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Physical realization of an environment with squeezed quantum fluctuations via quantum-nondemolition-mediated feedback

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We show how a squeezed environment can be obtained by means of a suitable feedback of the output signal corresponding to a quantum-nondemolition (QND) measurement of an observable. As an example we show how the variance of a field quadrature of a cavity mode subject to QND-mediated feedback can be squeezed below the standard quantum limit and that this actually means that applying feedback is equivalent to coupling the mode to a squeezed environment.

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I. INTRODUCTION

The influence of dissipation on macroscopic coherence has been discussed by several authors during the past decade after the pioneering work of Caldeira and Leggett [1]. We will only consider the quantum-optical domain [2,3]. According to this formulation, the global system is usually assumed to consist of a subsystem coupled to a bath. The subsystem is then referred to as an open quantum system, which dissipates its energy into the bath or environment. The open system is then described by a reduced density operator ρ, which evolves according to a master equation, obtained by contracting (or tracing over) the environment variables. The contraction of environment variables leads to the nonunitary evolution subsystem. Moreover, which subsystem observable is measured is determined by the coupling between the environment and the subsystem. The destruction of quantum-mechanical interference phenomena is usually related to dissipation consequences induced by the measurement apparatus. Zurek [4] showed how the large number of degrees of freedom of a measurement apparatus could be included in a quantum-mechanical formulation. Finally, following Zurek, Walls, Collett, and Milburn [5] introduced a model of a quantum limited measurement. They considered the interaction of one subsystem observable with a meter and the coupling between the meter and the environment. They showed that the off-diagonal elements of the meter state, as a consequence of nonunitary dissipative evolution, rapidly decay, determining a preferred or pointer basis. Kennedy and Walls [6] suggested using a squeezed environment to model a measurement apparatus able to preserve the interference fringes of a macroscopic superposition, confirming previous predictions based on a heuristic model [7]. They also observed that, in the optical regime, squeezing the environmental degrees of freedom enhances the diagonalization in the pointer basis. To have a squeezed environment, means considering a bath with nonstandard quantum fluctuations.

II. THE HAMILTONIAN MODEL

Let us consider as a particular example an electromagnetic field mode of frequency ω within a resonant ring cavity. The Hamiltonian of the system coupled to an external radiation bath is

$$H=\hbar\omega a^{\dagger}a+\hbar(a^{\dagger}\Gamma+a\Gamma^{\dagger})+H_{\text{bath}} , \tag{1}$$

where a, $a^{\dagger}$ are the boson annihilation and creation operators for the field mode, H_{bath} is the bath Hamiltonian, and Γ, $\Gamma^{\dagger}$ are bath operators. A squeezed environment is that characterized by the following correlation functions [8]:

$$\langle\Gamma^{\dagger}(t)\Gamma(t')\rangle=\gamma N\delta(t-t') , \tag{2a}$$

$$\langle\Gamma(t)\Gamma^{\dagger}(t')\rangle=\gamma(N+1)\delta(t-t') , \tag{2b}$$

$$\langle\Gamma(t)\Gamma(t')\rangle=\gamma Me^{-2i\omega t}\delta(t-t') , \tag{2c}$$

$$\langle\Gamma^{\dagger}(t)\Gamma^{\dagger}(t')\rangle=\gamma M^{*}e^{2i\omega t}\delta(t-t') , \tag{2d}$$

where γ is the cavity linewidth and $M=|M|e^{i\psi}$ is the parameter characterizing the nonstandard quantum fluctuation of the squeezed bath. The Heisenberg uncertainty relation imposes the constraint $|M|^2\le N(N+1)$. The effect of these squeezed fluctuations can be seen by making a contraction over the bath degrees of freedom and deriving the Markovian master equation for the density operator of the field mode [8]. In the interaction picture this is given by

$$\begin{aligned}\dot{\rho}=&\frac{\gamma}{2}(N+1)(2a\rho a^{\dagger}-a^{\dagger}a\rho-\rho a^{\dagger}a)\\&+\frac{\gamma}{2}N(2a^{\dagger}\rho a-aa^{\dagger}\rho-\rho aa^{\dagger})\\&-\frac{\gamma}{2}M(2a^{\dagger}\rho a^{\dagger}-a^{\dagger}a^{\dagger}\rho-\rho a^{\dagger}a^{\dagger})\\&-\frac{\gamma}{2}M^{*}(2a\rho a-aa\rho-\rho aa) .\end{aligned} \tag{3}$$

