Electron-beam-induced super-radiant emission from a grating

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(Received 1 June 1993; revised manuscript received 17 February 1994)

An alternative mechanism for Smith-Purcell radiation is proposed. This mechanism may have relevance to recent reports of higher radiation power. The electron beam excites resonant transitions of atomic quantum levels in the optical grating material by the fields of the traversing electrons. The dipole moments of all the atoms which are excited by the same electron radiate in phase with each other and produce "super-radiant radiation." To calculate the radiant intensity due to this process we first calculate the dipole moments of the atoms excited by the classical electrical field of the traversing electron. Assuming that the dipole oscillations are dominated by a collision time T_2 we calculate the classical radiant intensity from the optical gratings due to this process. Sample numerical calculations based on a ruby grating result in substantial radiation levels.

PACS number(s): 42.50.Fx, 41.60.Cr, 52.75.Ms

I. INTRODUCTION

The radiation produced by the interaction of an electron beam and a grating was first measured by Smith and Purcell [1]. They suggested that the radiation was caused by oscillating charge induced in the metal gratings by the electron beam.

In 1961, Salisbury [2] reported a measurement of Smith-Purcell radiation with an intensity that exceeded substantially those of other experimental reports as well as predictions of theoretical calculations. Recently, Shih *et al.* [3] also reported measurements of higher intensities than expected according to classical Smith-Purcell calculations. These prompted suggestions for alternative theoretical explanations for the measured radiation. A model based on interaction of the electrons with the space-charge field produced by other electrons scattered from the grating was suggested by Salisbury [4]. Recently Chang and McDaniel [5] proposed another mechanism based on bremsstrahlung amplified by interaction with the diffracted electron wave function. Reservations concerning the validity of these models were raised by Gover *et al.* [6] and Gover and Halperin [7].

In an attempt to find an explanation for enhanced emission of Smith-Purcell radiation (beyond the basic model of diffraction off the gratings), we explore here theoretically a scheme in which the Smith-Purcell radiation is enhanced by resonant excitation of radiative transitions in the grating material. According to this model the electric field of an electron traversing near the grating excites electronic quantum transitions of atoms in the grating. The dipole moments of atoms excited by the same traversing electron radiate with a definite relative phase that is determined by the velocity of this electron. Consequently the traversing electrons emit coherent super-radiant [8] radiation in directions determined by the grating periodicity according to the Smith-Purcell radiation condition.

For simplification we use in the present analysis a model of a two-level atom. This model would be rigorous only for a situation in which the grating material is characterized by distinct quantum levels where only the lower level is occupied (e.g., a three-level laser material like ruby and possibly direct band gap semiconductor material like Ga:As). In amorphous or polycrystalline materials and metals (from which commercial gratings are usually made) an extension of this model may be required.

An electron passing over the grating creates a transient electric field experienced by the atoms of the grating material (Fig. 1). The atom is much smaller than a characteristic length in which the field changes. Therefore the atom is affected by a spatially constant time-dependent electric field. This electric field excites the atom which is assumed to be initially in the ground state. The excitation of the atom is calculated by using time-dependent first-order quantum-mechanical perturbation theory. The electric excitation is used to calculate the induced dipole moment in the grating and the overall radiation.

II. PERTURBATION CALCULATION

The expression used for the electric field of the traversing electron is the classical field relativistic expression [9] (mks units):

$$E_z = \frac{1}{4\pi\varepsilon_0} \frac{-q\gamma vt}{(D^2+\gamma^2 v^2 t^2)^{3/2}} , \tag{1}$$

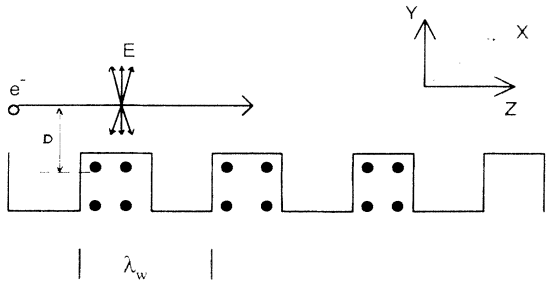

FIG. 1. Schematic diagram of radiation process.

