Using a Penning trap to weigh antiprotons

V. Lagomarsino, V. Lia,* G. Manuzio, and G. Testera
National Institute of Nuclear Physics, Via Dodecaneso 33, 16146 Genoa, Italy
(Received 17 January 1994)

A method for measuring the Earth's gravitational acceleration g on antiprotons is proposed. The value of g is obtained by measuring the gravity-induced shift of the center of the radial orbits of antiprotons that are stored in a Penning trap. Such a shift is a measurable effect for particles of very low energy ($\simeq 1$ μeV) if the point of injection lies near the axis of a big Penning trap in which the magnetic field is perpendicular to the direction of g. A possible experimental setup is described and several sources of error are analyzed. A comparison with a time-of-flight method, originally proposed to measure g on antiprotons (and now under experimental investigation by an international collaboration), indicates the advantages of the present proposal. The experimental feasibility of the method of measurement that we propose relies on the practical limit that can be achieved in minimizing the antiprotons' energy and on controlling the radial coordinates of the particles before injection into the g-sensitive trap.

PACS number(s): 06.30.Dr, 04.80.−y, 07.77.+p

I. INTRODUCTION

The equivalence principle is a cornerstone of general relativity and, more generally, of all metric theories of gravity. Different versions of this principle, restricting their applications to different classes of phenomena, are presently considered [1]. The weak equivalence principle (WEP) applies to mechanical quantities, whereas the Einstein equivalence principle includes WEP and all nongravitational phenomena.

Tests of WEP began with experiments of Galileo, achieved a significant level of precision with Eötvös's experiments [2], and recently showed that different macroscopic bodies fall in a gravitational field with the same acceleration within one part in 10^{11} [3].

Evidence that matter in the form of elementary particles still obeys WEP is far less conclusive. Although Eötvös-type experiments strongly suggest that neutrons, protons, and electrons obey WEP with a precision of one part in 10^8 [4], very few direct tests (with quite low precision) have been performed with massive elementary particles [5,6]. Moreover, the gravitational behavior of antimatter has never been experimentally investigated. WEP has never been tested directly with stable and massive antiparticles like positrons or antiprotons. A measurement of the gravitational acceleration of antimatter could provide further evidence for WEP and useful information for contemporary research in fundamental physics. Relevant issues include the low-energy behavior of gravity in unified field theories and open problems related to matter-antimatter asymmetry in the universe (a broad scenario is outlined in [7]).

In this paper we describe a method for measuring the acceleration g of antiprotons subjected to the gravitational field of the Earth. The experimental techniques which have been proposed to measure the gravitational force on antiprotons (or on charged particles) are essentially of two types. One is known as TOF (time-of-flight technique) and has already been employed in order to measure the value of the gravitational force acting on electrons [6]. The value of g is obtained by measuring the distribution of the transit times of particles which drift along a vertical tube where all the nongravitational forces can be neglected. In order to reach the top of the tube, where they can be detected, entering particles must have a minimum velocity. Therefore, the time distribution for upward launching will show a cutoff time t_c given by

$$t_c=\left[\frac{2L}{g}\right]^{1/2}, \tag{1}$$

where L is the length of the drift tube. The value of g is calculated from the above relation after t_c has been obtained by fitting the measured time distribution near the point of cutoff. This method has also been proposed in order to measure the gravitational force on antiprotons (particles will be extracted from the LEAR machine at CERN [8] and the experimental research on this subject is now actively pursued [9].

The other approach takes advantage of the fact that the combined action of the gravitational field and a steady transverse magnetic field produces a drift motion of the center of the cyclotron orbits of the charged particles which is simply related to the magnitude of the gravitational force. An application of this principle to the determination of g for antiprotons had originally been proposed by some of the authors and has been briefly described [10]. A description of a modified version of the same idea has also appeared in the literature [11]. The present article is a study of the experimental feasibility of a measurement of g on antiprotons by following the

*Present address: Massachusetts Institute of Technology, Cambridge, MA 02139.

second of these principles and implementing it with the use of a Penning trap.

II. FIRST PRINCIPLES OF THE METHOD

The easiest way to describe the principle of this method is to consider an idealized situation in which a particle of mass m and charge q is moving under the influence of the force of gravity and the force produced by a uniform magnetic field $\mathbf{B}_g$ directed along the z axis and perpendicular to $\mathbf{g}$. The resulting motion of the center of the cyclotron oscillations is the superposition of a uniform rectilinear motion along z and a drift motion along the direction of $\mathbf{g}\times\mathbf{B}_g$ with constant speed v_d given by

$$v_d=\frac{mg}{qB_g} . \tag{2}$$

As a consequence, a measurement of the displacement Δ of the center of the cyclotron orbits

$$\Delta=\frac{mg}{qB_g}\Delta t , \tag{3}$$

in a given interval of time Δt, can provide a value for g. However, the value of v_d is so small that even for a measurement at the level of a few percent one needs values of Δt which correspond to drift tubes that are too long to be considered in practice. A possible solution to this problem could be to confine the motion of the particles along the z direction by using "the walls" produced by an electrostatic potential superimposed to the magnetic field. However, Maxwell's equations prescribe that a confining electrostatic field $\mathcal{E}_z$ pointing along z has also radial components $\mathcal{E}_x$, $\mathcal{E}_y$ (whose existence has been ignored in [11]). The effect of these components cannot be neglected because they cause an additional drift motion of the center of the cyclotron oscillations. With the simplest choice one can take

$$\mathcal{E}_z=-\frac{k}{q}z . \tag{4}$$

Then for the radial components one gets

$$\mathcal{E}_x=\frac{k}{2q}x , \tag{5}$$

$$\mathcal{E}_y=\frac{k}{2q}y . \tag{6}$$

This field's configuration, that is, a constant magnetic field pointing along the z axis and the electric field described by (4), (5), and (6), is exactly the field configuration used in a Penning trap (see Fig. 1). This is a well-known device commonly used to confine charged particles [12,13].

The motion of a charged particle stored in a Penning trap is fully discussed in the literature [12]; in the present paper we will focus our attention on the possibility of using such a device to weigh charged particles, especially antiprotons. The trajectory of a charged particle stored in a Penning trap and subjected to the force of gravity is found to be very similar to the motion which results when the influence of gravity is completely neglected; the axial motion and the radial motion are in both cases fully decoupled and the force of gravity does not influence the axial motion, which is harmonic with frequency Ω_z,

$$\Omega_z=\left[\frac{k}{m}\right]^{1/2}=\left[\frac{2qV_g}{mZ_0^2}\right]^{1/2} , \tag{7}$$

where Z_0 is the geometrical dimension defined in Fig. 1 and V_g is the depth of the axial potential well which depends in a simple way on the values of the electric potentials that are applied to the electrodes of the trap.

