Dynamical quantum Zeno effect

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A purely dynamical explanation for the quantum Zeno effect is proposed. It is argued that a quantum system undergoes a quantum-Zeno-type dynamics as a consequence of a particular type of evolution involving a series of frequent spectral decompositions. The role of quantum measurements and of the "collapse of the wave function" is investigated and it is clarified that, provided a final observation is performed, the *dynamical* quantum Zeno effect can be obtained without making use of von Neumann's projection postulate. The meaning of *infinitely* frequent measurements is critically discussed and it is argued that it should be regarded as a mathematical idealization, impossible to realize from a physical point of view.

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I. INTRODUCTION

It is widely known that a quantum system, initially prepared in one of the eigenstates of the unperturbed Hamiltonian, undergoes a temporal evolution composed of three steps: a Gaussian-type decay for an early period, a Breit-Wigner exponential decay for intermediate times, and finally a power decay for longer times [1]. Khalfin [2] and Misra and Sudarshan [3] discovered that, if the short-time behavior is Gaussian and not exponential, it is possible to inhibit the decay of unstable quantum-mechanical systems by performing frequent "observations" in rapid succession. This phenomenon was named the quantum Zeno paradox, or quantum Zeno effect (QZE), after the Greek philosopher Zeno whose arrows, although darted, did not move. However, the experiment was very difficult to perform.

Recently, Cook [4] proposed an experimental test that makes use of a two-level atom whose transition is not of the exponential type but of the cosine type (i.e., Gaussian for a short period). It should be noted, however, that the proposed test did not involve observation of a naturally unstable quantum system, as in the original idea by Misra and Sudarshan. Following Cook's proposal, Itano *et al.* [5] carried out this experiment and claimed, by obtaining the same result as theoretically predicted by Cook, that the QZE had been proven experimentally. This conclusion has provoked an interesting debate [6–11] on whether this effect is really rooted in frequent observations, each of which is described by the *naive* wave-function collapse (i.e., von Neumann's projection rule), or rather in a purely dynamical process. In particular, Petrosky, Tasaki, and Prigogine [6] strongly claimed the latter point of view.

In this context, it is important to notice that in fact Itano *et al.* observed only one photon at the final step, but not at every step. Therefore, strictly speaking, they performed a different experiment from the original proposal put forward by Cook [4]. Thus we are led to an interesting question: Why did their experiment (in which one photon was observed only at the final step) yield the same result as the one theoretically derived under the assumption that the naive wave-function collapse (i.e., the simple projection) takes place many times? The main objective of the present paper is to answer this question.

In previous papers, first one [10] and then both of the present authors [12] proposed to use neutron spin-flip processes, instead of atomic transitions, in order to simplify and clarify the discussion. We drew conclusions essentially similar to those of Petrosky *et al.*

As the whole class of phenomena hinging upon the controversial issue of wave-function collapse [13, 14], the QZE is very interesting from the point of view of quantum measurements. In particular, it seems worth clarifying that the notion of "collapse," as given by von Neumann's projection rule, is not a fundamental prerequisite for the occurrence of a QZE. Therefore, in our opinion, the widespread belief that the QZE is clear-cut evidence in support of the wave function collapse, as given by von Neumann's projection rule, is a misunderstanding: Indeed, we shall endeavor to show that the quantum Zeno phenomenon is a pure dynamical process, always governed by strictly unitary evolutions.

II. THE FORMULATION OF MISRA AND SUDARSHAN

We shall first formulate the quantum Zeno effect by following the seminal procedure by Misra and Sudarshan [3], which is entirely based on von Neumann's projection rule. Let Q be an unstable quantum system whose states are vectors in the Hilbert space $\mathcal{H}$ and whose evolution is described by the unitary operator $U(t) = \exp(-iHt)$, where H is a semibounded Hamiltonian. The initial density matrix of system Q is assumed to be an undecayed state ρ_0 and E is the projection operator over the subspace of the undecayed states. By definition,

$$\rho_0 = E\rho_0 E, \qquad \mathrm{Tr}[\rho_0 E] = 1. \tag{2.1}$$

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Assume that we perform a measurement at time t, denoted by the projection operator E, in order to check whether Q is still undecayed. The measurement is idealized to be *instantaneous*. Accordingly, the state of Q changes into

$$\rho_0 \to \rho(t) = EU(t)\rho_0 U^\dagger(t)E, \tag{2.2}$$

so that the probability of finding the system undecayed is given by

$$p(t) = \mathrm{Tr}[U(t)\rho_0 U^\dagger(t)E]. \tag{2.3}$$

We shall refer to the process (2.2) as "naive wave-function collapse." We are now ready to review briefly Misra and Sudarshan's original formulation of what they named the "quantum Zeno paradox." We prepare Q in the initial state ρ_0 at time 0 (this is formally accomplished by performing an initial "preparatory" measurement of E) and perform a series of observations at times $T/N, 2T/N, \ldots, (N-1)T/N, T$. The state $\rho^{(N)}(T)$ of Q after the preparation and the above-mentioned N measurements reads

$$\begin{aligned} \rho^{(N)}(T) &= V_N(T)\rho_0 V_N^\dagger(T), \\ V_N(T) &\equiv [EU(T/N)E]^N \end{aligned} \tag{2.4}$$

and the probability to find the system undecayed is given by

$$P^{(N)}(T) = \mathrm{Tr}\left[V_N(T)\rho_0 V_N^\dagger(T)\right]. \tag{2.5}$$

Equations (2.4) and (2.5) display what will be referred to as the "quantum Zeno effect": Repeated observations in succession modify the dynamics of the quantum system, by slowing down the decay process, as we shall see in a particular example in Sec. III.

In the $N \to \infty$ limit (continuous observation), one defines

$$\mathcal{V}(T) \equiv \lim_{N\to\infty} V_N(T), \tag{2.6}$$

provided that the above limit exists in the strong sense. The final state is then

$$\tilde{\rho}(T) = \mathcal{V}(T)\rho_0\mathcal{V}^\dagger(T) \tag{2.7}$$

and the probability of finding the system undecayed is

$$\mathcal{P}(T) \equiv \lim_{N\to\infty} P^{(N)}(T) = \mathrm{Tr}\left[\mathcal{V}(T)\rho_0\mathcal{V}^\dagger(T)\right]. \tag{2.8}$$

Misra and Sudarshan assumed, on physical grounds, the continuity of $\mathcal{V}(t)$,

$$\lim_{t\to 0^+} \mathcal{V}(t) = E, \tag{2.9}$$

strongly and proved that under general conditions the operators $\mathcal{V}(T)$ (exist for all real T and) form a semigroup labeled by the time parameter T. Moreover, $\mathcal{V}^\dagger(T) = \mathcal{V}(-T)$, so that $\mathcal{V}^\dagger(T)\mathcal{V}(T) = E$. This implies by virtue of Eq. (2.1) that

$$\mathcal{P}(T) = \mathrm{Tr}\left[\rho_0 E\right] = 1. \tag{2.10}$$

If the particle is continuously observed (to check whether it decays or not), it is "frozen" in its initial state and will never be found to decay. This is the essence of the quantum Zeno paradox.

