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One-dimensional coherent-state representation on a circle in phase space

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The representation of the states of a single electromagnetic field mode by coherent states of an identical mean number of photons, i.e., the representation on a circle in phase space, is presented. The connection between the circle representation and the analytic representation is given.

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I. INTRODUCTION

Recently, much attention has been devoted to the one-dimensional representation of various nonclassical states of a single mode electromagnetic field. A wide class of states can be expanded into the continuous superposition of coherent states lying on a one-dimensional manifold in the α plane. For example, the superposition of coherent states with a complex Gaussian amplitude function on a straight line produces a squeezed coherent state [1], and a similar Gaussian distribution on a circle leads to an amplitude squeezed state [2].

The coherent states have features as classical as possible. On the other hand, even the most simple discrete superpositions of coherent states can realize quantum fields with nonclassical properties due to the quantum interference [3–6]. The one-dimensional coherent representation of quantum states can be arbitrarily well approximated by discrete superpositions. The quantum interference emerging in a discrete superposition that approximates a given state can reveal the origin of characteristic nonclassical properties of the state. Amplitude squeezing and quadrature squeezing has been analyzed in this manner [3,4].

Recently, various experimental schemes have been proposed in quantum optics, where discrete superpositions of coherent states are generated. Nonlinear optical processes [6,7], quantum nondemolition, and back-action evading measurements [8,9] lead to discrete superposition of coherent states. Atomic interference methods seem to be especially promising ideas for generating required superposition states. Appropriately prepared Rydberg atoms sent through a microwave cavity can transform the initial coherent state into the superposition of coherent states having the same amplitude as the initial one [9], i.e., they are on a circle in the α plane. An analogous experiment can be envisioned in the optical domain [10]. The required superposition of coherent states on a circle can be generated by designing the parameters of the apparatus [11]. In order to determine the required discrete superposition approximating a given quantum state, we need its one-dimensional representation.

Previously, the one-dimensional representation forms of various nonclassical states have been found intuitively. In a recent paper [12], a complete orthonormal basis set on a straight line has been presented, which makes it possible to obtain the coherent-state expansion on a straight line for a given state. In this paper, we give the systematic construction of the circle representation of quantum states.

The paper is organized as follows: The circle representation is defined in Sec. II and we determine the subset of the Hilbert space of which elements can be represented by the given form. The question of unicity of the circle representation is also considered in this section.

The main result of our paper is to present the connection between the analytic and the circle representations of a state in Sec. III. Additionally, we obtain an equivalent condition for the subset of the Hilbert space of which elements can be represented on a circle. In order to describe more general states of the field by coherent states with identical amplitude, the circle representation of density operators is considered. We conclude by summarizing our main results in Sec. IV.

II. CIRCLE REPRESENTATION

It was shown that there is a one-to-one correspondence between the states of the quantum harmonic oscillator and the entire functions, i.e., the complex functions that are analytic in the whole complex plane. Every quantum state of the harmonic oscillator can be expressed as a superposition of coherent states in the form [13,14]

$$| f\rangle = \frac{1}{\pi}\int e^{-\frac{|\alpha|^2}{2}} f(\alpha^*)\,|\,\alpha\rangle d^2\alpha\ , \tag{1}$$

where the expansion function can be derived in an arbitrary representation

$$f(\alpha^*) = e^{\frac{|\alpha|^2}{2}}\langle\alpha\,|\,f\rangle\ , \tag{2}$$

which is a complex entire function. The coherent state representation defined by Eq. (1), including the expansion function in (2), is called the analytic representation.

The coherent states form an overcomplete set in the Hilbert space, hence, there is an infinite way of expanding an arbitrary state in terms of them. Being confined to some given class of expansion functions, a unique representation of the states can be constructed. For example, in the case of the analytic representation, the expansion function was required to be an entire complex function. Cahill's theorem [15] implies that the coherent states lying on a one-dimensional manifold in the α plane form a complete set in the Hilbert space. Thus we can consider expansion functions that differ from zero only on a one-dimensional manifold of the α plane. Such a manifold, being of interest, can be a circle around the origin with radius R, where the coherent states have an identical mean number of photons equal to R^2.

