

Reply to “Analysis of the lifetimes and fractions of antiprotons trapped in metastable states of antiprotonic helium”

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Shimamura and Kimura have submitted the preceding Comment [Phys. Rev. A **50**, 5346 (1994)], (to be referred to as SK), which criticizes our recent paper [T. Yamazaki and K. Ohtsuki, Phys. Rev. A **45**, 7782 (1992)] (to be referred to as YO). In this Reply the authors clarify the purpose of YO and complement its essential points, thus showing that the criticisms of SK are inappropriate.

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The paper by Yamazaki and Ohtsuki (YO) [1] was aimed at discussing some aspects related to the metastability of hadronic helium atoms which had been discovered when negative kaons [2], negative pions [3], and antiprotons [4] were stopped in liquid helium. The delayed fraction, time spectrum shape, and lifetimes were the observables. Further experimental studies are in progress [5], and as of today there is no successful explanation for these interesting phenomena. So, YO tried to give brief and rather qualitative estimates for the observation in an intuitive way, considering only the leading terms. The following problems are discussed in as simple a manner as possible, starting from the exotic-atom viewpoints of Condo [6] and Russell [7]: (i) the atomic core-polarization effect, (ii) the structure and radiative lifetimes, (iii) the nonstatistical distribution of the angular momentum and an estimate of the delayed fraction, and (iv) the isotope effect, though the title of [1] refers only to (i).

To respond to the Comment of Shimamura and Kimura (SK) [15], it is important to consider the correspondence between the atomic approach and the molecular approach for the metastable antiprotonic helium atom of Condo-Russell. We therefore begin this Reply with a discussion of this aspect.

I. CORRESPONDENCE BETWEEN THE ATOMIC APPROACH AND THE MOLECULAR APPROACH

The usual two-body exotic atom consisting of a nucleus and an exotic particle X^- of reduced mass M without any electron contribution is described by the one-dimensional Schrödinger equation with the potential

$$U(r) = -Z/r + l(l+1)/2Mr^2. \quad (1)$$

$U(r)$ consists of an attractive long-range Coulomb potential and a repulsive short-range centrifugal potential. The solution of this equation gives the well-known Bohr energies, which are

$$E(n, l) = -\frac{Z^2}{n^2} \frac{M}{m_e} \text{Ry}. \quad (2)$$

The multiplet ($l=0, 1, \dots, n-1$) for each n is degenerate in energy.

The potential $U(r)$ resembles an interatomic potential, when the exotic particle has a large angular momentum (hence, large n); the system thus looks like a molecule, where the rotational (J) and vibrational (v) quantum numbers could be uniquely assigned as $J=l$, $v=n-l-1$, in the same way as Shimamura [8] showed in the case of the three-body system $X^- \text{He}^+$. Namely, a circular orbit is a rotating state with a nodeless radial wave function, which corresponds to a vibrational quantum number $v=0$, a next-circular state ($l=n-2$) with a single node, corresponding to $v=1$, and so on.

The Bohr orbits do not, however, possess the usual properties of rotation and vibration; the “vibrational motion” here is far from harmonicity and the “rotational” energy is not proportional to $J(J+1)$ at all (the “moment of inertia” is J dependent). Here, the rotation-vibration coupling is maximal ($\hbar\omega \sim J\hbar^2/\mathcal{I}_{\text{rot}}$) and the moment of inertia keeps increasing with J (large centrifugal stretching). This peculiar “rotation-vibration mode” reflects the fact that the system is nothing but a Coulomb atom. In spite of this, the assignment of rotational-vibrational quantum numbers to a two-body exotic atom is a quite sufficient and useful description, although, of course, it is not a necessary one.

Now, when an electron is coupled to large- l circular (or near-circular) orbitals, the electron motion can be treated adiabatically and the molecular approach (thus the assignment of rotation-vibration quantum numbers) does play an important role, as fully discussed by Shimamura [8]. In the adiabatic approximation the potential is now given by

$$V(R) = U(R) + \epsilon_\gamma(R), \quad (3)$$

where $\epsilon_\gamma(R)$ arises from the electron involved. Shimamura [8] and (previously) Ahlrichs *et al.* [9] as well as (more recently) Greenland and Thürwächter [10] solved this equation. Certainly, this molecular approach is more elegant and powerful than any other procedure starting from the atomic configurations which ought inevitably to take into account various configuration mixing effects among them.

