PHYSICAL REVIEW A VOLUME 50, NUMBER 4 OCTOBER 1994

Non-Markovian analysis of coherence in a driven two-level atom

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(Received 17 February 1994; revised manuscript received 24 May 1994)

Recent femtosecond processes [Phys. Rev. Lett. **66**, 2464 (1991)] confirm the importance of non-Markovian effects since the dynamical characteristic times of the system become of the same time scale as the reservoir. We analyze here the memory effects in the density-matrix coherence for a driven two-level atom in the stationary regime. The coherence parameter η is written in terms of the detuning, the strength of the external driving force, and the temperature as the reservoir. We verify that the memory effects, present in the density matrix, are enhanced by the external driving force. We introduce the Shanon information entropy and we verify that, while in the Markovian approximation the entropy decreases monotonically as function of the detuning, in the non-Markovian approach it is characterized by a minimum, which enables us to differentiate between the two processes.

PACS number(s): 42.50.Gy, 42.50.Ar

I. INTRODUCTION

Non-Markovian effects have received special attention in the past years, mainly in optics and radiation-matter interaction subjects, either in predicting novel effects or due to the necessity to go beyond the Markovian approximation in experiments involving femtosecond processes. Among the experimental papers we can cite recent ones. Tchénio *et al.* [1] prepared a non-Markovian atomic excitation process, with adjustable memory time, using correlated laser pulses and they verified that under strong-field conditions the atoms are not able to keep memory of the field phase and amplitude over a time interval larger than the coherence time. Considering femtosecond experiments [2–6], non-Markovian behavior appears in the optical dephasing of molecules in solution, since the dynamics of the thermalized environment may occur on the same time scale of the system.

Concerning the theoretical approach, Lewenstein, Mossberg, and Glauber [7] predicted the suppression of spontaneous emission related to the decay of cavity atoms in the presence of a strong driving field, thus modifying the spectrum of resonance fluorescence. Villaeys, Vallet, and Lin [8] studied the non-Markovian effects in the atomic absorption band shape for the transient and steady-state regimes; they conclude that in the steady state the appearance of the non-Markovian effects are washed out and therefore they cannot be probed, but in the transient regime these effects are perceptible. In this same line Gangopadhyay and Ray [9] constructed a non-Markovian master equation by considering density matrices with small delay time τ and then expanding up to first power in this parameter; they obtained the same qualitative results as in Ref. [8] in the transient regime; furthermore, they extend their formalism to nonlinear systems.

In the present work we are interested in analyzing the non-Markovian coherence in a driven two-level atom coupled to a thermal reservoir in the steady state and for nonzero temperature. For such we investigate the coherence parameter as well as the Shannon information entropy (SIE) as a function of the detuning and intensity of the probe field for two different memory functions of the reservoir.

We verify that the coherence parameter and the SIE are sensible in their shape to the correlation time of the reservoir and the memory effects are enhanced by the strength of the probe field. Even when the initial density matrix has zero coherence it asymptotically has a finite coherence that will depend on the detuning, the intensity of the external field, the correlation time τ, and the temperature of the reservoir. More importantly, one must pay attention to the fact that only the SIE permits one to differentiate between the Markovian and the non-Markovian processes due to a distinct feature of the latter.

The paper is organized as follows. In Sec. II the generalized master equation is introduced. In Sec. III the asymptotic density matrix is obtained for two memory functions of the reservoir. Section IV presents and analyzes the numerical results for the occupation probabilities, the coherence parameter, and the entropy. Section V is devoted to the conclusions.

II. GENERALIZED MASTER EQUATION

For the kind of Hamiltonian

$$\hat{H}=\hat{H}_{0\mathcal{S}}+\hat{H}_{0\mathcal{R}}+\hat{V}^1+\hat{V}^2(t) , \tag{1}$$