It is worth stressing the profound difference between Eqs. (2.5) and (2.8): To perform an experiment with N finite is only a practical problem, from the physical point of view. On the other hand, the $N \to \infty$ case is physically unattainable and is rather to be regarded as a mathematical limit (although a very interesting one). In this sense, we shall say that the quantum Zeno *effect*, with N finite, becomes a quantum Zeno *paradox* when $N \to \infty$.

Finally, we notice that if the Q system is allowed to follow its "free" evolution under the action of the Hamiltonian H, its final state at time T reads

$$\rho(T) = U(T)\rho_0 U^\dagger(T) \tag{2.11}$$

and the probability that the system is still undecayed at time T is

$$P(T) = \mathrm{Tr}[U(T)\rho_0 U^\dagger(T)E]. \tag{2.12}$$

We shall now endeavor to show that the consequences of the above theorem are liable to a pure *dynamical* explanation that *does not make use of projection operators*. In this sense, we believe that the quantum Zeno effect is just a consequence (although a peculiar one) of the Schrödinger equation. In the following we shall show that one needs only a particular dynamics in order to "freeze" the Q system in its initial state.

It is necessary to stress again that the observations (measurements) schematized via the operator E are *instantaneous*. This is a rather general characteristic of von Neumann–like descriptions of a measurement process: The Q system *instantaneously* makes the transition (2.2) by measurement (naive wave-function collapse). Even though such a picture is often accepted among physicists, it is *misleading*. Indeed a measurement process, as a physical process, takes place during a *very long time* on a *microscopic* scale, although we can regard it as if it happened *instantaneously* on a *macroscopic* scale.

III. QUANTUM ZENO EFFECT WITH NEUTRON SPIN

We set now the basis for a general analysis by discussing a particular solvable example [12, 15] which, in spite of its simplicity, yields rich physical insight and turns out to be very useful for the general analysis of Sec. IV. We shall show how the same "Zeno-type" evolution can be obtained both by making use of the technique of the preceding section and by means of a purely dynamical process. The following analysis is different from the one presented in Ref. [12], in an attempt to emphasize some points which are particularly important in the present context, and gives a guideline for the general treatment of Sec. IV.

The example we want to consider is a neutron spin in a magnetic field [12]. (A situation analogous to the one described in this section was outlined by Peres [16] with photons.) We shall consider two different experiments: Refer to Figs. 1(a) and 1(b). In the case schematized in Fig. 1(a), the neutron interacts with several identical regions in which there is a static magnetic field B, oriented along the x direction. We neglect any losses and assume that the interaction is given by the Hamiltonian

$$H = \mu B \sigma_1, \tag{3.1}$$

μ being the (modulus of the) neutron magnetic moment and σ_i $(i = 1,2,3)$ the Pauli matrices. We denote the spin states of the neutron along the z axis by $|\uparrow\rangle$ and $|\downarrow\rangle$: These can be identified with the undecayed and decayed states of Sec. II, respectively.

Let the initial neutron state be $\rho_0 = \rho_{\uparrow\uparrow} \equiv |\uparrow\rangle\langle\uparrow|$. The interaction with the first region simply provokes a rotation of the initial state around the x direction:

$$\begin{aligned}\rho_0 \to \rho(t) &= e^{-iHt/\hbar}\rho_{\uparrow\uparrow}e^{iHt/\hbar}\\ &= \cos^2\frac{\omega t}{2}\rho_{\uparrow\uparrow} + \sin^2\frac{\omega t}{2}\rho_{\downarrow\downarrow}\\ &\quad -i\cos\frac{\omega t}{2}\sin\frac{\omega t}{2}\rho_{\uparrow\downarrow} + \text{H.c.},\end{aligned} \tag{3.2}$$

where $\omega = 2\mu B/\hbar$, $t = \ell/v$ (ℓ is the length of the region where B is present and v the neutron speed) and the other notation is obvious. (In this section we do not set $\hbar = 1$.) We repeat the process N times, as shown in Fig. 1(a). The final density matrix at time $T = Nt$ is

$$\begin{aligned}\rho(T) &\equiv e^{-iHT/\hbar}\rho_0 e^{iHT/\hbar}\\ &= \cos^2\frac{\omega T}{2}\rho_{\uparrow\uparrow} + \sin^2\frac{\omega T}{2}\rho_{\downarrow\downarrow}\\ &\quad -i\cos\frac{\omega T}{2}\sin\frac{\omega T}{2}\rho_{\uparrow\downarrow} + \text{H.c.}\end{aligned} \tag{3.3}$$

We call this a "free" evolution, during which the system evolves under the sole influence of H. Note the presence of the off-diagonal terms with respect to the spin states. If T is chosen so as to satisfy the "matching" condition $\cos\omega T/2 = 0$ (notice that this can also be viewed as a fine-tuning requirement, similar to the one experimentally realized by Itano *et al.* [5]), we obtain

$$\rho(T) = \rho_{\downarrow\downarrow} \qquad \left(T = (2m+1)\frac{\pi}{\omega},\ m \in \mathbb{N}\right), \tag{3.4}$$

so that the probability that the neutron spin is down at time T is

$$P_\downarrow(T) = 1 \qquad \left(T = (2m+1)\frac{\pi}{\omega},\ m \in \mathbb{N}\right). \tag{3.5}$$

The above two equations correspond to Eqs. (2.11) and (2.12), for a specially chosen T: In our example, H is such that if the system is initially prepared in the up state, it will evolve to the down state after time T. Notice that, within our approximations, the experimental setup described in Fig. 1(a) is equivalent to a single region of length $L = N\ell$ with magnetic field B.

Let us now modify the experiment just described by inserting at every step a device able to select and detect

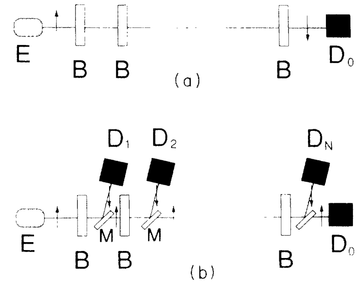

FIG. 1. (a) "Free" evolution of the neutron spin under the action of a magnetic field. An emitter E sends a spin-up neutron through several regions where a magnetic field B is present. The detector D_0 detects a spin-down neutron: No Zeno effect occurs. (b) The neutron spin is "monitored" at every step by selecting and detecting the spin-down component. D_0 detects a spin-up neutron: The Zeno effect takes place.

one component [say the down ($\downarrow$) one] of the neutron spin. This can be accomplished by a magnetic mirror M and a detector D. The former acts as a "decomposer" by splitting a neutron wave with indefinite spin (a superposed state of up and down spin) into two branch waves each of which is in a definite spin state (up *or* down) along the z axis. The down state is then forwarded to a detector, as shown in Fig. 1(b).

The magnetic mirror yields a spectral decomposition [17, 14] with respect to the spin states and can be compared to the inhomogeneous magnetic field in a typical Stern-Gerlach experiment. It is very important, in connection with the QZE, to bear in mind that the magnetic mirror does *not* destroy the coherence between the two branch waves: Indeed, the two branch waves corresponding to different spin states can be split coherently and brought back to interfere [18].