The radial motion (irrespectively of gravity) is described by the superposition of two oscillations: the cyclotron motion with frequency Ω_c and the slower magnetron motion with frequency Ω_m. The relevant result is that the only effect produced by gravity is a shift of the (equilibrium point) (see Fig. 2) of these radial oscillations with respect to the geometrical center of the trap. The new position of equilibrium rests at a point C where the force of gravity is exactly balanced by the electric force. With respect to the axis of Fig. 1, the point C has Cartesian coordinates $(C_g,0,0)$, where

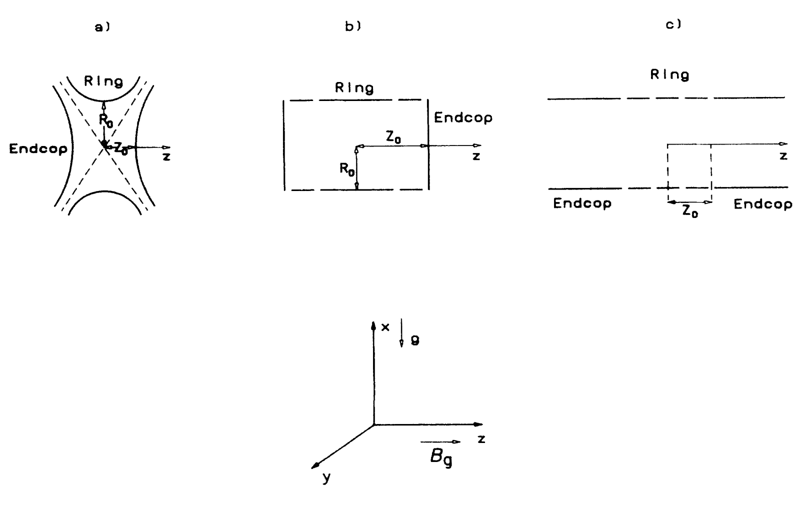

FIG. 1. Typical cross sections of Penning traps. The shape of the electrodes is obtained by rotating the figure around the z axis. (a) shows a regular Penning trap with three electrodes (two endcaps and one ring) shaped as revolution hyperboloids. The electric field described in Sec. II can be obtained in the trap when a constant potential difference is applied between the endcaps and the ring. (b) and (c) show cylindrical Penning traps. The prescribed field can be obtained (near the geometrical center) by applying appropriate constant potentials to the electrodes.

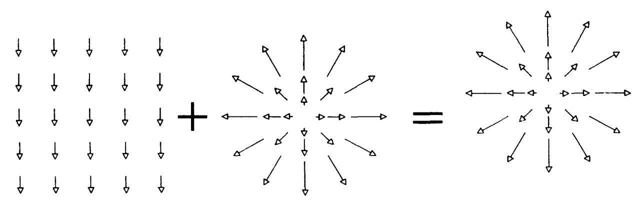

FIG. 2. Vector diagram showing that the addition of the constant gravitational field to the radial electric field of a Penning trap simply shifts the equilibrium point but does not change the shape of the force field.

$$C_g = 2\frac{mg}{k} \ . \tag{8}$$

The time dependence of the radial coordinates of the particle is given by

$$X(t)+iY(t)=(X_m+iY_m)e^{-i\Omega_m t}+(X_c+iY_c)e^{-i\Omega_c t}+C_g, \tag{9}$$

where

$$\Omega_m = \frac{\Omega_z^2}{2\Omega_c} \ , \tag{10}$$

$$\Omega_c = \frac{qB_g}{m} - \Omega_m \ . \tag{11}$$

The quantities R_m and R_c,

$$R_{m,c} = \sqrt{X_{m,c}^2 + Y_{m,c}^2} \ , \tag{12}$$

are the magnetron and cyclotron radii and their values are related to the particle initial position (x_0 and y_0) and initial velocity (v_{x0} and v_{y0})

$$R_m = \left[\frac{(v_{y0}+\Omega_c x_0 - \Omega_c C_g)^2 + (\Omega_c y_0 - v_{x0})^2}{(\Omega_c - \Omega_m)^2} \right]^{1/2}, \tag{13}$$

$$R_c = \left[\frac{(v_{y0}+\Omega_m x_0 - \Omega_m C_g)^2 + (\Omega_m y_0 - v_{x0})^2}{(\Omega_c - \Omega_m)^2} \right]^{1/2}. \tag{14}$$

The radial motion of a particle stored in a Penning trap and subjected to the force of gravity can then be described by the motion of the guiding center of the cyclotron orbits; the trajectory of this point is a circle of radius R_m centered at $C=(C_g,0,0)$. Figure 3 shows how radii R_m and R_c are related to the initial conditions of motion for a particle that, before undergoing capture into a Penning trap, travels along the z axis under the influence of a magnetic field whose strength is identical to the intensity of the field that is present inside the trap. Before entering the trap (left-hand side of Fig. 3), the particle cyclotron motion has radius $\tilde{r}_c$ and is centered at point A of radial coordinates (a_x,a_y). The right-hand side of Fig. 3 shows the particle radial orbit after capture. The two geometrical parameters R_m and R_c, indicated in Fig. 3, are given by the following formulas:

$$R_c \simeq \frac{m\sqrt{v_{x0}^2+v_{y0}^2}}{qB} = \tilde{r}_c \ , \tag{15}$$

$$R_m^2 = (a_x - C_g)^2 + a_y^2 \ . \tag{16}$$

These are obtained from (13) and (14) using

$$x_0 = a_x + r_c \cos(\tilde{\theta}_c) \ , \tag{17}$$

$$y_0 = a_x - r_c \sin(\tilde{\theta}_c) \ , \tag{18}$$

$$v_{x0} = -\frac{qB_g}{m} r_c \sin(\tilde{\theta}_c) \ , \tag{19}$$

$$v_{y0} = -\frac{qB_g}{m} r_c \cos(\tilde{\theta}_c) \tag{20}$$

($\tilde{\theta}_c$ is a proper phase), and

$$\Omega_m \ll \Omega_c \tag{21}$$

[the values of the magnetic field and the electric potentials can be properly chosen in order to satisfy (21)].