The peculiar aspect of a squeezed bath is the presence of the last two phase-sensitive terms in Eq. (3); they can have important effects on some properties of the cavity mode. For example, in the stationary state, the variance of a quadrature of the mode a can be squeezed below the standard quantum limit, when the parameter M is such that $N<|M|\le\sqrt{N(N+1)}$. Moreover, when the cavity mode is in a macroscopic superposition of two coherent states that can be created in a Kerr medium, while they are almost immediately destroyed by a standard bath [9], they may be preserved for a significant time if the two last terms in Eq. (3) are present [6]. Despite these important properties, it is not yet clear at all how it could be possible to reproduce a squeezed environment in practice. In this paper we show that a physical realization of a squeezed environment can be obtained by applying to the cavity mode an appropriate feedback loop.

III. FEEDBACK MECHANISM

Feedback means applying to a system part of an output signal obtained form a continuous measurement of one of its observables. Here we shall essentially follow the spirit of the Wiseman and Milburn approach to the continuous measurement problem [10] and to the description of optical feedback [11,12]. Anyway, we shall elaborate and slightly generalize their arguments.

These authors have developed a theory of optical feedback mediated by homodyne detection; an important result of their analysis is that homodyne-mediated feedback or any other feedback scheme implying an *extracavity* measurements (i.e., a measurement that does not change the master equation) cannot produce nonclassical light unless the cavity dynamics can do so without feedback [12]. On the other hand, a cavity mode in a squeezed bath can generally show squeezing of a field quadrature, and therefore it is clear that if one wants to obtain a squeezed environment, and all its peculiar properties, one does not have to use any feedback relying on these "extracavity" measurements. A viable way to obtain a squeezed bath through feedback is using a quantum-nondemolition (QND) scheme for the continuous measurement step of the feedback loop (this is in fact an "intracavity measurement" in the Wiseman and Milburn scheme, and is not subject to their "no-go" theorems [12]).

Let us consider a radiation mode and a generic QND measurement of one of its observables C; this is described by the coupling with a pseudoclassical meter via an interaction of the form

$$H_{\text{int}} = \hbar \chi C P_{\text{meter}} . \tag{4}$$

where χ is the coupling constant and P_{meter} is the conjugate momentum of a macroscopic "pointer observable" of the meter. It is often assumed that the meter momentum is strongly damped and therefore characterized by a very short relaxation time; in this case, one can perform an adiabatic elimination of the meter and obtain the following master equation for the mode of interest alone:

$$\dot{\rho} = \mathcal{L}\rho - \chi^2 D[C,[C,\rho]] , \tag{5}$$

where $\mathcal{L}$ is the unperturbed Liouvillian of the mode and

$$D = \int_0^{\infty} dt \langle P_{\text{meter}}(t) P_{\text{meter}}(0) \rangle \simeq \int_0^{\infty} dt \langle P_{\text{meter}}(0) P_{\text{meter}}(t) \rangle . \tag{6}$$

Equation (5) is the equation of motion for the density matrix describing the unconditioned state of the system of interest. In the presence of the continuous QND measurement process of Eq. (4), one can also consider the state that is a condition of the result of the measurement, which is described by the conditioned density matrix ρ_c. The outcome of a generic quantum measurement process is a stochastic variable and, therefore, the time evolution of the conditioned density matrix is generally driven by a stochastic differential equation. In the particular case of a continuous position QND measurement, it has been shown in Ref. [5] that the "pointer observable" Q_{meter} of the meter (which is the coordinate conjugated to P_{meter}) can be described as a Gaussian continuous stochastic process and that ρ_c is driven by an Ito stochastic nonlinear equation. If we apply the results of Ref. [5] to our generic continuous QND measurement of the observable C, we get that the pointer observable Q_{meter} evolves according to the stochastic equation

$$\dot{Q}_{\text{meter}} = \hbar\chi \left[\langle C \rangle_c(t) + \frac{1}{2\chi\sqrt{2D}} \xi(t) \right] , \tag{7}$$

and that the Ito stochastic equation for the conditioned density matrix is

$$\dot{\rho}_c = \mathcal{L}\rho_c - \chi^2 D[C,[C,\rho_c]] + \chi\sqrt{2D}\, \xi(t)(C\rho_c + \rho_c C - 2\langle C \rangle_c \rho_c) , \tag{8}$$

where $\xi(t)$ is a Gaussian c-number white noise with $\langle \xi(t)\xi(t') \rangle = \delta(t-t')$ and $\langle \cdots \rangle_c$ represents the trace with respect to the conditioned density matrix ρ_c. For example, $\dot{Q}_{\text{meter}}$ can represent the photocurrent obtained from the QND measurement of a field quadrature, as in the model proposed in Ref. [13]. The equation of motion for the unconditioned density matrix ρ is obtained from Eq. (8) by simply averaging over the results of measurements, i.e., over the distribution of the white noise $\xi(t)$.