$$E_\perp = \frac{1}{4\pi\varepsilon_0}\frac{\gamma q D}{(D^2+\gamma^2 v^2 t^2)^{3/2}} , \tag{2}$$

$$D = \sqrt{x^2+y^2} , \tag{3}$$

where it is assumed that the electron of charge $q=-|q|$ and velocity v traverses along the z axis [coordinates $(0,0,vt)$] with an impact parameter D relative to an atom which is located at coordinates $(x,y,0)$ (see Fig. 1). The impact parameter is the minimal distance between the atom and the electron trajectory. For simplicity we consider in our analysis only the dominant transverse component of the electric field $E_\perp$. We first calculate the excitation amplitudes of the quantum states in the atom. The calculated amplitudes will be used in the next step for calculation of the radiation from the different atoms.

The wave function of the excited atom is expanded in terms of the eigenfunctions of the unperturbed atom u_k:

$$\psi = \sum_k a_k(t) u_k e^{-i\omega_k t} , \tag{4}$$

where $\omega_k \equiv E_k/\hbar$, in our case, $k=1,2$.

The excitation level of the atom is calculated by solving the time-dependent perturbation equation for a two-level atom:

$$\dot{a}_2(t) = \frac{-i}{\hbar}\langle 2|H'|1\rangle a_1(t) e^{i\omega_{21}t} , \tag{5}$$

$$H' = q\mathbf{E}\cdot\mathbf{r} , \tag{6}$$

where H' is the perturbation interaction Hamiltonian.

$$a_2(t) = a_2(-\infty) + \frac{-i}{\hbar}\int_{-\infty}^{t}\langle 2|q\mathbf{E}\cdot\mathbf{r}|1\rangle a_1(t) e^{i\omega_{21}t} dt . \tag{7}$$

This equation is solved under the initial condition of an unperturbed atom whose electron is at the lower energy level as shown in Fig. 2, where

$$\omega_{21} = \frac{E_2 - E_1}{\hbar} . \tag{8}$$

The complex excitation amplitudes of the states are in a first-order approximation:

$$a_1(t) = a_1(-\infty) = 1 , \tag{9}$$

$$a_2(t) = \frac{-i}{\hbar}\int_{-\infty}^{t}\langle 2|q\mathbf{E}\cdot\mathbf{r}|1\rangle e^{i\omega_{21}t} dt . \tag{10}$$

We insert Eq. (2) for the electric field in Eq. (10). The longitudinal (z) component of the electric field is neglected because it changes sign during the passage of the electron above the atom (though in certain cases this may produce a resonant effect). The excitation level of the atom due to the passage of the traversing electron can now be calculated explicitly by performing the integration. In order to be able to perform the integration analytically, we replace the Lorentzian shape of the electric excitation impulse [Eq. (2)] with a Gaussian shape impulse of the same width $\tau = D/\gamma v$:

$$E'_\perp = \xi\frac{1}{4\pi\varepsilon_0}\frac{\gamma q}{D^2} e^{-(t/\tau)^2} , \tag{11}$$

where the normalization factor ξ is chosen so that $\int_{-\infty}^{\infty} E'_\perp dt = \int_{-\infty}^{\infty} E_\perp dt$, which results in $\xi = 2/\sqrt{\pi} \approx 1$. This approximation makes it possible to obtain an explicit analytical expression for the excitation amplitude of the upper level:

$$a_{2f} = \frac{1}{4\pi\varepsilon_0}\frac{ie\,\mathbf{r}_{21}\cdot\mathbf{d}}{\hbar D^2 v} e^{-(D\omega_{21}/\gamma v)^2} , \tag{12}$$

where $\mathbf{r}_{ij} \equiv \langle i|\mathbf{r}|j\rangle$, and $\hat{\mathbf{d}} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}}$ is a unit vector pointing from the electron trajectory (z) axis to the atom.

The interaction time between the traversing electron and the atom in the grating is much shorter than the characteristic time of radiation by the excited atom. The radiation during the time of passage of the electron is, therefore, neglected. We assume that after the electron passes, the atom's excitation level (and therefore the overall dipole moment) decays exponentially with a relatively long time constant T_2. This is the characteristic time after which atoms excited by the same electron lose coherence with one another due to phonon interaction (this terminology is borrowed from common laser-physics terminology [8]).