We choose the same initial state for Q as in the previous experiment [Fig. 1(a)]. The interaction with B in the first region still provokes the evolution (3.2). The spectral decomposition and the detection of a spin-down neutron, provoked by M and D, respectively, are (formally) globally represented by the operator $E \equiv \rho_{\uparrow\uparrow}$ [remember that we follow the evolution along the horizontal direction in Fig. 1(b), which corresponds to spin up], so that Eq. (2.2) yields

$$\rho_0 \to \rho(t) = EU(t)\rho_0 U^\dagger(t)E = \left(\cos^2\frac{\omega t}{2}\right)\rho_{\uparrow\uparrow}, \tag{3.6}$$

where $U = \exp(-iHt/\hbar)$. If the process is repeated N times, as in Fig. 1(b), we obtain

$$\begin{aligned}\rho^{(N)}(T) &= V_N(T)\rho_0 V_N^\dagger(T)\\ &= \left(\cos^2\frac{\omega t}{2}\right)^N \rho_{\uparrow\uparrow} = \left(\cos^2\frac{\pi}{2N}\right)^N \rho_{\uparrow\uparrow},\end{aligned} \tag{3.7}$$

where the "matching" condition for $T = Nt$ [see Eq. (3.4)] has been required again. The probability that the neutron spin is up at time T, if N observations have been made at time intervals t ($Nt = T$), is

$$P_{\uparrow}^{(N)}(T) = \left(\cos^2 \frac{\pi}{2N}\right)^N . \tag{3.8}$$

This discloses the occurrence of a QZE: Indeed, $P_{\uparrow}^{(N)}(T) > P_{\uparrow}^{(N-1)}(T)$ for $N \geq 2$, so that the evolution is "slowed down" as N increases. Moreover, in the limit of infinitely many observations

$$\rho^{(N)}(T) \stackrel{N\to\infty}{\longrightarrow} \widetilde{\rho}(T) = \rho_{\uparrow\uparrow} \tag{3.9}$$

and

$$\mathcal{P}_{\uparrow}(T) \equiv \lim_{N\to\infty} P_{\uparrow}^{(N)}(T) = 1. \tag{3.10}$$

Frequent observations freeze the neutron spin in its initial state by inhibiting ($N \geq 2$) and eventually hindering ($N \to \infty$) transitions to other states. Notice the difference with Eqs. (3.4) and (3.5): The situation is completely reversed.

The above result, peculiar as it may seem, is a straightforward consequence of the quantum formalism. It is worth stressing that [by setting for simplicity $m = 0$ in Eq. (3.4)] the condition $\omega T = \omega N t = \pi$, which is to be met at every step in Fig. 1(b), means that

$$B\ell = \frac{\pi \hbar v}{2\mu N} = O(N^{-1}), \tag{3.11}$$

where ℓ and v were defined after Eq. (3.2). This implies that, as N increases in the above equations, the practical realization of the experiment becomes increasingly difficult.

Notice that the final results Eqs. (3.9) and (3.10) are *automatically properly normalized*: The probability of detecting a spin-down neutron vanishes in the $N \to \infty$ limit. This is the essence and the peculiarity of the Zeno argument. We stress that in the above analysis we have neglected any losses and reflections at the mirrors.

We contend now that the *same* result can be obtained without making use of projection operators by simply performing a different analysis involving *only unitary processes*. Observe first that, so far, only the Q states have been taken into account. If the state of the total (neutron plus detectors) system is taken into account, we can write the initial state as

$$\Xi_I^{\rm tot} = (\xi_0 \otimes \rho_{\uparrow\uparrow}) \otimes \sigma_I^{D_0} \otimes \prod_{j=1}^{N} \sigma_I^{D_j}, \tag{3.12}$$

where $\xi_0 = |\phi_0\rangle\langle\phi_0|$, ϕ_0 being the neutron wave packet traveling along the horizontal direction in Fig. 1(b), and $\sigma_I^{D_\ell}$ is the initial density matrix of detector D_ℓ (displaying no result). A trivial, if lengthy, calculation yields the following final state:

$$\begin{aligned}\Xi_F^{\rm tot} &= \left(\cos^{2N} \frac{\pi}{2N}\right) (\xi_0 \otimes \rho_{\uparrow\uparrow}) \otimes \sigma_F^{D_0} \otimes \prod_{j=1}^{N} \sigma_I^{D_j} \\ &\quad + \sin^2 \frac{\pi}{2N} \sum_{k=1}^{N} \left(\cos^{2(k-1)} \frac{\pi}{2N}\right) (\xi_k \otimes \rho_{\downarrow\downarrow}) \\ &\quad \otimes \sigma_I^{D_0} \otimes \sigma_F^{D_k} \otimes \prod_{j\neq k} \sigma_I^{D_j},\end{aligned} \tag{3.13}$$

where $\xi_k = |\phi_k\rangle\langle\phi_k|$, ϕ_k being the neutron wave packet traveling towards the kth detector, and $\sigma_F^{D_\ell}$ is the final density matrix of detector D_ℓ (displaying neutron detection). Observe that in the above expression the total density matrix $\Xi_F^{\rm tot}$ has no off-diagonal components as a consequence of the wave-function collapse by measurement.

In conclusion, in the channel representation, we obtain the density matrix

$$\widetilde{\Xi}_{ij} \equiv \begin{pmatrix} c^{2N} & & & & \underline{0} \\ & s^2c^{2N-2} & & & \\ & & s^2c^{2N-4} & & \\ & & & \ddots & \\ \underline{0} & & & & s^2 \end{pmatrix}, \qquad i,j = 0,1,\ldots,N \tag{3.14}$$

where $c = \cos(\pi/2N)$ and $s = \sin(\pi/2N)$. This corresponds to the case of frequent observations, in which we confirmed, at every step, the neutron route among the various possibilities $0,1,\ldots,N$. Notice that the $i = j = 0$ component corresponds to detection by D_0, while the $i = j = n$ ($n = 1,\ldots,N$) component corresponds to detection in channel $N - n + 1$.

Assume now that $D_1,\ldots,D_N$ are removed in Fig. 1(b): In other words, we make *no* observation of the neutron route, except the final one performed by D_0. We start from the initial state

$$\psi_I^{\rm tot} = |\phi_0\rangle \otimes |\uparrow\rangle, \tag{3.15}$$

and a calculation analogous to the one sketched above yields the following final state:

$$\begin{aligned}\psi_F^{\rm tot} &= c^N|\phi_0\rangle \otimes |\uparrow\rangle \\ &\quad + \Big(-isc^{N-1}|\phi_N\rangle - isc^{N-2}|\phi_{N-1}\rangle \\ &\quad - \cdots - is|\phi_1\rangle\Big) \otimes |\downarrow\rangle.\end{aligned} \tag{3.16}$$

The correspondent density matrix is

$$\Xi_{ij} \equiv \begin{pmatrix} c^{2N} & isc^{2N-1} & isc^{2N-2} & \cdots & isc^{N} \\ -isc^{2N-1} & s^2c^{2N-2} & s^2c^{2N-3} & \cdots & s^2c^{N-1} \\ -isc^{2N-2} & s^2c^{2N-3} & s^2c^{2N-4} & \cdots & s^2c^{N-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -isc^{N} & s^2c^{N-1} & s^2c^{N-2} & \cdots & s^2 \end{pmatrix}, \qquad i,j = 0,1,\ldots,N. \tag{3.17}$$

Equations (3.14) and (3.17) clearly show that we have the *same* probability $P_{\uparrow}^{(N)} = [\cos^2(\pi/2N)]^N$ of detecting a spin-up neutron at D_0 *both* if the detectors $D_1, \ldots, D_N$ are present or absent in Fig. 1(b). It appears therefore that *no projection rule is necessary* in this context.

