Photon noise reduction by reflection from a movable mirror

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We study the statistics of photons reflected by a harmonically suspended mirror. The noise modification can be interpreted in terms of temporal redistribution of the photon flux. Due to radiation pressure, the mirror is displaced and a variable delay is applied to the photons. The statistical properties of the reflected light are studied by Monte Carlo simulation with a corpuscular photon model and by analytic derivation. In the latter derivation, the effect of thermal fluctuations is included. The results indicate that a movable mirror appears as a passive quantum-noise eater at frequencies about the mechanical eigenfrequency of the mirror suspension.

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I. INTRODUCTION

Squeezed states of light are usually generated by using optical nonlinear media [1,2]. They can also be obtained by optomechanical coupling between light and mirrors. Mirrors may indeed move in response to radiation pressure exerted by the light [3]. This leads to an intensity-dependent phase shift for the light field equivalent to an optical Kerr effect, with a dynamic response depending on the mechanical susceptibility of the mirror. This coupling has been studied in detail in connection with the problem of quantum limits in interferometric measurements [4,5].

It has recently been proposed to use this optomechanical coupling in systems composed of an empty cavity with a movable mirror to generate sub-Poissonian light [6] or to realize QND measurements [7]. These systems are similar to bistable devices composed of a cavity containing an optical Kerr medium, which have been extensively considered for squeezing generation [8–10] or QND measurements [11,12]. The statistical properties of these systems can be understood in a linear analysis of the field and position fluctuations. Radiation pressure couples the phase and intensity fluctuations of the intracavity field. The cavity couples back the intracavity phase fluctuations to intensity fluctuations and thus plays an important role to generate sub-Poissonian light at the cavity output.

In this paper, we propose a regulation mechanism of the photon flux which appears conceptually simpler. It consists in a single movable mirror irradiated by a monochromatic laser beam (see Fig. 1). If the mirror is not allowed to move, the intensity fluctuations of the reflected beam are exactly the same as those of the incident beam. For a mirror harmonically suspended, the radiation pressure produces a mirror displacement and therefore a modification of the optical path depending on the incident intensity. This leads to a temporal redistribution of the photons between the incident and reflected beams, which may regulate the photon flux. In this case, the intensity of the reflected beam is less noisy than the incident one and the movable mirror appears as a passive photon-noise eater. In this system the reflected intensity is directly coupled to the mirror position and to the incident intensity (see Fig. 1). The phase fluctuations are not coupled back to the intensity fluctuations. The intensity-noise reduction is thus based on a different physical mechanism than the Kerr effect.

As a consequence the quantum-noise reduction can be simply interpreted as a photon flux regulation. In accordance with this interpretation, we will use a corpuscular model in which the photon flux is regarded as a point process [13–15]. A photon is treated as a discrete event localized in time and the light intensity is defined as the rate of such events. This model has been used to demonstrate the possibility of photon-noise reduction by a control mechanism in which the photon flux is regulated with a variable delay line [16]. The corpuscular approach is reserved, of course, for systems in which only the intensity of the light field plays a role, that is, if the phase fluctuations are not coupled back to the intensity fluctuations. This model is readily amenable to Monte Carlo simulation. The results of these numerical simulations are presented in Sec. II, where the basic statistical properties of the system are delineated.

Section III is devoted to an analytical derivation of the statistical properties of this system. We make use of a semiclassical analysis to determine an input-output transformation of the intensities. A linearization procedure leads to simple analytical expressions for the noise reduc-

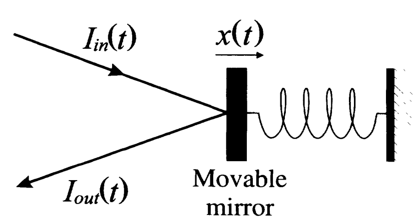

FIG. 1. Optomechanical system studied in this paper: a monochromatic laser beam is reflected on a harmonically suspended mirror.

tion of the reflected intensity. This treatment is valid for high photon fluxes, whereas the numerical simulation is limited to low photon fluxes. The semiclassical treatment can also be used to determine other properties of the reflected beam, such as the phase noise, which has to be increased when the intensity noise is reduced. We use this analysis to include the thermal fluctuations of the mirror (Sec. IV).

