# Propagation of partially polarized light

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We present a general formalism describing the evolution of partially polarized light propagating in a medium with cubic nonlinearity. Within the ergodic approximation we found that totally polarized coherent single-mode light remains totally polarized, irrespective of the type of optical medium. Partially polarized light consisting of a mixture of a coherent component with a random one preserves its degree of polarization in a dissipation-free medium with isotropic nonlinearity, while only linear or nonlinear dissipation may lead to alteration of the degree of polarization. We also found that in some crystal symmetry point groups, a time-nonreversible optical response may be the only reason for an alteration of the degree of polarization.

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One of the main differences between natural light and coherent laser radiation is that natural light is polarized only partially, i.e., it contains a component whose polarization state changes randomly in the time domain, while coherent radiation has a fixed polarization state. Traditionally, nonlinear optics deals with polarized light. However, we believe that nonlinear optics of partially polarized and unpolarized light also deserves attention since it has not yet been properly developed and is closely related to the polarization instability problem, which has attracted a lot of attention during the past few years [1]. In linear optics it is known that unpolarized light may change its degree of polarization as the result of an interaction with an absorbing material: polaroid film, for example.

In nonlinear optics the evolution of the degree of polarization is evidently more complicated than in the linear case. Nonlinear propagation of multimode laser radiation in an absorbtionless medium with isotropic nonlinearity under the assumption of an input polarization ellipse fluctuating in magnitude but not in ellipticity and orientation has been considered by Crosignani, Daino, and Di Porto [2]. It was found that the polarization changes in the medium from totally polarized to partially polarized and the polarization degree tends to a limit value which is determined by the input elipticity. This result was later generalized by Chernov and Zon [3] to the case of a nonlinear medium with isotropic nonlinearity and low linear birefringence. The results obtained in [2,3] are suitable for describing propagation of a fluctuating multimode laser having Gaussian distribution of the fields with zero mean values and fixed polarization when the orthogonal polarization components are strictly correlated. This sort of input radiation, however, is totally polarized in the input of the medium and the results of [2,3] could not be used for description of natural light (one can readily see that by passing through the appropriate birefringent retarder the polarization state considered in [2,3] may be transformed into linear polarization with random intensity). In order to describe propagation of genuine partially polarized (natural) light one shall consider the more complicated and physically absolutely different case of a wave with fluctuations not only in intensity (as in [2,3]), but also in the elipticity and polarization azimuth, i.e., it shall be presumed that all the "instant" wave parameters may have random components; i.e., the polarization state treated in [2,3] is not a specific case of a partially polarized light wave. It is known that a partially polarized light wave may be uniquely decomposed as the sum of a coherent, totally polarized component and an incoherent, totally depolarized one [4]; i.e., in the case of partially polarized light the mean field values of the wave might be nonzero. The partially polarized light propagation problem is an important practical question. Partially polarized light may be created by scattering of ideal single-mode coherent laser light. A realistic single-mode laser whose output is spoiled by spontaneous transitions in the active medium is another example of a partially polarized light source [5]. Narrow-band partially polarized light may also be prepared from the light of a thermal light source (the Sun, for instance) by polarization-sensitive reflection or scattering combined with passing through a narrow-band spectral filter. In this paper we intend to address the problem of propagation of partially polarized light considering self-action phenomena.

Let us consider plane-wave propagation along the $z$ direction of a Cartesian coordinate frame. Under the assumption that the wave forms a narrow band about a large central frequency $\omega$, we introduce the wave vector $\mathbf{k}$ and will use the following material equation of the medium:

$$P_i(\omega,\mathbf{k})=\chi_{ij}^{(1)}\mathcal{E}_j+ik_m\gamma_{ijm}^{(1)}\mathcal{E}_j+\chi_{ijkl}^{(3)}\mathcal{E}_j\mathcal{E}_k\mathcal{E}_l^* +ik_m\gamma_{ijklm}^{(3)}\mathcal{E}_j\mathcal{E}_k\mathcal{E}_l^*+\cdots , \quad (1)$$