Equations (7) and (8) are the two main ingredients needed to derive a theory of feedback mediated by QND measurement. In fact, applying feedback to a system means taking part of the output signal $\dot{Q}_{\text{meter}}$ associated to the measured observable C and feeding it back to the system. Following the suggestions of Wiseman and Milburn, this means adding the following term, linear in $\dot{Q}_{\text{meter}}$, to the equation of motion for $\rho_c(t)$ [11,12]:

$$[\dot{\rho}_c(t)]_{fb} = \dot{Q}_{\text{meter}}(t-\tau_d)\mathcal{K}\rho_c(t) , \tag{9}$$

where τ_d is the time delay in the feedback loop and $\mathcal{K}$ is a superoperator describing the way in which the feedback signal acts on the system of interest, and it usually has [12] the Hamiltonian form $\mathcal{K}\rho = -(i/\hbar)[A,\rho]$. The presence of delay makes the evolution of the conditioned state a non-Markovian process; this prevents the description of the dynamics of a system under the influence of a QND-measurement-mediated feedback in terms of a simple master equation. Despite this, Wiseman and Milburn [12], by showing that the feedback term (9) has to be interpreted in the Stratonvich sense, transforming the resulting equation for ρ_c in the Ito form, averaging over the white noise, and finally considering the limit of time delay τ_d much shorter than the characteristic time of the system, i.e., γ^{-1}, have been able to show that a linear Markovian master equation for the unconditioned density matrix ρ of radiation mode subject to feedback can be generally obtained. By applying their arguments to our generic QND feedback, we get

$$\dot{\rho}=\mathcal{L}\rho-\chi^2 D[C,[C,\rho]]+\frac{\hbar\chi}{2}\mathcal{K}(C\rho+\rho C)+\frac{\hbar^2}{16D}\mathcal{K}^2\rho \ . \tag{10}$$

This master equation describes the time evolution of a radiation mode, with unperturbed Liouvillian $\mathcal{L}$, when one of its observables C, is continuously monitored via a QND measurement and part of the signal output obtained from this measurement is sent back in a feedback loop. The second term in Eq. (10) describes the effect of the QND measurement, the third term is the feedback term itself, and the fourth term is a diffusionlike term, inevitably induced by the noise introduced in the measurement step of the feedback loop. An analogous equation describing QND-mediated feedback has been already derived by Wiseman and Milburn in Ref. [12]. They have considered a QND measurement of a field quadrature performed via a bilinear coupling with the quadrature of another strongly damped mode supported by the cavity, playing the role of the pseudoclassical meter. In our opinion the QND-mediated feedback scheme of Eq. (10) is more general and it can be applied to a large class of QND measurements, as, for example, the one considered in Ref. [14].

IV. SQUEEZED ENVIRONMENT

Let us now apply the general equation (10) to show our central argument: a squeezed environment acting on a system of interest can be effectively reproduced by an appropriately chosen QND-mediated feedback mechanism. We shall show this with a simple example: a QND measurement of the quadrature X of an electromagnetic mode of frequency ω in a cavity with linewidth γ. We use the interaction picture, so that, neglecting the mean number of thermal photons, which is negligibly small at optical frequencies, one has

$$\mathcal{L}\rho=\frac{\gamma}{2}(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a) \tag{11}$$

and

$$C\rightarrow X=\tfrac{1}{2}(a+a^\dagger) \ . \tag{12}$$

The output signal $\dot{Q}_{\rm meter}$ obtained from the continuous monitoring of the quadrature X is then fed back to the cavity involving another phase-dependent quadrature operator X_θ, i.e.,

$$\mathcal{K}\rho=-\frac{iG}{\hbar}[X_\theta,\rho]=\frac{iG}{2\hbar}[ae^{i\theta}+a^\dagger e^{-i\theta},\rho] \ , \tag{13}$$

where G is the gain or "efficiency" of the feedback loop and θ is controlled by the experimenter. Equation (13) describes a feedback mechanism which implies adding to the Hamiltonian a driving term of the form $H_{fb}(t)=G\dot{Q}_{\rm meter}(t)X_\theta$, where the output signal $\dot{Q}_{\rm meter}(t)$ plays the role of the driving force and it can be seen as feedback current. If we insert Eqs. (11)–(13) in the general equation (10), we obtain the following master equation for the cavity mode, subject to QND-mediated feedback:

$$\dot{\rho}=\mathcal{L}_1\rho+\mathcal{L}_2\rho+\mathcal{L}_3\rho \ , \tag{14}$$

where

$$\mathcal{L}_1\rho=\frac{\gamma}{2}\left[1+\frac{\chi^2 D}{2\gamma}+\frac{G^2}{32D\gamma}\right](2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a)+\frac{\gamma}{2}\left[\frac{\chi^2 D}{2\gamma}+\frac{G^2}{32D\gamma}\right](2a^\dagger\rho a-aa^\dagger\rho-\rho aa^\dagger) \ , \tag{15a}$$

$$\mathcal{L}_2\rho=\frac{\gamma}{2}\left[\frac{\chi^2 D}{2\gamma}+\frac{G^2}{32D\gamma}e^{-2i\theta}\right](2a^\dagger\rho a^\dagger-a^\dagger a^\dagger\rho-\rho a^\dagger a^\dagger)+\frac{\gamma}{2}\left[\frac{\chi^2 D}{2\gamma}+\frac{G^2}{32D\gamma}e^{2i\theta}\right](2a\rho a-aa\rho-\rho aa) \ , \tag{15b}$$

$$\mathcal{L}_3\rho=-\frac{i\chi G}{8}[aae^{i\theta}+a^\dagger a^\dagger e^{-i\theta},\rho]-\frac{i\chi G}{8}e^{i\theta}(a\rho a^\dagger-a^\dagger\rho a+aa^\dagger\rho-\rho a^\dagger a)-\frac{i\chi G}{8}e^{-i\theta}(a^\dagger\rho a-a\rho a^\dagger+a^\dagger a\rho-\rho aa^\dagger) \ . \tag{15c}$$

The most interesting aspect of this master equation is that it is very similar to Eq. (3), describing a cavity mode in a *squeezed environment;* in fact, the first two terms [(15a) and (15b)] exactly reproduce the dissipative terms associated with a bath with squeezed quantum fluctuations. This is due to the fact that the term associated with the QND measurement and the feedback-induced diffusionlike term in Eq. (10) produce nonstandard quantum fluctuations, which add to the usual vacuum fluctuations, thereby simulating a squeezed environment. Anyway, the equivalence is not complete because of the presence of the extra term $\mathcal{L}_3$ of Eq. (15c), which also results from the presence of feedback. This means that the dynamics of the electromagnetic mode subject to the QND-mediated feedback of Eq. (14) is generally different from that induced by the presence of a squeezed bath, as in Eq. (3); in spite of this, we are now able to show that at least *stationary properties* of the cavity mode with QND feedback can be interpreted in terms of a radiation mode interacting with a squeezed vacuum.

Let us consider the equation of motion for $\langle X\rangle$ and $\langle X^2\rangle$ generated by the master equation (14):

$$\langle\dot{X}\rangle=-\frac{\gamma}{2}\left[1+\frac{\chi G}{\gamma}\sin\theta\right]\langle X\rangle \ , \tag{16a}$$

$$\langle\dot{X}^2\rangle=-\gamma\left[1+\frac{\chi G}{\gamma}\sin\theta\right]\langle X^2\rangle+\frac{\gamma}{4}+\frac{G^2}{32D}\sin^2\theta \ . \tag{16b}$$

Both $\langle X\rangle$ and $\langle X^2\rangle$ asymptotically reach a stationary value, for any value of the phase θ, only when the condition

$$\frac{\chi G}{\gamma}<1 \tag{17}$$

is satisfied. This stability condition actually means that the energy introduced by the feedback loop cannot overcome a critical value if instabilities have to be avoided. In the stationary state, the variance of the continuously monitored quadrature $\langle \Delta X^2 \rangle$ has the following asymptotic value:

$$\langle \Delta X^2 \rangle_{\rm st} = \frac{1}{4} \frac{1+\frac{G^2}{8\gamma D}\sin^2\theta}{1+\frac{\chi G}{\gamma}\sin\theta} . \tag{18}$$

It is easy to see from this equation that this variance can be squeezed below the standard quantum limit $\langle \Delta X^2 \rangle < \frac{1}{4}$, if a suitable choice of the feedback parameters G and θ is made:

$$0 < \frac{G}{8\chi D}\sin\theta \le 1 . \tag{19}$$

It is worth noting that the extra term $\mathcal{L}_3\rho$ in Eq. (14) is actually necessary to obtain quadrature squeezing. In fact, if this term were absent, one would have a phase-sensitive bath, as that of Eq. (3), but which is unable to produce squeezing, because the parameters G,D,χ,θ are such that the coefficients M and N satisfy $|M| \le N$. Therefore, the above QND feedback is necessary to obtain quadrature squeezing; without feedback, one would simply get Eq. (5), which is formally equivalent to the master equation of the squeezed bath, Eq. (3), but with $N=M$, and therefore unable to produce squeezing.