II. MONTE CARLO SIMULATION

We use a corpuscular model in which a light beam is considered as a stream of photons. The instantaneous intensity is given by

$$I(t)=\sum_k \delta(t-t_k) , \tag{1}$$

where t_k is the event time of the photon labeled k. Intensity is defined as a photon flux (number of photons per second) and the mean intensity is equal to the inverse of the average delay $\overline{\tau}_P$ between successive photons. The intensity power spectrum $S_I[\Omega]$ normalized to the shot noise is related to the Fourier transform of the intensity $I[\Omega]$ by the equation

$$2\pi \overline{I}\delta(\Omega+\Omega')S_I[\Omega]=\langle I[\Omega],I[\Omega']\rangle , \tag{2}$$

where the mean intensity $\overline{I}$ and the variance $\langle I[\Omega],I[\Omega']\rangle$ are obtained by a statistical average over all possible realizations of photon streams.

The intensity statistics for the reflected beam is evaluated through a Monte Carlo simulation of the path followed by each photon from its generation to its detection. To begin, we produce the generation times t_k in order to mimic the intensity I_{in} of the incident beam. Assuming that the photon statistics are Poissonian, the delays τ_P between successive photons are independent stochastic variables characterized by an exponential probability distribution. Photon generation times are thus obtained from the recurrence relation

$$t_k=t_{k-1}+\tau_P , \tag{3a}$$

$$\tau_P=-\overline{\tau}_P \ln(1-u) , \tag{3b}$$

where u is a uniform deviate distributed between 0 and 1.

To determine the modification of the photon statistics due to reflection on the mirror, we treat the reflection of every photon as a collision between the photon (energy $\hbar\omega_0$ and momentum $\hbar\omega_0/c$) and the mirror [position $x(t)$, speed $\dot{x}(t)$, and mass M]. In the time interval between two photon reflections, the mirror obeys the free equation of motion

$$\ddot{x}(t)+\gamma_M \dot{x}(t)+\Omega_M^2 x(t)=0 , \tag{4}$$

where γ_M is the damping rate of the mirror suspension and Ω_M its mechanical eigenfrequency. We have neglected all fluctuating force, in particular the thermal fluctuations associated with the damping. Those additional noise sources will be considered in Sec. IV.

For a photon arriving at time t_k at position $x=0$, the reflection time $t_{M,k}$ on the mirror is related to the position of the mirror by

$$t_{M,k}=t_k+\frac{x(t_{M,k})}{c} . \tag{5}$$

The evolution of the system after a reflection is deduced from the conservation of energy and momentum. We find that both the speed $\dot{x}(t)$ of the mirror and the photon energy $\hbar\omega_0$ are modified by quantities $\Delta\dot{x}$ and $\hbar\Delta\omega$, respectively, according to the equations (valid at the first order in $\dot{x}/c$)

$$\frac{\Delta\dot{x}(t_{M,k})}{c}=\varepsilon_r\left[2-\frac{\Delta\omega}{\omega_0}\right] , \tag{6a}$$

$$\frac{\Delta\omega}{\omega_0}=\frac{2\dot{x}(t_{M,k})+\Delta\dot{x}(t_{M,k})}{c}\approx 2\left[\frac{\dot{x}(t_{M,k})}{c}+\varepsilon_r\right] , \tag{6b}$$

where ε_r is the ratio between the energies of the photon and the mirror at rest

$$\varepsilon_r=\frac{\hbar\omega_0}{Mc^2} . \tag{7}$$

Equation (6a) expresses the recoil effect of the mirror associated with momentum conservation, taking into account the photon frequency shift $\Delta\omega$. Equation (6b) is nothing but the Doppler effect experienced by the photon on the moving mirror, taking into account the variation $\Delta\dot{x}$ of the mirror speed.

For each reflected photon, the event time t'_k, defined as the time when the reflected photon is at position $x=0$, is obtained by addition of a variable delay proportional to the position of the mirror:

$$t'_k=t_k+2x(t_{M,k})/c . \tag{8}$$

In the simulation, the mirror motion is computed between two photon reflection times according to Eq. (4) and the mirror speed is instantaneously modified at the reflection time according to Eq. (6a). The intensity I_{out} of the reflected beam is given by the delayed event times t'_k [Eq. (8)] and the output intensity-noise spectrum is computed by applying the standard technique of periodograms [17].