Physically, the situation just described corresponds to performing a coincidence experiment between the emitter E and D_0, as was explicitly shown in Ref. [12]. One can also consider this result as a consequence of a negative-result measurement [19] of the neutron spin. This shows that the final observation by D_0 plays a special role in the present analysis, introducing the stochastic element that is typical of quantum measurements. See Ref. [12].

Notice that the *diagonal elements* of the two density matrices (3.14) and (3.17) are the same. Moreover, in both cases, in the $N \to \infty$ limit, only the c^{2N} ($i = j = 0$) element survives: All other terms disappear because they contain at least an s factor. Remember that the $N \to \infty$ limit is only mathematical and is *impossible* to realize *physically* because the elapse of time T/N, even though it can be considered very short on a macroscopic scale, is in fact the time spent by the neutron in each B region. We have to keep in mind this remark throughout this paper. In fact, the uncertainty principle imposes a limit on the maximum value that N can attain in a given experimental situation. This is at present under investigation [15].

The main objective of this paper is to generalize this conclusion. We shall also see that the experiment performed by Itano *et al.* [5], in which a photon was observed *only* at the final step, will appear as a particular case of our analysis. Indeed, if one identifies the effect of the mirrors in Fig. 1(b) with the effect provoked by the laser pulses in Ref. [5], the above discussion inequivocally implies that the same result would be obtained whether or not Itano *et al.* observed the intermediate photons. Their experiment is just equivalent to a series of spectral decompositions performed by the laser pulses. This point will be discussed in detail in Sec. V.

IV. QUANTUM ZENO EFFECT AS A PURELY DYNAMICAL PROCESS

Our purpose is to give a general formulation of the QZE as a dynamical process, without making use of von Neumann's projection postulate. We shall see that when a system evolves under particular conditions and undergoes a "generalized spectral decomposition" (the precise meaning of which will be explained in a while), its initial state becomes frozen, in the sense explained in the preceding sections.

The evolution we are going to consider is peculiar, in that it involves the creation of several quantum correlations in rapid succession. We start (without loss of generality and in line with the formalism of the preceding sections) from a two-level quantum system Q, living in a Hilbert space $\mathcal{H}_Q$, and embed the latter in the larger space $\mathcal{H} \equiv \mathcal{H}_Q \otimes \mathcal{H}_U$, where the subscript U stands for Universe. Our "universe" can be anything: the spatial component of the total wave function of Q (as in Sec. III), a quantized electromagnetic field interacting with Q (as will be shown in Sec. V), some detectors surrounding Q, or any atmosphere in which Q exists. Two points are worth noticing. First, it is essential that the universe be treated quantum mechanically: If a "classical" behavior were postulated, we would conceptually go back to von Neumann's projection rule and to Misra and Sudarshan's seminal idea. Second, the only purpose for introducing the universe is to "follow" the quantum correlations engendered by Q, namely, to monitor their "spreading" towards other degrees of freedom of the total wave function ("leakage" and "environment" would maybe be better words than "spreading" and "universe," respectively, but we prefer the former expressions because, as we shall see, the quantum coherence can eventually be recovered).

We start from the initial state

$$|\Psi_I\rangle \equiv |\pm\rangle \bigotimes_{n=0}^{N} \bigotimes_{j=1}^{2^n} |0_j^{(n)}\rangle \equiv |\pm\rangle|0\rangle_N \ , \qquad (4.1)$$

where $|\pm\rangle$ denotes any state of the two-level system Q, $0_j^{(n)}$'s are occupation numbers, and $|0\rangle_N$ is the ground state of the universe. The state $+$ $(-)$ plays the same role as the state $\uparrow$ $(\downarrow)$ in Sec. III, and the undecayed (decayed) state in Sec. II. The structure of $\mathcal{H}_U$ can be understood by looking at Fig. 2: The universe consists of many channels (labeled j, n), which can be in either of the two possible states $0_j^{(n)}, 1_j^{(n)}$. The state $0_j^{(n)}$ $(1_j^{(n)})$ denotes the absence (presence) of an excitation in the corresponding channel. The index n labels the "step" (horizontal direction in Fig. 2), while the index j labels the "branch" (vertical direction) at step n. After n steps there are 2^n branches.

For the sake of simplicity, the following analysis is performed in terms of wave functions instead of density matrices.

A. "Free" evolution

We assume that the system undergoes two different types of evolutions at every step in Fig. 2. The first evolution is governed by the Hamiltonian H_0 and takes place in the rectangular regions. The second evolution is governed by the Hamiltonian H' and takes place in the circular regions.

The effect of the two evolutions is profoundly different. The evolution engendered by H_0 is identical at every step and will be written as

$$\begin{aligned} U_0(t)|+\rangle \otimes |\cdots\rangle &\equiv e^{-iH_0t}|+\rangle \otimes |\cdots\rangle \\ &= [\gamma_+(t)|+\rangle + \epsilon_-(t)|-\rangle] \otimes |\cdots\rangle, \\ U_0(t)|-\rangle \otimes |\cdots\rangle &\equiv e^{-iH_0t}|+\rangle \otimes |\cdots\rangle \\ &= [\epsilon_+(t)|+\rangle + \gamma_-(t)|-\rangle] \otimes |\cdots\rangle \ , \end{aligned} \qquad (4.2)$$

where the dots denote any state of the universe. Notice that this leaves the universe unaltered. We shall be interested in the behavior for small t, which is, in general,

$$\epsilon_\pm(t) = O(t), \quad \gamma_\pm(t) = 1 - O(t^2). \qquad (4.3)$$

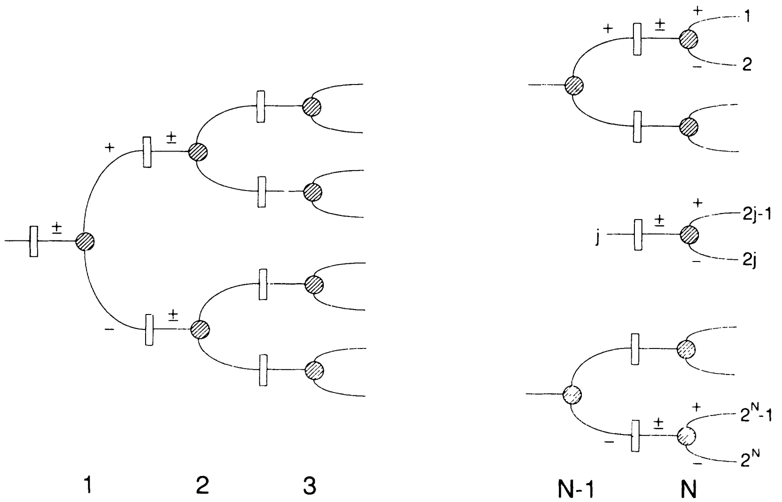

FIG. 2. The quantum correlation "tree." After N steps there are 2^N branches. The value of n ($n = 1, \ldots, N$) is indicated below.

The Hamiltonian H_0 plays the same role as H in Eq. (3.1). In some sense, we can consider the above evolution as "free," where "free" simply means that the behavior of the Q system is not "monitored" and Q is allowed to follow a natural smooth evolution under the action of H_0.