Let us consider the circle representation of an arbitrary state $| g\rangle$ in the form of a complex path integral along a circle with radius $| z |= R$ in the α plane

$$| g\rangle = \frac{e^{\frac{R^2}{2}}}{2\pi i} \oint_{|z|=R} g(z) \mid z\rangle dz , \tag{3}$$

where $g(z)$ is an analytic function in the vicinity of the circle in order to ensure that the integral can be evaluated. First we consider the question of unicity of the circle representation. The coherent states $| z\rangle$ lying on a circle ($| z |= R$) still form an overcomplete set, hence several analytic expansion functions $g(z)$ in Eq. (3) can produce the same state $| g\rangle$. A restrictive condition has to be imposed on the class of circle expansion functions in order to construct an unique representation.

Due to the analicity of $g(z)$ on the circle, it can be expanded into a Laurent series

$$g(z) = \sum_{k=1}^{\infty} \frac{g_{-k}}{z^k} + \sum_{k=0}^{\infty} g_k z^k . \tag{4}$$

We prove that setting the regular part of this Laurent series

$$g_{\text{reg}}(z) = \sum_{k=0}^{\infty} g_k z^k \tag{5}$$

into the circle integral in Eq. (3) produces the zero element of the Hilbert space

$$\oint g_{\text{reg}}(z) \mid z\rangle dz = 0 . \tag{6}$$

This can be proved by applying simple transformations as follows:

$$\begin{aligned}\oint g_{\text{reg}}(z) \mid z\rangle dz &= \oint \left(\sum_{k=0}^{\infty} g_k z^k\right) \left(e^{-\frac{R^2}{2}} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{n!}} \mid n\rangle\right) dz \\ &= \sum_n \sum_k e^{-\frac{R^2}{2}} \frac{g_k}{\sqrt{n!}} \mid n\rangle \oint z^{k+n} dz \\ &= 0 ,\end{aligned} \tag{7}$$

where we have made use of the well-known identity

$$\frac{1}{2\pi i} \oint z^m dz = \delta_{m,-1} , \tag{8}$$

and in our case $k+n$ cannot be -1, thus the circle integrals in Eq. (7) are zero.

As a consequence, an infinite number of weight functions that differ only in the regular part of their Laurent series produce the same state by the circle integral in Eq. (3). The addition of an arbitrary function $h(z)$ to a given weight function $g(z)$ leaves the integral in Eq. (3) invariant provided $h(z)$ is analytic inside the circle since in this case its Laurent series contains only the regular part.

Due to the uncertainty of the representation defined by Eq. (3), we have to specify a class of weight functions where there is only one function representing a given state. We show that unique representation can be constructed if the regular part of the Laurent series of the weight function is required to be zero. This restriction means that we cut out that part of the weight function that produces the zero element of the Hilbert space in accordance with Eq. (6). Only the main part of the Laurent series is retained. That is the weight function $g(z)$, which can be written in the form

$$g(z) = \sum_{k=1}^{\infty} \frac{g_k}{z^k} . \tag{9}$$

To prove the unicity of the representation in this class of weight functions it is enough to verify the unicity of the expansion of an arbitrary number state.