which allows us to treat local and first-order nonlocal, linear and nonlinear self-action effects. Summation over repeated indices is presumed here and below. In the slowly varying envelope approximation, i.e., when considering the electric-field strength in the form $\mathcal{E}=\mathbf{E}(z)e^{-i\omega t+ikz}+\text{c.c.}$, the wave equation reduces to

$$\frac{dE_i}{d\zeta}=i(u_{ij}E_j+\kappa_{ijkl}E_jE_kE_l^*+\cdots),\quad \zeta=\frac{2\pi z\omega^2}{kc^2}, \tag{2}$$

where $k^2=(\omega/c)^2[1+(4\pi/3)\mathrm{Tr}(\mathrm{Re}\chi_{ij}^{(1)})]$ and using the Kronecker symbol $\delta_{ij}$ we introduce the following material tensors:

$$\begin{aligned} u_{ij}&=4\pi\chi_{ij}^{(1)}-\frac{4\pi}{3}\mathrm{Tr}(\mathrm{Re}\chi_{ij}^{(1)})\delta_{ij}+4\pi ik\gamma_{ijz}^{(1)},\\ \kappa_{ijkl}&=4\pi(\chi_{ijkl}^{(3)}+ik\gamma_{ijklz}^{(3)}). \end{aligned} \tag{3}$$

The use of Eq. (2) presumes that the light spectrum remains a narrow band around the central frequency and correspondingly we exclude from consideration those situations where the spectrum expands dramatically, i.e., we exclude processes such as stimulated Raman scattering.

After algebraic rearrangements [6] one can transform (2) into the evolution equation for the following quadratic forms of the electric-field strength components, which will be referred to below as the s vector, $s^\alpha=\sigma_{ij}^{(\alpha)}E_i^*E_j$ ($\alpha=0,\ldots,3$), where $\sigma^{(\alpha)}$ are the Pauli matrices [7]:

$$\frac{ds_\alpha}{d\zeta}=-\mathrm{Im}(\Omega_0 s_\alpha+\Omega_\alpha s_0)-\mathrm{Re}(e_{\alpha\beta\gamma}\Omega_\beta s_\gamma),\quad \alpha=1,2,3, \tag{4}$$

$$\frac{ds_0}{d\zeta}=-\mathrm{Im}(\Omega_\alpha s_\alpha+\Omega_0 s_0),\quad \alpha=1,2,3. \tag{5}$$

Here $e_{\alpha\beta\gamma}$ is the Levi-Civita symbol,

$$\begin{aligned} &v^\alpha=\sigma_{ji}^\alpha u_{ij},\quad u_{ij}=\tfrac{1}{2}v^\alpha\sigma_{ij}^{(\alpha)},\\ &w^{\alpha\beta}=\sigma_{ji}^{(\alpha)}\kappa_{ijkl}\sigma_{kl}^{(\beta)},\quad \kappa_{ijkl}=\tfrac{1}{4}w^{\alpha\beta}\sigma_{ij}^{(\alpha)}\sigma_{lk}^{(\beta)}, \end{aligned} \tag{6}$$

and we have introduced the self-action vector $\Omega_\alpha=v^\alpha+\frac{1}{2}w^{\alpha\beta}s_\beta$, $\alpha,\beta=0,\ldots,3$.