The effect of the force of gravity is completely negligible at typical experimental conditions for Penning traps ($Z_0 \simeq 1$ cm, $V_0 \simeq$ a few volts). However, a Penning trap can be used to weigh an antiproton if the well depth gets close to the value of the difference in gravitational potential energy and if the trap dimensions are large enough. Explicitly, using (7) and (8), one finds

$$C_g = \frac{mgZ_0^2}{qV_g} \ , \tag{22}$$

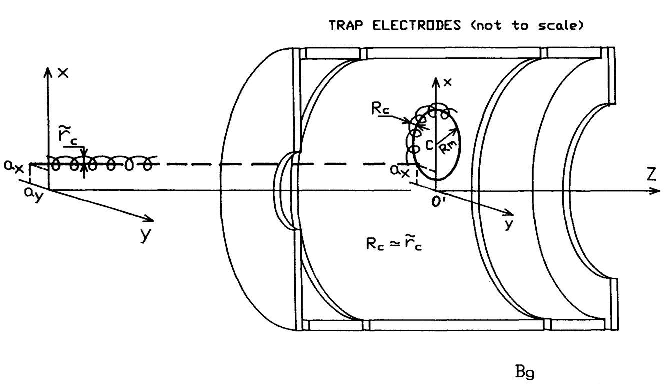

FIG. 3. The relation between the radial orbit parameters of a charged particle moving inside a Penning trap (and subjected to the force of gravity) and the radial initial conditions of motion preceding the particle capture. For simplicity the axial motion inside the trap is not displayed. With the gravity field pointing in the x direction, the center of the cyclotron motion inside the trap describes a circle centered around the point $C=(C_g,0,0)$. For an ideal capture process, the cyclotron radius R_c inside the trap is practically equal to the cyclotron radius $\tilde{r}_c$ before capture and the radius R_m is linked directly to the particle radial position (a_x,a_y) before capture. R_m is close to C_g if the values of a_x and a_y are small.

that is

$$C_g = 10^{-1} \frac{[Z_0^2\ (\mathrm{m}^2)]}{[V_g\ (\mu\mathrm{V})]}\ \mathrm{m}\ . \tag{23}$$

A measurement of g can therefore be obtained from that of C_g. This in turn can be obtained after measuring the radial coordinates of many trapped particles (the particles will be extracted along the z axis of the trap and sent to impinge on a detector which measures their positions) under two different conditions, namely, (i) a very short time after capture into the trap, and (ii) a time T_g [equal to one-half of the magnetron period $T_m = (2\pi)/\Omega_m$] after capture into the trap (other choices for the value of T_g are possible). The mean value of the first class of measurements should yield $\langle X \rangle = 0$ and $\langle Y \rangle = 0$, defining the position of the point O' in Fig. 3. The standard deviations about these values will measure the fluctuations due to the unknown parameters a_x, a_y, and $\tilde{r}_c$. The mean value of the second class of measurements will define another point placed at a distance $2\langle R_m \rangle = 2C_g$ from O'. These data should fluctuate around the mean value with the same standard deviation obtained from the first group of measurements. The value of C_g and its statistical error can therefore be measured.

III. EXPERIMENTAL SETUP

In order to measure g with the method that we propose, very low-energy antiprotons ($\simeq 1\ \mu$eV) need to be injected near the axis of a big Penning trap ($Z_0 \simeq 1$ m). After a given time has elapsed, particles have to be extracted and their radial coordinates measured.

A possible schematic representation of the experimental apparatus is shown in Fig. 4. Three different traps are employed:

(i) The first one, which will be called "storage trap," can be used to store a large number of antiprotons with a thermal energy of 4.2 K ($\simeq 350\ \mu$eV).

(ii) The second trap, which will be called "cooling trap," can be used to cool very small bunches of antiprotons (extracted from the storage trap) down to energies of a few μeV.

(iii) The last trap (which has larger dimensions) can be used to perform the gravity measurement and will be called a "g trap."

Recent experiments at CERN have demonstrated the possibility of capturing 10^4–10^6 antiprotons in a Penning trap; the particles had been extracted from LEAR [9,14]. Antiprotons have subsequently been cooled down to 4.2 K by using electron and resistive cooling. The storage trap of the apparatus we propose should perform in such a fashion.

Due to space-charge effects, the volume occupied by a large number of particles in this trap would be relatively high. To reduce the dimension of the radial orbits of the particles before injecting them into the g trap, a few particles at a time will be transferred from the storage to the cooling trap. The operations to be performed in the cooling trap are the following: (a) reduce the number of trapped particles to 1 by expelling the rest; (b) reduce the magnetron radius of the remaining particle to the minimum possible value; and (c) reduce the particle energy to the optimum value for injection into the g trap.

Techniques that can be used in order to perform operations (a) and (b) are very well known and have already been employed with success. To detect and to measure the number of particles present in the cooling trap one can proceed as described in [15]. In order to decrease the dimensions of the magnetron orbits it is possible to use the sideband cooling technique, which has been studied and has already been employed in several experiments [12]. This last can be used, in particular, to couple the cyclotron motion with the magnetron motion while keeping the cyclotron motion at low temperature: the final equilibrium condition should be

$$r_c = r_m\ , \tag{24}$$

where r_c and r_m are the cyclotron and the magnetron radii in the cooling trap. Therefore if the limiting energy E_l of the cyclotron motion is 4.2 K, the values of the radii at equilibrium should be

$$r_c \simeq r_m \simeq \sqrt{2} \left[\frac{m}{q} \right]^{1/2} \frac{\sqrt{[E_l\ (\mathrm{eV})]}}{B_c}\ , \tag{25}$$

where B_c is the magnetic field in the cooling trap. For values of B_c of the order of a few teslas, one obtains radii corresponding to fractions of 1 μm.

To achieve goal (c), that is, further reduction of the antiprotons's energy to values of the order $\simeq 1\ \mu$eV, we plan to use adiabatic cooling by decreasing the values of the confining potential of the cooling trap V_i and the value of the magnetic field B_c. If V_i is reduced adiabatically to the final value V_f, the initial axial energy E_{zi} decreases to E_{zf}, where

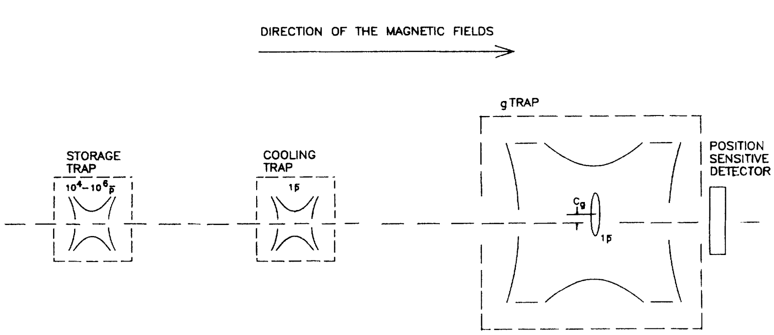

FIG. 4. Schematic drawing of the experimental apparatus.

$$E_{zf} = E_{zi} \left[\frac{V_f}{V_i} \right]^{1/2} . \qquad (26)$$

For energies in the range relevant to the experiment, the confining condition

$$E_{zf} < qV_f \qquad (27)$$

is always fulfilled if we start with V_i at several hundreds volts and E_{zi} at about 350 μeV.

