Lasing without inversion with a fluctuating pump: Gain dependence on pump noise and frequency

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(Received 24 November 1993)

Recently it was reported that replacing an incoherent pumping mechanism by a spectrally colored (partially coherent) pump can lead to significant gain enhancement in lasing without inversion schemes [Phys. Rev. A **48**, R4055 (1993)]. We extend that study further by considering the effects of the noise parameters (strength and bandwidth) of the pump on the gain in two types of three-level atomic energy-level schemes—the ladder and the Λ models. The calculations presented in this paper show a way to continuously "tune" the bandwidth of the pump and make a direct comparison between the gain obtained for coherent, incoherent, and partially coherent pumping. We find that for a given set of atom-field parameters, the gain for coherent pumping is larger than for incoherent pumping, and that maximum gain is obtained for a *partially* coherent pump. Increasing the strength of the noise also leads to an increase in the gain. The frequency of the colored pump plays a critical role in determining the maximum gain that can be obtained.

PACS number(s): 42.50.Md, 42.65.Vh

I. INTRODUCTION

Several lasing without inversion (LWI) schemes have been reported in the literature in the past few years [1–6]. Inversionless gain in two-level atoms has been investigated [7,8] and much attention has been focused on three-level atomic schemes, which include the ladder, Λ, and the V models [9–11]. While several of the schemes for LWI originally proposed had no inversion in the bare atomic states, there was hidden population inversion in the dressed atomic states. Imamoglu, Field, and Harris [10] reported a three-level Λ model which showed lasing without population inversion in any basis set and this gain was analyzed by Agarwal [10] and shown to be arising from coherence between the dressed states. The effect of various atom-field parameters on the gain and other properties of these systems have also been thoroughly investigated. More recently, a four-level LWI scheme with a single coherent pump has also been reported [12] and some workers have reported on the quantum properties of LWI systems. These lasers possess narrower linewidths due to reduced spontaneous emission and the radiation exhibits other interesting quantum statistical properties such as squeezing and sub-Poissonian statistics [12–14]. One reason for such intense study of LWI schemes is the possibility of obtaining enhanced refractive indices with minimal absorption, as pointed out by Scully and co-workers [15]. Lasers based on LWI schemes also offer the tempting possibility of generating radiation in regions of the spectrum where conventional lasers are difficult to operate. While several of these LWI schemes have relied on an incoherent pumping mechanism to place the population in the upper state of the relevant lasing transition, recently Agarwal, Vemuri, and Mossberg reported that the use of a partially coherent pump for the population pumping process can lead to a significant enhancement in gain [16]. (We will refer to a pump with linewidth much broader than the transition width as an incoherent pump and one with linewidth comparable to the transition width as colored or partially coherent.) The motivation for replacing an incoherent pump by a partially coherent pump arises from experimental considerations where it is often more convenient to use a laser to transfer population to the upper lasing levels. Since lasers typically have linewidths comparable to or less than the linewidths of the atomic transitions of interest in LWI studies, it is of interest to investigate the effects of such a pump on the gain in LWI systems. The study in [16] was done on a three-level ladder scheme, relevant to the ^{138}Ba atom, and showed that using a partially coherent pump instead of an incoherent pump, one can realize gain increase by a factor of 2 to 3, for a given set of atom-field parameters.

Replacing an incoherent (broadband) pump by a partially coherent (colored) pump introduces some additional complexities into the density-matrix equations of the atom-field system. For incoherent pumping one can use traditional techniques developed in the context of treating Markovian processes and thus use the decorrelation approximation to decorrelate the atom and field parameters. In this case one is in the rate equation regime where only the strength of the noise enters into the problem and the gain can be calculated analytically, at least in principle (the complexity of the equations for three-level systems may still make numerical calculations necessary). For purely coherent pumps the gain can again be calculated analytically. The problem becomes severe when one has to deal with partially coherent pumps, since the

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decorrelation approximation is no longer valid. One has to explicitly incorporate the strength and the bandwidth of the noise in the calculations and we have found that Monte Carlo techniques are especially useful in this context [17]. An additional aspect of introducing the colored pump is that one has to account for its frequency and determine the optimum frequency at which the colored pump is most effective. For incoherent pumps, the frequency of the pump is irrelevant since the spectrum of the pump is much wider than the transition width of interest and one cannot meaningfully talk of the pump being on resonance or being detuned from the transition.

In this paper we conduct a systematic study of the effect of the noise parameters of the fluctuating pump on the gain in LWI schemes. We focus attention on two types of three-level atoms, the ladder and the Λ systems. The noise parameters that enter into our discussion are the strength of the noise and the bandwidth of the noise. We also show that the frequency of the colored pump is an important factor in determining the maximum gain that can be obtained.