Here, we make some remarks concerning the molecu-

lar approach. Although the potential $V(R)$ looks like an interatomic potential in shape, it is essentially different from the case of normal molecules. In the present exotic helium case, the long-range attractive part comes from the Coulomb interaction and the short-range repulsion is from the centrifugal barrier, while in real molecules the long-range part comes from the molecular interatomic binding force mediated by the electron(s) and the short-range component is from the internuclear repulsion. In the former, although both the centrifugal and Coulomb interactions together form a potential minimum where bound states are accommodated, the electron energy $\epsilon_\gamma(R)$ is a slowly varying function which plays no essential role in binding the two objects, but only modifies the binding energies so as to remove the degeneracy of different- l states for each n . Physically, the binding is essentially of "atomic origin," and as shown below, most of the results of the molecular approach, as derived by Shimamura, already exist in a two-body system without an electron. The theoretical fact that the large- l orbitals behave like bound states in a "rovibrational" potential has not been recognized experimentally, but the present metastable exotic helium atom is an exceptional case.

(i) The propensity rule ($\Delta v=0$ dominance) that Shimamura found already exists in a two-body atom, and was described by YO as the selection rule $\Delta n = \Delta l = 1$, which conserves the number of radial node. To show this explicitly, YO presented Eq. (YO-5) and (YO-6).

(ii) Similarity of the wave functions. The essential character of the wave functions (radial dependence, node, etc.) comes from the two-body atom. The rovibrational states are not characteristic of a "molecule," but already exist in two-body atoms having large angular momenta.

(iii) The level structure is essentially of the atomic type, though the degeneracy is removed by the e^-X^- interaction. It is very different from that of normal molecules.

It is thus very interesting to consider the correspondence between the atomic approach and the molecular approach. Although the former is physically transparent, it involves a large number of configuration mixings and is mathematically troublesome. In order to account for Auger processes, however, it is a useful procedure. The molecular approach is mathematically elegant, and gives a convenient description of the energy levels and wave functions, though the structure is essentially of atomic origin. YO attempted some possible discussions of the phenomena in a rather qualitative and intuitive way based on the atomic picture. The essential point lies in the correlation between \bar{p} and e^- , which we call atomic core polarization. The results starting from the atomic approach are (and must be) consistent with the molecular approach, and thus YO are connecting the two views. The rigorous arguments on the validity of configuration mixing and detailed quantitative treatments, however, were outside the scope of YO. They will be described in a forthcoming paper [11].

II. ATOMIC CORE POLARIZATION

The term "atomic core polarization" comes from nuclear core polarization, which is a generic concept in nu-

clear physics, symbolizing various phenomena in which low-energy transitions (or static moments) of multipolarity $E\lambda$ (or $M\lambda$) are either enhanced or suppressed due to the coherent coupling with core excitation of the same multipolarity, even if the core excitation energy is high. Here, the core in its physical essence is not a "spatially inner" region, but, rather, an "energetically deeper" region (in a nuclear system all of the nucleons are confined within the same nuclear boundary, where the core implies deeper bound states, not spatially inner). In this sense, the simplest atomic core polarization is seen in the present antiprotonic helium atom, where the $E1$ transition energies (≈ 2 eV) are by an order of magnitude smaller compared with the electronic excitation energy (≈ 18 eV). Here, the $1s$ electron constitutes the simplest core with respect to \bar{p} . For the concept of core polarization, YO referred to Arima and Horie [12] in the case of the magnetic moments, and to Leon and Seki [13] in the case of exotic atoms.