If the magnetic field B_c is decreased adiabatically, the action integrals corresponding to the radial motion are conserved. Their values are E_c/ω_c and E_m/ω_m where E_c and E_m are the energies of the cyclotron and magnetron motion respectively, and ω_m and ω_c are the magnetron and cyclotron frequencies in the cooling trap. Therefore the radial energy of the particle decreases almost linearly when the value of the magnetic field goes from B_c to the final value B_g. The cyclotron and magnetron radii increase according to the relations

$$r_{mf} = r_{mi} \left[\frac{B_c}{B_g} \right]^{1/2} , \qquad (28)$$

$$r_{cf} = r_{ci} \left[\frac{B_c}{B_g} \right]^{1/2} \qquad (29)$$

where $r_{mf,cf}$ and $r_{mi,ci}$ are the values of the radii when the magnetic field is B_g and B_c, respectively. Once the adiabatic cooling is completed, the magnetic field in the storage trap will be the same as in the g trap.

It is to be noted that reducing the value of the magnetic field increases the uncertainty of the measurement [see (15) and (16)]; however, this adjustment is necessary in order to properly set the length of the period T_m of the magnetron motion in the final trap and therefore the value of the measuring time T_g. T_m is directly proportional to the value of the magnetic field as can be verified considering (7), (10), (11), and (21):

$$T_m = \frac{2\pi}{\Omega_m} \simeq 2\pi \times 10^6 \frac{[B\ (\mathrm{T})][Z_0^2\ (\mathrm{m}^2)]}{[V_b\ (\mu\mathrm{V})]} \ \mathrm{sec} . \qquad (30)$$

An upper bound for the period of time available to perform the measurement is set by the typical frequency at which significantly perturbing events can occur. Major disturbances are in principle connected to processes which can alter the particle trajectory, increase its energy, or cause its loss. The confining period is basically limited by the processes of annihilations of the antiprotons with the molecules of the residual gas. However, experimental data reported in [14] show that antiprotons of energy corresponding to 4.2 K can remain confined for several months. More serious problems can be caused by processes which can increase the energy of the antiprotons. In particular, an increase of the radial energy corresponds to an increase of the cyclotron radius and consequently tends to decrease the precision of the measurements; furthermore, heating of the axial motion can cause the loss of the particle.

Elastic collisions with the molecules of the residual gas (its temperature is 4.2 K for a cryogenic apparatus) constitute the major contribution to the heating of the antiprotons (other processes will be discussed in the following section). If an elastic collision with the residual gas changes the antiproton energy by an amount ΔE, the conditions for particle confinement read

$$E_{zg} + \Delta E < qV_g , \qquad (31)$$

where $E_{zg} \simeq E_{zf}$ is the particle axial energy in the g trap [see relation (26)] and

$$\Delta E \ll E_{cg} , \qquad (32)$$

where E_{cg} is the cyclotron energy in the g trap.

The mean time τ between two successive collisions with helium atoms, when the exchange energy is ΔE, has been evaluated in [16]. For an apparatus at 4.2 K we have

$$\tau = 168 \frac{10^{-12}}{[P\ (\mathrm{torr})]} [\Delta E\ (\mu\mathrm{eV})]^{1/4} \ \mathrm{sec} . \qquad (33)$$

The condition that has to be fulfilled is therefore

$$\tau \gg T_g , \qquad (34)$$

with ΔE satisfying (31) and (32). From Eqs. (33), (34), and (30) it can be inferred that the magnetic field in the g trap cannot be the same as in the storage trap (in order to achieve an efficient electron cooling, the value of the magnetic field in the storage trap has to be approximately 6 T). Feasible experimental conditions can be obtained if the value of the magnetic field in the g trap is of the order of a few gauss. The resulting magnetron periods are of the order of hundreds of seconds and are therefore compatible with condition (34) when the temperature is 4.2 K.

A possible set of working conditions, which in our opinion could be used to perform the measurement, appears in Table I (see also the next section). Actual values will basically depend on the experimental limits that can be reached for V_g and T_g.

The statistical error $(\Delta_g/g)_s$, which affects the measurement of g, is related to the statistical error of C_g; therefore it is largely dominated by the uncertainty in the initial conditions of the particles at the instant of injection into the g trap. The measurement of the radial position of N particles yields

$$\left[\frac{\Delta g}{g} \right]_s = \left[\frac{\Delta C_g}{C_g} \right]_s = \frac{(r_{mf} + r_{cf})}{\sqrt{N}} \frac{q}{m} \frac{V_b}{g Z_0^2} , \qquad (35)$$

where r_{mf} and r_{cf} [defined by (28) (29)] are related to a_y, a_y, and $\tilde{r}_c$ by

$$r_{cf} \simeq \tilde{r}_c \qquad (36)$$

and

$$r_{mf} \simeq \sqrt{a_x^2 + a_y^2} . \qquad (37)$$

Relations (36) and (37) hold only if we assume ideal conditions during the transfer of the particles from the cooling trap to the g trap. Using the data of Table I, we get

TABLE I. A possible set of working conditions that can be used to perform the measurement of g. The first value of δ_{px}, δ_{py}, and C_g/C_p corresponds to the actual experimental limit on the patch effect (which is $w\Phi_{rms}=10^{-4}$ cm V) and the second one corresponds to the patch field that would be obtained with the coating technique of [19].

