Role of phase coherence in the transition dynamics of a periodically driven two-level system

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Some aspects of quantum tunneling of a particle in a double-well potential periodically driven by an external force are studied within the two-level approximation. A closed expression for the temporal evolution of the occupation probability is obtained in the limit of large-amplitude oscillation by the transfer-matrix formalism. The mechanism of coherent destruction of tunneling found by Grossmann *et al.* [Phys. Rev. Lett. **67**, 516 (1991)] is made clear from a viewpoint of interference at periodic level crossings.

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The quantum dynamics of a particle in a double-well potential periodically driven by an external force has been a subject of considerable interest in recent years. The attenuation has mainly focused on the possible chaotic behavior or "quantum chaos" that the system exhibits, and its relationship with that of its classical counterpart [1–3]. The effect of the external modulation on quantum tunneling through the classically impenetrable region has been another, related, theme of investigation [4–6]. An archetypal model Hamiltonian is given in the form

$$H(t)=H_0+Sx\cos(\omega t)\ , \tag{1}$$

$$H_0=\frac{p^2}{2M}+V(x)\ , \tag{2}$$

where $V(x)$ is a symmetric double-well potential, and S is a coupling constant with an oscillating field of frequency ω.

In this connection, Grossmann and *et al.* [5,6] found a peculiar behavior in the tunneling dynamics of a driven system which they termed "a coherent destruction of tunneling:" A Gaussian wave packet initially located in one of the potential wells never transfers to the other well, as if the quantum tunneling is frozen by the periodic modulation. They reported that this phenomenon occurs only for parameter values of S and ω restricted to one-dimensional manifolds in two-dimensional parameter plane. Earlier than this, Lin and Ballentine [3] had noticed that the tunneling probability is highly *enhanced* due to the periodic modulation on the basis of numerical calculation for just the same models as that of Grossmann *et al.* but in a different parameter region. These observations strongly suggest that the tunneling dynamics in the driven system is governed by a mechanism quite different from that of the static potential system.

The low-lying eigenstates of the unperturbed Hamiltonian H_0 form nearly degenerate doublets which are composed of the linear combinations of pairs of low-lying states localized in the left and the right wells, respectively, and split by the barrier tunneling. Under the condition that the tunnel splitting is small enough, and the modulation amplitude of the energies of the left and right wells is much smaller than the representative excitation energy in a single well, the transition dynamics can well be described by the two-level model for the lowest doublet. The suppression of tunneling has been discussed within the two-level model by applying Floquet formalism [6,7]. The purpose of the present report is to propose a slightly different point of view for this problem, from which one can obtain further insight into the mechanism of the suppression of tunneling.

Let us consider a two-level Hamiltonian,

$$H_{\rm TL}(t)=\tfrac{1}{2}A\cos(\omega t)(|1\rangle\langle 1|-|2\rangle\langle 2|)+\Delta(|1\rangle\langle 2|+|2\rangle\langle 1|)\ , \tag{3}$$

where $|1\rangle$ and $|2\rangle$ represent, say, the left and the right localized states, respectively. For the state vector written in the form

$$|\Psi(t)\rangle=C_1(t)\exp[-i(A/2\omega)\sin(\omega t)]|1\rangle+C_2(t)\exp[i(A/2\omega)\sin(\omega t)]|2\rangle\ ,$$

the Schrödinger equation is given by

$$\begin{aligned} i\frac{d}{dt}C_1(t)&=\Delta\exp[i(A/\omega)\sin(\omega t)]C_2(t)\ ,\\ i\frac{d}{dt}C_2(t)&=\Delta\exp[-i(A/\omega)\sin(\omega t)]C_1(t)\ . \end{aligned} \tag{4}$$

The unit $\hbar=1$ is used here and hereafter. The probability that the system is in $|2\rangle$ at time t under the condition that it starts from $|1\rangle$ at $t=0$ will be denoted as $P(t)$. Although it is an easy matter to calculate $P(t)$ numerically by direct integration or by the technique of Floquet mapping, our purpose is to obtain analytical expressions. This can be done in some limiting cases in the parameter space (Δ, A, ω).