YO pointed out that there exists an effect that is similar to nuclear core polarization. The important and essential point is that the residual interaction causes a special type of mixture of configurations which gives a coherent contribution to the transition rates to the first order of the mixing amplitude. For electric dipole transitions the mixing of wave functions of $\Delta L = 1$, as defined by (YO-8) and (YO-9), is most important. This $1s$ - np mixing is caused by the dipole-dipole part of the Coulomb interaction $V(\mathbf{r}_e, \mathbf{R}_p)$. What YO discussed is the effect of $1s$ - np mixing caused by the dipole-dipole correlation which was not taken into account in Russell's calculation. Thus we taken up only the correlation part using the basis wave functions which have readily taken into account the main part of the Coulomb interaction. This procedure assures that we can see only the core polarization effect on the dipole transitions, since the $1s$ - np mixing does not affect the transition energies very much. Note that the term "perturbation" in YO applies only to the dipole-dipole part of the interaction. In other words, the unperturbed wave functions include the screening term, but exclude $1s$ - np mixing.

SK seem to misunderstand this point; we never thought of using the $Z=2$ hydrogenic wave functions as the unperturbed wave functions. What YO paid attention to is the effect of $\Delta L = 1$ mixing of the wave function, which does not affect the energy intervals so much, but, rather, changes the $E1$ transition rates drastically. To see the essence the first-order perturbation is good enough and instructive.

Specifically, the mixing coefficients as defined in Eq. (YO-8) and (YO-9), are expressed by

$$c_L = - \left[\frac{L}{3(2L+1)} \right]^{1/2} \left\langle \frac{r_{<}}{r_{>}^2} \right\rangle \frac{1}{\Delta E_{2p-1s}} \quad (4)$$

and

$$d_{L-1} = \left[\frac{L}{3(2L-1)} \right]^{1/2} \left\langle \frac{r_{<}}{r_{>}^2} \right\rangle \frac{1}{\Delta E_{2p-1s}} \quad (5)$$

The typical numerical values, though not presented in the paper, are $c_{37} = -0.127$ and $d_{36} = 0.128$. The magnitudes of these mixings are of the order of 1%. Neverthe-

less, the mixing causes a dramatic change in the transition rate. The transition matrix element is generally

$$\begin{aligned} \langle f || \mathbf{r}^{(1)} || i \rangle &= \langle (I_e^f I_{\bar{p}}^f) L' || \mathbf{r}_e^{(1)} + \mathbf{r}_{\bar{p}}^{(1)} || (I_e^i I_{\bar{p}}^i) L \rangle \\ &= (-1)^{I_e^f + I_{\bar{p}}^i + L + 1} \begin{Bmatrix} I_e^f & 1 & I_e^i \\ L & I_{\bar{p}}^i & L' \end{Bmatrix} \sqrt{(2L+1)(2L'+1)} \langle I_e^f || \mathbf{r}_e^{(1)} || I_e^i \rangle \langle I_{\bar{p}}^i || I_{\bar{p}}^i \rangle \\ &\quad + (-1)^{I_e^f + I_{\bar{p}}^i + L' + 1} \begin{Bmatrix} I_{\bar{p}}^f & 1 & I_{\bar{p}}^i \\ L & I_e^i & L' \end{Bmatrix} \sqrt{(2L+1)(2L'+1)} \langle I_{\bar{p}}^f || \mathbf{r}_{\bar{p}}^{(1)} || I_{\bar{p}}^i \rangle \langle I_e^f || I_e^i \rangle, \end{aligned} \quad (6)$$

where

$$\langle l' || \mathbf{r}^{(1)} || l \rangle = (-1)^{l'} \sqrt{(2l'+1)(2l+1)} \begin{Bmatrix} l' & 1 & l \\ 0 & 0 & 0 \end{Bmatrix} \langle f'(r) | r | f(r) \rangle \delta(l' \pm 1, l). \quad (7)$$