$H_{0\mathcal{S}}$ and $H_{0\mathcal{R}}$ are the Hamiltonians of the system of interest ($\mathcal{S}$) and environment ($\mathcal{R}$), respectively, V^1 is the system-environment interaction, and $V^2(t)$ comes from a time-dependent external force acting only on $\mathcal{S}$. The corresponding quantum Liouville equation is

$$i\frac{\partial\hat{\rho}(t)}{\partial t}=[\hat{H}_0,\hat{\rho}(t)]+[\hat{V}^1,\hat{\rho}(t)]+[\hat{V}^2(t),\hat{\rho}(t)]\ , \qquad (2)$$

where $H_0=H_{0\mathcal{S}}+H_{0\mathcal{R}}$. In the interaction picture this equation becomes

$$i\frac{\partial\hat{\rho}_{\mathcal{I}}(t)}{\partial t}=[e^{iL_0 t}\hat{V}^1,\hat{\rho}_{\mathcal{I}}(t)]+[e^{iL_{0\mathcal{S}}t}\hat{V}^2(t),\hat{\rho}_{\mathcal{I}}(t)]\ , \qquad (3)$$

where

$$\hat{\rho}_{\mathcal{I}}(t)=e^{iL_0 t}\hat{\rho}(t)$$

and $\hat{L}_{0\mathcal{S}}$ and $\hat{L}_0$ are Liouville operators,

$$\hat{L}_{0\mathcal{S}}\cdot=[\hat{H}_{0\mathcal{S}},\cdot]\ ,$$

$$\hat{L}_0\cdot=[\hat{H}_{0\mathcal{S}}+\hat{H}_{0\mathcal{R}},\cdot]\ .$$

After integrating and iterating, Eq. (3) can be written as

$$\begin{aligned} i\frac{\partial\hat{\rho}_{\mathcal{I}}(t)}{\partial t}&=[\hat{V}^1_{\mathcal{I}}(t),\hat{\rho}_{\mathcal{I}}(0)]+[\hat{V}^2_{\mathcal{I}}(t),\hat{\rho}_{\mathcal{I}}(t)]\\ &\quad -i\int_0^t dt'[\hat{V}^1_{\mathcal{I}}(t),[\hat{V}^1_{\mathcal{I}}(t'),\hat{\rho}_{\mathcal{I}}(t')]]\\ &\quad -i\int_0^t dt'[\hat{V}^1_{\mathcal{I}}(t),[\hat{V}^2_{\mathcal{I}}(t'),\hat{\rho}_{\mathcal{I}}(t')]]\ , \end{aligned} \qquad (4)$$

where

$$\hat{V}^1_{\mathcal{I}}(t)=e^{iL_0 t}\hat{V}^1$$

and

$$\hat{V}^2_{\mathcal{I}}(t)=e^{iL_{0\mathcal{S}}t}\hat{V}^2(t)\ .$$

Note that Eq. (4) is exact; no approximations were assumed for its derivation. Since we are interested in the time evolution of the system $\mathcal{S}$, irrespective of the environment, we take the trace over the environment variables and then we get the reduced density operator

$$\hat{\rho}_{\mathcal{I}\mathcal{S}}(t)=\mathrm{Tr}_{\mathcal{R}}\hat{\rho}_{\mathcal{I}}(t)\ ;$$

moreover, assuming the conditions (1) V^1 has nondiagonal terms in the chosen representation, (2) $\rho_{\mathcal{I}}(0)=\rho_{\mathcal{I}\mathcal{S}}(0)\rho_{\mathcal{R}}(0)$, where $\rho_{\mathcal{R}}(0)=e^{-H_{0\mathcal{R}}/kT}/\mathrm{Tr}_{\mathcal{R}}e^{-H_{0\mathcal{R}}/kT}$ is diagonal in the representation in which we take the trace, and (3) for all times $\rho_{\mathcal{I}}(t)=\rho_{\mathcal{I}\mathcal{S}}(t)\rho_{\mathcal{R}}(0)$, i.e., the environment behaves as an ideal reservoir that remains undisturbed independently of the strength of the interaction V^1, Eq. (4) reduces to the generalized master equation (GME)

$$\frac{\partial\hat{\rho}_{\mathcal{S}\mathcal{I}}(t)}{\partial t}=-i[\hat{V}^2_{\mathcal{I}}(t),\hat{\rho}_{\mathcal{S}\mathcal{I}}(t)]-\int_0^t dt'\,\mathrm{Tr}_{\mathcal{R}}[\hat{V}^1_{\mathcal{I}}(t),[\hat{V}^1_{\mathcal{I}}(t'),\hat{\rho}_{\mathcal{S}\mathcal{I}}(t')\hat{\rho}_{\mathcal{R}}(0)]]\ . \qquad (5)$$