Quantity	Value
V_g	100 μV
Z_0	1 m
R_0	1 m
C_g	1 mm
$\left[\frac{\Delta g}{g}\right]_s$	$\frac{3\%}{\sqrt{N}}$
$E_{cg}\simeq E_{cf}$	0.3 μeV (3.5 mK)
$E_{zg}\simeq E_{zf}$	0.35 μeV (4.2 mK)
B_g	50 G
T_g	314 sec
r_0, z_0	$\simeq$1 cm
V_i	100 V
V_f	100 μV
B_c	6 T
r_{ci}, r_{mi}	0.4 μm
r_{cf}, r_{mf}	15 μm
δ_{px}, δ_{py}	2 mm, 1 μm
C_g/C_p	$5\times10^{-1}, 10^3$

$$\left[\frac{\Delta g}{g}\right]_s \simeq \frac{3\%}{\sqrt{N}} . \tag{38}$$

This result shows that the method we propose is very sensitive and allows, in principle, the performance of a 1% measurement by using a very small number of cold antiprotons (in order to reach the same level of precision by using the TOF technique one needs many more particles, i.e., 10^6 antiprotons thermalized at $T=4.2$ K). The final precision of the g measurement will be largely dominated by systematic errors, as we will show in the next paragraph, we expect that some of them may actually introduce uncertainties whose values will be comparable with C_g. It is then avoidable to perform a differential measure by calibrating the experimental apparatus with other particles, for instance, H^- ions. The following section will be devoted to the discussion of what we expect to be the most important effects in contributing to the systematic error of the measurement. Other possible sources of systematic error, like those related to the particle transfer between traps or the effect of the presence of the position detector, will not be discussed.

IV. SOURCES OF ERRORS AND DISTURBANCES FOR THE MEASUREMENT

In this section we will discuss what we expect to be (i) the main problems in obtaining antiprotons with the projected energy and orbit dimensions, and (ii) the main factors which can affect the final uncertainty of the measurement if the requirements of point (i) are fulfilled.

We will often refer to [16], where an analysis of the sources of error for a gravity measurement on antiprotons with the TOF technique is presented. Several considerations therein expressed are also pertinent in this context.

The source of errors and disturbances that will be considered here (the interaction of the antiprotons with the residual gas has already been discussed) are the following:

(i) Stray electric fields due to patch effect.

(ii) Vertical electric fields (in the g trap) induced by thermal gradients and gravity.

(iii) Forces produced by image charges and by stray charges.

(iv) Precision of the electric and magnetic fields in the measuring trap.

(v) Any process which can heat up the particle and bring it back to 4.2 K.

A. Patch effect

Among the forces which can affect the final result of the measurement, one of the most important is due to stray electric fields that are caused by patch effect. Such an electric field is present in the vicinity of any conductor and is due to the potential gradient which results from the spatial variation of the work function along its surface. This variation may be due to the random orientation of the crystal domains of the surface. The average value of this electric field is zero but fluctuations can be detected on a small scale. In particular, along the axis of a cylinder of radius R_0, the rms value of the radial component of the field is

$$\mathcal{E}_{pr}^{\rm rms}=0.2\frac{w}{R_0^2}\Phi^{\rm rms} , \tag{39}$$

[17] while the fluctuations of the axial component are described by

$$\mathcal{E}_{pz}^{\rm rms}=0.4\frac{w}{R_0^2}\Phi^{\rm rms} . \tag{40}$$

In the above relations w is the typical dimension of the patches and $\Phi^{\rm rms}$ describes the fluctuations of the potential along the axis of the cylinder.

The patch effect is a major source of disturbance for the cooling and the measurement procedure. The g measurement is affected because a radial component of the patch electric field introduces a force which superimposes on the force of gravity. If the particles move in a region which is close to the axis of the g trap we can assume that the field $\mathcal{E}_{pr}$ is approximately constant and that its effect is to cause a displacement C_p of the center of the radial orbits, which adds to the displacement produced by the force of gravity. The ratio between these displacements is

$$\frac{C_g}{C_p}=\frac{mg}{q\mathcal{E}_{pr}^{\rm rms}}=5\times10^{-5}\frac{[R_0^2\ ({\rm m}^2)]}{w[\Phi^{\rm rms}\ ({\rm cm\,V})]} . \tag{41}$$

It can be noted that with large values of R_0 ($R_0\simeq1$ m) a relative precision of the order of 1 can be obtained assuming the lowest value which has ever been measured for $w\Phi^{\rm rms}$ ($w\Phi^{\rm rms}<10^{-4}$ cm V [18]). At this time research is in progress to investigate coating techniques which are expected to reduce the value of $w\Phi^{\rm rms}$ down to $(5-10)\times 10^{-8}$ V cm [19,20]. If these efforts will be successful, it will be possible to make a measurement at the level of 10^{-3} or better. In the cooling trap the mean value of the electric field due to patch effect is much higher than the mean field in the g trap since the dimensions of the cooling trap are of the order of 1 cm (dimensions cannot be increased for the resistive cooling to be effective).

The particle trajectory in the g trap is influenced by the patch effect in the cooling trap because stray electric fields produce random displacements of the center of the radial oscillations in the cooling trap, which in turn reflect as uncertainties in the position of the antiprotons at the moment they enter the g trap. If δ_{px} and δ_{py} represent the displacement of the center of the radial oscillations in the cooling trap after the adiabatic cooling process has taken place, we have

$$\delta_{px}\simeq\delta_{py}=\frac{\mathcal{E}_{pr}^{\rm rms}}{V_f}z_0^2\ , \tag{42}$$

that is,

$$\delta_{px}\simeq\delta_{py}=0.2w\frac{\Phi^{\rm rms}}{V_f}\frac{z_0^2}{r_0^2}\ , \tag{43}$$

where r_0 and z_0 are the dimensions of the cooling trap.

The radial coordinates of the antiprotons, at the instant they leave the cooling trap, can be parametrized as

$$x=r_{mf}\cos(\theta_m)+r_{cf}\cos(\theta_c)+\delta_{px}\ , \tag{44}$$

$$y=-r_{mf}\sin(\theta_m)-r_{cf}\sin(\theta_c)+\delta_{py}\ . \tag{45}$$

Assuming that the transfer takes place in a homogeneous magnetic field we can write $X(T_g)$, $Y(T_g)$ as functions of C_g, r_{mf}, r_{cf}, δ_{px}, δ_{py}, θ_m, and θ_c. Since phases θ_m and θ_c are randomly distributed, if we neglect the difference between the cyclotron frequencies and qB/m, we find that the mean values of the radial coordinates in the g trap are

$$\langle X(T_g)\rangle=(\delta_{px}-C_g)\cos(\Omega_m T_g)+\delta_{py}\sin(\Omega_m T_g)+C_g\ , \tag{46}$$

$$\langle Y(T_g)\rangle=(\delta_{px}-C_g)\sin(\Omega_m T_g)+\delta_{py}\cos(\Omega_m T_g)\ . \tag{47}$$

These relations show how the measurement of C_g is affected by the displacement of the center of the radial oscillations in the cooling trap. However, by measuring $\langle X(T_g)\rangle$ and $\langle Y(T_g)\rangle$ for different values of T_g and fitting the data with (46) and (47), the values of δ_{px}, δ_{py}, C_g, and Ω_m can be separately obtained.