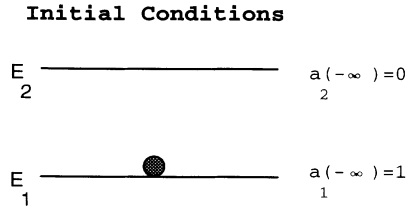

FIG. 2. Illustration of a grating atom's initial conditions.

III. POLARIZATION CALCULATION

We now use the excitation amplitude of the two quantum states for calculating the radiation process. In order to do that, we compute the dipole moment of each atom with the initial condition of a coherent state in the sense defined by Scully *et al.* [10]. The quantum states have distinct phase relations between their complex amplitudes. These amplitudes were calculated in the previous step where we calculated the atom excitation by first-order perturbation theory [$a_1(0^+)=1$ and $a_2(0^+)=a_{2f}$ as given in Eqs. (9) and (12)]. We neglect any contribution to the radiation process during the short excitation time.

The dipole moment of a single atom is calculated to be

$$p = -e\langle\psi|\mathbf{r}|\psi\rangle , \tag{13}$$

where $\mathbf{r}$ is the position operator of the electron in the atom. In our simplified model we take the collision time into consideration by multiplying the upper-state wave function by a time decaying exponent. This is a widely accepted approximation (a more rigorous approach would have been to do a statistical calculation of the distribution of collisions over time and calculate the result according to the mean value). The coherent state of the atom after the excitation stage is thus given by

$$\psi \cong e^{-i\omega_1 t}u_1 + a_{2f}e^{-i\omega_2 t}e^{-t/T_2}u_2 . \tag{14}$$

We note that in this approximation the normalization of the wave function is not exact.

The time-dependent dipole moment of the excited atom is given as the expectation value of $-e\mathbf{r}$ using the wave function of the excited atom (in the coordinate picture) [Eq. (14)].

$$\begin{aligned}\mathbf{p}&=-e\left\langle e^{-i\omega_1 t}u_1+a_{2f}e^{-i\omega_2 t}e^{-t/T_2}u_2\left|\mathbf{r}\right|1e^{-i\omega_1 t}u_1+a_{2f}e^{-i\omega_2 t}e^{-t/T_2}u_2\right\rangle\\&=-e(\mathbf{r}_{11}+|a_{2f}|^2\mathbf{r}_{22}e^{-2t/T_2}+a_{2f}^{*}e^{i\omega_{2t}t}e^{-t/T_2}\mathbf{r}_{21}+a_{2f}e^{-i\omega_{21}t}e^{-t/T_2}\mathbf{r}_{12})\\&=-e\mathbf{r}_{11}-e|a_{2f}|^2\mathbf{r}_{22}e^{-2t/T_2}-a_{2f}^{*}\mathbf{r}_{21}e^{i\omega_{21}t}e^{-t/T_2}-ea_{2f}\mathbf{r}_{12}e^{-i\omega_{21}t}e^{-t/T_2}\ .\end{aligned}\tag{15}$$

We may view the process as consisting of two main stages: (a) an excitation period starting at $t=0$ in which the electron passes next to the atom and excites it, lasting a time duration $\approx\tau=D/\gamma v$; and (b) a radiation period following the excitation during which the atom radiates until losing coherence due to collisions in a time duration $T_2\gg\tau$. We assume that the two stages are distinct and that no radiation is produced during the excitation period. While this assumption is not fully accurate, it should not affect the final results due to the very short time length of the excitation stage. Consequently we multiply the dipole moment calculated above with a step function $\eta(t)$ where $t=0$ is the time when the atom is excited (presumably instantaneously). We then take the time derivative of the polarization and consider only the significant part, which is the one oscillating at the optical frequency ω_{21}:

$$\begin{aligned}\frac{d\mathbf{p}}{dt}&=\frac{2}{T_2}ea_{2f}^2\mathbf{r}_{22}e^{-2t/T_2}+ie\omega_{21}a_{2f}\mathbf{r}_{12}e^{-i\omega_{21}t}e^{-t/T_2}+\text{c.c.}\\&\cong ie\omega_{21}a_{2f}\mathbf{r}_{12}e^{-i\omega_{21}t}e^{-t/T_2}+\text{c.c}\ .\end{aligned}\tag{16}$$