Figure 2 shows the simulation results for the time evolution of the photon streams and of the mirror position and speed. Time is divided in small intervals (equal to $\overline{\tau}_P/5$) and the number of incident photons in every time interval is counted and plotted in curve (a), which reproduces the characteristic fluctuations of Poissonian statistics. Curves (b) and (c) represent, respectively, the position and speed of the mirror, for a mirror suspension without damping ($\gamma_M=0$). The speed evolution presents small irregularities at each reflection time, corresponding to the momentum exchange between mirror and photon. It appears clearly in these curves that the fluctuations of the radiation pressure induce fluctuations of the mirror motion. It is also visible that the mirror motion is damped, although there is no suspension damping. This damping is actually connected to radiation pressure fluctuations through fluctuation-dissipation relations. It can

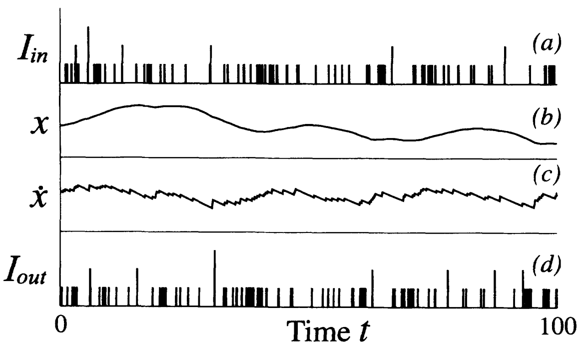

FIG. 2. Simulated evolution of the incident photon flux (a), mirror position (b), mirror speed (c), and reflected photon flux (d). Time t is normalized to the mean delay $\overline{\tau}_P$ between successive photons.

be shown that the mirror is submitted to a viscous force with a damping constant $\gamma_{\rm rad}$ equal to [18],

$$\gamma_{\rm rad}=4\varepsilon_r \overline{I}_{\rm in} \tag{9}$$

It has to be emphasized that this dissipation results in the simulation as the cumulative effect of successive photon collisions treated as discrete events.

The parameters are selected so that the quality factor Q of the mirror is small:

$$\gamma_{\rm rad}\overline{\tau}_P=\tfrac{1}{20},\quad \Omega_M\overline{\tau}_P=\tfrac{1}{40},\quad Q=\Omega_M/\gamma_{\rm rad}=0.5\ . \tag{10}$$

As a consequence, the mirror position [curve (b) in Fig. 2] is smoothed on a time interval of the order of $20\overline{\tau}_P$ and the delay added to every photon [Eq. (8)] does not vary appreciably from one photon to the next one. Curve (d) shows the photon flux for the reflected beam. Even though the event times are changed on reflection, the photon flux after reflection does not appear more regular on this time scale than the incident one [curve (a)].

The small delays added to the event times have, however, important consequences for the noise spectrum. Figure 3 presents the intensity noise spectra deduced from the event times obtained in the simulation (Fourier transform of sample data of 10^7 photons). The power spectrum of the incident beam [curve (a)] is at the shot-noise level, as expected for Poissonian statistics. The power spectra of the reflected beam [curves (b)–(d)] are obtained for different ratios $\gamma_{\rm rad}/\gamma$, where γ is the total damping rate

$$\gamma=\gamma_M+\gamma_{\rm rad}\ . \tag{11}$$

Curve (d) exhibits remarkable noise reduction at frequencies about the mechanical eigenfrequency Ω_M of the mirror. The shape of the noise spectrum essentially depends on two parameters. The first one is the ratio between the two damping constants $\gamma_{\rm rad}$ and γ_M, which determines the amplitude of noise reduction. Figure 4 shows the optimum noise reduction, that is, the noise at frequency Ω_M, as a function of the ratio $\gamma_{\rm rad}/\gamma$. The intensity noise after reflection is more reduced as $\gamma_{\rm rad}$ becomes larger than γ_M ($\gamma_{\rm rad}/\gamma\to 1$). Best results are obtained when the mirror is only damped by the radiation pressure ($\gamma_{\rm rad}\gg\gamma_M$).

The second important parameter is the quality factor Q related to the total damping constant γ by

$$Q=\Omega_M/\gamma\ . \tag{12}$$

This parameter determines the width of noise reduction. For large Q, the mirror evolution frequencies are mainly limited to a small range about the eigenfrequency Ω_M. As shown in Fig. 5, the noise reduction in the reflected beam is limited to those frequencies.