Again we stress that no hypotheses were made about the strengths of the interactions. In the Schrödinger picture we have

$$\frac{\partial\hat{\rho}_{\mathcal{S}}(t)}{\partial t}=-i[\hat{H}_{0\mathcal{S}}+\hat{V}^2(t),\hat{\rho}_{\mathcal{S}}(t)]-\int_0^t dt'\,\mathrm{Tr}_{\mathcal{R}}[\hat{V}^1,[e^{-iL_0(t-t')}\hat{V}^1,\hat{\rho}_{\mathcal{S}}(t')\hat{\rho}_{\mathcal{R}}(0)]]\ . \qquad (6)$$

Our physical problem is a two-level atom ($\mathcal{S}$) coupled to a reservoir ($\mathcal{R}$), represented by a very large number of harmonic oscillators (HO's) in thermal equilibrium at some temperature T, and driven by a classical (c-number) monochromatic electric field. For this case we have ($\hbar=1$)

$$\begin{aligned} \hat{H}_{0\mathcal{S}}&=\frac{\omega_0}{2}(|\uparrow\rangle\langle\uparrow|-|\downarrow\rangle\langle\downarrow|)\ ,\\ \hat{H}_{0\mathcal{R}}&=\sum_n\omega_n b_n^\dagger b_n\ ,\\ \hat{V}^1&=\sum_n(K_n^* b_n^\dagger|\downarrow\rangle\langle\uparrow|+K_n b_n|\uparrow\rangle\langle\downarrow|)\ , \end{aligned} \qquad (7)$$

and

$$\hat{V}^2(t)=(F_0 e^{i\omega_c t}|\downarrow\rangle\langle\uparrow|+F_0^* c^{-i\omega_c t}|\uparrow\rangle\langle\downarrow|)\ .$$

In the above expressions $|\uparrow\rangle$ and $|\downarrow\rangle$ are the states of the two-level atom and ω_0 is the transition frequency. The operators $b_n^\dagger$ and b_n, associated with the reservoir's HO's, satisfy the bosonic commutation relations and the ω_n are the frequencies of the oscillators. K_n (K_n^*) are the coupling parameters related to the atom-reservoir interaction, while F_0 (F_0^*) are the coupling parameters concerned with the interaction between the atom and the electric field. These latter parameters are given by $F_0=-\boldsymbol{\mu}\cdot\mathbf{E}$, where μ is the atomic dipole matrix element and $\mathbf{E}$ the electric field amplitude multiplied by the unit vector in its direction; further, ω_c is the electric field oscillating frequency.

In order to get a solution for the reduced matrix operator we work in the interaction picture and we do an expansion of $\hat{\rho}_{\mathcal{S}\mathcal{I}}(t)$ in terms of a complete set of operators,

$$\hat{\rho}_{\mathcal{S}\mathcal{I}}(t)=\sum_{j=1}^{4}W_j(t)\hat{O}_j\ , \qquad (8)$$

where

$$\hat{O}_1=|\uparrow\rangle\langle\uparrow|\ ,$$

$$\hat{O}_2=|\downarrow\rangle\langle\downarrow|\ ,$$

$$\hat{O}_3=|\uparrow\rangle\langle\downarrow|\ ,$$

$$\hat{O}_4=|\downarrow\rangle\langle\uparrow|\ ,$$

being these operators orthonormal with respect to the inner product

$$\langle \hat{O}_i|\hat{O}_j\rangle = \mathrm{Tr}_{\mathcal{S}}\hat{O}_i^\dagger\hat{O}_j=\delta_{ij} \ .$$

Thus, the expansion (8) may be written as

$$\hat{\rho}_{\mathcal{S}\mathcal{J}}(t)=W_1(t)|\uparrow\rangle\langle\uparrow|+W_2(t)|\downarrow\rangle\langle\downarrow|+W_3(t)|\uparrow\rangle\langle\downarrow| + W_4(t)|\downarrow\rangle\langle\uparrow| \ . \tag{9}$$

The normalization condition $\mathrm{Tr}_{\mathcal{S}}\hat{\rho}_{\mathcal{S}\mathcal{J}}(t)=1$ leads to the relation

$$W_1(t)+W_2(t)=1 \ ,$$

for the occupation probabilities, while for the coherence coefficients we have $W_3(t)=W_4(t)^*$.