We now substitute the result of Eq. (12):

$$\begin{aligned}\frac{d\mathbf{p}}{dt}=\frac{1}{4\pi\epsilon_0}\hat{\mathbf{d}}\frac{ie^3\omega_{21}|r_{21}|^2}{\hbar D^3 v}e^{-(D\omega_{21}/\gamma v)^2}\\\times e^{i\omega_{21}t-t/T_2}\eta(t)+\text{c.c.}\end{aligned}\tag{17}$$

Note that the above expression was calculated with the simplifying assumption of an atom with isotropic optical response, so that the polarization $\mathbf{p}$ is in the direction of the field $\mathbf{E}_\perp(\hat{\mathbf{d}})$. We now make an additional approximation by neglecting the lateral component (x) of $\mathbf{p}$, asserting that the lateral polarization components produced on the atom by the different electrons in a finite-width electron beam cancel each other on the average. Also note that the above expression was calculated for an atom located at position $(x,y,0)$ and excited at time $t=0$. For an atom located at coordinate $\mathbf{r}=(x,y,z)$ and excited at time $t=z/v$ we calculate

$$\begin{aligned}\frac{d\mathbf{p}_y(\mathbf{r},t)}{dt}=\frac{1}{4\pi\epsilon_0}\frac{ie^3\omega_{21}|r_{21}|^2}{\hbar D^2 v}\cos\varphi e^{-(D\omega_{21}/\gamma v)^2}\\\times e^{i\omega_{21}(t-z/v)}e^{(t-z/v)/T_2}\\\times\eta\left(t-\frac{z}{v}\right)+\text{c.c.}\ ,\end{aligned}\tag{18}$$

$$\cos\varphi\equiv\frac{y}{D}\ ,\tag{19}$$

$$\begin{aligned}\frac{dp_y(\mathbf{r},t)}{dt}=\frac{1}{4\pi\epsilon_0}\frac{ie^3\omega_{21}|r_{21}|^2 y}{\hbar D^3 v}e^{-(D\omega_{21}/\gamma v)^2}\\\times e^{i\omega_{21}(t-z/v)}e^{-(t-z/v)/T_2}\\\times\eta\left(t-\frac{z}{v}\right)+\text{c.c.}\end{aligned}\tag{20}$$

Note that $D=D(x,y)$. We now use the definition of polarization density as follows:

$$\mathbf{P}(\mathbf{r},t)\equiv n(\mathbf{r})\mathbf{p}(\mathbf{r},t)\ ,\tag{21}$$

where $n(\mathbf{r})$ is the density of radiators (atoms) in the matter. The polarization current density $\mathbf{J}(\mathbf{r},t)$ is then

$$\begin{aligned}\mathbf{J}(\mathbf{r},t)&\equiv\frac{d\mathbf{P}}{dt}\\&=n(\mathbf{r})\hat{\mathbf{y}}\frac{ie^3\omega_{21}r_{21}^2 y}{\hbar D^3 v}e^{-(D\omega_{21}/\gamma v)^2}e^{i\omega_{21}(t-z/v)}\\&\quad\times e^{(t-z/v)/T_2}\eta\left(t-\frac{z}{v}\right)+\text{c.c.}\end{aligned}\tag{22}$$

The density of radiators $n(\mathbf{r})$ is a function of the following form:

$$n(\mathbf{r})=n_0 q(\mathbf{r})\ ,$$

$$q(\mathbf{r})=\begin{cases}1, & j\lambda_w\le z<(j+\frac{1}{2})\lambda_w,\ y_1-h\le y\le y_1\\0, & \text{otherwise}\ ,\end{cases}\tag{23}$$

where $j=0,1,2,\ldots$, and h is the grating rulings' depth. The grating base ($y<y_1-h$) is neglected even though there are certainly radiators there, because that part of the grating is not periodic and therefore cannot contribute to the overall radiation.