All particular optical single-wave propagation effects in linear and nonlinear media, including linear and circular birefringence and dichroism, gyrotropic linear and circular birefringence and dichroism, polarization ellipse self-rotation, nonlinear optical activity, nonlinear absorption, as well as polarization symmetry breaking effects, can be described by (4) and (5) presuming steady-state conditions. Evidently, the four-dimensional s vector in (4) and (5) is similar to the Stokes vector except that the Stokes vector S is defined for steady-state stochastical processes and thus involves averaging over time $\mathbf{S}=\langle \mathbf{s}\rangle$, denoted here by $\langle\ \rangle$. It is attractive to adapt Eqs. (4) and (5) for use with the Stokes parameters, which conveniently allow the description of partially polarized light. This may be undertaken by using the ergodic approximation (the ergodic approximation was also used in the analysis reported in [3]). In accordance with the individual ergodic theorem the time average may be replaced by an ensemble average in phase space if the process is metrically *indecomposable* (see [8], for example). The phase space is said to be indecomposable if it cannot be split into two invariant parts. An invariant part of the space means that an arbitrary trajectory in that part remains within that part during all natural motion. Applied to the problem of propagation of partially polarized light, this means that the ergodic approximation may be used to derive the Stokes vector evolution equations if in any point of the nonlinear medium the trajectory on the Poincaré sphere remains within one closed area, which, however, does not necessary cover all the sphere. Here by the Poincaré sphere we mean the energy surface in the $\{s_1;s_2;s_3\}$ coordinate frame.

In approach to the propagation problem we make use of the fact that incident partially polarized light may be uniquely decomposed as a totally polarized component $\mathbf{E}_0$ and an unpolarized component $\delta\mathbf{E}$ with Gaussian statistics [4,9]. It is clear that light with Gaussian statistics complies with the necessary condition of applicability of the ergodic approach. Indeed the evolution of the incident light polarization state on the Poincaré sphere remains within an invariant part (this part is located around the fixed point which corresponds to the state of the polarized component) and this area is metrically indecomposable. Now, in order to use the ergodic idea for the nonlinear propagation process we have to prove that the phase space remains metrically indecomposable everywhere inside the nonlinear media. This directly follows from the fact that the solution of the polarization dynamics of a single wave in the scope of the third-order nonlinear processes in any anisotropic nonlinear medium cannot exhibit multistability or chaotic behavior and in most cases can be reduced to quadratures [10]. Therefore, single-wave evolution is described by a continuous and single-valued function of the initial conditions. This inevitably means that during propagation the initial invariant area on the Poincaré sphere which encloses the process trajectory may suffer from nonlinear deformation but remains metrically indecomposable. That is, it does not split into several different areas, since the splitting would contradict the single-valued and continuous nature of the transformation. Consequently the ergodic hypothesis can be used to average the single-wave polarization evolution equations (4) and (5) with partially polarized incident light. Here we shall note that in the problem of nonlinear interaction of two waves the metrically indecomposability of the phase space is not secured [11] and the use of the ergodic approximation is questionable for two-wave interaction.

Now we shall consider the averaging procedure. In order to work out the Stokes vector components $\langle s_\alpha\rangle$ and the biquadratic structures $\langle s_\alpha s_\beta\rangle$, which appear when the averaging procedure is used with respect to Eqs. (4) and (5), we start with representation of the light wave electric field in the following form:

$$\mathbf{E}=\mathbf{E}_0+\delta\mathbf{E}, \tag{7}$$

i.e., we traditionally decompose the partially polarized light into a totally polarized, coherent component $\mathbf{E}_0$ and a totally unpolarized, random component $\delta\mathbf{E}$ with zero mean values. The $S_0$ component of the Stokes vector which represents the light wave intensity in these terms may be written as

$$S_0=\langle (E_{0i}^*+\delta E_i^*)\sigma_{ij}^{(0)}(E_{0j}+\delta E_j)\rangle$$
$$=(\mathbf{E}_0\mathbf{E}_0^*)+\langle (\delta\mathbf{E}\delta\mathbf{E}^*)\rangle \ . \quad (8)$$

By using the following intrinsic property of totally unpolarized light:

$$\langle \delta E_i^*\delta E_j\rangle=\tfrac{1}{2}\langle (\delta\mathbf{E}\delta\mathbf{E}^*)\rangle\delta_{ij} \quad (9)$$

one can readily obtain the expected result that the remaining components $S_1,\ldots,S_3$ of the Stokes vector depend only on the coherent totally polarized contribution of the light wave:

$$S_\alpha=\langle (E_{0i}^*+\delta E_i^*)\sigma_{ij}^{(\alpha)}(E_{0j}+\delta E_j)\rangle=E_{0i}^*\sigma_{ij}^{(\alpha)}E_{0j} \ . \quad (10)$$