The unperturbed amplitude for a typical transition $(n, l) \rightarrow (n-1, l-1)$ is

$$\langle (s \ L-1) L-1 || \mathbf{r}_{\bar{p}}^{(1)} || (sL) L \rangle = -\sqrt{L} \langle f_{L-1}(r) | r | f_L(r) \rangle_{\bar{p}} \quad (8)$$

and the first-order contribution is

$$\begin{aligned} &c_L \langle (s \ L-1) L-1 || \mathbf{r}_e^{(1)} || (p \ L-1) L \rangle + d_{L-1} \langle (pL) L-1 || \mathbf{r}_e^{(1)} || (sL) L \rangle \\ &= c_L \left[\frac{2L+1}{3} \right]^{1/2} \langle s || \mathbf{r}_e^{(1)} || p \rangle + d_{L-1} \left[\frac{2L-1}{3} \right]^{1/2} \langle p || \mathbf{r}_e^{(1)} || s \rangle \\ &= \left[-c_L \left[\frac{2L+1}{3} \right]^{1/2} + d_{L-1} \left[\frac{2L-1}{3} \right]^{1/2} \right] \langle f_{2p}(r) | r | f_{1s}(r) \rangle_e, \end{aligned} \quad (9)$$

which has a destructive contribution to the unperturbed amplitude.

The typical numerical value for the unperturbed amplitude is

$$\langle 37, 36 || E1 || 38, 37 \rangle_{\bar{p}} = -3.07, \quad (10)$$

while the first-order amplitude is

$$\left[-c_{37} \left[\frac{2L+1}{3} \right]^{1/2} + d_{36} \left[\frac{2L-1}{3} \right]^{1/2} \right] \langle f_{1s}(r) | r | f_{2p}(r) \rangle_e = +1.20. \quad (11)$$

[In Eq. (YO-10) there was an error in the sign of c_L , but the conclusion is unchanged.] The radial matrix element was evaluated by using the calculated basis functions. The second term gives rise to a substantial reduction in the total transition amplitude, yielding a suppression factor of $1 - \alpha \approx (1.87/3.07)^2 \approx 0.37$.

Of course, we do not have to use the perturbation method. We confirmed the results of the perturbation approach by comparing the diagonalization results on the energies, wave functions, and transition rates in the two cases, with and without $1s-np$ mixing. Note that the energy level intervals do not depend on the mixing, while the transition rates do change by a factor of 3.

The large reduction of the $E1$ transition strength which has been shown above from the viewpoint of the $\Delta L = 1$ configuration mixing is physically equivalent to the compensation of the \bar{p} dipole moment by that of the

e^- dipole moment in the language of the molecular approach. The two views are not mutually exclusive at all.

III. DISTRIBUTION OF THE ANGULAR MOMENTUM

The angular momentum to be brought in exotic-atom formation is given essentially by a classical estimate, namely, the momentum times the impact parameter, because the wavelength of the incoming X^- is much smaller than the collision size. The maximum angular momentum L at an incident center-of-mass (c.m.) energy E is

$$L(E) = a\sqrt{2ME}, \quad (12)$$

with a being the maximum impact parameter. Since the bound X^- is formed most likely at a radius which is around the electron radius, we can set $a = a_e$.

Exotic-atom formation takes place when the energy of particle X^- reaches somewhere below the ionization energy I_0 after the ionization loss process. The beam intensity $w(E)$ at the formation is expected to distribute continuously from 0 to E_{\max} and thus the angular momentum has an upper limit

$$l_{\max} = a\sqrt{2ME_{\max}}. \quad (13)$$

The maximum value E_{\max} may probably lie between the lowest excitation energy, i.e., $E_1 = 21$ eV, and the ionization energy $I_0 = 24.6$ eV, because above I_0 the ionization process dominates.

The initial population of l , $P(l)$, is a convolution of the

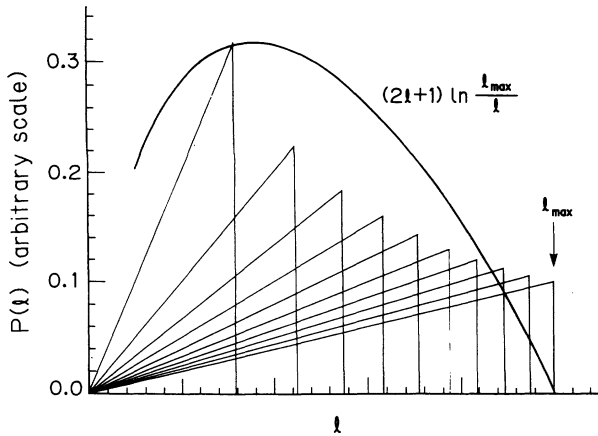


FIG. 1. A schematic presentation of a convolution effect of normalized statistical shapes with cutoff at $l=L(E)$ which corresponds to the \bar{p} energy at capture. The yield $\sigma(E)w(E)$ is assumed to be independent of E .