We have also computed the energy flux (or field power) of the reflected beam defined as

$$E(t)=\sum_k \hbar(\omega_0+\Delta\omega)\delta(t-t'_k)\ , \tag{13}$$

where $\Delta\omega$ and t'_k are, respectively, the frequency shift [Eq. (6b)] and the event time [Eq. (8)] of the reflected photon k. Figure 6 presents the simulated spectra for the energy flux of the incident beam [curve (a)] and of the reflected beam, for different values of the ratio $\gamma_{\rm rad}/\gamma$

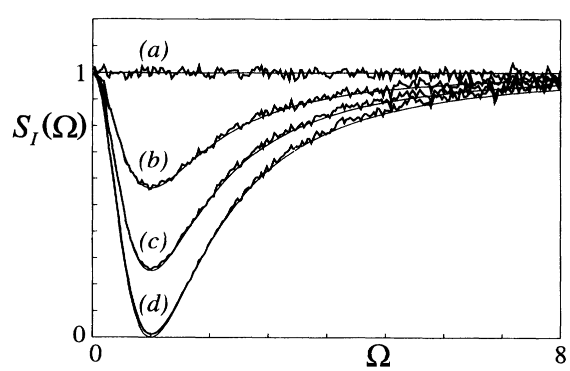

FIG. 3. Simulated intensity spectra for the incident (a) and reflected beams, for different damping rate $\gamma_{\rm rad}$ due to radiation pressure. Curves (b), (c), and (d) correspond to $\gamma_{\rm rad}/\gamma=\frac{1}{4}$, $\frac{1}{2}$, and 1, respectively (γ total damping rate). Frequencies are normalized to the mechanical eigenfrequency Ω_M. Smooth curves represent analytical results.

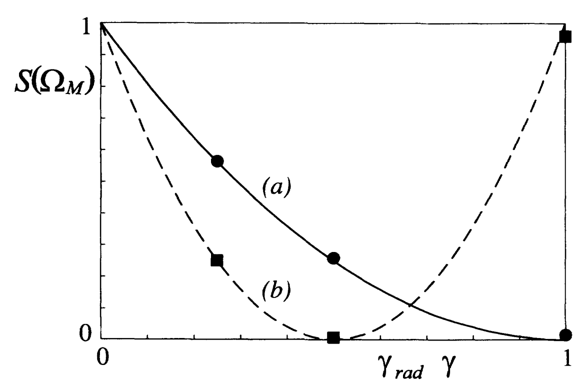

FIG. 4. Optimum noise (noise at the mechanical eigenfrequency Ω_M) as a function of the ratio between the damping $\gamma_{\rm rad}$ due to radiation pressure and the total damping γ, for the intensity [curve (a)] and for the energy flux [curve (b)]. Points and curves are simulated and analytical results, respectively.

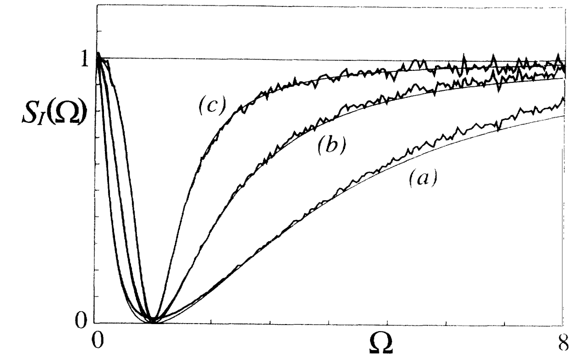

FIG. 5. Effect of the quality factor Q on the intensity spectrum of the reflected beam (for $\gamma_M=0$). Curves (a), (b), and (c) are simulated results for $Q=\frac{1}{4}$, $\frac{1}{2}$, and 1, respectively. Frequencies are normalized to the mechanical eigenfrequency Ω_M. Smooth curves represent analytical results.