B. The generalized spectral decomposition

The second interaction, governed by H', is a sort of *spectral decomposition*: Different states of the Q system become *entangled* with different states of the universe. One can think, for example, of a sort of Stern-Gerlach decomposition of an initial spin state (so that every component of the spin becomes associated with a different wave packet, as in Sec. III) or of an entanglement of a two-level atomic system with a photonic state (see the analysis of the experiment by Itano *et al.* in Sec. V and the lucid discussion of Petrosky *et al.* on this point) or, more generally, of an entanglement of each state of Q with different degrees of freedom of the universe. In this sense, we shall speak of *generalized spectral decomposition* (GSD) of the Q states.

In order to describe this situation, refer to Fig. 3 and assume the following Hamiltonian:

$$H'(t) \equiv g(t) \left[|+\rangle\langle+|\sigma_\beta + |-\rangle\langle-|\sigma_\gamma \right] \sigma_\alpha$$
$$\equiv g(t) H' , \quad \int_0^\tau g(t) dt = b \in \mathbb{R} \tag{4.4}$$

where the interaction is switched on during the time interval $[0, \tau]$, g is a real function, $\sigma_\mu^\dagger = \sigma_\mu$ (the index $\mu = \alpha, \beta, \gamma$ labels the channel), and the effect of σ_μ is defined by

$$\sigma_\mu |0_\mu\rangle = |1_\mu\rangle, \quad \sigma_\mu |1_\mu\rangle = |0_\mu\rangle, \tag{4.5}$$

so that if there is a "particle" in channel μ the operator σ_μ destroys it, while if there is no particle, σ_μ creates one. The effect of σ_μ $(\forall \mu)$ is therefore identical to that of the first Pauli matrix. (We are implicitly assuming that there cannot be more than one particle in channel μ.) We set

$$[\sigma_\mu, \sigma_\nu] = 0 \qquad \forall\, \mu, \nu \, . \tag{4.6}$$

The action of the Hamiltonian H' is

$$H' \left(c_+ |+\rangle + c_- |-\rangle \right) \otimes |1_\alpha, 0_\beta, 0_\gamma\rangle$$
$$= \left(c_+ |+\rangle \otimes |0_\alpha, 1_\beta, 0_\gamma\rangle + c_- |-\rangle \otimes |0_\alpha, 0_\beta, 1_\gamma\rangle \right) . \tag{4.7}$$

H' acts therefore by sending the $+$ $(-)$ state of the Q particle in the upper (lower) channel in Fig. 3, performing in this way a GSD.

In general, the only effect of a GSD is to set up a correlation between the two states of Q and different states of the universe. Obviously, for the purpose of our analysis, we are interested in obtaining a perfect GSD (namely, a univocal and unambiguous correspondence between different states of Q and of the universe) of the type described in Eq. (4.7). As will be shown in Appendix A, this can be easily accomplished by setting $b = \pi/2$ in Eq. (4.4): This is a sort of fine tuning and corresponds to the so-called π-pulse condition, widely used for electromagnetic cavities [20]. This condition was experimentally realized in Ref. [5]. As explained in Appendix A, $b = \pi/2$ can also be viewed as the requirement that the GSD be "reflectionless." Notice also that all losses are neglected.

The evolution engendered by H' is explicitly calculated in Appendix A. The result is

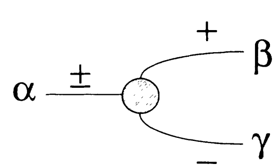

FIG. 3. The generalized spectral decomposition.

$$U'(t)\,(c_+|+\rangle + c_-|-\rangle)\otimes|1_\alpha,0_\beta,0_\gamma\rangle \equiv \exp\left(-i\int_0^\tau H'(t')dt'\right)(c_+|+\rangle + c_-|-\rangle)\otimes|1_\alpha,0_\beta,0_\gamma\rangle$$
$$= -i\,(c_+|+\rangle\otimes|0_\alpha,1_\beta,0_\gamma\rangle + c_-|-\rangle\otimes|0_\alpha,0_\beta,1_\gamma\rangle)\quad (t>\tau) \tag{4.8}$$

and yields a genuine GSD.

Our idea is to get a Zeno-type dynamics without making use of nonunitary evolutions (projection operators). In Sec. II, the operator E represented a measurement that was assumed to be *instantaneous.* As already emphasized, this is clearly an idealized situation that cannot correspond to a real physical process, taking place at a microscopic level. The problem is therefore to understand how we can simulate such an instantaneous and unphysical process in our analysis, which makes use *only* of unitary evolutions.

We observe that, in general, a GSD must take place in a very short time. Obviously, the term "very short time" must be understood at a macroscopic level of description because microscopically the time required to efficaciously perform a GSD is very long. Therefore, if we restrict our analysis to a macroscopic level of description, we can describe an (almost) instantaneous GSD by means of the so-called impulse approximation

$$\int_0^\tau g(t)dt = \pi/2, \qquad \tau\to 0+\ , \tag{4.9}$$

which roughly amounts to setting $g(t)\to(\pi/2)\delta(t)$ as $\tau\to 0$, where δ is the Dirac function $\int_0^\tau \delta(t)=1$. This is our alternative description of a von Neumann–like instantaneous projection.

C. Evolution at the nth step

We can now tackle the general case. Refer to Fig. 2 and assume the following Hamiltonian at the nth step:

$$H'_{(n)}(t)\equiv g(t)H'_{(n)}$$
$$\equiv g(t)\sum_{j=1}^{2^{n-1}}\{[|+\rangle\langle+|\sigma_{2j-1}^{(n)} + |-\rangle\langle-|\sigma_{2j}^{(n)}]\sigma_j^{(n-1)}\}$$
$$\equiv g(t)\sum_{j=1}^{2^{n-1}} H'_{(n),j}\ ,\quad \int_0^\tau g(t)dt = b\in\mathbb{R}, \tag{4.10}$$

where the interaction is switched on during the time interval $[0,\tau]$ and the action of $\sigma_i^{(n)}$ is defined by

$$\sigma_i^{(n)}|0_i^{(n)}\rangle = |1_i^{(n)}\rangle,\quad \sigma_i^{(n)}|1_i^{(n)}\rangle = |0_i^{(n)}\rangle, \tag{4.11}$$

so that if there is a particle in the ith channel at the nth step, the operator $\sigma_i^{(n)}$ destroys it, while if there is no particle, $\sigma_i^{(n)}$ creates one. We set

$$\left[\sigma_i^{(n)},\sigma_k^{(m)}\right]=0 \qquad \forall\, i,k,n,m\ . \tag{4.12}$$

The action of the Hamiltonian $H'_{(n),j}$ is given by

$$H'_{(n),j}\,(c_+|+\rangle + c_-|-\rangle)\otimes|\cdots,1_j^{(n-1)},\cdots\rangle$$
$$= \left(c_+|+\rangle\otimes|\cdots,1_{2j-1}^{(n)},\cdots\rangle + c_-|-\rangle\otimes|\cdots,1_{2j}^{(n)},\cdots\rangle\right), \tag{4.13}$$

where the dots will henceforth signify that all other occupation numbers are 0: $H'_{(n),j}$ acts therefore on the + (−) state of the Q particle in the jth channel at the $(n-1)$th step, by sending it into the $(2j-1)$th ($2j$th) channel at step n, performing in this way a GSD.