The intensity of the totally unpolarized part of the light wave is often conveniently presented in terms of the parameter known as the degree of polarization $r$, which is given by the ratio of the coherent, polarized component intensity to the total wave intensity:

$$r^2=\frac{S_1^2+S_2^2+S_3^2}{S_0^2} \ . \quad (11)$$

Thus

$$(\delta\mathbf{E}\delta\mathbf{E}^*)=S_0(1-r) \quad (12)$$

and $r=1$ refers to totally polarized and $r=0$ to totally unpolarized light.

The use of Eqs. (8)–(12) leads to the following expressions for the average values of the biquadratic structure $s_\alpha s_\beta$, which will appear if one averages the evolution equations (4) and (5):

$$\langle s_\alpha s_\beta\rangle=\langle (E_{0i}^*+\delta E_i^*)\sigma_{ij}^{(\alpha)}(E_{0j}+\delta E_j)(E_{0k}^*+\delta E_k^*)\sigma_{kl}^{(\beta)}(E_{0l}+\delta E_{0l})\rangle$$
$$=S_\alpha S_\beta+\sigma_{ij}^{(\alpha)}\sigma_{kl}^{(\beta)}\langle (E_{0i}^*E_{0l}\delta E_k^*\delta E_j)+E_{0k}^*E_{0j}\delta E_i^*\delta E_l+\delta E_i^*\delta E_j\delta E_k^*\delta E_l\rangle$$
$$=S_\alpha S_\beta+(1-r)S_0E_{0i}\sigma_{ij}^{(\alpha)}\sigma_{jl}^{(\beta)}E_{0l}+\langle \delta E_i^*\delta E_j\delta E_k^*\delta E_l\rangle \ . \quad (13)$$

By making use of the following property of the Pauli matrices:

$$\sigma_{ij}^{(\alpha)}\sigma_{jk}^{(\beta)}=\sigma_{ik}^{(0)}\delta_{\alpha\beta}+ie_{\alpha\beta\gamma}\sigma_{ik}^{(\gamma)} \quad (14)$$

the second term of the last line of Eq. (13) may be rearranged resulting in

$$\langle s_\alpha s_\beta\rangle=S_\alpha S_\beta+(1-r)rS_0^2\delta_{\alpha\beta}+\langle \delta E_i^*\delta E_j\delta E_k^*\delta E_l\rangle \ . \quad (15)$$

Equation (15) does not presume any specific statistical properties of light, but in order to evaluate the last term in the equation above we need to make use of the fact that the unpolarized component has Gaussian statistics *in the input to the medium.* Of course, in general the nonlinear propagation process may change the statistics within the medium. However, the alteration of the statistics is intensity dependent since a linear interaction does not affect the statistical properties of a wave. Intensity-dependent change in the statistics may be taken into account by including a nonlinearity proportional correction term when the statistical moments are calculated:

$$\langle \delta E_i\delta E_j^*\delta E_k\delta E_l^*\rangle$$
$$=\langle \delta E_i\delta E_j^*\rangle\langle \delta E_k\delta E_l^*\rangle+\langle \delta E_i\delta E_l^*\rangle\langle \delta E_k\delta E_j^*\rangle$$
$$+O(\kappa^{(3)}|\mathbf{E}|^2)O(\delta\mathbf{E}^4) \ . \quad (16)$$

Here $\kappa$ is the nonlinear parameter from Eq. (6) which incorporates the local and nonlocal contributions to the third-order nonlinearity. In (16) the first and second terms of the right-hand side match the Gaussian statistics. The last term represents the nonlinear correction of the statistics and $O(x)$ stands for the terms of the order of $x$ or higher. Thus the $O$ term in (16) reflects the difference between the initial statistics and actual statistics inside the nonlinear medium.