[curves (b)–(d)]. It appears that the energy fluctuations can also be reduced for frequencies about the mechanical eigenfrequency Ω_M. The dependence of the reduction on the ratio $\gamma_{\text{rad}}/\gamma$ is, however, different from the one obtained for the photon flux. In particular, there is no noise reduction when γ_{rad} is much larger than γ_M, that is, when the photon noise reduction is optimized (see Fig. 4). The Doppler shift $\Delta\omega$ experienced by every photon exactly compensates in this case the modification of the photon flux. The optimum energy-noise reduction is reached when the two damping constants are equal, that is, for $\gamma_{\text{rad}}=\gamma_M$ [curve (c) in Fig. 6].

The computation time evolves as the number of photons and it is not possible to indefinitely increase this number. Although these numerical simulations give a simple interpretation of noise reduction in terms of temporal redistribution of photons, they are limited to small photon fluxes. In the case of high photon fluxes, the dynamic equations can be approximated to obtain compact analytical expressions.

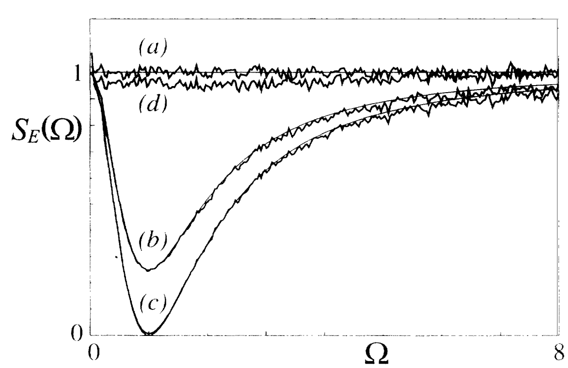

FIG. 6. Simulated spectra for the energy flux of the incident (a) and reflected beams. Curves (b), (c), and (d) correspond to $\gamma_{\text{rad}}/\gamma=\frac{1}{4}$, $\frac{1}{2}$, and 1, respectively. Frequencies are normalized to the mechanical eigenfrequency Ω_M. Smooth curves represent analytical results.

III. ANALYTIC DERIVATION

In this section, we derive analytic expressions for the noise spectra of the reflected intensity and energy flux. We assume that the mirror is not submitted to fluctuations, except those associated with radiation pressure (this assumption will be given up in Sec. IV). Its evolution equation is then given by

$$\ddot{x}(t)+\gamma_M\dot{x}(t)+\Omega_M^2 x(t)=\frac{1}{M}F_{\text{rad}}(t)\ , \tag{14}$$

where γ_M is the damping rate of the mirror suspension, Ω_M its mechanical eigenfrequency, and F_{rad} the radiation pressure force. This force is proportional to the momentum exchange during the reflection of a single photon and to the number of photons reflected per second:

$$F_{\text{rad}}(t)=2\frac{\hbar\omega_0}{c}\frac{1-v(t)}{1+v(t)}I_{\text{in}}[t-x(t)/c] \tag{15a}$$

with

$$v(t)=\dot{x}(t)/c\ . \tag{15b}$$

The velocity dependence in this equation is a consequence of the Doppler shift and of the modification of the field amplitudes by reflection on a moving mirror [18]. The transformation of the intensities may be deduced from the fact that the number of photons is preserved by reflection. More precisely, the photon number is an adiabatic invariant [18] and the instantaneous incident and reflected intensities, normalized as numbers of photons per second, are equal in the mirror's proper frame. Using the Lorentz transformations from the proper frame to the laboratory frame, one deduces that the instantaneous intensity $I_{\text{out}}(t)$ of the reflected beam in the laboratory frame, measured as a number of photons per second, is related to the incident intensity $I_{\text{in}}(t)$ by

$$I_{\text{out}}[t+x(t)/c]=\frac{1-v(t)}{1+v(t)}I_{\text{in}}[t-x(t)/c]\ . \tag{16}$$

This relation is closely connected to the transformation of frequencies by Doppler effect on a moving mirror. We note that this effect is usually ignored by standard quantum optics approaches. If the optomechanical coupling is, for example, described as a Kerr effect, $v(t)$ is considered as negligible and the reflected intensity is found to be equal to the incident one. Although $v(t)$ is small [see Eq. (15b)], the relation (16) indicates that there is a temporal modification of the intensities. As we will see in the following, $v(t)$ can be of the same order as the relative intensity fluctuations $\delta I_{\text{in}}/\bar{I}_{\text{in}}$, thus leading to a significant modification of the reflected intensity fluctuations.