II. LANGEVIN EQUATIONS FOR LWI SYSTEMS

Ladder systems

The three-level ladder system that we consider has been described in [16] and we briefly summarize the main points here. The energy-level scheme is shown in Fig. 1(a), where a strong, coherent, monochromatic pump at frequency ω_2 couples states $|2\rangle$ and $|3\rangle$. The upper transition $|1\rangle \leftrightarrow |2\rangle$ is coupled by the colored pump of frequency ν and a weak probe at frequency ω_1 probes the gain on the upper transition. The levels $|1\rangle$ and $|2\rangle$ decay to the next-lower-lying levels and have radiative

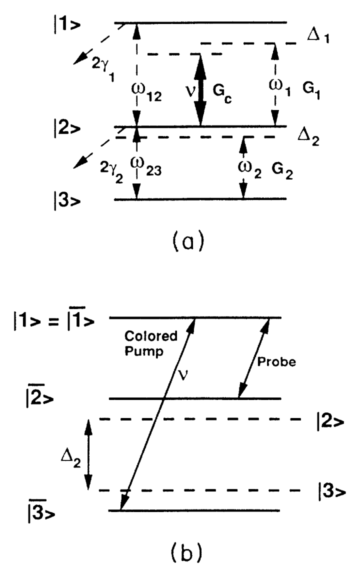

FIG. 1. (a) Atomic-energy-level diagram for the ladder system. The thick arrow represents the colored pump on the lasing transition. (b) Semiclassical dressed states corresponding to the bare states of (a).

widths of $2\gamma_1$ and $2\gamma_2$. The $|1\rangle \leftrightarrow |3\rangle$ transition is forbidden. This energy-level scheme is relevant to the ^{138}Ba atom, where the levels $|1\rangle$, $|2\rangle$, and $|3\rangle$ can be identified with the $6s6d\,^1D_2$, $6s6p\,^1P_1$, and $6s^2\,^1S_0$ states, respectively. It is fairly straightforward to write the Hamiltonian for this atom-field system [16] and after transforming to rotating frames at appropriate frequencies, the resulting equations for the time evolution of the density-matrix elements are

$$\dot\rho_{11} = -2\gamma_1\rho_{11} + ig(t)\rho_{21} - ig^*(t)\rho_{12} , \tag{2.1a}$$

$$\dot\rho_{12} = -(\gamma_1+\gamma_2+i\Delta_1)\rho_{12} + ig(t)(\rho_{22}-\rho_{11}) - iG_2^*\rho_{13} , \tag{2.1b}$$

$$\dot\rho_{13} = -(\gamma_1+i\Delta_2+i\Delta_1)\rho_{13} + ig(t)\rho_{23} - iG_2\rho_{12} , \tag{2.1c}$$

$$\dot\rho_{22} = 2\gamma_1\rho_{11} - 2\gamma_2\rho_{22} - ig(t)\rho_{21} + ig^*(t)\rho_{12} + iG_2\rho_{32} - iG_2^*\rho_{23} , \tag{2.1d}$$

$$\dot\rho_{23} = -(\gamma_2+i\Delta_2)\rho_{23} + ig^*(t)\rho_{13} + iG_2(\rho_{33}-\rho_{22}) , \tag{2.1e}$$

$$\dot\rho_{33} = 2\gamma_2\rho_{22} - iG_2\rho_{32} + iG_2^*\rho_{23} . \tag{2.1f}$$

In these equations $\Delta_1 = \omega_{12} - \omega_1$ is the detuning of the probe from the upper transition, $\Delta_2 = \omega_{23} - \omega_2$ is the detuning of the strong pump on the lower transition (ω_{12} and ω_{23} are the transition frequencies between levels $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, respectively), and

$$g(t) = G_1 + G_c(t)e^{-i(\nu-\omega_1)t} ,$$

where

$$G_1 = \frac{\mathbf{d}_{12}\cdot\boldsymbol{\varepsilon}_1}{\hbar}, \quad G_2 = \frac{\mathbf{d}_{23}\cdot\boldsymbol{\varepsilon}_2}{\hbar}, \quad G_c(t) = \frac{\mathbf{d}_{12}\cdot\boldsymbol{\varepsilon}_c(t)}{\hbar} . \tag{2.2}$$

In Eq. (2.2) $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_2$, and $\boldsymbol{\varepsilon}_c$ are the electric fields associated with the probe, strong pump, and the fluctuating field, respectively, and $d_{\alpha\beta}$ are the transition matrix elements between the states $|\alpha\rangle$ and $|\beta\rangle$. The colored pump enters the density-matrix equations (2.1) through the $g(t)$ term. It is obvious that the density-matrix equations are Langevin equations with multiplicative noise. The colored pump is taken to be a Gaussian-Markovian random process that obeys chaotic modulation and so has zero mean [$\langle \varepsilon_c(t) \rangle = 0$]) and an autocorrelation function of the form

$$\langle G_c^*(t)G_c(t+\tau)\rangle = D\Gamma\exp(-\Gamma|\tau|) . \tag{2.3}$$

Equation (2.3) implies that the random process has a variance of $D\Gamma$, where D is the strength of the noise and Γ is the bandwidth of the noise (this kind of exponentially correlated noise process is also referred to as an Ornstein-Uhlenbeck process). Physically, the product $D\Gamma$ describes the intensity of the colored pump. The colored pump as defined here has a Lorentzian spectral profile with a full width at half maximum of 2Γ. The incoherent pump limit can be obtained by letting $\Gamma \rightarrow \infty$ for which Eq. (2.3) reduces to