1. Limit of rapid oscillation

When the condition

$$\Delta\ll\omega \tag{5}$$

is satisfied, the amount of change of the state vector during a period of oscillation $2\pi/\omega$ can be regarded as being infinitesimal, at most of order of Δ/ω. In this case, it can be easily shown that Eq. (4) is approximated to the lowest order of Δ/ω by

$$i\frac{d}{dt}C_1(t)=J_0(A/\omega)\Delta C_2(t)\ ,$$
$$i\frac{d}{dt}C_2(t)=J_0(A/\omega)\Delta C_1(t)\ , \tag{6}$$

where

$$J_0(A/\omega)\equiv\frac{\omega}{2\pi}\int_0^{t+2\pi/\omega}\exp[i(A/\omega)\sin(\omega\tau)]d\tau$$

is the zeroth-order Bessel function. Therefore, $P(t)$ is given by the formula for the static two-level system;

$$P(t)=\sin^2\{J_0(A/\omega)\Delta t\}\ . \tag{7}$$

Essentially the same result has been obtained by Grossmann and Hänggi [6] and by Gomez and Plata [7]. The effective tunneling parameter is reduced by the factor $|J_0(A/\omega)|(\leq 1)$ and even vanishes for A/ω satisfying $J_0(A/\omega)=0$, as noted by the above authors. The suppression of tunneling is due to the interference between transition paths as can be clearly seen from the argument to follow.

2. *Limit of large-amplitude oscillation*

The problem can be seen from the viewpoint of nonadiabatic transition at level crossings. Since the off-diagonal coupling works effectively only for energy difference or order Δ, and since the velocity of change for the energy difference is of order $A\omega$, the time duration τ_{tr} that the system exists in the transition region is estimated to be of order $\Delta/A\omega$ for a moderate value of Δ. In the limit of small Δ, τ_{tr} is estimated to be of order $1/\sqrt{A\omega}$ by the argument of the convergence domain of Fresnel's integral, which appears in the lowest-order perturbation of the transition probability [8]. Therefore, in the case that the condition

$$\max(\Delta,\omega)\ll A \tag{8}$$

is satisfied, where max means that the largest value should be chosen, the inequality $\tau_{tr}\ll 2\pi/\omega$ follows, which means that the transition is well localized around the times $t_n\equiv(n-\frac{1}{2})\pi/\omega$, $(n=1,2,\ldots)$ at which $|1\rangle$ and $|2\rangle$ cross. Aside from impulsive transitions at t_n, the system propagates almost freely between each crossing time. By averaging out the rapid oscillation with small amplitude in $P(t)$ around the turning point of the oscillation, one can define the probability P_n that the system exists in $|2\rangle$ after the nth crossing.

The present author has shown that the transition dynamics in such a situation can be described by the transfer-matrix formalism [9]. The transfer matrix at each crossing is determined by the adiabaticity parameter $\delta\equiv\Delta^2/A\omega$. It has been shown that the probability P_n is given by

$$P_{2m-1}=1-q|\beta_{2m-1}|^2\ , \tag{9}$$
$$P_{2m}=q|\beta_{2m}|^2,\quad m=1,2,\ldots,$$

where $q\equiv\exp(-2\pi\delta)$, and β_n is defined by the recursive relation

$$\beta_{n+1}+2i\sqrt{1-q}\sin(\alpha+\phi)\beta_n-\beta_{n-1}=0\ , \tag{10}$$

with β_0 and $\beta_1=1$. In the above equation, α is the relative phase acquired during the propagation between crossings;

$$\alpha=\int_0^{\pi/2\omega}\sqrt{A^2\cos^2(\omega t)+4\Delta^2}dt\quad(\simeq A/\omega)\ ,$$

and ϕ is the Stokes phase given by

$$\phi=\pi/4+\arg\Gamma(1-i\delta)+\delta(\ln\delta-1)\ ,$$

in which $\Gamma(z)$ is the Γ function. The Stokes phase is a decreasing function of δ which takes the limiting values $\phi(\delta\to 0)=\pi/4$ and $\phi(\delta\to\infty)=0$.