Equations (14)–(16) allow us to write an input-output relation for the intensities that gives the reflected intensity as a function of the incident one. From these equations, it is possible to determine the statistical properties of the reflected intensity, that is, the cumulants of I_{out} at any order. Assuming that the incident beam is intense, the intensity fluctuations δI_{in} are small compared to the mean intensity $\bar{I}_{\text{in}}$ and simple expressions can be obtained by linearizing the dynamical equations about the mean intensity. At the first order in v and $\delta I_{\rm in}/\bar{I}_{\rm in}$ (assumed to be small), the radiation force [Eq. (15a)] can be decomposed into two parts, a damping force proportional to $\dot{x}(t)$ and a fluctuating force. The evolution equation of the mirror [Eq. (14)] can then be written

$$\ddot{x}(t)+\gamma\dot{x}(t)+\Omega_M^2 x(t)=\frac{c}{2}\gamma_{\rm rad}\frac{\delta I_{\rm in}(t)}{\bar{I}_{\rm in}} , \tag{17}$$

where γ is the total damping rate of the mirror including the damping $\gamma_{\rm rad}$ due to radiation pressure [Eqs. (9) and (11)]. Note that the mirror position $x(t)$ is now defined relatively to the new equilibrium position, taking into account a constant term in the radiation force $(c\gamma_{\rm rad}/2)$ proportional to the mean intensity.

From this equation of motion, the Fourier transform $v[\Omega]$ of the mirror speed is found to be proportional to the incident intensity fluctuations

$$v[\Omega]=\frac{1}{2}\frac{\gamma_{\rm rad}}{\gamma}Y[\Omega]\frac{\delta I_{\rm in}[\Omega]}{\bar{I}_{\rm in}} , \tag{18}$$

where $Y[\Omega]$ is the mechanical admittance of the mirror (normalized to 1 at resonance)

$$Y[\Omega]=\frac{-i\gamma\Omega}{\Omega_M^2-\Omega^2-i\gamma\Omega} . \tag{19}$$

At the first order in v and $\delta I_{\rm in}/\bar{I}_{\rm in}$, the input-output relation for the intensities [Eq. (16)] leads to

$$\bar{I}_{\rm out}=\bar{I}_{\rm in} , \tag{20a}$$

$$\delta I_{\rm out}[\Omega]=\delta I_{\rm in}[\Omega]-2\bar{I}_{\rm in}v[\Omega] . \tag{20b}$$

The mean reflected intensity is equal to the incident one as a consequence of the fact that all photons are reflected by the mirror. The intensity fluctuations are modified, however, due to a temporal redistribution of the photons. From Eqs. (18) and (20b), we obtain

$$\delta I_{\rm out}[\Omega]=\left[1-\frac{\gamma_{\rm rad}}{\gamma}Y[\Omega]\right]\delta I_{\rm in}[\Omega] . \tag{21}$$

The intensity fluctuations are completely suppressed if $(\gamma_{\rm rad}/\gamma)Y[\Omega]=1$ (that is, at frequency Ω_M, if $\gamma_{\rm rad}=\gamma$). From Eqs. (2) and (21), the normalized noise spectrum of the reflected intensity can be written as (the incident beam is at the shot-noise level)

$$S_I^{\rm out}[\Omega]=\left|1-\frac{\gamma_{\rm rad}}{\gamma}Y[\Omega]\right|^2 . \tag{22}$$

Using the expression of the admittance function Y [Eq. (19)], we eventually obtain

$$S_I^{\rm out}[\Omega]=1-\frac{\gamma_{\rm rad}(2\gamma-\gamma_{\rm rad})}{\gamma^2}|Y[\Omega]|^2 . \tag{23}$$

The modification of the intensity noise appears as a product of two terms. The function $Y[\Omega]$ characterizes the frequency response of the mirror. Noise modification is significant only about the mechanical eigenfrequency Ω_M, with a relative bandwidth of the order of the inverse of the quality factor. The fraction in Eq. (23) determines the amplitude of noise reduction. Starting from the shot-noise level for $\gamma_{\rm rad}\ll\gamma_M$ ($\gamma_{\rm rad}/\gamma\approx 0$), the noise of the reflected beam is more reduced as $\gamma_{\rm rad}$ is increased ($\gamma_{\rm rad}/\gamma\to 1$; see Fig. 4). The noise is completely suppressed at the mechanical eigenfrequency if the damping $\gamma_{\rm rad}$ due to the radiation pressure is much larger than the damping γ_M of the mirror suspension. We find the same dependence of the noise reduction on the two parameters $\gamma_{\rm rad}/\gamma$ and Q as the one obtained by the numerical simulation.