$$\langle G_c^*(t) G_c(t+\tau) \rangle = 2D\delta(\tau) . \quad (2.4)$$

The gain G on the upper lasing transition is given by [16]

$$G = -\mathrm{Im}\left[\frac{\langle \rho_{12} \rangle \gamma_1}{G_1}\right] , \quad (2.5)$$

where ρ_{12} is the ensemble average of the steady-state value of ρ_{12} (averaged over the fluctuations of the colored pump). In [16] it is shown that the optimum frequency of the colored pump ν can be determined by diagonalizing the Hamiltonian for this system in the presence of the strong pump on the lower transition. This gives rise to two new semiclassical dressed states $|\bar{2}\rangle$ and $|\bar{3}\rangle$, which are linear combinations of the states $|2\rangle$ and $|3\rangle$ and are shown in Fig. 1(b). The colored pump is chosen to be resonant with the $|\bar{1}\rangle \leftrightarrow |\bar{3}\rangle$ transition and hence is given by (setting energy of $|3\rangle$ equal to zero)

$$\nu = \omega_{12} + \Delta_2 + \left[\frac{\Delta_2}{2} - \left[\frac{\Delta_2^2}{4} + G_2^2\right]^{1/2}\right] . \quad (2.6)$$

With the colored-pump frequency set in this manner, the probe is now tuned across the $|\bar{1}\rangle \leftrightarrow |\bar{2}\rangle$ transition. Since most of the population is expected to be in level $|\bar{3}\rangle$ (the most ground-state-like), when the probe frequency is close to the $|\bar{1}\rangle \leftrightarrow |\bar{2}\rangle$ transition, it is amplified due to a stimulated Raman process.

In this paper we fix the colored-pump frequency as given by Eq. (2.6) and scan the probe across the $|\bar{1}\rangle \leftrightarrow |\bar{2}\rangle$ transition. The gain G as defined by Eq. (2.5) is then studied as a function of Δ_1.

Λ systems

In this section we discuss the theoretical basis for our calculations dealing with the Λ model. The energy-level scheme is shown in Fig. 2(a). A strong, monochromatic, coherent pump at frequency ω_2 couples states $|1\rangle$ and $|2\rangle$ and the colored pump at frequency ν couples the lasing transition $|1\rangle \leftrightarrow |3\rangle$. A weak probe at frequency ω_1 is then scanned across the $|1\rangle \leftrightarrow |3\rangle$ transition. The radiative decay rate of $|1\rangle \leftrightarrow |3\rangle$ is taken to be $2\gamma_1$ and of $|1\rangle \leftrightarrow |2\rangle$ as $2\gamma_2$. The Hamiltonian for this system in the appropriately rotating frames is given by

$$\frac{H}{\hbar} = \Delta_1 |3\rangle\langle 3| + \Delta_2 |2\rangle\langle 2| + [g(t)|1\rangle\langle 3| + G_2 |1\rangle\langle 2| + \text{H.c.}] , \quad (2.7)$$

where $\Delta_1 = \omega_1 - \omega_{13}$, $\Delta_2 = \omega_2 - \omega_{12}$ ($\omega_{\alpha\beta}$ is the transition frequency between states $|\alpha\rangle$ and $|\beta\rangle$), and

$$g(t) = G_1 + G_c(t) e^{-i(\nu - \omega_1)t} .$$

The parameters G_1, G_2, and $G_c(t)$ are defined in a manner similar to that of Eq. (2.2) for the ladder system. The resulting density-matrix equations are given by (treating G_1 and G_2 as real)

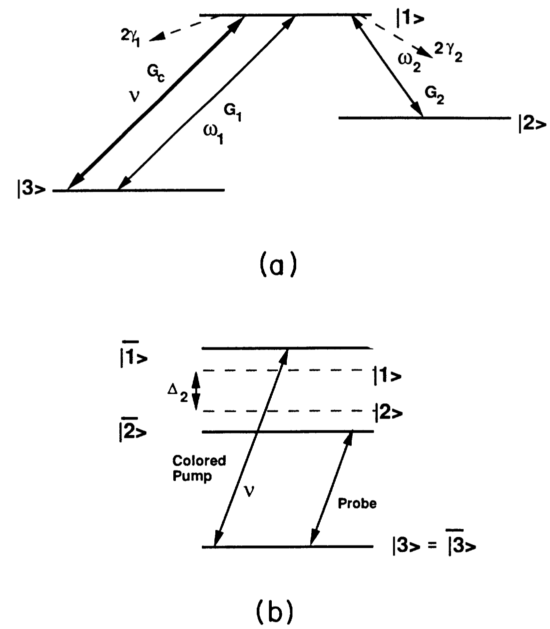

FIG. 2. (a) Atomic-energy-level diagram for the Λ system. The thick arrow is the colored pump on the lasing transition. (b) Semiclassical dressed states corresponding to the bare states of (a).