We take the Fourier transform of the current density:

$$\tilde{\mathbf{J}}_\omega(\mathbf{r},\omega)\equiv\int\mathbf{J}(\mathbf{r},t)e^{-i\omega t}dt\ ,\tag{24}$$

$$\begin{aligned}\tilde{\mathbf{J}}_\omega(\mathbf{r},\omega)=\int_{z/v}^{\infty}\left[n(\mathbf{r})\hat{\mathbf{y}}\frac{1}{4\pi\epsilon_0}\frac{ie^3\omega_{21}r_{21}^2 y}{\hbar D^3 v}e^{-(D\omega_{21}/\gamma v)^2}\right.\\\left.\times e^{i\omega_{21}(t-z/v)}e^{-(t-z/v)/T_2}+\text{c.c.}\right]e^{-i\omega t}dt\ .\end{aligned}\tag{25}$$

We define

$$t' \equiv t - z/v \ . \tag{26}$$

$$\tilde{\mathbf{J}}_\omega(\mathbf{r},\omega) = n(\mathbf{r})\hat{\mathbf{y}} \frac{1}{4\pi\varepsilon_0} \frac{ie^3\omega_{21}r_{21}^2 y}{\hbar D^3 v} e^{-(D\omega_{21}/\gamma v)^2} e^{-i\omega z/v} \times \int_0^\infty (e^{i\omega_{21}t'} e^{-t'/T_2} + \text{c.c.}) e^{i\omega t'} dt' \ , \tag{27}$$

$$\tilde{\mathbf{J}}_\omega(\mathbf{r},\omega) = n(\mathbf{r})\hat{\mathbf{y}} \frac{1}{4\pi\varepsilon_0} \frac{ie^3\omega_{21}r_{21}^2 y}{\hbar D^3 v} \times \left[\frac{1}{\omega_{21}-\omega-i/T_2} e^{-(\omega_{21}\tau)^2} e^{-i\omega z/v} e^{-z/vT_2} + \frac{1}{\omega_{21}+\omega-i/T_2} e^{-(\omega_{21}\tau)^2} \times e^{-i\omega z/v} e^{-z/vT_2} \right] , \tag{28}$$

where

$$\tau = \frac{D(x,y)}{\gamma v} \ .$$

IV. RADIATION CALCULATION

The overall spectral energy radiated by a single electron can now be calculated by inserting the Fourier transformed current density into the classical radiation formula [11].

$$\frac{\partial U_\omega}{\partial\Omega} d\omega = \frac{1}{4\pi} \left[\frac{\mu_0}{\varepsilon_0} \right]^{1/2} \left| \int (\tilde{\mathbf{J}}_\omega \times \mathbf{k}) e^{-i\mathbf{k}\cdot\mathbf{r}} dV \right|^2 d\omega \ , \tag{29}$$

where $\tilde{J}_\omega$ is the Fourier transform of the current density and $\mathbf{k}$ is the radiation wave-vector. The integration is extended to all the atoms in the grating—the entire radiating volume. Note that $\tau = \tau(x,y)$ in (28), and it limits the effective interaction volume, and $n(\mathbf{r}) = n_0 q(\mathbf{r})$ [Eq. (23)] introduces the effect of the grating periodicity:

$$\frac{\partial U_\omega}{\partial\Omega} d\omega = \frac{e^2\omega_{21}^2}{36\pi^3 c\varepsilon_0^3\hbar^2 v^2} n_0^2 \times \left| \frac{k\cos\theta}{\omega_{21}-\omega-i/T_2} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \frac{y}{D^3} e^{-i(k_x x + k_y y)} e^{-(\omega_{21}\tau)^2} \int_0^L dz q(\mathbf{r}) e^{i(\omega - k_z v + i/T_2)(z/v)} + \frac{k\cos\theta}{\omega_{21}+\omega-i/T_2} \int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy \frac{y}{D^3} e^{-i(k_x x + k_y y)} e^{-i(k_x x + k_y y)} e^{-(\omega_{21}\tau)^2} \int_0^L dz\, q(\mathbf{r}) e^{i(\omega - k_z v + i/T_2)(z/v)} \right|^2 d\omega \ . \tag{30}$$