Let us suppose that the state $| n\rangle$ is represented on the circle by $g^{(n)}(z) = \sum_1^{\infty} \frac{g_k^{(n)}}{z^k}$ in accordance with Eq. (9). Then

$$\begin{aligned}| n\rangle &= e^{\frac{R^2}{2}} \frac{1}{2\pi i} \oint g^{(n)}(z) \mid z\rangle dz \\ &= e^{\frac{R^2}{2}} \frac{1}{2\pi i} \oint g^{(n)}(z) \left(e^{-\frac{R^2}{2}} \sum_{l=0}^{\infty} \frac{z^l}{\sqrt{l!}} \mid l\rangle\right) dz .\end{aligned} \tag{10}$$

Inserting the Laurent series of $g^{(n)}(z)$ and exchanging the order of integration and summation we obtain

$$| n\rangle = \sum_l \frac{1}{\sqrt{l!}} \mid l\rangle \sum_{k=1}^{\infty} g_k^{(n)} \frac{1}{2\pi i} \oint z^{l-k} dz . \tag{11}$$

Using the identity in Eq. (8) we find

$$| n\rangle = \sum_{k=1}^{\infty} \frac{g_k^{(n)}}{\sqrt{(k-1)!}} \mid k-1\rangle . \tag{12}$$

Consequently, the coefficients in the Laurent series can be recognized to be

$$g_k^{(n)} = \delta_{k,n+1} \sqrt{n!} , \tag{13}$$

which provides the circle representation of the number state $| n\rangle$ without uncertainty

$$| n\rangle = \frac{e^{\frac{R^2}{2}}}{2\pi i} \oint_{|z|=R} \frac{\sqrt{n!}}{z^{n+1}} \mid z\rangle dz , \tag{14}$$

Thus if the class of weight functions of the form determined in Eq. (9) is considered, the unique circle representation of an arbitrary state $| g\rangle = \sum_n c_n \mid n\rangle$ has the weight function

$$g(z) = \sum_{n=0} \frac{c_n \sqrt{n!}}{z^{n+1}} . \tag{15}$$

We note that the complex path integral representation form in Eq. (3) can be transformed into an integral with respect to an angle θ by using the substitution $z = Re^{i\theta}$ and the differential element along an arc $dz = iRe^{i\theta}d\theta$. On evaluating this transformation, Eq. (14) furnishes the well-known expression for Fock states on a circle [3,16]

$$| n\rangle = \frac{e^{\frac{R^2}{2}}}{2\pi} R^{-n}\sqrt{n!} \int_0^{2\pi} e^{-in\theta} \mid Re^{i\theta}\rangle d\theta \,. \tag{16}$$

It may be surprising that an arbitrarily large photon number state can be produced by superposition of coherent states comprising arbitrarily small mean numbers of photons.

The circle representation function belonging to a state of which the number state expansion is known has been determined. Inversely, the circle representation can furnish directly the coefficients of the number state expansion

$$\begin{aligned}\frac{1}{2\pi i}\oint_{|z|=R} g(z)\frac{z^n}{\sqrt{n!}}dz &= \frac{1}{2\pi i}\oint_{|z|=R}\sum_{k=0}^{\infty} c_k\frac{\sqrt{k!}}{z^{k+1}}\frac{z^n}{\sqrt{n!}}dz \\ &= \sum_k c_k\frac{\sqrt{k!}}{\sqrt{n!}}\frac{1}{2\pi i}\oint z^{n-k-1}dz \\ &= \sum_k c_k\frac{\sqrt{k!}}{\sqrt{n!}}\delta_{n,k} = c_n \,. \end{aligned} \tag{17}$$

This expression can be considered the inverse transformation of Eq. (15).

By this point of our analysis we could derive a weight function $g(z)$ in its Laurent-series form exploiting tacitly that an appropriate function associated with the state $\mid g\rangle$ exists at all. For being self-consistent the obtained form of $g(z)$ in Eq. (15) must be analytic in the vicinity of the circle. A necessary and sufficient condition is that the series on the right side of Eq. (15) is required to be absolutely convergent for every value of the variable z, i.e.,

$$\sum_n \frac{\sqrt{n!} \mid c_n \mid}{R^{n+1}} < \infty \,. \tag{18}$$

Thus, Eq. (18) serves as a condition for the existence of the circle representation: if the number of state coefficients c_n of a state satisfy this condition, then the state has the circle representation of the form of Eq. (3) with the weight function appearing in Eq. (15). This condition for the convergence of the series c_n is stronger than the condition for the normalization $\sum_n \mid c_n \mid^2 = 1$. Hence not all the states in the Hilbert space have the circle representation form of Eq. (3). The subset depends on the radius of the circle. For any state there exists a circle with large enough radius on which the state can be represented. In the $R \to \infty$ limit, every state can be represented by a $g(\theta)$ function given by Eq. (3).