$$\dot{\rho}_{11} = -(2\gamma_1 + 2\gamma_2)\rho_{11} - ig(t)\rho_{31} + ig^*(t)\rho_{13} + iG_2(\rho_{12} - \rho_{21}) , \quad (2.8a)$$

$$\dot{\rho}_{12} = -(\gamma_1 + \gamma_2)\rho_{12} - ig(t)\rho_{32} + i\Delta_2\rho_{12} + iG_2(\rho_{11} - \rho_{22}) , \quad (2.8b)$$

$$\dot{\rho}_{22} = 2\gamma_2\rho_{11} + iG_2(\rho_{21} - \rho_{12}) , \quad (2.8c)$$

$$\dot{\rho}_{33} = 2\gamma_1\rho_{11} - ig^*(t)\rho_{13} + ig(t)\rho_{31} , \quad (2.8d)$$

$$\dot{\rho}_{13} = -(\gamma_1 + \gamma_2)\rho_{13} - iG_2\rho_{23} + i\Delta_1\rho_{13} + ig(t)(\rho_{11} - \rho_{33}) , \quad (2.8e)$$

$$\dot{\rho}_{23} = -i(\Delta_2 - \Delta_1)\rho_{23} - iG_2\rho_{13} + ig(t)\rho_{21} . \quad (2.8f)$$

Once again we have Langevin equations with multiplicative noise, where the colored pump $G_c(t)$ has zero mean and an autocorrelation function given by Eq. (2.3). The gain on the lasing transition is now given by

$$G = \mathrm{Im}\left[\frac{\langle \rho_{13} \rangle \gamma_1}{G_1}\right] , \quad (2.9)$$

where $\langle \rho_{13} \rangle$ is the ensemble average of the steady-state value of the density-matrix element ρ_{13} averaged over the field fluctuations.

Following a procedure similar to the ladder systems, we diagonalize the Hamiltonian in the presence of G_2 to get the semiclassical dressed states $|\bar{1}\rangle$ and $|\bar{2}\rangle$ which are linear combinations of the bare states $|1\rangle$ and $|2\rangle$. The eigenvalues of the Hamiltonian in the presence of the strong pump at ω_2 are given by

$$\frac{\Delta_2}{2} \pm \left[\frac{\Delta_2^2}{4} + G_2^2\right]^{1/2} . \quad (2.10)$$

These new states are shown in Fig. 2(b). Following an argument similar to that for ladder systems [16], one can choose the colored-pump frequency as

$$\nu=\omega_{13}+\left[\frac{\Delta_2}{2}+\left[\frac{\Delta_2^2}{4}+G_2^2\right]^{1/2}\right] \tag{2.11}$$

and the probe frequency as

$$\omega_1\sim\omega_{13}-\left[\frac{\Delta_2}{2}-\left[\frac{\Delta_2^2}{4}+G_2^2\right]^{1/2}\right] . \tag{2.12}$$

It has been shown by Imamoglu, Field, and Harris [10] that for appropriate atom and field parameters, the Λ system can exhibit inversionless gain when the fields G_1 and G_2 are on-resonance with their respective transitions. Thus, we restrict our discussion to the case when $\Delta_1=\Delta_2=0$ for which the colored-pump frequency ν given by Eq. (2.11) becomes

$$\nu=\omega_{13}+G_2 . \tag{2.13}$$

Since the frequency of the colored pump appears in the form $G_c(t)e^{-i(\nu-\omega_1)t}$ in the density-matrix equations, we can set the oscillating term associated with $G_c(t)$, i.e., $(\nu-\omega_1)$, as being equal to G_2 (for $\Delta_1=0$). This choice corresponds to setting the colored-pump frequency in a manner identical to the ladder system. However, to understand the role of ν in the LWI process and its effect on the gain, we will vary ν for the Λ system and study the gain.

As stated earlier, the presence of a colored pump in these calculations makes it necessary for us to resort to numerical calculations. We use Monte Carlo methods for obtaining the gain in this work. The use of these Monte Carlo methods in the study of laser-atom interactions has been described in detail in other publications [17] and we will give only a brief discussion of it in Sec. III.