We can also derive simple analytical expressions for the energy flux. The energy flux of the reflected beam is related to the incident energy flux and to the mirror speed by the relation

$$E_{\rm out}[t+x(t)/c]=\left[\frac{1-v(t)}{1+v(t)}\right]^2 E_{\rm in}[t-x(t)/c] . \tag{24}$$

Comparing to the input-output relation for the intensities at the first order in v [Eq. (20b)], it appears that the velocity dependence is doubled as a consequence of the Doppler frequency shift

$$\delta E_{\rm out}[\Omega]=\delta E_{\rm in}[\Omega]-4\bar{E}_{\rm in}v[\Omega] . \tag{25}$$

From Eq. (18), we obtain the normalized spectrum for the energy flux of the reflected beam (the incident beam is monochromatic)

$$S_E^{\rm out}[\Omega]=\left|1-2\frac{\gamma_{\rm rad}}{\gamma}Y[\Omega]\right|^2=1-\frac{4\gamma_{\rm rad}\gamma_M}{\gamma^2}|Y[\Omega]|^2 . \tag{26}$$

The shape of the noise spectrum is also given by the admittance function $Y[\Omega]$ [compare to Eq. (23)]. The dependence on the ratio $\gamma_{\rm rad}/\gamma$ is different (Fig. 4) and explains the results obtained in the numerical simulations. There is no noise reduction when $\gamma_{\rm rad}$ is much larger than γ_M, and the optimum noise reduction is reached when the two damping constants are equal (perfect noise reduction at frequency Ω_M for $\gamma_{\rm rad}/\gamma=\frac{1}{2}$). It appears that the energy fluctuations of the light may be transferred to the mirror and dissipated in the presence of the viscous force associated with the damping constant γ_M.

The smooth curves in Figs. 3–6 represent the noise spectra of the reflected intensity and energy flux for the same parameters as in Monte Carlo simulations. The results are observed to be in excellent agreement. The slight discrepancies may be attributed to nonlinear effects associated with the fact that the simulations are not in the high-intensity regime. Better agreement would be obtained by computing the next-order term in the analytical derivation. We note, however, that other effects may appear at this order, in particular those related to the quantum fluctuations of the mirror and to the vacuum radiation pressure [19]. These effects have been ignored in the numerical simulation as well as in the analytical derivation.

The semiclassical model described in this section can be used to determine the phase fluctuations of the reflected field. As the mirror motion is not coupled to the phase of the incident field, the fluctuations added to the phase by the mirror motion are not correlated with the incident phase fluctuations (in the case considered in this paper of a coherent input field) and the reflected phase noise is always larger than the incident one.

IV. THERMAL FLUCTUATIONS

Up to now, we have considered that the mirror motion is sensitive only to the intensity fluctuations. Other fluctuations may be taken into account, in particular thermal fluctuations associated with mirror damping. Quantum fluctuations of the mirror and vacuum radiation pressure effects [19] can be disregarded since they are usually negligible in comparison with thermal fluctuations (the condition $k_B T \gg \hbar\Omega_M$ is fulfilled for usual temperatures). Thermal fluctuations can be accounted for by adding a fluctuating force $F_T(t)$ to the evolution equation [Eq. (17)]. The power spectrum of this force is related to the damping rate γ_M of the mirror suspension and to the temperature T (k_B is the Boltzmann constant)

$$S_T[\Omega]=2M\gamma_M k_B T \ . \tag{27}$$

Solving the evolution equation in the Fourier space, we find that the mirror speed acquires additional fluctuations proportional to F_T,

$$v[\Omega]=Y[\Omega]\left\{\frac{1}{2}\frac{\gamma_{\rm rad}}{\gamma}\frac{\delta I_{\rm in}[\Omega]}{\bar{I}_{\rm in}}+\frac{1}{Mc\gamma}F_T[\Omega]\right\} . \tag{28}$$