Another way to account for the systematic error described by δ_{px} and δ_{py} is to take measurements with different values of V_g (C_g depends on V_g but δ_{px} and δ_{py} do not). It is worth pointing out that the previous estimates are realistic only if the particle is confined near the center of the trap.

Possible values of δ_{px} and δ_{py} and the relevant data from which they were calculated are tabulated in Table I. For the patch effect field we have used values which range from the present lowest limit to values which could be achieved after expected future developments.

B. Thermoelectric fields

Problems related to the presence of thermoelectric fields are discussed in detail in Ref. [16]. If the temperature of the surface of a metal is not uniform along direction s, the electric field near the surface is

$$\mathcal{E}_T=\left[\frac{1}{q}\frac{\partial W}{\partial T}+S\right]\frac{dT}{ds}\ , \tag{48}$$

where $\partial W/\partial T$ is the variation of the work function with temperature and S is the thermoelectric coefficient. Typical values for S are of the order of 1 μeV/K. The term $\partial W/\partial T$ is presently not well understood. In particular, it has been observed that its value is strongly affected by gas adsorption (especially helium); as a consequence, the rate of variation of W with respect to temperature can be extremely high (up to a few mV/K) [16].

For the experiment that we propose, temperature gradients along the axis of the g trap do not interfere with gravity measurements; the only effect of the resulting field is to displace the center of the axial oscillations. Temperature variations along the x direction produce electric fields which overlap to gravity and are therefore a source of systematic error. Their effect is to displace the center of the radial oscillations in the cooling and the g trap as well as along the transfer channel which connects the two traps.

The most important effect occurs inside the g trap. The ratio between the displacement of the radial oscillations C_T and C_g is

$$\frac{C_T}{C_g}=\frac{q\mathcal{E}_T}{mg}\ , \tag{49}$$

that is,

$$\frac{C_T}{C_g}\simeq 10^4[S\ (\mathrm{mV/K})]\left[\frac{dT}{dx}\ (\mathrm{K/m})\right]\ . \tag{50}$$

The control of temperature gradients is therefore crucial to the success of the experiment. Temperature gradients as low as 10^{-5} K/m have already been obtained [16].

C. Gravity-induced electric field

A metallic structure placed in a gravitational field undergoes slight deformations of the crystal lattice. Moreover, the electron's distribution inside the solid is affected by the presence of a gravitational field. The resultant electric field inside a cylinder of vertical axis has been widely discussed in the literature [16]. The value of this field $\mathcal{E}_i$ is given by

$$\mathcal{E}_i=\frac{m_e g}{q}+\frac{\partial W}{\partial z}\ , \tag{51}$$

where m_e is the mass of the electron and W is the work function of the metal. The first term is due to the electron redistribution, the second, which gives a bigger contribution, is due to the crystal deformation. Recent measurements [21] have shown that the ratio between the gravity-induced electric fields and the weight of antiprotons is close to 1. A measurement of gold electrodes has yielded a ratio less than 0.18. In the apparatus that we propose the axis of the cylinder is perpendicular to the direction of gravity; however, we expect that the order of magnitude for the above ratio will still be given by Eq. (51). As for other sources of systematic errors, it is crucial to calibrate the apparatus with different kinds of particles.

D. Forces due to image charges

The field created by the image charges induced on the walls of the trap affects the motion of the antiprotons. An estimate for the force $F_{\rm im}$ near the axis of the trap can be obtained from

$$F_{\rm im}=\frac{C_2}{4\pi\epsilon_0}q^2\frac{r}{R_0^3}\ , \tag{52}$$

where R_0 is the radius of the apparatus and C_2 is 1.0027 [16] (this formula is exact for a cylinder of infinite length). The above contribution has to be added to the force due to the confining fields and therefore affects the value of C_g. The x coordinate of the center of the radial oscillations in the g trap becomes

$$C_{\rm im}=\frac{mg}{\frac{k}{2}+\frac{C_2q^2}{4\pi\epsilon_0R_0^3}} \tag{53}$$

and the relative variation is

$$\frac{C_g-C_{\rm im}}{C_g}=\frac{q}{4\pi\epsilon_0}\frac{Z_0^2}{R_0^3V_b}\ , \tag{54}$$

or, by inserting the numerical factors,

$$\frac{C_g-C_{\rm im}}{C_g}=\frac{1.44\times10^{-3}[Z_0^2\ ({\rm m}^2)]}{[V_b\ (\mu{\rm V})][R_0^3\ ({\rm m}^3)]}\ . \tag{55}$$

For the given values of the potentials and the geometrical dimensions, this effect introduces a negligible error.

E. Stray charges effect

In principle the surface of the trap electrodes should not be considered as an ideal conducting surface. Indeed, an amount of stray charges could build up on thin layers of nonconducting matter. This process generates an extra stray electric field inside the trap that could disturb the measure. However, the named effect is very difficult to quantify because it depends on the particular experimental configuration and history. Here we limit ourselves to state the existence of the problem.

F. Precision of the fields in the g trap

The trajectory described in Sec. II has been calculated assuming ideal behavior of the electric and magnetic fields. In real conditions the effect of manufacturing imperfections of the electrodes, nonhomogeneous magnetic fields, and errors in the alignment between the trap axis and the direction of the magnetic field are effects whose contributions to the particle trajectory and therefore to the error in the final measurement have to be evaluated.

A nonhomogeneous magnetic field, and therefore a gradient of $\mathbf{B}_g$, introduces a motion of drift of the center of the cyclotron orbits which superimposes on the motion of drift due to the force of gravity [22]. The drift velocity v_∇ due to this gradient is given by

$$v_\nabla=(E_\perp+2E_\parallel)\frac{\mathbf{B}_g\times\nabla B_g}{qB_g^3}\ , \tag{56}$$

where $E_\perp$ and $E_\parallel$, for each point on the trajectory, represent, respectively, the energy of the motion along the transverse and parallel direction with respect to the magnetic field. The tolerance of ∇B_g can be obtained observing that in ideal conditions (if $R_m\simeq C_g$) the center of the cyclotron orbits moves with a velocity which is given by (2); we therefore require that

$$v_\nabla\ll\frac{mg}{qB_g}\ . \tag{57}$$

Using (56) and neglecting the difference between $\mathbf{B}_g$ and its z component, we have

$$\frac{\partial B_g}{\partial s}\frac{1}{B_g}=\frac{mg}{E_\perp+2E_\parallel} \tag{58}$$

($\partial B_g/\partial s$ stays for either $\partial B_g/\partial x$ or $\partial B_g/\partial y$), that is,

$$\frac{\partial B_g}{\partial s}\frac{1}{B_g}=\frac{10^{-3}}{[(E_\perp+2E_\parallel)\ (\mu{\rm eV})]}\ {\rm cm}^{-1}\ . \tag{59}$$

Therefore, for a measurement of g with a precision of 1%, an accuracy of at least 10^{-5}–10^{-4} cm^{-1} is required.