III. PROCEDURE FOR NUMERICAL SOLUTION OF DENSITY-MATRIX EQUATIONS

The first step in the numerical solution of the density-matrix equations given by Eqs. (2.1) and (2.8) is to produce the colored noise $G_c(t)$ which has zero mean and satisfies the autocorrelation function of Eq. (2.3). To do this, we initially obtain the Gaussian, δ-correlated (white) noise g_w with zero mean, which provides the source term for the colored noise. g_w can be obtained from the Box-Mueller algorithm [18] and is given by

$$g_w=\sqrt{-2D\,\Delta t\,\ln(a)}\,e^{2\pi ib} , \tag{3.1}$$

where a and b are uniformly random numbers between 0 and 1 and Δt is the integration time step used in the numerical integration of Eqs. (2.1) and (2.8). The colored noise is obtained from the solution of the equation

$$\frac{dG_c(t)}{dt}=-\Gamma G_c(t)+\Gamma g_w , \tag{3.2}$$

in which g_w is the Gaussian white noise of Eq. (3.1). We have shown in a previous publication [17] that the integration of (3.2) gives

$$G_c(t+\Delta t)=G_c(t)e^{-\Gamma\Delta t}+h(t) , \tag{3.3}$$

where h is Gaussian and depends on g_w. h has zero mean and a second moment given by

$$\left\langle |h(t,\Delta t)|^2\right\rangle=D\Gamma[1-e^{-2\Gamma\Delta t}] . \tag{3.4}$$

To generate the colored noise $G_c(t)$, we first produce h from the formula

$$h=\sqrt{-D\Gamma[1-e^{-2\Gamma\Delta t}]\ln(a)}\,e^{2\pi ib} , \tag{3.5}$$

where a and b are again random numbers uniformly distribution on the unit interval. The exponentially correlated noise is then obtained from Eq. (3.3).

The Langevin equations [Eqs. (2.1) or (2.8)] were solved numerically using the colored noise generated above. A Euler method was used for the integration with a typical time step Δt of 10^{-5}. The accuracy of the numerical results was checked by trying smaller time steps and also using a fourth-order Runge-Kutta method and noting that the results were identical to those obtained with the time step of 10^{-5}. The Euler method was chosen over the Runge-Kutta method due to the faster speed of the former. The integration was carried out for 100 units of dimensionless time and the steady-state values of the relevant density matrix element (ρ_{12} or ρ_{13}) were stored. The integration procedure was carried out for 500 to 1000 iterations typically, with each iteration having a completely different set of random numbers. This ensured that the numerical results were not affected by small number statistics. The final result was an average of the steady-state values of ρ_{12} (or ρ_{13}) over all the iterations. The uncertainty in our results is better than 3–4 % for the small-Γ values ($\Gamma\le 10$) and about 15% for the large-Γ values ($\Gamma\ge 50$). The values of D and Γ can be input easily into the algorithm for generating the colored noise. For $\Gamma\gg\gamma_1$ the algorithm generates an incoherent pump and for $\Gamma\ll\gamma_1$ it generates a coherent pump. Thus, by varying the values of Γ relative to γ_1, one can continuously "tune" the bandwidth of the pump.

It is worth noting some of the constraints imposed by the numerical algorithm for generating colored noise described here. To simulate monochromatic pumps would require that Γ be zero. A zero value for Γ is, however, not possible in our algorithm since h, as given by Eq. (3.5), becomes zero. In our calculations we have restricted ourselves to values of Γ of $0.1\gamma_1$ or higher and believe that such small values of Γ can be taken as a fair representation of monochromatic pumps. Small values of Γ also make the time taken for the numerical computations longer, since one has to integrate for a greater number of points for a given value of Δt. This is necessary to ensure that the system of equations evolves over a sufficient number of correlation times. Similarly, making Γ very large to study the incoherent pump limit requires that one reduce the time step (for the numerical integration to work accurately, $\Delta t\ll$ all other time scales in the problem and so $\Delta t\ll\Gamma^{-1}$) and hence again integrate for more number of points. We have thus restricted ourselves to values of $\Gamma \leq 100\gamma_1$.

IV. RESULTS

We now report our results on dependence of the gain in LWI systems on colored-pump parameters. The first results are for the ladder system that has been discussed in [16]. For the ladder system, we fix the colored-pump frequency ν as given by Eq. (2.6) which corresponds identically to the situation described in [16]. The units of all rates in our results are in terms of γ_1, i.e., we set γ_1 equal to 1 and all other rates in the problem are in units of γ_1. We have chosen the ^{138}Ba atom as our prototype system and so we choose γ_2 to be $5.4\gamma_1$ (for ^{138}Ba, $2\gamma_1=3.7$ MHZ and $2\gamma_2=20$ MHZ). The detuning Δ_2 is fixed at $25.1\gamma_1$, $G_1=0.2\gamma_1$, and $G_2=14.3\gamma_1$. For the colored-pump noise parameters, we consider two cases, one where the product of D and Γ is $30\gamma_1^2$ and the other where the product is $60\gamma_1^2$ (note that both D and Γ are in units of γ_1) but vary Γ. This implies that the colored-pump intensity is being kept constant while the bandwidth is being varied. For a given value of D and Γ, we calculate the gain given by Eq. (2.5) as a function of Δ_1. The maximum value of the gain for each set of values of D and Γ is then plotted as a function of the bandwidth Γ. The results are shown in Fig. 3 where the gain is in units of the weak-field resonant absorptivity when the atom is initially prepared in the bare state $|2\rangle$. A value of zero for the bandwidth corresponds to a purely coherent, monochromatic pump, while the larger values of Γ tend towards an incoherent pump. We see very clearly from this figure that the gain for a coherent pump is greater than the gain obtained with an incoherent pump. A surprising feature, however, is that the maximum gain is obtained for a partially coherent pump. This behavior is seen in both curves of Fig. 3(a), which are for two different colored-pump intensities, indicating that this may be a fairly general property of LWI systems. The gain enhancement that one can obtain by changing from an incoherent pump to a partially coherent one is close to a factor of 4 for $D\Gamma$ equal to both $30\gamma_1^2$ and $60\gamma_1^2$. This indicates that there is an optimum bandwidth of the colored pump where even greater gain enhancement can be obtained as compared to a coherent or incoherent pump. These factors of 3 or 4 in gain enhancement, as reported here and in [16], can become significant in LWI experiments since the gain levels are usually very small.