The calculated result for the spectral radiated energy is

$$\frac{\partial U_\omega}{\partial\Omega} = \frac{e^2}{36\pi^3 c\varepsilon_0^3\hbar^2} \frac{|r_{21}|^4}{(\omega-\omega_{21})^2 + \frac{1}{T_2^2}} n_0^2 N_w (kd\cos\theta)^2 \times |I_\perp(k_x,k_y)|^2 |I_z(k_z)|^2 \ , \tag{31}$$

$$I_\perp(k_x,k_y) \equiv \int_{-\infty}^{\infty} dx \int_{y_1-h}^{y} dy \frac{y}{D^3} e^{-i(k_x x + k_y y)} e^{-(\omega_{21}\tau)^2} \ , \tag{32}$$

$$I_z(k_z) \equiv \int_0^L dz q(\mathbf{r}) e^{i(\omega - k_z v + i/T_2)(z/v)} = \sum_{j=0}^{N_w} \int_{j\lambda_w}^{(j+1/2)\lambda_w} dz\, e^{i(\omega - k_z v + i/T_2)(z/v)} \ , \tag{33}$$

where θ is the elevation angle of radiation and N_w is the number of grating periods involved in a coherent radiation process.

The integrals I_z and $I_\perp$ are considered next:

$$|I_z(k_z)| = \frac{1}{N_w} \frac{\sin\left[\left[\frac{N_w}{2} \right] \left[\omega\frac{\lambda_w}{v} - k_z\lambda_w + i\frac{\lambda_w}{vT_2} \right] \right]}{\sin\left[\frac{1}{2} \left[\omega\frac{\lambda_w}{v} - k_z\lambda_w + i\frac{\lambda_w}{vT_2} \right] \right]} \ . \tag{34}$$

We assume that T_2 is sufficiently long so that the imaginary part of the argument of Eq. (34) is negligible relative to the real part.

$$I_z(k_z) \cong \frac{1}{N_w} \frac{\sin\left[\left[\frac{N_w}{2} \right] \left[\omega\frac{\lambda_w}{v} - k_z\lambda_w \right] \right]}{\sin\left[\frac{1}{2} \left[\omega\frac{\lambda_w}{v} - k_z\lambda_w \right] \right]} \ . \tag{35}$$

$I_z(k_z)$ reaches its maximal value when

$$\left[\frac{\omega}{v} - k_z \right] \lambda_w = 2m\pi \ ,$$

$$\lambda_m = \frac{\lambda_w}{m} \left[\frac{c}{v} - \cos\theta \right] \ , \quad m = 1,2,\ldots \ . \tag{36}$$

This is the Smith-Purcell radiation condition.

The integral of $I_\perp$ cannot be calculated analytically in a straightforward manner. A numerical calculation was performed. In order to perform the numerical calculation, the above equation was written with variables that are dimensionless:

$$I_\perp(K_x,K_y) = \int_{-\infty}^{\infty} du \int_{y_1'-h'}^{y_1'} dv \frac{v}{(u^2+v^2)^{3/2}} e^{i(uK_x + vK_y)} \times e^{-(u^2+v^2)(\gamma\beta)^2} \ ,$$

$$u = kx, \quad v = ky, \quad h' = kh \ , \quad y_1' = ky \ ,$$

$$K_x = \frac{k_x}{k}, \quad K_y = \frac{k_y}{k} \ , \quad k = 2\pi/\lambda_{21} \ .$$

The results are as shown in the graph of Fig. 3 for parameters $\beta=0.5$, $y_1=0$ (grazing electron). It may be seen that the results of the numerical calculations have an upper bound of the order of 1 and the dependence on transverse radiation angles K_x, K_y is small.