normalized statistical shape with a cutoff parameter $L(E)$ that depends on E (see Fig. 1), namely,

$$P(l) \propto \int_{E_1}^{E_{\max}} \frac{2l+1}{[L(E)]^2} \sigma(E)w(E) dE$$

$$\propto (2l+1) \int_{E_1}^{E_{\max}} \frac{1}{E} \sigma(E)w(E) dE, \quad (14)$$

with

$$E_1 = \left(\frac{l}{a} \right)^2 \frac{1}{2M}. \quad (15)$$

For a crude estimate of $P(l)$, let us assume that the formation rate $\sigma(E)w(E)$ is independent of E . Then, we obtain the simple formula

$$P(l) \propto (2l+1) \ln(l_{\max}/l). \quad (16)$$

As shown in Fig. 1, this gives a bell-shaped distribution. The purpose of deriving such a formula was to understand the phenomena in a semiempirical and semitheoretical parametrization. Since the energy distributes from 0 to E_{\max} , there is a certain limit of l , and the statistical distribution of l for unlimited l is inconceivable. No experimental information concerning the initial population of l is available from any earlier experiments on exotic atoms, since they are not directly related to the initial distribution of l . A nonstatistical distribution with a cutoff parameter, $P(l) = (2l+1)\exp(-\alpha l)$, is often introduced without any real justification.

Although the above bell-shaped distribution formula results from definite physical assumptions, the value of l_{\max} should be regarded as being a somewhat adjustable parameter in view of the crudeness of Eq. (12). Theoretically, a better-founded formula may certainly be possible and would, indeed, be welcome.

The l distribution has a scaling factor \sqrt{M} as do n_0 and l_{\max} , and thus the shape of $P(l)$ is a universal function of l/l_{\max} . The critical angular momentum L_0 which divides metastable and "prompt" states (namely, a state with $l > L_0$ is metastable) can be crudely estimated as fol-

lows: (i) The binding energy of the formed atomic levels of $\bar{p}\text{He}^+$ at $n=n_0$ (with respect to He^{2+}) is assumed to be that of He^0 , 79 eV. The critical angular momentum is given by setting the binding energy of circular orbitals of ionized $\bar{p}\text{He}^{2+}$,

$$\frac{4}{(l+1)^2} \frac{M}{m} \text{Ry}, \quad (17)$$

to 79 eV, which yields

$$\frac{L_0}{n_0} \approx 0.83. \quad (18)$$

Although the actual critical angular momentum may be somewhat larger than this crude estimate, what is important here is that L_0 is scaled by $M^{1/2}$ (or, equivalently, by n_0). This means that the delayed fraction is nearly independent of the mass of X^- . This is consistent with the observation that the delayed fractions for π^- , K^- , and \bar{p} are all around 3% [2-4].

SK's criticisms seem to be mostly based on misunderstanding and confusion. In the above derivation YO assume the energy dependence of the beam intensity (not the number density) and the capture cross section to be uniform. The criticism (1) of SK is incorrect, because we use $w(E)$ as the beam intensity spectrum, not the density spectrum, and so no velocity comes in explicitly. This is a normal procedure which experimentalists use [14]. We can say more generally that the same formula results from the assumption that the total yield of exotic atoms, namely, $\sigma(E)w(E)$, is independent of E below a certain limit (say, E_1 or I_0). There is no inconsistency in YO; Eq. (YO-14) is a formula for a maximum l when E is arbitrarily given, while the l_{\max} in the expression of $P(l)$, Eq. (YO-17), is that for a distributed E with a definite upper limit E_{\max} . Whether $w(E)$ extends to E_1 or I_0 gives a different l_{\max} , (48 or 52, respectively). The value of the parameter should, however, be taken as being somewhat arbitrary because of the too simple assumptions in Eq. (YO-14).