As in Sec. III, we derive the output intensity fluctuations at the first order by linearizing the input-output relation for the intensities with respect to the fluctuations $\delta I_{\rm in}$ and F_T. From Eq. (20b) we obtain

$$\delta I_{\rm out}[\Omega]=\left[1-\frac{\gamma_{\rm rad}}{\gamma}Y[\Omega]\right]\delta I_{\rm in}[\Omega] -\frac{2\bar{I}_{\rm in}}{Mc\gamma}Y[\Omega]F_T[\Omega] \ . \tag{29}$$

The first term is proportional to the incident intensity fluctuations. It describes the direct reflection of those fluctuations and the modification of the photon flux due to the displacement of the mirror associated with radiation pressure. This term does not depend on thermal fluctuations and has the same form as in Sec. III [Eq. (21)]. The last term in Eq. (29) describes the modification of the photon flux due to the thermal Brownian motion of the mirror. Since the fluctuations $\delta I_{\rm in}$ and F_T are independent, the normalized noise spectrum of the reflected beam is obtained from Eq. (2) as

$$S_I^{\rm out}[\Omega]=\left|1-\frac{\gamma_{\rm rad}}{\gamma}Y[\Omega]\right|^2+\frac{4\bar{I}_{\rm in}}{M^2c^2\gamma^2}|Y[\Omega]|^2 S_T[\Omega] \ . \tag{30}$$

From the expressions of Y and S_T we eventually obtain

$$S_I^{\rm out}[\Omega]=1-\frac{\gamma_{\rm rad}(2\gamma-\gamma_{\rm rad})}{\gamma^2}|Y[\Omega]|^2 +2\frac{\gamma_{\rm rad}\gamma_M}{\gamma^2}|Y[\Omega]|^2\frac{k_B T}{\hbar\omega_0} \ . \tag{31}$$

Comparing to the equation obtained without thermal fluctuations [Eq. (23)], one finds that the shape of the noise spectrum is not changed and is still determined by the admittance function $Y[\Omega]$. The amplitude of noise reduction is changed however, and the intensity noise is always increased by thermal fluctuations [the last term in Eq. (31) is positive]. This correction is small in practice since $k_B T$ can be made much smaller than $\hbar\omega_0$ (ω_0 is the optical frequency). Furthermore, this correction disappears when the squeezing is optimized, that is, when $\gamma_{\rm rad} \gg \gamma_M$. The effect of quantum fluctuations can be estimated as a corrective term of the order of $\hbar\Omega_M/\hbar\omega_0$, which is even smaller in practice.

V. CONCLUSION

We have shown that the beam reflected by a harmonically suspended mirror exhibits sub-Poissonian photon statistics. We have attributed the noise reduction to a temporal redistribution of photons between the incident and reflected beams. The movable mirror thus appears as a passive photon-noise eater, based on an optomechanical coupling between the field and the mirror.

The efficiency of this system strongly depends on the relative values of the damping rate γ_M of the mirror suspension and the damping $\gamma_{\rm rad}$ due to radiation pressure. Since the existence of damping leads to the presence of associated fluctuations, we have included thermal fluctuations in the analytical derivation. These fluctuations have a negligible effect on the intensity-noise reduction as long as $k_B T$ remains much smaller than the photon energy $\hbar\omega_0$.

The noise reduction is optimum when $\gamma_{\rm rad}$ is much larger than γ_M. This condition seems difficult to achieve in practice since the damping rate of the mirror suspension is generally much larger than the one due to radiation pressure ($\gamma_M \gg \gamma_{\rm rad}$). We may note, however, that the mirror has two distinct functions in this system. First, the mirror position measures the intensity fluctuations of the incident beam. Second, the mirror adds a variable delay to every photon. This system can be considered as a passive correction mechanism [16] in which both the measurement and the correction are realized by a single mirror. More elaborate systems can be envisioned to improve the experimental feasibility. One can split the two functions of the control mechanism by using two different mirrors, one mirror measuring the incident intensity fluctuations and providing a correction signal that drives the position of a second mirror. The condition on the damping rates is then replaced by a condition on the electronic or electromechanical gain of the correction signal. Another possible approach would consist in using a high-finesse resonant cavity built with a second mirror, thus exalting the noise reduction by the cavity finesse [6]. This system is equivalent to a bistable device containing a Kerr medium and it can be described by a Kerr model, in contrast with the system studied in this paper for which a Kerr model does not appropriately describe the photon flux regulation.

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