Figure 3(b) shows the results when the colored-pump bandwidth is fixed at $\Gamma=\gamma_1$ and the strength of the noise D is varied. We find a monotonic increase in gain with increase in D. This result is expected since as we increase D for a fixed Γ, we are simply making the colored pump stronger and hence transferring more population from one level to another. Constraints on computer time prevent us from investigating the effect of higher values of D, but it is reasonable to expect that eventually the gain will saturate or even decrease with further increase in D. As the colored pump gets stronger, eventually it will start to dress the upper transition, thus mixing up the levels and contributing to decreased gain.

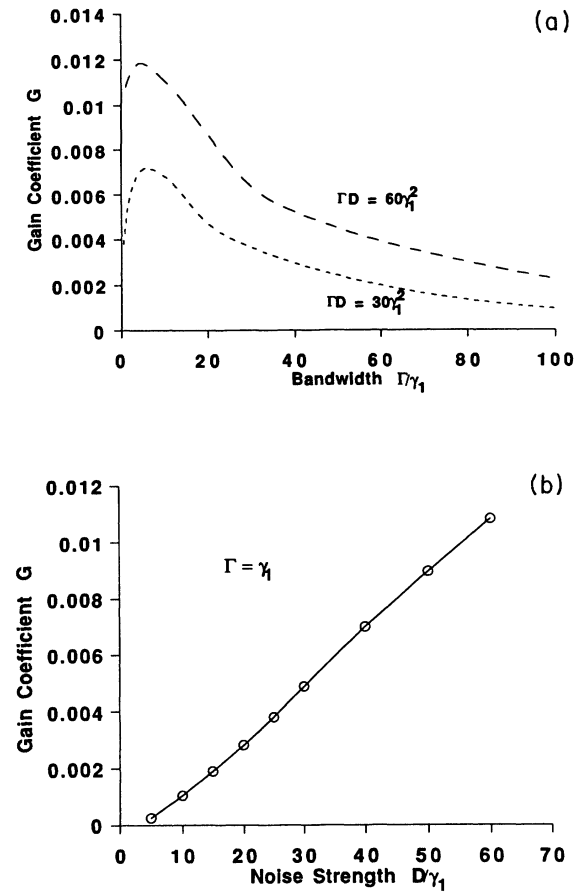

FIG. 3. (a) Maximum gain versus colored-pump bandwidth for the ladder system. Curves are for $D\Gamma=60\gamma_1^2$ and $D\Gamma=30\gamma_1^2$. D and Γ are in units of γ_1. $G_1=0.2\gamma_1$, $G_2=14.3\gamma_2$, and $\Delta_2=25.1\gamma_1$. ν is chosen as determined by Eq. (2.6). (b) Maximum gain versus strength of noise D for ladder system. $\Gamma=\gamma_1$, $G_1=0.2\gamma_1$, $G_2=14.3\gamma_1$, and $\Delta_2=25.1\gamma_1$. ν is chosen as determined by Eq. (2.6). The open circles represent gain calculated from Monte Carlo methods (not all calculated points are shown; curve drawn through all calculated points).

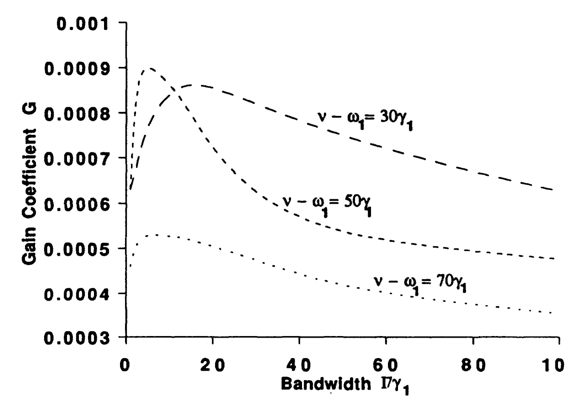

FIG. 4. Gain versus colored pump bandwidth for the Λ system. All curves are for $D\Gamma=30\gamma_1^2$, $G_1=0.2\gamma_1$, $G_2=50\gamma_1$, and $\Delta_1=\Delta_2=0$. The curves are for $\nu-\omega_1=50\gamma_1$, $\nu-\omega_1=30\gamma_1$, and $\nu-\omega_1=70\gamma_1$.