In comparison, one does not need a very high precision in aligning the direction of the magnetic field with the trap axis z. If we introduce a finite angle Θ_B between these two directions, the new equations of motion show that the radial and the axial motions are coupled; furthermore, the radial orbits are noncircular and the three associated frequencies are not the same as those given by Eqs. (7), (10), and (11); however, the point of equilibrium of the orbits does not change with respect to the ideal case. The coupling between the radial and the axial motion causes the axial energy of a particle to depend on the cyclotron radius. As a consequence, the radial dimensions of the orbits in the g trap increase with the difference of the average energy of the motion along z and the average energy of the radial motion. Since

$$R_c\simeq\frac{mv_\perp}{qB_g}=\frac{m\left|\mathbf{v}-\mathbf{v}\cdot\frac{\mathbf{B}_g}{B_g}\right|}{qB_g}\ , \tag{60}$$

the change in the radius of the cyclotron motion due to a finite value of Θ_B (with $\Theta_B\ll1$) is readily obtained:

$$\frac{\Delta R_c}{R_c}=-\Theta_B+\frac{E_{zg}}{E_{cg}}\frac{\Theta_B^4}{8}\ . \tag{61}$$

Therefore, a misalignment of a few degrees can be tolerated as long as the value of E_{zg} is close enough to the value of E_{cg}.

In order to extract the value of g from a measurement of C_g, it is crucial to minimize nonideal behavior of the elastic field introduced by imperfections of the shape of the electrodes. We have estimated the order of magnitude of the tolerance in the construction of the electrodes by considering an elliptical shape in place of the ideal circular one. Equations (5) and (6) become

$$\mathcal{E}'_x = \frac{k_x}{2q} x \ , \tag{62}$$

$$\mathcal{E}'_y = \frac{k_y}{2q} y \ , \tag{63}$$

where (referring for simplicity to a Penning trap having electrodes shaped as hyperboloids)

$$k_x = -\frac{4q(V_{\text{ring}} - V_{\text{endcap}})}{R_X^2 + [1+(1+\epsilon)^2]Z_0^2} \ , \tag{64}$$

$$k_y = -\frac{4q(V_{\text{ring}} - V_{\text{endcap}})}{R_X^2 + [1+(1+\epsilon)^2]Z_0^2}(1+\epsilon)^2 \ . \tag{65}$$

R_X and R_0 are the lengths of the semiaxis of the ellipse and

$$R_X = (1+\epsilon)R_0 \ . \tag{66}$$

Since

$$g = \frac{C_g k_x}{2m} \ , \tag{67}$$

the error in k_x contributes to the error in g:

$$\frac{\Delta g}{g} = \frac{\Delta C_g}{C_g} + \frac{\Delta k}{k} \ . \tag{68}$$

Using (59) we get

$$\frac{\Delta k}{k} = \frac{(k_x - k)}{k} = \frac{1+2Z_0^2/R_0^2}{(1+\epsilon)^2 + (2+2\epsilon+\epsilon^2)Z_0^2/R_0^2} - 1 \ . \tag{69}$$

If for the sake of simplicity we let $R_0 = Z_0$ (for $\epsilon \ll 1$) we obtain

$$\frac{(k_x - k)}{k} = 1.33\epsilon \ . \tag{70}$$

Therefore, the relative error introduced by axially asymmetric mechanical imperfections is expected to be of the same order of magnitude as the degree of asymmetry in the shape of the electrodes.

Other mechanical imperfections occurring in the manufacture of the electrodes, if they do not break the azimuthal symmetry, perturb the harmonic electric potentials by introducing terms which are proportional to even powers of the coordinates. Therefore, near the trap center, the electric potential in polar coordinates takes the form [12]

$$V \propto \sum c_k (r/d)^k P_k(\cos\theta) \ , \tag{71}$$

where

$$d = \tfrac{1}{2}\left[Z_0^2 + \frac{R_0^2}{2} \right] \tag{72}$$

and $P_k(\cos\theta)$ are the Legendre polynomials. If we neglect terms of order 6 or higher, the potential of an antiproton near the center of the g trap is

$$qV(x,y,z) = -\frac{qV_0}{4d^2}(r^2 - 2z^2) + \frac{c_4 q V_0}{16 d^4}(8z^4 + 3r^4 - 24r^2 z^4) + mgx \ . \tag{73}$$

As a consequence, the x coordinate of the point of equilibrium of the radial oscillations has to be modified [with respect to Eq. (18)] by adding a term D_4, where

$$\frac{D_4}{C_g} = \tfrac{3}{2} c_4 \left[\frac{C_g}{d} \right]^2 \ . \tag{74}$$

Typical values of c_4 are of the order of 10^{-1}–10^{-2} [12] and using "compensating" electrodes this value can be lowered by at least an order of magnitude. Even without the employment of sophisticated techniques the systematic error due to an anharmonic potential can be reduced to a negligible value. However, it has to be pointed out that the above estimates apply only if we assume small oscillation amplitudes. Furthermore, anharmonic terms can introduce a statistical error, for the dimensions of radial orbits would also depend on the axial energy.

Finally we observe that a misalignment $\delta'_{x,y}$ between the axis of the cooling trap and the axis of the g trap produces an effect which can be described by relations which are formally identical to (46) and (47).

G. Sources of heat for antiprotons

Once the adiabatic process of cooling is completed, the temperature of antiprotons will be lower than the temperature of the surrounding environment by several orders of magnitude. It is therefore important to evaluate the typical time scale involved before significant changes of the antiprotons's temperature begin to take place. The processes that we considered are the following: (i) elastic collisions with the residual gas, and (ii) resistive heating.

Process (i) has already been considered in Sec. III. Process (ii) can be investigated considering the effect of a resistance R connected to an electrode of the trap. A resistive load is produced by the tuned circuits that are employed to detect particles in the cooling trap. The resistance R acts as a generator of random potential across the trap. The power spectrum is related to the equivalent noise temperature T_{eff} by $4KT_{\text{eff}}R$ ($T_{\text{eff}} \simeq 4.2$ K). The effect of the corresponding random electric field is to set a lower limit to the process of resistive cooling if the initial energy of the particles is greater than KT_{eff} or to increase the temperature of the particles if their energy is lower than KT_{eff}.