The evolution engendered by $H'_{(n)}(t)$ is explicitly calculated in Appendix B. If the condition $b=\pi/2$ is used again, one obtains

$$U'_{(n)}(t)\,(c_+|+\rangle + c_-|-\rangle)\otimes|\cdots,\otimes_{j=1}^{2^{n-1}}1_j^{(n-1)},\cdots\rangle$$
$$=\exp\left(-i\int_0^\tau H'_{(n)}(t')dt'\right)(c_+|+\rangle + c_-|-\rangle)\otimes|\cdots,\otimes_{j=1}^{2^{n-1}}1_j^{(n-1)},\cdots\rangle$$
$$=(-i)^{2^{n-1}}\left(c_+|+\rangle\otimes|\cdots,\otimes_{j=1}^{2^{n-1}}1_{2j-1}^{(n)},\cdots\rangle + c_-|-\rangle\otimes|\cdots,\otimes_{j=1}^{2^{n-1}}1_{2j}^{(n)},\cdots\rangle\right)\ . \tag{4.14}$$

Summarizing, at every step of the process described in Fig. 2, the system evolves according to

$$U_{(n)}(\tau,t)\equiv U'_{(n)}(\tau)U_0(t)\ , \tag{4.15}$$

where U_0 was defined in Eq. (4.2).

Our interest will be focused on the evolution engendered by the limiting operator

$$\lim_{N\to\infty} U_{\rm tot}^{[N]}(T)\equiv \lim_{N\to\infty}\left(\prod_{n=1}^N U_{(n)}(T_1/N,T_2/N)\right)U_I\ , \tag{4.16}$$

where $T=T_1+T_2$ and the operator U_I is introduced here in order to set up the initial incoming Q state at $t=0$. This is formally accomplished by setting

$$H_I \equiv \frac{\pi}{2}\delta(t)\,[|+\rangle\langle+| + |-\rangle\langle-|] \otimes \sigma_1^{(0)}$$
$$= \frac{\pi}{2}\delta(t)\,\mathbf{1} \otimes \sigma_1^{(0)} \ , \tag{4.17}$$
$$U_I(t) \equiv \exp\left(-i\int_0^\tau H_I(t')dt'\right) = -i\mathbf{1}\otimes\sigma_1^{(0)} \ , \quad t > \tau,$$

where δ is the Dirac function $[\int_0^\tau \delta(t) = 1]$ and we assumed for simplicity that the incoming state is set up at $t = 0$. Notice that $T = T_1 + T_2$, the total duration of the "experiment," is kept finite in taking the above limit.

D. The $N \to \infty$ limit

Let us study the action of the operator $U_{\rm tot}^{[N]}(T)$ on *any* initial state of the Q system. [The universe is initially taken to be in the ground state. See Eq. (4.1).] If, for instance, the initial Q state is $|+\rangle$, the final state is readily computed from Eqs. (4.2), (4.3), and (4.14)–(4.16) as

$$U_{\rm tot}^{[N]}(T)|+\rangle \otimes |0\rangle_N$$
$$= (-i)^{2^N}\left[\gamma_+(T_2/N)\right]^N |+\rangle \otimes |\cdots, 1_1^{(N)}, \cdots\rangle$$
$$+O(1/N). \tag{4.18}$$

On the other hand, if the initial state is $|-\rangle$, the final state reads

$$U_{\rm tot}^{[N]}(T)|-\rangle \otimes |0\rangle_N$$
$$= (-i)^{2^N}\left[\gamma_-(T_2/N)\right]^N |-\rangle \otimes |\cdots, 1_{2^N}^{(N)}\rangle$$
$$+O(1/N). \tag{4.19}$$

This clearly displays a QZE for N finite and a quantum Zeno paradox in the $N \to \infty$ limit. Indeed, observe that

$$\lim_{N\to\infty}\left[\gamma_\pm(T_2/N)\right]^N = \lim_{N\to\infty}\left[1 - O(1/N^2)\right]^N = 1 \ , \tag{4.20}$$

where we made use of Eq. (4.3). In other words, in the limit of infinitely many GSD's, both states of Q are frozen, while the universe evolves into the uppermost or lowermost state in Fig. 2, according to whether the initial Q state is $+$ or $-$, respectively. Moreover, the final result does not depend on T_1, the total time needed to perform GSD's. In particular, in the impulse approximation (4.9), we can get $T_1 = 0$ and the total duration of the experiment is just $T = T_2$. This is interesting and reflects, in our opinion, the essence of the "dynamical QZE": The final state does not depend on the GSD time, which can be made arbitrarily small in order to *mimic* the effect of an instantaneous projection E as in Sec. II. Notice, however, that it is *not* necessary to take the $T_1 \to 0$ limit, because we would obtain a dynamical QZE even if $T_1 = N\tau$ would be kept finite. On the other hand, if the total duration of the experiment $T = T_1 + T_2$ (with $T_1 = N\tau$) is kept finite, the $N \to \infty$ limit can be taken only in the impulse approximation $\tau \to 0$.

We stress again that the $\tau \to 0$ limit is unphysical and impossible to realize, in practice. Indeed, as already emphasized in Sec. III, even though τ can be considered very short in a macroscopic sense, it is in fact a very long time on a microscopic scale. In the case considered in Sec. III, for instance, τ is the time elapsed during the interaction between the neutron and a magnetic mirror M, which is of the order of $10^{-6} - 10^{-7}$ s.

But there is more to this: Experimentally, even the requirement that the total time spent in free evolutions T_2 be finite appears prohibitive. Indeed, such a total time should be divided into many small time intervals whose duration T_2/N vanishes as $N \to \infty$. This additional problem is manifest when one looks at the examples of Figs. 1(a) and 1(b): There is no conceptual problem related to the experiment in Fig. 1(a) because, within our approximation, the experimental arrangement is equivalent to a magnetic field B in a region of length $L = N\ell$ [see definitions after Eq. (3.2)]. On the other hand, in the experiment sketched in Fig. 1(b), each single region containing B must have a length $\ell = L/N$, which vanishes in the $N \to \infty$ limit. Consequently, also the time T_2/N spent by the neutron in each single region should vanish. This is clearly impossible to realize, in practice.