The coefficients $W_j(t)$ are obtained from

$$W_j(t)=\mathrm{Tr}_{\mathcal{S}}[\hat{O}_j^\dagger\hat{\rho}_{\mathcal{S}\mathcal{J}}(t)] \ ; \tag{10}$$

then, taking the time derivative of Eq. (10) together with Eq. (5), we get the following set of differential equations:

$$\dot{W}_j(t)=\sum_{k=1}^{2}\int_0^t dt' Q_{jk}(t,t')W_k(t') -(-1)^j[iF_0\overline{W}_3(t)-iF_0^*\overline{W}_4(t)] \ , \quad j=1,2 \ , \tag{11a}$$

and

$$\dot{\overline{W}}_j(t)=\int_0^t dt'\overline{Q}_{jj}(t,t')W_j(t') -(-1)^j[iF_0^*W_1(t)-iF_0^*W_2(t)+i\Delta\omega\overline{W}_j(t)] \ , \quad j=3,4 \ . \tag{11b}$$

In the above expressions,

$$\begin{aligned}
Q_{jk}(t,t')&=-(-1)^{j+k}[e^{i\omega_0(t-t')}\xi_k(t-t')+\text{c.c.}] \ , \quad j,k=1,2 \ ,\\
\overline{Q}_{33}(t,t')&=e^{i\omega_c(t-t')}[\xi_1(t-t')+\xi_2(t-t')] \ ,\\
\overline{Q}_{44}(t,t')&=\overline{Q}_{33}(t,t')^* \ ,\\
\overline{W}_3(t)&=e^{-i\Delta\omega t}W_3(t) \ ,\\
\overline{W}_4(t)&=e^{i\Delta\omega t}W_4(t) \ ,
\end{aligned} \tag{12}$$

with $\Delta\omega=\omega_0-\omega_c$ the detuning between the atomic and the driving field frequencies. The functions $\xi_1(t-t')$ and $\xi_2(t-t')$ are correlation functions of the reservoir operators at different times, given by

$$\xi_k(t-t')=\sum_m |K_m(\omega_m)|^2 e^{-i\omega_m(t-t')} \times[\bar{n}(\omega_m)+\delta_{k1}], \quad k=1,2 \ , \tag{13}$$

where

$$\bar{n}(\omega_m)=\langle b_m^\dagger b_m\rangle=(e^{\omega_m/k_BT}-1)^{-1} \ ,$$

stands for the quanta mean value of the mth HO, k_B is the Boltzmann constant, and T is the absolute temperature.

To solve the system (11) we use the Laplace-transform method, which leads to a system of algebraic equations for the transforms $\widetilde{W}_j(p)$ and $\widetilde{\overline{W}}_j(p)$. The solution of this algebraic system yields

$$\widetilde{W}_j(p)=\sum_{k=1}^{2}X_{jk}(p)W_k(0), \quad j=1,2 \ , \tag{14a}$$

where

$$X_{jk}(p)=\frac{p\delta_{jk}+(\widetilde{Q}_{12}\delta_{j1}+\widetilde{Q}_{21}\delta_{j2})+|F_0|^2h(p)}{p(p+\widetilde{Q}_{12}(p)+\widetilde{Q}_{21}(p)+2|F_0|^2h(p))} \ , \tag{14b}$$

whereas

$$\widetilde{\overline{W}}_3(p)=iF_0^*\frac{\widetilde{W}_1(p)-\widetilde{W}_2(p)}{p+i\Delta\omega-\widetilde{\overline{Q}}_{33}(p)} \tag{14c}$$

and

$$\widetilde{\overline{W}}_4(p)=\widetilde{\overline{W}}_3(p)^* \ . \tag{14d}$$

In these expressions it was assumed that initially $W_3(0)=W_4(0)=0$, meaning an absolute lack of coherence. The quantities $\widetilde{Q}_{jk}(p)$ and $\widetilde{\overline{Q}}_{33}(p)$ are the transforms of $Q_{jk}(t)$ and $\overline{Q}_{33}(t)$ and their expressions are