III. DERIVATION OF THE CIRCLE REPRESENTATION

An expression for the circle representation based on the number state expansion of the state has already been presented in Eq. (15). Unfortunately, sometimes the summation in Eq. (15) cannot be evaluated to obtain an explicit closed form. The analytic coherent representation of a state is shown to be sufficient to derive the circle representation. The analytic representation is a very convenient tool to describe the states of the harmonic oscillator, since the expansion function is analytic in the whole complex plane and it can be derived easily according to Eq. (2). The circle representation can be transformed to produce the analytic one, which is also presented in this section.

First, the analytic representation is transformed to obtain the circle representation. Let us substitute the number state expansion of coherent states and the circle representation of the number states into the analytic representation of a state $\mid f\rangle$,

$$\begin{aligned}\mid f\rangle &= \frac{1}{\pi}\int e^{-\frac{|\alpha|^2}{2}} f(\alpha^*) \mid \alpha\rangle d^2\alpha \\ &= \frac{1}{\pi}\int e^{-|\alpha|^2} f(\alpha^*)\left(\sum \frac{\alpha^n}{\sqrt{n!}} \mid n\rangle\right) d^2\alpha \\ &= \frac{1}{\pi}\int e^{-|\alpha|^2} f(\alpha^*)\sum_n \frac{\alpha^n}{\sqrt{n!}} \\ &\quad\times \left(e^{\frac{R^2}{2}}\frac{1}{2\pi i}\oint_{|z|=R}\frac{\sqrt{n!}}{z^{n+1}} \mid z\rangle dz\right) d^2\alpha \,. \end{aligned} \tag{19}$$

After exchanging the order of integration and summation we find

$$\begin{aligned}\mid f\rangle = e^{\frac{R^2}{2}}\frac{1}{2\pi i}\oint_{|z|=R}\left(\frac{1}{\pi}\sum_n\int e^{-|\alpha|^2} f(\alpha^*)\frac{\alpha^n}{z^{n+1}}d^2\alpha\right) \\ \times \mid z\rangle dz \,. \end{aligned} \tag{20}$$

The expression in the brackets in Eq. (20) has to be the circle representation $g(z)$ of the state $\mid f\rangle$. Let us evaluate the integral with respect to the variable α in the entire complex plane in a polar coordinate system. With the substitution $\alpha = \rho e^{i\varphi}$ and with the correspondent differential element $d^2\alpha = \rho d\rho d\varphi$, the expression in the brackets in Eq. (20) becomes

$$g(z) = \frac{1}{\pi}\sum_n\int e^{-\rho^2}\frac{\rho^n e^{in\phi}}{z^{n+1}} f(\rho e^{-i\varphi})\rho d\rho d\varphi \,. \tag{21}$$

We can use the number state expansion of the analytic representation $f(z)$ [13], which is

$$f(\rho e^{-i\varphi}) = \sum_0^{\infty} c_k\frac{\rho^k}{\sqrt{k!}}e^{-ik\varphi} \,, \tag{22}$$

so it follows that

$$\begin{aligned} g(z) = \frac{1}{\pi}\sum_{n=0}^{\infty}\int_0^{\infty} e^{-\rho^2}\frac{\rho^n}{z^{n+1}}\sum_{k=0}^{\infty} c_k\frac{\rho^k}{\sqrt{k!}} \\ \times \left(\int_0^{2\pi} e^{i(n-k)\varphi}d\varphi\right)\rho d\rho \,. \end{aligned} \tag{23}$$