Observe that the final state, at time T, is *fully coherent*: The evolution is obviously *unitary* and *no "collapse" of the wave function* has taken place. Needless to say, this result holds true for any possible state of Q: Indeed, application of the superposition principle yields

$$U_{\rm tot}^{[N]}(T)\,(c_+|+\rangle + c_-|-\rangle) \otimes |0\rangle_N = c_+(-i)^{2^N}\left[\gamma_+(T_2/N)\right]^N |+\rangle \otimes |\cdots, 1_1^{(N)}, \cdots\rangle$$
$$+c_-(-i)^{2^N}\left[\gamma_-(T_2/N)\right]^N |+\rangle \otimes |\cdots, 1_{2^N}^{(N)}\rangle + O(1/N) \ , \tag{4.21}$$

which is still a pure state. At last, the underlying quantum coherence can be simply brought to light by "recombining the two beams" (the uppermost and lowermost states in Fig. 2), by means of the formal operator

$$H_F \equiv \frac{\pi}{2}\delta(t)\left[|+\rangle\langle+|\sigma_1^{(N)} + |-\rangle\langle-|\sigma_{2^N}^{(N)}\right]\sigma_1^{(0)}, \tag{4.22}$$
$$U_F \equiv \exp\left(-i\int_0^\tau H_F(t')dt'\right)$$
$$= -i\left[|+\rangle\langle+|\sigma_1^{(N)} + |-\rangle\langle-|\sigma_{2^N}^{(N)}\right]\sigma_1^{(0)},$$

where again $\int_0^\tau \delta(t) = 1$ [notice that, for the sake of simplicity, we are identifying the final and initial states (channels) of the universe]. By defining

$$U_{\rm tot}^{[N]\prime}(T) \equiv U_F U_{\rm tot}^{[N]}(T) \tag{4.23}$$

we obtain

$$U_{\rm tot}^{[N]\prime}(T)\,(c_+|+\rangle + c_-|-\rangle) \otimes |0\rangle_N$$
$$= (-i)^{2^N+1}\{c_+\left[\gamma_+(T_2/N)\right]^N |+\rangle$$
$$+c_-\left[\gamma_-(T_2/N)\right]^N |-\rangle\} \otimes |0\rangle_N + O(1/N). \tag{4.24}$$

In conclusion, by denoting $|0\rangle$ the ground state of the universe in the $N \to \infty$ limit, we get

$$\lim_{N\to\infty} U_{\mathrm{tot}}^{[N]\prime}(T)\,(c_+|+\rangle + c_-|-\rangle) \otimes |0\rangle_N \propto (c_+|+\rangle + c_-|-\rangle) \otimes |0\rangle, \quad (4.25)$$

up to a phase factor. This result is stronger than the one obtained in Ref. [3] and outlined in Sec. II: Indeed we have shown that it is possible, by making use of a dynamical process, to freeze *any* initial Q state and not only a suitably chosen initial Q state.

V. THE EXPERIMENT BY ITANO *et al.*

The recent experiment performed by Itano *et al.* [5] has provoked a renewed interest and a lively debate on the meaning of the QZE. The above-mentioned authors claimed to have observed experimentally the quantum Zeno effect by making use of atomic transitions, on the basis of Cook's proposal [4].

This conclusion was challenged by, among others, Petrosky, Tasaki, and Prigogine [6], who proved, via a detailed theoretical calculation, that the experimental results in Ref. [5] are liable to a dynamical explanation and therefore need not be ascribed to any collapse of the wave function. Itano *et al.* replied to the above criticisms [9], without anyway withdrawing their original conclusion.

Let us therefore briefly review this experiment and discuss its meaning and implications from the point of view outlined in this paper. Itano *et al.* put $^9Be^+$ ions in a rf cavity. The ion energy level configuration was such that $E_1 < E_2 < E_3$ and the resonating rf-field frequency $\omega_2 = (E_2 - E_1)/\hbar$ created a coherent superposition state of the two lower levels. Upon measurement, the ion can be found in level 1 *or* 2, but never in both levels at the same time.

We denote the probability of finding the atom in level 1 (2) at time t by $P_1(t)$ $[P_2(t)]$. If the initial condition $P_1(0) = 1$ is chosen, it is possible, by making use of a "π pulse" [20], to find a time T such that $P_1(T) = 0$. Notice that the π-pulse condition is essentially similar to that described in Sec. III [see, in particular, Eqs. (3.4) and (3.5)] and can be viewed as a fine-tuning condition, as explained in Sec. IV B.

In order to observe the state of the atom, Itano *et al.* irradiated it with very short optical pulses of frequency $\omega_3 = (E_3 - E_1)/\hbar$ and chose the level configuration in such a way that the spontaneous decay $3 \to 1$ was strongly favored, while the decay $3 \to 2$ was forbidden. In this way, the atom is known to be in the first level if a photon of frequency ω_3 is observed while it is in the second level if no photon is observed.

According to the quantum measurement theory, the wave-function collapse takes place as a consequence of observation and consequently the density matrix of the atom loses its off-diagonal components (with respect to the first and second atomic states). If N observations are performed during the time interval $(0, T)$, the probability of finding the atom in the first level is given by

$$P_1^{(N)}(T) = \cos^{2N} \frac{\pi}{2N}. \quad (5.1)$$

This displays the quantum Zeno effect, because $P_1^{(N)}(T) > P_1^{(N-1)}(T)$ for $N \geq 2$ and $\lim_{N\to\infty} P_1^{(N)}(T) = 1$. Itano *et al.* experimentally confirmed the above prediction [21] by sending N optical pulses and then claimed to have observed the quantum Zeno effect. A dynamical explanation, involving no collapse of the wave function, was suggested in Ref. [6] and is, in our opinion, very convincing.

By making use of the techniques used in Sec. IV, we propose the following purely dynamical explanation: Let the Q system be the atom, while the universe is the Fock space of the photons absorbed and then reemitted in the $1 \leftrightarrow 3$ transition. The initial state of the total system is

$$|\Psi_I\rangle \equiv |\phi_1\rangle \otimes |0\rangle, \quad (5.2)$$

where $|\phi_i\rangle$ represents the atomic level i $(i = 1, 2)$ and $|0\rangle$ is the ground state of the Fock space.

The free evolution yields simply the Rabi oscillations between the atomic levels 1 and 2 and is obviously in agreement with the general behavior (4.3):

$$|\Psi_I\rangle \to |\Psi_{(1)}\rangle = \left(\cos\frac{\Omega t}{2}|\phi_1\rangle + i\sin\frac{\Omega t}{2}|\phi_2\rangle\right) \otimes |0\rangle, \quad (5.3)$$

where Ω is the frequency of the Rabi oscillations between levels 1 and 2. Notice that we are not mentioning the atomic level 3.

The ω_3 pulse yields, in a very short time τ, the evolution

$$|\Psi_{(1)}\rangle \to |\Psi'_{(1)}\rangle = \cos\frac{\Omega t}{2}|\phi_1\rangle \otimes |1\rangle + i\sin\frac{\Omega t}{2}|\phi_2\rangle \otimes |0\rangle, \quad (5.4)$$

where $|1\rangle$ denotes a one-photon state. Equation (5.4) is a generalized spectral decomposition, in the sense explained in Sec. IV B (channels α and β coincide and correspond to the vacuum, while channel γ represents the one-photon state). The analysis can now proceed along the lines sketched in Sec. IV.

Observe also that, by repeating the reasoning outlined in Sec. III, the same result is obtained independently of whether a photon of frequency ω_3 is observed *only* at the final step, after the Nth optical pulse was irradiated (as in Ref. [5]), or after every pulse irradiation. This was discussed at length in Ref. [12].

VI. DISCUSSION

We have shown that the QZE is liable to a purely dynamical explanation, which does not involve any projection operator. We claim therefore, contrary to widespread belief, that a quantum Zeno-type dynamics is *not* an argument in support of the collapse of the wave function, provided we observe the same state as the initial one at the final detector D_0. The Schrödinger equation alone can yield a satisfactory explanation of the phenomenon.

Even though we do not question in the least the effectiveness and the practical validity of the projection postulate, we have critically discussed its physical meaning on several occasions [14]: We believe that a projection does not correspond to any *physical* operation and therefore should be regarded only as a convenient expedient (a "working rule") in order to account for the loss of quantum-mechanical coherence (the "collapse" of the wave function). In this sense, von Neumann's projection rule is to be considered as purely mathematical and no physical meaning should be ascribed to it. In our opinion, the projection technique is artificial and extraneous to quantum mechanics as a *physical* theory.