The above equation is solved by an elementary arithmetic to yield an explicit formula;

$$P_{2m-1}=1-q\left[\frac{\cos[(2m-1)\theta]}{\cos\theta}\right]^2\ ,$$
$$P_{2m}=q\left[\frac{\sin(2m\theta)}{\cos\theta}\right]^2\ , \tag{11}$$

in which θ is defined by

$$\sin\theta=\sqrt{1-q}\sin(\alpha+\phi)\ . \tag{12}$$

In the case that θ/π is an irrational number, and the sequence $\{P_n\}$ distributes uniformly and densely over the interval [0,1]. If θ/π is a rational number, $\{P_n\}$ has a periodic structure. Especially, for parameter values satisfying $\theta=k\pi$ with an integer k, the sequence $\{P_n\}$ simply oscillates between 0 (for n even) and $1-q$ (for n odd): The interference between the transition paths to reach $|2\rangle$ after $2m$th crossing completely destroys the transfer to $|2\rangle$ at every period of oscillation. Note that the value $1-q$ is nothing but that given by the Landau-Zener formula [10]. See Kayanuma [9] for a comparison of formula (11) with numerical calculations.

In the diabatic limit $\delta\ll 1$, we set $q\to 1$, $\phi\to\pi/4$, and $\alpha\to A/\omega$ in Eqs. (11) and (12) and find

$$P_n=\sin^2\left\{\sqrt{2\pi/A\omega}\Delta n\sin\left[\frac{A}{\omega}+\frac{\pi}{4}\right]\right\}\ . \tag{13}$$

For a gross structure in the long-time scale, we may set $n\pi=\omega t$ and recover the continuous time dependence

$$P(t)=\sin^2\left\{\sqrt{2\omega/A\pi}\Delta t\sin\left[\frac{A}{\omega}+\frac{\pi}{4}\right]\right\}\ . \tag{14}$$

Since $J_0(x)\simeq\sqrt{2/\pi x}\sin[x+(\pi/4)]$ for $x\gg 1$, the above formula reproduces formula (7) in the limit $A/\omega\gg 1$ as it should. Actually, formula (14) agrees with (7) fairly well even for a value of A/ω not much greater than unity. For example, the first critical value of A/ω at which the tunneling is completely suppressed is predicted as $A/\omega=2.4048\ldots$ by formula (7), while (14) predicts $A/\omega=3\pi/4=2.3561\ldots$. Formula (14) has the advantage that it clearly shows the role of phase coherence in the transition dynamics of the driven two-level system. The phase factor $\pi/4$ is nothing but the Stokes phase at the level crossing in the diabatic limit, and A/ω is the phase acquired during the free propagation.

To summarize, we have obtained analytical expressions of $P(t)$ in the two asymptotic regions $\Delta \ll \omega$ and $\Delta, \omega \ll A$ in the parameter space (Δ, ω, A). They coincide in the common subregion $\Delta \ll \omega \ll A$, given in Eq. (14). The mechanism of suppression of the tunneling has been clarified as due to the destuctive interference between transition paths.

The situation treated by Lin and Ballentine [3] is out of the scope of the present analysis, since the oscillation amplitude of the driven energy levels is rather large in their case: Several localized levels in the left and right wells contribute to the crossing event. However, one may well expect that the phase coherence at multilevel crossings also plays an important role in this case. It is left for future works to extend the transfer-matrix analysis to multilevel systems.

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