$$\begin{aligned}
\widetilde{Q}_{12}(p)&=\int_0^\infty d\omega\, g(\omega)|K(\omega)|^2\bar{n}(\omega)\frac{2p}{p^2+(\omega_0-\omega)^2} \ ,\\
\widetilde{Q}_{21}(p)&=\int_0^\infty d\omega\, g(\omega)|K(\omega)|^2[\bar{n}(\omega)+1] \times\frac{2p}{p^2+(\omega_0-\omega)^2} \ ,\\
\widetilde{\overline{Q}}_{33}(p)&=-\int_0^\infty d\omega\, g(\omega)|K(\omega)|^2[2\bar{n}(\omega)+1] \times\frac{1}{p-i(\omega_c-\omega)} \ ,
\end{aligned}$$

in which we introduce the density distribution function $g(\omega)$ for the frequencies of the reservoir's HO's. In Eq. (14b), $h(p)$ is defined as

$$h(p)=\frac{1}{p+i\Delta\omega-\widetilde{\overline{Q}}_{33}(p)}+\text{c.c.}$$

The coefficients $W_j(t)$ are finally obtained by applying the inverse Laplace transform to $\widetilde{W}_j(p)$, $j=1,2$, and $\widetilde{\overline{W}}_j(p)$, $j=3,4$, that is,

$$W_j(t)=\frac{1}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}dp\, e^{pt}\widetilde{W}_j(p), \quad j=1,2 \ , \tag{15a}$$

and

$$W_3(t)=e^{+i\Delta\omega t}\frac{1}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}dp\, e^{pt}\widetilde{\overline{W}}_3(p) \ , \tag{15b}$$

III. ASYMPTOTIC DENSITY MATRIX

We are interested in the steady-state situation ($t\rightarrow\infty$) and this is achieved by calculating the residue of the integrand at the pole $p=0$ in Eqs. (15); so we get in the Schrödinger picture

$$W_j^\infty=\frac{[\tilde{Q}_{12}(0)\delta_{j1}+\tilde{Q}_{21}(0)\delta_{j2}]+|F_0|^2h(0)}{\tilde{Q}_{12}(0)+\tilde{Q}_{21}(0)+2|F_0|^2h(0)},\quad j=1,2 \tag{16a}$$

and

$$W_3^\infty=e^{-i\omega_c t}\frac{iF_0^*[\tilde{Q}_{12}(0)-\tilde{Q}_{21}(0)]}{[\tilde{Q}_{12}(0)+\tilde{Q}_{21}(0)+2|F_0|^2h(0)][+i\Delta\omega-\tilde{\tilde{Q}}_{33}(0)]}\,. \tag{16b}$$

In order to get analytical results for the steady-state diagonal and nondiagonal elements of $\rho_{\mathcal{S}}$, we have to introduce shapes for the distribution function $g(\omega)$. Initially considering $g(\omega)=g_0$ constant for all values of the frequencies, we get the Markov approximation (MA) in which the W_j^∞'s acquire the form

$$W_j^\infty=\frac{[\bar{n}(\omega_0)+\delta_{j2}]\{\gamma^2[\bar{n}(\omega_0)+\frac{1}{2}]^2+\Delta\omega^2\}+2|F_0|^2[\bar{n}(\omega_0)+\frac{1}{2}]}{2[\bar{n}(\omega_0)+\frac{1}{2}]\{\gamma^2[\bar{n}(\omega_0)+\frac{1}{2}]^2+\Delta\omega^2+2|F_0|^2\}},\quad j=1,2 \tag{17a}$$

and

$$W_3^\infty=e^{-i\omega_c t}\frac{-F_0^*(\Delta\omega+i\gamma[\bar{n}(\omega_0)+\frac{1}{2}])}{2[\bar{n}(\omega_0)+\frac{1}{2}]\{\gamma^2[\bar{n}(\omega_0)+\frac{1}{2}]^2+\Delta\omega^2+2|F_0|^2\}}\,, \tag{17b}$$

where $\gamma=2\pi g_0|K(\omega_0)|^2$.

To characterize a non-Markovian process the function $g(\omega)$ is assumed to have some nontrivial structure, distinct from the flat one. In the following we will consider two types of shapes for the distribution function, the square and the Cauchy, that depend on the heat-bath correlation time τ.