Equation (18) shows that a field quadrature can be arbitrarily squeezed by performing a QND measurement on it and by feeding back part of the signal output obtained from this measurement with an appropriate feedback loop. The possibility of quadrature squeezing via QND feedback has been already shown by Wiseman and Milburn in Ref. [12], where they consider a feedback scheme very similar to that proposed above; anyway, this feedback-induced squeezing has a much deeper meaning in our opinion. In fact, the stationary squeezed state asymptotically reached by the cavity mode can be considered as ultimately due to the fact that the mode is surrounded by a *squeezed environment*, as in Eq. (3). Therefore, we can say that the QND-mediated feedback described by Eq. (10) actually reproduces a squeezed environment. To be more precise, even though the dynamical properties of the mode are not exactly the same because of the differences between master equations (3) and (14), its steady-state properties, when it is inside a cavity subject to the QND feedback, are actually the same as it would have if it were interacting with a bath with squeezed quantum fluctuations. In fact, the stationary solution of master equation (3) is the generalized Gaussian density matrix

$$\rho_{\rm st} = Z^{-1}\exp\left[-na^\dagger a - \frac{m}{2}a^\dagger a^\dagger - \frac{m^*}{2}aa\right] \tag{20}$$

(with Z a normalization constant), and it is possible to prove that this "nondiagonal" density matrix is also the exact solution for the stationary state of the cavity mode subject to the QND feedback described by Eqs. (14) and (15), provided that m and n are given by

$$n = \frac{\nu+\frac{1}{2}}{\sqrt{(\nu+\frac{1}{2})^2-|\mu|^2}} \times \ln\left\{\frac{[\sqrt{(\nu+\frac{1}{2})^2-|\mu|^2}+\frac{1}{2}]^2}{\nu(\nu+1)-|\mu|^2}\right\} , \tag{21a}$$

$$m = \frac{\mu}{\nu+\frac{1}{2}} n , \tag{21b}$$

where

$$\nu = \frac{\frac{G^2}{32D\gamma}-\frac{\chi G}{4\gamma\sin\theta}}{1+\frac{\chi G}{\gamma}\sin\theta} + \frac{\chi^2 D}{2\gamma} + \frac{\chi G\cos^2\theta}{4\gamma\sin\theta}\frac{1}{1+\frac{\chi G}{2\gamma}\sin\theta} , \tag{22a}$$

$$\mu = -e^{-2i\theta}\frac{\frac{G^2}{32D\gamma}-\frac{\chi G}{4\gamma\sin\theta}}{1+\frac{\chi G}{\gamma}\sin\theta} - \frac{\chi^2 D}{2\gamma} - e^{-i\theta}\frac{\chi G\cos\theta}{4\gamma\sin\theta}\frac{1}{1+\frac{\chi G}{2\gamma}\sin\theta} . \tag{22b}$$

This equivalence between the application of a QND-mediated feedback loop and a squeezed environment is based on an indirect argument because we have only considered cavity mode properties. An exhaustive proof would have shown that feedback actually means modifying not only the cavity mode properties but also those of the external environment (consisting of both the vacuum and the whole measuring apparatus), in such a way as to get squeezed quantum fluctuations as those of Eqs. (2). Anyway, this is impossible to achieve within the Wiseman and Milburn approach, which from the beginning focuses only on the effects of feedback on the system of interest.

V. CONCLUSIONS

The main result of this paper is not only the possibility of producing squeezed light with an appropriate feedback loop, there are in fact practical sources of squeezed light more reliable than the one proposed here, even if we have to remark that QND measurements similar to those adopted here are now beginning to be performed [15]. What we want to stress in this paper is that a suitable feedback loop can be interpreted as a way of simulating the presence of a bath with squeezed quantum fluctuations. In our opinion, this fact has very interesting consequences because of the physical properties of a squeezed bath. For example, we shall show in a forthcoming paper [16] that it is possible to realize a QND-mediated feedback reproducing a squeezed environment, which is able to preserve for a certain time the interference pattern associated with linear superpositions of macroscopically distinguishable states, as those generated in Kerr media.

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