Further calculations were done to see the effect of the electron beam width on the radiation. These effects can be seen through the dependence of the integral $I_\perp$ on the distance of the electron above the grating: y_1. $I_\perp$ is a decaying function of y_1 with a decay length $\gamma\beta\lambda_{21}/2\pi$. This means that traversing electrons which pass above the grating with $y_1 \gg \gamma\beta\lambda_{21}/2\pi$ make a negligible contribution to the integral and to the measured radiation. The spectral radiant intensity which would be emitted from an electron beam which fills up uniformly a layer of thickness w above the grating is proportional to the square of the integral of $I_\perp$ over the beam thickness (from 0 to w). This is displayed in Fig. 4, where the parameter $\int_0^{w'} I_\perp dy_1'$ is drawn as a function of $w'=w2\pi/\lambda_{21}$, and in Fig. 5 where the parameter $(1/w')\int_0^{w'} I_\perp dy_1'$ is drawn. In both cases, $K_x=K_y=0$ and $\beta=0.5$. In an experiment in which a uniform density sheet electron beam traverses above the grating, filling up the space from the grating face to a thickness w (controlled by an aperture), the radiant intensity is proportional to the curve in Fig. 4. In an experiment in which the beam thickness w is varied by refocusing the electron beam, keeping the total current in the beam constant, the radiant intensity would be proportional to the curve in Fig. 5. In both cases it is shown that, like other Smith-Purcell radiation mechanisms, the effective range of electron interaction with the grating is $\gamma\beta\lambda_{21}/2\pi$.

V. RESULTS

The spectral radiant energy intensity [Eq. (30)] includes a term which is resonant in the energy-level difference between the two atomic levels. The "explosion" of the resonance is damped by the coherence lifetime of the excited atom T_2. The radiated energy is dependent on the number of radiators squared, which is a typical result for super-radiant processes. It is also proportional to the number of grating periods which are involved in the process. This is a result which is typical of Smith-Purcell processes. As shown in Eq. (36), the resonance is achieved when the radiation angle, wavelength, and grating period fulfill the Smith-Purcell radiation condition.

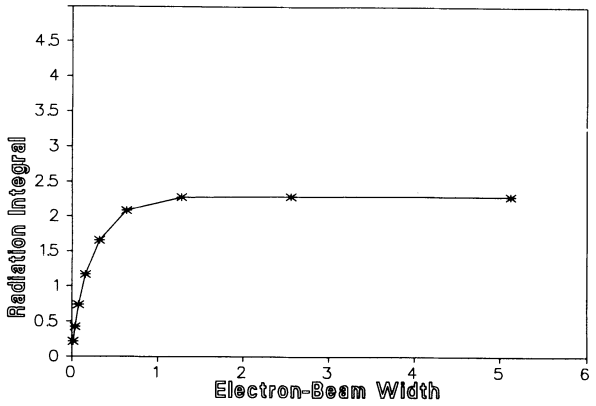

FIG. 4. Graph of radiation integral $\int_0^{w'} I_\perp(K_x,K_y)dy_1'$ vs w'—the electron beam width in units of $\lambda_{21}/2\pi$. Here $K_x=K_y=0$, $\beta=0.5$.

Super-radiant radiation is coherent radiation from atoms that emit in phase with each other either because they are close to each other (within a wavelength range) or because of the periodic structure that has a "phase matching" effect. It is instructive to estimate the number of atoms that are made to emit coherently with each other in this process in order to assess the enhancement relative to incoherent spontaneous emission.

We proceed to calculate the number of radiators involved in the process. The characteristic volume of the grating that is involved in the radiation process is in the shape of half a cylinder. The radius of the cylinder is given by the penetration depth of the electron stimulation into the grating. This can be seen from Eq. (12):

$$R \simeq \frac{\gamma v}{\omega_{21}} , \tag{37}$$

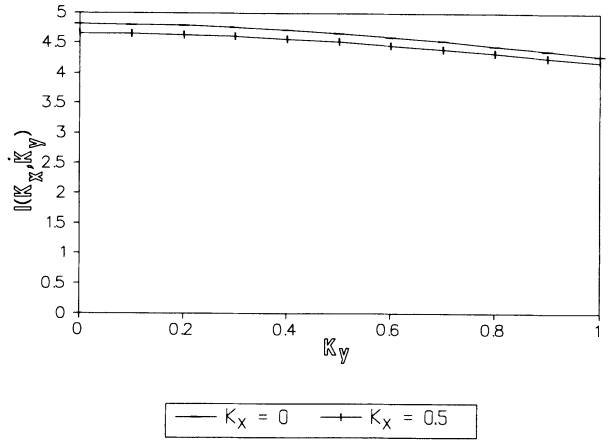

FIG. 3. Graph of angular distribution of radiation $I(K_x,K_y)$.