It is not difficult to demonstrate that the characteristic time of this heating process is identical to the characteristic time τ_r of resistive cooling, where

$$\tau_r = \frac{4ml_0^2}{(\alpha q)^2 R} , \tag{75}$$

l_0 stands for the radial or axial dimension of the trap, and α is a geometrical factor ($\alpha < 1$) [12]. In order to estimate the characteristic time scales involved, we considered the interval of time Δt_2 which is necessary to double the initial energy (here called $E_{r,z}^0$) of the particles. Since the axial and radial energy of the antiprotons increase with time according to

$$E_{r,z}(t) \simeq (E_{r,z}^0 - KT_{\rm eff})e^{-t/\tau_r} + KT_{\rm eff} , \tag{76}$$

we get

$$\Delta t_2 = 2.8 \times 10^9 [E_{r,z}^0 \ (\mu\text{eV})] \frac{[l_0^2 \ (\text{m}^2)]}{[R \ (\text{ohm})]} \ \text{sec} , \tag{77}$$

where we assumed $T_{\rm eff} = 4.2$ K and $\alpha = 0.5$. Values of $R \simeq 10^5$ Ω which are typical for Penning traps connected to a tuned detection circuit, would cause problems. However, for our purposes, this effect is negligible. In the g trap here is no detection circuit and in the cooling trap, while the adiabatic cooling process is taking place, the detection circuit is detuned.

V. CONCLUSIONS

The method that we propose in order to measure the gravitational acceleration on antiprotons can represent a possible alternative to the TOF technique (whose applicability is presently under experimental investigation).

The main difficulty with the method that we propose regards the preparation of well localized single particles, of very low energy, and small cyclotron and magnetron radii. If (before injection into the g trap) this delicate operation can be completed successfully, the accuracy of the gravity measurement becomes completely independent of the original energy and radius distribution of the particles. This aspect represents an improvement with respect to the TOF technique in the sense that no hypothesis on the initial velocity distribution of the launched particles needs to be made [8]. Furthermore, though the measuring time per particle is longer, each particle provides an independent measurement of g; repeated measurements serve the only purpose of decreasing the statistical error by the usual factor $1/\sqrt{N}$.

Finally, we find that the TOF technique and the method we propose have different sensitivities to patch effect fields. As discussed in [16], taking into account the field caused by the patch effect, the time of flight t for a particle of velocity v_0 moving along the axis of a vertical tube of length L is

$$t = \left[\frac{m}{2}\right]^{1/2} \int_0^L \left[\frac{mv_0^2}{2} - mgz + qV_p(z)\right]^{-1/2} dz , \tag{78}$$

where $V_p(z)$ is the patch potential along the axis. It follows that in order to perform the measurement it must be $qV_p(z) \ll mgz$; the patch potential not only causes a systematic error (as is the case for the method that we propose) but also strongly affects the statistical error. The rms value of V_p on the tube axis is

$$V_p^{\rm rms} = 0.2 \frac{w\Phi^{\rm rms}}{a_d} , \tag{79}$$

where a_d is the radius of the drift tube. If $w\Phi^{\rm rms} = 10^{-4}$ cm V (see Sec. IV A), considering a drift tube with the length to diameter ratio assumed in [8], the condition $qV_p^{\rm rms} \ll mgL$ implies $La_d \gg 2$ m^2 (or $a_d \gg 15$ cm and $L \gg 15$ m). These dimensions are not realistic for a TOF measurement, mainly because of the strict requirements on the strength and homogeneity of the magnetic field. As a consequence, the feasibility of the TOF experiment relies strongly on the success of future techniques which are expected to reduce the value of the patch effect field. As we mentioned in Sec. IV, the method that we propose would allow a differential measurement for g, with the value of the patch fields actually measured. Progress in lowering the values of the patch fields would contribute anyway to an increase in the precision of the measurement and a decrease in the difficulty of the experiment.

ACKNOWLEDGMENT

The authors wish to thank Michael Holzscheiter for useful discussions and suggestions.

[1] C. M. Mc Will, *Theory and Experiments in Gravitational Physics* (Cambridge University Press, Cambridge, 1991).
[2] R. Eötvös, D. Pekar, and E. Fekete, Ann. Phys. (N.Y.) **68**, 11 (1922).
[3] E. G. Adelberger, Phys. Rev. D **42**, 3267 (1990).
[4] T. E. O. Ericson and A. Richter, Europhys. Lett. **11**, 295 (1990).
[5] W. Paul, Rev. Mod. Phys. **62**, 531 (1990).
[6] (a) J. M. Lockart *et al.*, Phys. Rev. Lett. **38**, 1120 (1977); (b) F. C. Witteborn, Rev. Sci. Instrum. **48**, 1 (1977).
[7] M. M. Nieto and T. Goldman, Phys. Rep. **205**, 5 (1991).
[8] N. Beverini *et al.*, Los Alamos Report No. LA-UR **86**, 260, 1986 (unpublished).
[9] (a) M. Holzscheiter *et al.*, Nucl. Phys. A **558**, 709 (1993); (b) M. Holzscheiter *et al.* (unpublished).
[10] J. Eades *et al.* (unpublished).
[11] D. Hajdukovic, Phys. Lett. B **226**, **352** (1989).
[12] L. S. Brown and G. Gabrielse, Rev. Mod. Phys. **58**, 1 (1986).
[13] (a) G. Gabrielse, L. Haarsma, and S. L. Rolston, Int. J. Mass Spectrom. Ion Processes **88**, 319 (1989); (b) J. Tau and G. Gabrielse, Appl. Phys. Lett. **55**, 2144 (1989).
[14] G. Gabrielse *et al.*, Phys. Rev. Lett. **65**, 1317 (1990).
[15] F. L. Moore, D. L. Farnham, P. B. Schwinberg, and R. S. Van Dyck, Nucl. Instrum. Methods Phys. Res. Sect. B **43**, 425 (1989).
[16] T. W. Darling, F. Rossi, G. I. Opat, and G. F. Moorhead, Rev. Mod. Phys. **64**, 237 (1992).

[17] M. S. Rzchowski and J. R. Henderson, Phys. Rev. A **38**, 4622 (1988).
[18] J. B. Camp *et al.*, J. Appl. Phys. **69**, 7126 (1991).
[19] J. B. Camp *et al.*, J. Appl. Phys. **71**, 783 (1992).
[20] J. B. Camp and R. Schwarz, Los Alamos Report No. LA-UR **92**, 3553 (1992) (unpublished).
[21] F. Rossi and G. I. Opat, Phys. Rev. B **45**, 249 (1992).
[22] B. Lehenert, *Dynamics of Charged Particles* (North-Holland, Amsterdam, 1964).