In. Fig. 4 we show the results of calculations of the Λ model. In these results we have fixed the colored-pump intensity, i.e., the product $D\Gamma$ equal to $30\gamma_1^2$ and varied the pump bandwidth. This calculation is analogous to the calculations for the results presented in Fig. 3(a). Once again, all rates are in units of γ_1 and we have taken $G_1=0.2\gamma_1$, $G_2=50\gamma_1$, and $\Delta_1=\Delta_2=0$. As pointed out by Harris, Imamoglu, Field, and [10], for a Λ system to exhibit inversionless gain, $\gamma_2 \gg \gamma_1$, and so we have chosen γ_2 equal to $10\gamma_1$. We first focus on the result for the case of the colored-pump frequency $\nu=\omega_{13}+G_2$ and hence $\nu-\omega_1=50\gamma_1$ (for $\Delta_1=0$). This choice of ν corresponds to picking ν as we did for the ladder system. Clearly, this curve is qualitatively very similar to the curves in Fig. 3, thus validating the fact that perhaps all inversionless systems based on incoherent pumping schemes will exhibit significant gain enhancement on being replaced by partially coherent pumping mechanisms. Once again, we find that while the coherent pump provides greater gain than the incoherent pump, the maximum gain is for an intermediate value of the pump bandwidth. The maximum gain we get in this situation is a factor of 2 greater than the gain for the incoherent pump. We expect that larger gains can be realized in this system by choosing appropriate values of the atom-field parameters.

We have also investigated the effect of the colored-pump frequency on the gain one can realize in the Λ systems. Figure 4 also displays the gain vs bandwidth results for $\nu-\omega_1=30\gamma_1$ and $\nu-\omega_1=70\gamma_1$ with all other parameters being the same as for the curve with $\nu-\omega_1=50\gamma_1$. It is clear from these curves that the choice of ν does have an observable impact on the gain in LWI systems where a colored pump is used. We note first of all that the maximum gain is observed when $\nu-\omega_1=G_2=50\gamma_1$. While one can still see that a partially coherent pump provides greater gain than an incoherent pump or a purely coherent pump when $\nu-\omega_1\neq G_2$, it is clear that the gain discrimination as a function of the bandwidth is no longer as pronounced. We find that the gain from a coherent pump can now become comparable to the gain from an incoherent pump, whereas for $\nu-\omega_1=50\gamma_1$, the gain from a coherent pump is always larger than from an incoherent pump. Furthermore, as the value of the colored-pump frequency ν is increased, we find that the bandwidth of the pump at which the maximum gain occurs moves towards smaller values.

As the colored-pump bandwidth gets much larger than γ_1, one expects that the gain would become independent of the value of ν. We have indeed checked this by calculating the gain for $\Gamma=1000\gamma_1$ and find that the gain is the same for any value of ν and is equal to what one would get from an incoherent pump (i.e., by replacing the colored pump in the density-matrix equations by an incoherent pumping term).

V. DISCUSSION

In this paper we have studied the impact of the noise parameters of a fluctuating pump in lasing without inversion systems when an incoherent pumping mechanism is replaced by a partially coherent pump. A numerical technique for "tuning" the bandwidth of the pump is shown which can be used to study other LWI systems and perhaps help understand the physical mechanisms at play in these systems.

We have presented results for two types of three-level atomic-energy-level schemes—the ladder model and the Λ model. Our calculations indicate very clearly that while a coherent pump provides greater gain in LWI systems than an incoherent pump, maximum gain is realized for a *partially* coherent pump. The physical mechanism of gain enhancement in three-level atoms with colored pumps has been elucidated in [16] and is based on a stimulated Stokes process between the semiclassical dressed states obtained from the interaction of a strong pump with two of the bare atomic levels. In other words, the gain in these systems is a consequence of the fact that a colored pump can transfer population preferentially among the relevant levels, while an incoherent pump cannot do this preferential population transfer.

It is difficult to get a physical picture from numerical calculations as to why a partially coherent pump is more effective than a coherent pump. We speculate that perhaps the maximum gain is obtained when the pump linewidth is comparable to the linewidth of the lasing transition and the entire spectral density under the pump field spectrum overlaps the atomic transition line shape. For an incoherent pump the effective linewidth of the $|1\rangle \leftrightarrow |3\rangle$ transition in the Λ model, for example, is broadened and becomes $2(\gamma_1+\gamma_2+D)$ from well-known substitution rules. Thus, if the linewidth of the lasing transition is being broadened in the presence of a partially coherent pump in some other manner, it is possible that a pump linewidth larger than $\Gamma=\gamma_1$ is required for optimum overlap of the pump and the transition linewidths and it is this pump linewidth of $\Gamma>\gamma_1$ that provides the maximum gain. It is clear that a rigorous elucidation of the mechanism for gain enhancement by partially coherent pumps is still needed.