Now we can disclose the high-order correlators in (13)

$$\sigma_{ij}^{(\alpha)}\sigma_{kl}^{(\beta)}\langle \delta E_i^*\delta E_j\delta E_k^*\delta E_l\rangle$$
$$=\frac{(1-r)^2}{2}S_0^2\delta_{\alpha\beta}+O(\kappa^{(3)}|\mathbf{E}|^2)O(\delta\mathbf{E}^4) \ , \quad (17)$$

where $O_{\alpha\beta\gamma\delta}$ is a nonlinear correction of the order of $w^{\mu\eta}S_\eta$. Finally we get

$$\langle s_\alpha s_\beta\rangle=S_\alpha S_\beta+\tfrac{1}{2}S_0^2(1-r^2)\delta_{\alpha\beta}+O_{\alpha\beta\gamma\delta}S_\gamma S_\delta \ ,$$
$$\alpha,\beta=1,2,3 \quad (18)$$

and similarly

$$\langle s_0^2\rangle=S_0^2+\tfrac{1}{2}S_0^2(1-r^2)+O_{00\gamma\delta}S_\gamma S_\delta \ ,$$
$$\langle s_0 s_\alpha\rangle=S_0S_\alpha(2-r)+O_{0\alpha\gamma\delta}S_\gamma S_\delta \ . \quad (19)$$

Time averaging of (4) and (5) and substitution of the biquadratic terms (18) and (19) into the averaged formula allows us finally to derive the set of equations for the evolution of the Stokes vector of partially polarized light in a nonlinear medium.

$$\frac{dS_\alpha}{d\zeta} = -\mathrm{Im}\{\Omega_0 S_\alpha + \Omega_\alpha S_0\} - \mathrm{Re}\{e_{\alpha\beta\gamma}\Omega_\beta S_\gamma\} - \mathrm{Im}\{\tfrac{1}{4}(w^{0\alpha}+w^{\alpha 0})S_0^2(1-r^2) + \tfrac{1}{2}(w^{\alpha\beta}S_\beta + w^{00}S_\alpha)S_0(1-r)\}$$
$$-\mathrm{Re}\{\tfrac{1}{2}e_{\alpha\beta\gamma}w^{\beta 0}S_\gamma S_0(1-r) + \tfrac{1}{4}e_{\alpha\beta\gamma}w^{\beta\gamma}S_0^2(1-r^2)\} + \cdots ,$$
$$\frac{dS_0}{d\zeta} = -\mathrm{Im}(\Omega_0 S_0 + \Omega_\alpha S_\alpha) - \mathrm{Im}\{\tfrac{1}{4}(w^{00}+w^{\alpha\alpha})S_0^2(1-r^2) + \tfrac{1}{2}(w^{\alpha 0}+w^{0\alpha})S_\alpha S_0(1-r)\} + \ldots, \quad \alpha,\beta,\gamma = 1,2,3 \tag{20}$$

Here $\Omega$ is now a function of the Stokes vector $\Omega_\alpha = v^\alpha + \frac{1}{2}w^{\alpha\beta}S_\beta$, and not of the s vector as in (4) and (5). Clearly, for totally polarized light ($r=1$) Eqs. (20) returns to (4) and (5).

The $O_{\alpha\beta\gamma\delta}$ terms in (18) and (19) which take into account the change of the light statistics and are proportional to the medium nonlinearity parameter $w^{\alpha\beta}$ and contribute to Eq. (20) only as terms of the order of $w^{\alpha\beta}w^{\gamma\delta}\ S_\beta S_\gamma S_\delta$ (i.e., of the order of $|\chi^{(3)}|^2$ and higher) and are not presented here. Since we are interested in the third-order nonlinear propagation phenomena only, they may and will be neglected in the following consideration. This, of course, imposes some limitation on the use of Eq. (10), which is valid if $w^{\alpha\beta}S_\beta \ll 1$ (i.e., $\chi^{(3)}E^2 \ll 1$). This is, however, a very good approximation for most nonlinear-optical problems. Consequently, any change of the initial Gaussian statistics in the single-wave propagation problem does not effect the averaged propagation equations (20) for the Stokes parameters within the scope of the third-order phenomena and affects only the light field statistic moments of higher order which are not essential for the calculation of the Stokes parameters. However, we admit that the light statistic alteration may have implications for the evolution of the degree of polarization if higher-order nonlinearities are taken into account (e.g., $\chi^{(5)}$). These problems, however, lie outside the scope of the present paper.