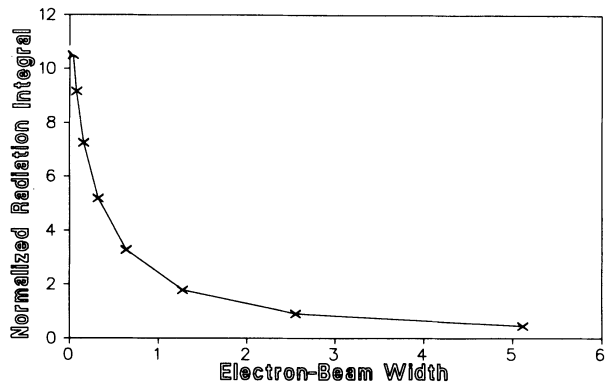

FIG. 5. Graph of the normalized radiation integral $\int_0^{w'} I(K_x,K_y)dy_1'/w$ vs w'—the beam width in units of $\lambda_{21}/2\pi$. Here $K_x=K_y=0$, $\beta=0.5$.

where R is the radius of the half cylinder or the penetration depth.

The length of the interaction region is $L=vT_2$. This is the distance the electron will travel during the time that the coherence of the excited atoms is maintained. Thus, when the electron reaches the end of this region, the atoms at its beginning just go out of coherence. Thus the volume V of radiators is

$$V=\tfrac{1}{2}\pi R^2 L \cong \frac{\pi\gamma^2 v^3 T_2}{2\omega_{21}^2}$$

and the characteristic number of radiators which emit coherently is

$$N \cong n_0 \frac{\pi\gamma^2 v^3 T_2}{2\omega_{21}^2} .$$

We see that the number of radiators involved in the radiation process and with it the radiated energy will be strongly dependent on the velocity of the stimulating electron and on the coherence lifetime of the grating material.

A sample calculation of such a process was done assuming a grating made of ruby. This calculation gave the result of relatively substantial radiation. To calculate the result for ruby we used the data

$$\lambda=0.69\ \mu\text{m} ,$$

$$n=1\times10^{25}\ \text{m}^{-3}\ \ (\text{doping level}\sim10\ \text{ppm}) ,$$

$$t_{\text{spont}}=3\times10^{-3}\ \text{sec} ,$$

and to get the value of r_{21} the following formula was used [8]:

$$\omega_{\text{spont}}=\frac{e^2\omega_0^3 n^3}{3\pi c^3\hbar\varepsilon}|r_{21}|^2\cong\frac{1}{t_{\text{spont}}} . \tag{38}$$

The result for the calculated radiation is

$$\frac{\partial\nu_q}{\partial\Omega}=1\times10^{-4}\ \text{photon/sterad-electron} \tag{39}$$

and the number of radiators involved is $N\cong1\times10^7$. This radiation level is of the same order of magnitude as that measured in the experiment conducted by Gover *et al.* [6] at Tel-Aviv University.

Thus quite a substantial radiation level results theoretically from this process due to the fact that the number of "super-radiant" radiators involved is indeed quite large.

VI. SUMMARY

We proposed a quantum-mechanical model for a process that includes quantum excitation of atoms by traversing electrons and "super-radiant" radiation of those atoms from a periodical grating. It was shown that such a process radiates according to the Smith-Purcell radiation rule. For this process the radiant intensity levels were derived and calculated through a classical formula. As expected in a "super-radiance" process, the radiation intensity is proportional to the square of the radiators' density. The number of radiators (excited atoms) that emit coherently with each other, as result of excitation by a single traversing electron, was calculated to be quite large. Radiation levels were calculated for a grating made of ruby and shown to be quite significant. For future study and experimentation, we suggest that it would be interesting to conduct such an experiment with a pure specimen of a two-quantum-level crystal such as gallium arsenide. Due to the square law dependence of the radiation intensity on the concentration of radiators [Eq. (31)] such an experiment may yield high radiation levels.

ACKNOWLEDGMENTS

This work was sponsored in part by grants from the U.S.–Israel Science Foundation and the Israeli Academy of Sciences. The authors would like to thank Professor Shlomo Rushin and Professor Aaron Kasher for fruitful discussions.

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