A. Square distribution

The distribution function is considered to have a flat shape for a limited interval of frequencies only,

$$g(\omega)=\begin{cases} g_0, & \omega_0-\tau^{-1}\le\omega\le\omega_0+\tau^{-1} \\ 0, & \text{otherwise}\,, \end{cases}$$

with width $2\tau^{-1}$. This distribution yields for W_j^∞, $j=1,2$,

$$W_j^\infty=\frac{[\bar{n}(\omega_0)+\delta_{j2}]\left\{\gamma^2[\bar{n}(\omega_0)+\frac{1}{2}]^2+\Delta\omega^2\left[1-\frac{k(\bar{n}(\omega_0)+\frac{1}{2})}{\Omega\pi}\ln\left[\frac{1+\Omega}{1-\Omega}\right]\right]^2\right\}+2|F_0|^2[\bar{n}(\omega_0)+\frac{1}{2}]}{2[\bar{n}(\omega_0)+\frac{1}{2}]\left\{\gamma^2[\bar{n}(\omega_0)+\frac{1}{2}]^2+\Delta\omega^2\left[1-\frac{k(\bar{n}(\omega_0)+\frac{1}{2})}{\Omega\pi}\ln\left[\frac{1+\Omega}{1-\Omega}\right]\right]^2+2|F_0|^2\right\}}\,, \tag{18a}$$

for $\Omega<1$, and

$$W_j^\infty=\frac{\bar{n}(\omega_0)+\delta_{j2}}{2[\bar{n}(\omega_0)+\frac{1}{2}]} \tag{18b}$$

if $\Omega>1$; and for W_3^∞,

$$W_3^\infty=e^{-i\omega_c t}\frac{-F_0^*\left[\Delta\omega\left[1-\frac{k(\bar{n}(\omega_0)+\frac{1}{2})}{\Omega\pi}\ln\left[\frac{1+\Omega}{1-\Omega}\right]\right]+i\gamma[\bar{n}(\omega_0)+\frac{1}{2}]\right]}{2[\bar{n}(\omega_0)+\frac{1}{2}]\left\{\gamma^2[\bar{n}(\omega_0)+\frac{1}{2}]^2+\Delta\omega^2\left[1-\frac{k(\bar{n}(\omega_0)+\frac{1}{2})}{\Omega\pi}\ln\left[\frac{1+\Omega}{1-\Omega}\right]\right]^2+2|F_0|^2\right\}}\,, \tag{18c}$$

for $\Omega<1$, and

$$W_3^\infty=e^{-i\omega_c t}\frac{-F_0^*}{2[\bar{n}(\omega_0)+\frac{1}{2}]\Delta\omega\left[1-\frac{k(\bar{n}(\omega_0)+\frac{1}{2})}{\Omega\pi}\ln\left[\frac{1+\Omega}{1-\Omega}\right]\right]}\,, \tag{18d}$$

if $\Omega>1$. In these expressions we introduced $k=\gamma\tau$ and $\Omega=\Delta\omega\tau$.

B. Cauchy distribution

The Cauchy distribution function is a smooth continuous function for all values of frequencies, i.e.,

$$g(\omega)=g_0\frac{1}{1+(\omega-\omega_0)^2\tau^2}\ ,$$

with a peak at $\omega=\omega_0$. For this distribution we obtain for W_j^∞, $j=1,2$,

$$W_j^\infty=\frac{[\bar{n}(\omega_0)+\delta_{j2}]\{\gamma^2[\bar{n}(\omega_0)+\frac{1}{2}]^2+\Delta\omega^2[1+\Omega^2-k(\bar{n}(\omega_0)+\frac{1}{2})]^2\}+2|F_0|^2[\bar{n}(\omega_0)+\frac{1}{2}](1+\Omega^2)}{2[\bar{n}(\omega_0)+\frac{1}{2}]\{\gamma^2[\bar{n}(\omega_0)+\frac{1}{2}]^2+\Delta\omega^2[1+\Omega^2-k(\bar{n}(\omega_0)+\frac{1}{2})]^2+2|F_0|^2(1+\Omega^2)\}}\ , \tag{19a}$$