The fact that a partially coherent pump is more effective than either an incoherent or a coherent pump is significant for experiments on LWI systems where the signals are usually meager. We have also found that the frequency of the colored pump is a very important factor in determining the optimum gain that can be obtained. Our calculations clearly show the procedure for picking the frequency ν in an experiment. It is obvious that one has to tune ν in a manner determined by Eq. (2.11) for a Λ system for maximum efficiency. While we have not reported the results for a ladder system in this context, we find results similar to the Λ model and the optimum frequency is determined by Eq. (2.6).

The predictions of this paper can be experimentally tested via techniques developed by Elliott and co-workers for controlling the linewidth and line shape of a laser [19]. These techniques permit precise control and variation of the laser linewidth through electro-optic and acousto-optic modulation and can incorporate the small and large pump linewidth regimes discussed in this work.

In conclusion, replacing an incoherent pump by a partially coherent pump, coupled with a judicious choice of its bandwidth and frequency as outlined in this paper, can produce significant gain enhancement over gain obtained with an incoherent pump. We anticipate that similar results will hold true for other LWI systems where an incoherent pump is replaced by a partially coherent pump.

ACKNOWLEDGMENTS

This work was partially supported by a IUPUI faculty development grant. We gratefully acknowledge the interest of, and discussions with, Professor G.S. Agarwal, and one of us (G.V.) acknowledges a stimulating discussion with Professor Lorenzo Narducci. D.M.W. received additional support from IUPUI.

[1] O. Kocharovskaya, P. Mandel, and Y. V. Radeonychev, Phys. Rev. A **45**, 1997 (1992); O. Kocharovskaya and P. Mandel, *ibid.* **42**, 523 (1990).

[2] S. E. Harris, Phys. Rev. Lett. **62**, 1033 (1989).

[3] V. G. Arkhipkin and Y. I. Heller, Phys. Lett. **98A**, 12 (1983).

[4] G. S. Agarwal, S. Ravi, and J. Cooper, Phys. Rev. A **41**, 4721 (1990); **41**, 4727 (1990).

[5] F. E. Fill, M. O. Scully, and S. Y. Zhu, Opt. Commun. **77**, 36 (1990); M. O. Scully, S. Y. Zhu, and A. Gavrielides, Phys. Rev. Lett. **62**, 2813 (1989).

[6] L. M. Narducci, H. M. Doss, P. Ru, M. O. Scully, S. Y. Zhu, and C. H. Keitel, Opt. Commun. **81**, 379 (1991); L. M. Narducci, M. O. Scully, C. H. Keitel, S. Y. Zhu, and H. M. Doss, *ibid.* **86**, 324 (1991); H. M. Doss, L. M. Narducci, M. O. Scully, and J. Gao, *ibid.* **95**, 57 (1993).

[7] G. S. Agarwal, Phys. Rev. A **42**, 686 (1990); N. Lu, Opt. Commun. **73**, 479 (1989).

[8] A. Lezama, Y. Zhu, M. Kanskar, and T. W. Mossberg, Phys. Rev. A **41**, 1576 (1990); M. H. Anderson, G. Vemuri, J. Cooper, P. Zoller, and S. J. Smith, *ibid.* **47**, 3202 (1993).

[9] G. B. Prasad and G. S. Agarwal, Opt. Commun. **86**, 409 (1991).

[10] A. Imamoglu, J. E. Field, and S. E. Harris, Phys. Rev. Lett. **66**, 1154 (1991); G. S. Agarwal, Phys. Rev. A **44**, R28 (1991).

[11] Y. Zhu, Phys. Rev. A **45**, R6149 (1992).

[12] Y. Zhu, Phys. Rev. A **47**, 495 (1993); Y. Zhu, O. C. Mullins, and M. Xiao, *ibid.* **47**, 602 (1993).

[13] G. S. Agarwal, Phys. Rev. Lett. **67**, 980 (1991).

[14] K. Gheri and D. F. Walls, Phys. Rev. Lett. **68**, 3428 (1992); Phys. Rev. A **45**, 6675 (1992).

[15] M. O. Scully, Phys. Rev. Lett. **67**, 1855 (1991); M. Fleischhauer, C. H. Keitel, M. O. Scully, C. Su, B. T. Ulrich, and S. Y. Zhu, Phys. Rev. A **46**, 1468 (1992).

[16] G. S. Agarwal, G. Vemuri, and T. W. Mossberg, Phys. Rev. A **48**, R4055 (1993).

[17] G. Vemuri, R. Roy, and G. S. Agarwal, Phys. Rev. A **41**, 2749 (1990); R. F. Fox, I. R. Gatland, R. Roy, and G. Vemuri, *ibid.* **38**, 5938 (1988).

[18] W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes: The Art of Scientific Computing* (Cambridge University Press, Cambridge, England, 1989).

[19] D. S. Elliott and S. J. Smith, J. Opt. Soc. Am. B **5**, 1927 (1988); C. Chen, D. S. Elliott, and M. W. Hamilton, Phys. Rev. Lett. **68**, 3531 (1992).