarrow_1\rangle-|2\rangle|\uparrow_1\rangle). \tag{8}$$

Analogously, since the second atom is also in its ground state, the state $|0\rangle|\downarrow_2\rangle$ remains unchanged during the interaction with the cavity mode. When the velocity is selected in such a way that the state $|2\rangle|\downarrow_2\rangle$ performs half a Rabi cycle, the state of the atom-1–plus–atom-2–plus–cavity mode becomes

$$|\Psi\rangle_2=\frac{1}{\sqrt{2}}(|0\rangle|\downarrow_1\rangle|\downarrow_2\rangle-|1\rangle|\uparrow_1\rangle|\uparrow_2\rangle). \tag{9}$$

Finally, by selecting the velocity of the third atom, one obtains for the system the state

$$\begin{aligned}|\Psi\rangle_3&=\frac{1}{\sqrt{2}}(|0\rangle|\downarrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle-|0\rangle|\uparrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle)\\&=\frac{1}{\sqrt{2}}(|\downarrow_1\rangle|\downarrow_2\rangle|\downarrow_3\rangle-|\uparrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle)|0\rangle\\&=|\Psi\rangle_{\rm GHZ}|0\rangle.\end{aligned} \tag{10}$$

We wish to point out that entangled states between four cavity modes and one three-level atom have been proposed recently [18]. In contrast to this model, our proposal provides entangled states between three atoms which are spatially separated.

This scheme can be easily generalized for the preparation of other entangled states of n particles. Provided one is able to prepare the cavity mode initially in the state

$$|\Psi\rangle_c = \frac{1}{\sqrt{2}}(|0\rangle - |n\rangle), \tag{11}$$

the state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|\downarrow_1\rangle|\downarrow_2\rangle\cdots|\downarrow_n\rangle - |\uparrow_1\rangle\uparrow_2\rangle\cdots|\uparrow_n\rangle) \tag{12}$$

is obtained by sending n atoms initially prepared in their ground state, and with controlled velocities.

These schemes to prepare entangled states are valid provided one can neglect dissipation in both the two-level atoms and in the cavity mode during the preparation procedure. For circular state Rydberg atoms and microwave cavities both radiative lifetimes and cavity decay times range from 0.01 to 0.1 s [14]. The required interaction time between each atom and the cavity mode is $\lesssim 10^{-5}$ s. Thus there is sufficient time to prepare the states (12) without any significant decoherence due to dissipation [note that, for example, to prepare the state (10) one has to send six atoms through the cavity].

As an aside, we note that all these states could, in principle, also be prepared using the adiabatic transfer technique recently proposed in Refs. [16,19] in the cavity QED context. This technique is independent of the atomic velocity, and it involves the adiabatic transfer between atomic and cavity mode coherences. Hence one does not need to control atomic velocities in order to prepare the entangled states between two or more atoms. We would like to point out that the state (7) for the cavity mode could be prepared, in this case, using the same technique.

In summary, we have proposed a scheme for the preparation of entangled states between atoms; in particular, singlet pairs and GHZ states. The scheme is based on the passage of two-level atoms through a resonant cavity. After the entangled state is prepared, the cavity is left in the vacuum which prevents the atomic state from being modified by any kind of noise in the cavity. The prepared state could serve to perform tests of Bell's inequality which circumvent some of the problems present in experiments realized with photons [5,4]. We believe that such a scheme is experimentally feasible with Rydberg atoms and microwave cavities. A more detailed analysis of the preparation of atomic entangled states, along with other possible applications of these states, will be presented in a forthcoming paper.

Note added. After submission of the manuscript we found out that Phoenix and Barnett [20] have proposed a scheme to prepare the singlet atomic state (1) that is similar to the one proposed in this paper.

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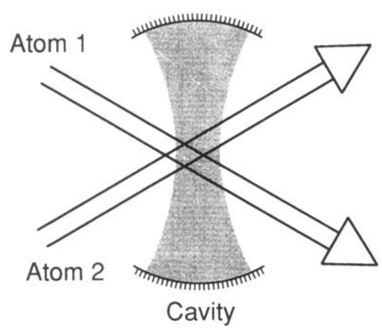

FIG. 1. Proposed configuration for the preparation of the entangled state (singlet state) of two two-level atoms. Initially the cavity is in the vacuum state. Atom 1 enters the cavity in its excited state and atom 2 in its ground state. After crossing the cavity, the atoms become entangled and the cavity is left in its original state: the vacuum.