The evolution of the degree of polarization itself is controlled by the following equation, which may be deduced from (20):

$$\frac{1}{2}\frac{d(r^2)}{d\zeta} = -\mathrm{Im}\left\{\frac{\Omega_\alpha S_\alpha}{S_0}\right\}(1-r^2) - \mathrm{Im}\left[\frac{1}{4}(w^{0\alpha}+w^{\alpha 0})S_\alpha - \frac{1}{4}w^{00}S_0\right](1-r)^2(1+2r)$$
$$-\mathrm{Im}\left\{\frac{1}{2}w^{\alpha\beta}\frac{S_\alpha S_\beta}{S_0}(1-r) + \frac{1}{4}w^{\alpha\alpha}S_0 r^2(1-r^2)\right\} - \mathrm{Re}\left[\frac{1}{4}e_{\alpha\beta\gamma}w^{\beta\gamma}S_\alpha\right](1-r^2), \quad \alpha,\beta,\gamma = 1,2,3 . \tag{21}$$

Equation (21) allows us to address the problem of the degree of polarization evolution in a very direct and convenient way. For example, in linear optics ($w^{\alpha\beta}=0$) the universal equation for the polarization degree appears to be quite simple

$$\frac{1}{2}\frac{d(r^2)}{d\zeta} = -\mathrm{Im}\left\{\frac{v^\alpha S_\alpha}{S_0}\right\}(1-r^2) . \tag{22}$$

Since the material equation (1) and the consideration above do not imply any reservations concerning the internal symmetry of the material tensors, Eqs. (20)–(22) may be used to describe not only conventional optical systems but also media with broken time reversibility. Time reversibility of optical response is broken in magnetic materials [12], presumed to be broken in some high-$T_c$ superconductors [13], in noncentrosymmetric crystals where spin-orbit interaction of optical electrons cannot be ignored [14] and also in a nonequilibrium state when the crystal is subjected to a transient excitation [17]. In terms of linear optical susceptibilities, broken time reversibility lifts the widely accepted restriction that the tensor $\chi_{ij}^{(1)}$ should be symmetric and $\gamma_{ijk}^{(1)}$ should be antisymmetric with respect to permutation of their first two indices [16,15]. The superscripts $s$ and $a$ below refer to the symmetric and antisymmetric parts of the tensors correspondingly, i.e., in the general case $\chi_{ij}^{(1)} = \chi_{ij}^{(1s)} + \chi_{ij}^{(1a)}$ and $\gamma_{ijk}^{(1)} = \gamma_{ijk}^{(1s)} + \gamma_{ijk}^{(1a)}$, while $\chi_{ij}^{(1a)}$ and $\gamma_{ijk}^{(1s)}$ are presumed to appear only in a time nonreversible state.

The following results concerning propagation of partially polarized light may be immediately derived by analysis of the right-hand side of (21) and (22).

*(a) Linear and nonlinear optics.* Totally polarized, single-mode, coherent light ($r=1$) in linear and nonlinear optics remains totally polarized in any circumstances, since the right-hand side of (21) becomes zero. Consequently, light self-action cannot lead to its depolarization.