We stress, in this context, that an alternative explanation for the loss of quantum coherence has been proposed [14], in which the decoherence (collapse) is viewed as a *physical* dephasing process, ascribable to the interaction of the quantum system with a macroscopic object. Notice that, in this approach, the macroscopic system is always treated quantum mechanically and the unitarity of the evolutions is always kept: Dephasing is viewed as a statistical effect, even though it can be shown to take place (and can be given a definite meaning) also for single events.

It must be emphasized, however, that the description of the quantum Zeno effect given in this paper implicitly requires an observation at the final step [such as, for instance, a detection by D_0 in Figs. 1(a) and 1(b)] and this introduces a statistical (probabilistic) element in our analysis. In this sense, the probabilistic aspect of the quantum phenomenon is always present and retains its fundamental ontological role.

It would be possible to prove other conditions (i.e., in the strong vector topology) stronger than the limit (4.25), but we preferred to limit our analysis to a particular choice of the initial state of the universe because our only purpose was to disclose the occurrence of the dynamical QZE (4.25). It is worth stressing that our analysis has been performed under the assumption of lossless and reflectionless GSD's. In order to realize practically this type of experiment, we have to estimate the effects of such losses and reflections on the final results. It would also be interesting to understand whether these effect would yield additional phases in the transmitted states. Indeed, in such a case, interesting links with "decoherence" effects [14] might come to light, due to (partial) phase randomization.

ACKNOWLEDGMENTS

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APPENDIX A

In this appendix we shall derive Eq. (4.8) and discuss in some detail the role of the impulse approximation and of the fine-tuning condition. We start from the Hamiltonian (4.4),

$$\begin{aligned} H'(t) &= g(t)\,[|+\rangle\langle+|\sigma_\beta + |-\rangle\langle-|\sigma_\gamma]\,\sigma_\alpha \\ &= g(t)H', \end{aligned} \tag{A1}$$

where $\sigma_\mu = \sigma_\mu^\dagger$ acts on the space spanned by the vectors $|0_\mu\rangle, |1_\mu\rangle$ like the first Pauli matrix [see Eq. (4.5)]. The interaction is switched on during the time interval $[0,\tau]$ and we set

$$\int_0^\tau g(t)dt = b \in \mathbb{R}. \tag{A2}$$

Observe that

$$[H']^\dagger = H', \quad [H']^2 = \mathbf{1}\ , \tag{A3}$$

which makes the behavior of H' itself very similar to that of a Pauli matrix.

The evolution operator is given by

$$\begin{aligned} U'(t) &\equiv \exp\left(-i\int_0^\tau H'(t')dt'\right) = e^{-ibH'} \\ &= \cos b - iH'\sin b \quad (t>\tau) \end{aligned} \tag{A4}$$

where we made use of Eqs. (A2) and (A3). This operator acts on an initial state of the type $(c_+|+\rangle + c_-|-\rangle)\otimes|1_\alpha,0_\beta,0_\gamma\rangle$ and it is easily evinced from the above expression and Eq. (4.7) that $U'(t)$ engenders a genuine GSD if and only if $b=\pi/2$. If the above fine-tuning condition is not met, there is a nonvanishing amplitude for the wave function of the total system to undergo a sort of reflection, namely, there would be a nonvanishing amplitude for the universe to still be in the $|1_\alpha,0_\beta,0_\gamma\rangle$ state.

As explained in Sec. IV B, the only effect of a GSD is to set up a correlation between the two states of Q and different states of the universe. Obviously, we are interested in obtaining a perfect GSD (namely, a univocal and unambiguous correspondence between states of Q and the universe) of the type described in Eq. (4.7). Let us therefore set $b=\pi/2$. This condition of fine tuning is equivalent to the one experimentally realized in Ref. [5]. We get

$$U'(t) = -iH' \quad (t>\tau) \tag{A5}$$

and by making use of Eq. (4.7) we obtain Eq. (4.8).

Incidentally, observe that the role of time τ is completely absorbed in the condition (A2). Since, in general, a GSD takes place in a very short time, we are naturally led to the impulse approximation, which roughly amounts to setting $g(t)\to(\pi/2)\delta(t)$ as $\tau\to 0$. See Eq. (4.9).

APPENDIX B

In this appendix we shall derive Eq. (4.14). We start from the Hamiltonian (4.10), which describes the interaction at the nth step in Fig. 2,

$$H'_{(n)}(t) = g(t) \sum_{j=1}^{2^{n-1}} \left\{ \left[|+\rangle\langle+|\sigma_{2j-1}^{(n)} + |-\rangle \right.\right. \\ \left.\left. \times \langle-|\sigma_{2j}^{(n)} \right] \sigma_j^{(n-1)} \right\} = g(t) \sum_{j=1}^{2^{n-1}} H'_{(n),j} , \qquad (B1)$$

where $\sigma_i^{(n)}$ is the first Pauli matrix, the interaction is switched on during the time interval $[0, \tau]$, and

$$\int_0^\tau g(t)dt = b. \qquad (B2)$$

Observe that

$$\left[H'_{(n),j}\right]^\dagger = H'_{(n),j} , \quad \left[H'_{(n),j}\right]^2 = \mathbf{1} , \qquad (B3)$$

which makes the behavior of $H'_{(n),j}$ very similar to that of a Pauli matrix. Moreover Eq. (4.12) implies

$$\left[H'_{(n),j}, H'_{(n),j'}\right]^\dagger = 0 \quad \forall j, j' . \qquad (B4)$$

The evolution operator is given by

$$U'_{(n)}(t) \equiv \exp\left(-i \int_0^\tau H'_{(n)}(t')dt'\right) \\ = \exp\left(-ib \sum_{j=1}^{2^{n-1}} H'_{(n),j}\right) \\ = \prod_{j=1}^{2^{n-1}} e^{-ibH'_{(n),j}} \\ = \prod_{j=1}^{2^{n-1}} \left(\cos b - iH'_{(n),j} \sin b\right) \quad (t > \tau), \qquad (B5)$$

where we made use of Eqs. (B2) and (B4).

Once again, the above evolution engenders a genuine GSD if and only if $b = \pi/2$. If the above fine-tuning condition is not met, there is a nonvanishing amplitude for the wave function of the total system to undergo a sort of reflection and the universe will have a nonvanishing amplitude of still being at the $(n-1)$th step. Therefore we set $b = \pi/2$ and get

$$U'_{(n)}(t) = (-i)^{2^{n-1}} \prod_{j=1}^{2^{n-1}} H'_{(n),j} \quad (t > \tau), \qquad (B6)$$

which, by making use of Eq. (4.13), yields Eq. (4.14).

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[21] Notice that Itano *et al.* checked the somewhat different formula $P_1^{(N)}(T) = [1 + \cos^N(\pi/N)]/2$. The two expressions have a similar behavior for large N. See Refs. [4] and [5].

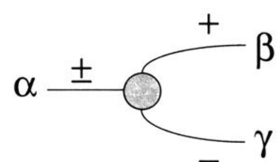

FIG. 3. The generalized spectral decomposition.