Since $\int_0^{2\pi} e^{i(n-k)\varphi}d\varphi = 2\pi\delta_{k,n}$ the integral with respect to φ and the summation with respect to k can be carried out

$$g(z)= \frac{2}{z}\int_0^\infty e^{-\rho^2}\sum_n c_n \frac{1}{\sqrt{n!}}\frac{\rho^{2n}}{z^n}\rho d\rho$$
$$= \frac{2}{z}\int_0^\infty e^{-\rho^2} f\left(\frac{\rho^2}{z}\right)\rho d\rho \, . \quad (24)$$

On substituting the positive, real variable ρ by its square root x ($x = \rho^2$) we get the final expression of transforming the two-dimensional analytic weight function $f(\alpha^*)$ into a one-dimensional circle weight function

$$g(z) = \frac{1}{z}\int_0^\infty e^{-x} f\left(\frac{x}{z}\right) dx \, , \quad (25)$$

which is one of the main results of this paper. This transformation formula serves for determining the circle representation of an arbitrary state, where Eq. (2) can be used

$$f\left(\frac{x}{z}\right) = e^{\frac{x^2}{2R^2}}\langle \frac{x}{z^*} \mid f\rangle \, . \quad (26)$$

This derivation leads to the same circle representation as has already been presented in Eq. (15). It is sufficient to verify the equivalence in the case of the number state $\mid n\rangle$,

$$g_n(z) = \frac{1}{z}\int_0^\infty \frac{x^n}{z^n\sqrt{n!}}e^{-x}dx = \frac{1}{z^{n+1}\sqrt{n!}}n! = \frac{\sqrt{n!}}{z^{n+1}} \, , \quad (27)$$

where we have inserted the analytic representation $f\left(\frac{x}{z}\right) = \frac{1}{\sqrt{n!}}\left(\frac{x}{z}\right)^n$ of the state $\mid n\rangle$ and we got back the circle representation given by Eq. (14).

The transformation formula in Eq. (25) holds if $g(z)$, i.e., the circle representation of the state described by its analytic coherent representation $f(\alpha^*)$ exists at all. Otherwise, the integral in Eq. (25) cannot be evaluated. Since all the states have analytic coherent representation, the convergence of the integral in Eq. (25) determines whether the state can be represented on a circle or not. This condition for the existence of the circle representation is equivalent to that given by Eq. (18) in the preceding section.

It is interesting to apply Eq. (25) in the case of a coherent state $\mid \beta\rangle$. Setting its analytic representation $f\left(\frac{x}{z}\right) = \exp\left(-\frac{|\beta|^2}{2} + \frac{x}{z}\beta\right)$ into Eq. (25) we find

$$g(z)= \frac{1}{z}\int_0^\infty e^{-x}e^{-\frac{|\beta|^2}{2}}e^{\frac{x}{z}\beta}dx$$
$$= \frac{e^{-\frac{|\beta|^2}{2}}}{z}\int_0^\infty e^{-(1-\frac{\beta}{z})x}dx$$
$$= e^{-\frac{|\beta|^2}{2}}\left[\frac{1}{\beta - z}e^{-(1-\frac{\beta}{z})x}\right]_0^\infty \, , \quad (28)$$

where the limit in the infinity can be evaluated for arbitrary z provided $\mid \beta \mid < \mid z \mid = R$. In this case we get the representation form of coherent state $\mid \beta\rangle$

$$\mid \beta\rangle = e^{\frac{R^2}{2}-\frac{|\beta|^2}{2}}\frac{1}{2\pi i}\oint \frac{1}{z-\beta}\mid z\rangle dz \, . \quad (29)$$

In the case of $\mid \beta \mid > R$ the integral divergent for some phase of the complex number z, i.e., the state cannot be represented by complex function belonging to the previously specified class. Consequently, only the coherent states inside the circle can be represented on it.