*(b) Linear optics ($w^{\alpha\beta}\equiv 0$, i.e., $\chi_{ijkl}^{(3)}\equiv 0$ and $\gamma_{ijklm}^{(3)}\equiv 0$).* In spectral areas close to absorption resonances, components of optical susceptibility tensors have complex values. A change in the degree of polarization may appear due to either $\mathrm{Im}\chi_{ij}^{(1s)}$ or $\mathrm{Im}\gamma_{ijk}^{(1a)}$. This is a well-known fact and such a change of the degree of polarization is the result of predominant absorption of one of the eigenmodes. For example, anisotropy of $\mathrm{Im}\chi_{ij}^{(1s)}$ ($\mathrm{Im}\chi_{xx}^{(1)} \neq \mathrm{Im}\chi_{yy}^{(1)}$) is widely used in polaroid films. However, time nonreversible $\mathrm{Im}\chi_{ij}^{(1a)}$ and $\mathrm{Im}\gamma_{ijk}^{(1s)}$ do not affect the degree of polarization. On the contrary $\mathrm{Re}\chi_{ij}^{(1a)}$ and $\mathrm{Re}\gamma_{ijk}^{(1s)}$ may cause anisotropic absorption and affect the degree of polarization. In media of some particular crys-

tal symmetries, time nonreversible contributions to the optical response may be the only reason for variation of $r$. This may be found for example in the $\bar{4}3m$ crystal class, where time reversible absorption is completely isotropic ($\chi_{ij}^{(1s)}=\chi\delta_{ij}$ and $\gamma_{ijk}^{(1a)}=0$) and no alteration of the degree of polarization degree appears due to it. However, $\mathrm{Re}\gamma_{xyz}^{(1s)}$ may lead to change of the degree of polarization. A similar situation, i.e., when the time nonreversible response is the only reason for alteration of the degree of polarization, may be found for light propagating along the optical axis, for example, in crystals of the 3, $\bar{3}$, 4, $\bar{4}$, $4/m$, $4mm$, and $\bar{4}2m$ point groups [18]. If all susceptibilities are real, the degree of polarization may change only due to $\mathrm{Re}\chi_{ij}^{(1a)}$ or $\mathrm{Re}\gamma_{ijk}^{(1s)}$, i.e., only due to the time nonreversible terms. However, the appearance of $\mathrm{Re}\chi_{ij}^{(1a)}$ or $\mathrm{Re}\gamma_{ijk}^{(1s)}$ in a spectral region far from absorption resonances is hardly realistic. Finally, in a dissipation-free medium ($\mathrm{Im}\chi_{ij}^{(1s)}\equiv 0$), $\mathrm{Im}\gamma_{ijk}^{(1a)}\equiv 0$, $\mathrm{Re}\chi_{ij}^{(1a)}\equiv 0$, and $\mathrm{Re}\gamma_{ijk}^{(1s)}\equiv 0$) the initial degree of polarization does not change.

*(c) Nonlinear optics.* An intensity-dependent change of the degree of polarization may be expected in a dissipative system with anisotropic nonlinear absorption. However, a less trivial result is that in an absorptionless system, with isotropic nonlinear response, i.e., where both $\chi_{ijkl}^{(3)}$ and $\gamma_{ijklm}^{(3)}$ have the same internal symmetry as for an isotropic medium, no intensity-dependent change of $r$ may be expected. This is true even if the linear response is not isotropic, i.e., the system exhibits birefringence. The model of isotropic nonlinearity is widely accepted in fiber optics. Here we specifically note that polarization symmetry breaking, i.e., loss of stability of a "fast" polarization eigenmode in a weakly birefringent fiber does not affect the degree of polarization of a mixture of a coherent and randomly polarized components. Moreover, as might be seen from Eq. (21), the degree of polarization does not change in any dissipation-free time reversible media where $e_{\alpha\beta\gamma}w^{\beta\gamma}S_{\alpha}=0$. Considering the straightforward result (a)–(c) as a whole, one can intuitively develop an assumption that in any dissipation-free medium, no change of polarization degree may be expected for any symmetry of linear or nonlinear response.

In conclusion, we have derived the analytical evolution equation for the degree of polarization (21) of partially polarized light consisting of a coherent and a random component with Gaussian statistics. We also presented the evolution equations (20) for all the Stokes parameters in a nonlinear medium with nonlinearities up to the third order. We believe that these equations may find wide use in modern nonlinear optics and spectroscopy. Of course observations (a)–(c) are only the tip of the iceberg which lies beneath this equation and are due to be investigated.

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