The above derivation of $P(l)$ does not depend on the population of n . Obviously, the formula is applicable when states of n larger than l are populated, as SK also point out. Empirically, there is evidence that the circular states of $l \sim n_0 - 1$ or more, are not highly populated; otherwise, a much longer-lived component than $\tau \sim 3 \mu\text{sec}$ would be revealed. This means that there is an empirical limit on l which nearly satisfies $l \leq n_0 - 1$. In conclusion, we learn empirically that l_{\max} is around n_0 . We need and wait for a rigorous theory for the individual population of both n and l .

IV. ISOTOPE EFFECT

YO considered the isotope effect on the overall lifetime of the $\bar{p}\text{He}^+$ atom on the following grounds. Let us define $R = M(\bar{p}^4\text{He})/M(\bar{p}^3\text{He}) = \frac{16}{15}$. In the atomic picture the energy spacing is approximately given by differentiating Eq. (YO-2) as

$$\Delta E \approx \frac{79}{n_0} \quad (19)$$

for the most probable $n = n_0 = (M/m)^{1/2}$. Here, the essential point is the inverse proportionality of ΔE to n_0 , which is defined separately for ${}^3\text{He}$ and ${}^4\text{He}$. We then obtain

$$\frac{\Delta E_{n_0}({}^3\text{He})}{\Delta E_{n_0}({}^4\text{He})} = R^{1/2} = 1.03. \quad (20)$$

A calculation using the molecular approach gives this ratio as 1.025, showing that the crude estimate is not bad. Although the $E1$ matrix element depends on the reduced mass, the unperturbed transition rates at the most likely n are the same for ${}^3\text{He}$ and ${}^4\text{He}$, as stressed in YO. In the first-order correction the transition rates for both ${}^3\text{He}$ and ${}^4\text{He}$ are reduced, but remain equal to each other because the reduction factor α is independent of M . YO therefore considered only the M dependence of $(\Delta E)^{-3}$, giving

$$\tau_{n_0}({}^4\text{He})/\tau_{n_0}({}^3\text{He}) = R^{3/2} = 1.09. \quad (21)$$

The molecular approach gives a value (1.09) equal to the above simple estimate. Do SK want to claim that this relation has a large error as much as 30%?

This ratio may be good enough to represent the ratio for the trapping time, but we made a slightly more accurate tuning. The overall lifetime should be the representative level lifetime multiplied by an average cascade number of metastable states at $n \sim n_0$. Since the metastable states are distributed over a given energy interval, the number of metastable cascades is expected to be proportional to the level density at $n \sim n_0$, namely, to $(\Delta E)^{-1}$.

Multiplying this factor, YO obtained

$$\tau \propto (\Delta E)^{-4} \propto M^2. \quad (22)$$

This procedure implicitly assumes that the number of the contributing metastable cascades is rather small. This view is consistent with the experimental fact that the overall trapping time is around $3 \mu\text{sec}$. The purpose of deriving such a crude estimate was to plan an experimental verification of the Condo-Russell model for the metastability. This crude estimate gave $\tau({}^4\text{He})/\tau({}^3\text{He}) = R^2 = 1.13$, a small but observable effect. Indeed, shortly after the derivation a dedicated experiment to observe the isotope effect was carried out [5]; the experimental value turned out to agree with the above crude estimate.

The present estimate would not be good if we were not allowed to neglect effects other than the radiative transitions. Better theories would take into account effects other than the radiative lifetimes.

SK seem to insist that when a theory has an uncertainty in explaining the absolute value of an experimental observable it cannot predict the isotope shift with better precision. This does not make sense; all of the isotope shifts, isomer shifts, hyperfine anomalies, etc., have very small, but nevertheless, meaningful effects.

In summary, the present authors stress that the purpose of YO was to give as simple estimates as possible for dominant effects consistently based on the atomic picture for the metastable antiprotonic helium atoms. The atomic approach is shown to be similar to the molecular approach. We believe that the two approaches are mutually consistent and helpful. We hope that the crude estimates given by YO may be improved by more comprehensive theoretical treatments of these interesting phenomena.

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