The analytic representation of a state $\mid g\rangle = \sum_n c_n \mid n\rangle$ can be derived from its circle representation $g(z)$. The number state expansion of the analytic representation can be exploited, where the coefficients are expressed by using Eq. (17),

$$f(\alpha^*)= \sum_{n=0}^\infty c_n\frac{\alpha^{*n}}{\sqrt{n!}} = \sum_n \frac{\alpha^{*n}}{\sqrt{n!}}\frac{1}{2\pi i}\oint_{|z|=R} g(z)\frac{z^n}{\sqrt{n!}}dz$$
$$= \frac{1}{2\pi i}\oint g(z)\left(\sum_{n=0}^\infty \frac{(\alpha^* z)^n}{n!}\right)dz \, , \quad (30)$$

where the order of summation and integral has been exchanged. The summation in the brackets provides the Taylor series of an exponential function, therefore,

$$f(\alpha^*) = \frac{1}{2\pi i}\oint_{|z|=R} g(z)e^{z\alpha^*}dz \, . \quad (31)$$

This formula and its inverse transformation given by Eq. (25) determines the connection between the circle and the analytic representation.

The entangled states of a quantum system, which are more general than the pure ones considered above, can be described by a density operator. The analytic representation can be generalized to represent any density operator in a unique way. The expansion function is required to be a function of two complex variables $R(\alpha^*,\beta)$ which is analytic throughout both of the finite α and β complex planes [13]. Then we find the following one-to-one correspondence between such entire $R(\alpha^*,\beta)$ functions and the density operators

$$\hat\rho = \frac{1}{\pi^2}\int \mid \alpha\rangle R(\alpha^*,\beta)\langle\beta \mid e^{-\frac{1}{2}(|\alpha|^2+|\beta|^2)}d^2\alpha d^2\beta \, , \quad (32)$$

where

$$R(\alpha^*,\beta) = e^{\frac{1}{2}(|\alpha|^2+|\beta|^2)}\langle\alpha \mid \hat\rho \mid \beta\rangle \, . \quad (33)$$

The representation of density operators on a circle can be constructed according to the preceding methods,

$$\hat\rho = \frac{1}{(2\pi i)^2}\oint_{|z_1|=R}\oint_{|z_1|=R} G(z_1,z_2)\mid z_1\rangle\langle z_2 \mid dz_1 dz_2 \, . \quad (34)$$

The same derivation leads to the connection between the circle and the analytic representation of density operators as in the case of pure states. The transformation formula between the two representations is

$$G(z_1,z_2) = \frac{1}{z_1 z_2} \int_0^\infty \int_0^\infty R\left(\frac{x_1}{z_1}, \frac{x_2}{z_2}\right) e^{-x_1-x_2} dx_1 dx_2 \;, \tag{35}$$

and its inverse is

$$R(\alpha^*,\beta) = \frac{1}{(2\pi i)^2} \oint_{|z_1|=R} \oint_{|z_1|=R} G(z_1,z_2) e^{z_1\alpha^* + z_2\beta} \times dz_1 dz_2 \;. \tag{36}$$

These formulas make it possible to describe the more general states of an electromagnetic field mode by coherent states with identical numbers of photons. Such a field can be prepared in optical processes, or in an interaction with an environment which leads to entangled states of the quantum system. Every density operator has an analytic representation form but the transformation in Eq. (35) cannot be evaluated in some cases. In these cases a circle having a large enough radius has to be chosen to represent the state.

IV. CONCLUSIONS

In conclusion, a one-dimensional coherent representation was constructed that can be widely used in quantum optics. The significance of the circle representation is that there are several proposed experiments where superposition of coherent states on a circle is generated. The set of states has been specified, which can be expanded into the superposition of coherent states lying on a circle in phase space. Considering a special class of functions the circle representation was proved to be unique. The derivation of the circle representation from the number state and the analytic representation has been presented. The inverse transformations associated with these derivations was also given in the paper.

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