and for W_3^∞,

$$W_3^\infty=e^{-i\omega_c t}\frac{-F_0^*(1+\Omega^2)\{\Delta\omega[1+\Omega^2-k(\bar{n}(\omega_0)+\frac{1}{2})]+i\gamma[\bar{n}(\omega_0)+\frac{1}{2}]\}}{2[\bar{n}(\omega_0)+\frac{1}{2}]\{\gamma^2[\bar{n}(\omega_0)+\frac{1}{2}]^2+\Delta\omega^2[1+\Omega^2-k(\bar{n}(\omega_0)+\frac{1}{2})]^2+2|F_0|^2(1+\Omega^2)\}} \tag{19b}$$

for any value of Ω.

IV. OCCUPATION PROBABILITIES, COHERENCE PARAMETER, AND ENTROPY

Besides the occupation probabilities of the levels, Eqs. (17a), (18a), (18b), and (19), we also introduce the coherence parameter and the Shannon information entropy to analyze the system asymptotically. The former is defined as

$$\eta=\frac{|W_3^\infty|}{[W_1^\infty W_2^\infty]^{1/2}} \tag{20}$$

and the entropy is

$$S=-\sum_{j=1}^{2}\lambda_j\ln\lambda_j\ , \tag{21}$$

where the

$$\lambda_j=\tfrac{1}{2}[1-(-1)^j\sqrt{1-4(W_1^\infty W_2^\infty-|W_3^\infty|^2)}] \tag{22}$$

are the eigenvalues of the reduced density matrix.

We are interested in the behavior of the occupation probabilities, the coherence parameter, and the entropy as function of the detuning $\Delta\omega$, the strength of the external force F_0, the temperature, and the correlation time τ of the reservoir. From now on in all figures the MA plot is represented by a solid line, the Cauchy distribution by a dashed line, and the square distribution by crosses.

A. Occupation probability

The occupation probability W_1^∞ is plotted as function of the detuning in Fig. 1 for three different sets of $(\bar{n},F_0)$ and fixed values of $\gamma=0.2$ and $\tau=0.1$. It displays the following features.

(1) For $\Delta\omega=0$, resonance, and $\Delta\omega\to\infty$ all curves coincide independently of the values of the parameters.

(2) The occupation probability for the MA is higher than the one for the non-Markovian Cauchy distribution.

(3) The square distribution curve decreases, in all plots, abruptly to a constant value, $W_1^\infty=\bar{n}/(2\bar{n}+1)$ at $\Delta\omega=\tau^{-1}$, due to the logarithm function in expression (18a).

(4) The external force strength F_0 stresses the memory effects, being the differences to the MA more significant for $\Delta\omega$ around the value τ^{-1}. For very strong forces, $(|F_0|\gg\gamma,\Delta\omega)$, $W_1^\infty=\frac{1}{2}$, except when $\Delta\omega>\tau^{-1}$ for the square distribution, as one can see from the expressions for W_1^∞.

(5) Keeping F_0 constant, as the temperature, or $\bar{n}$, increases the memory effects lessen.

B. Coherence parameter

The coherence parameter η, Eq. (20), displays essentially the quantum effects of the system since it compares the diagonal elements with the nondiagonal ones of the reduced density matrix. Even when initially the system is in a complete mixed state, $W_3(0)=W_4(0)=0$, asymptotically $W_3^\infty=W_4^{\infty *}\neq0$, Eqs. (17b), (18c), (18d), and (19b). This means that an external classical force induces a coherence in the system even when it is coupled to a reservoir. Obviously for $F_0=0$ then $W_3^\infty=0$ and there is no creation of coherence. Figure 2 shows the plots of η as function of $\Delta\omega$ for different sets of parameters and the main features are listed below.

(1) Again for $\Delta\omega=0$ and $\Delta\omega\to\infty$ all three curves coincide for any set of parameters.

(2) Here the MA curve presents lower coherence than the Cauchy distribution one, whereas for $\Delta\omega<\tau^{-1}$ the MA and the square distribution curves are almost totally coincident.

(3) At $\Delta\omega=\tau^{-1}$ the square distribution curve now exhibits a discontinuity: $\eta(\Delta\omega=\tau^{-1})=0$, although at the left and at the right of this point it assumes quite different values.

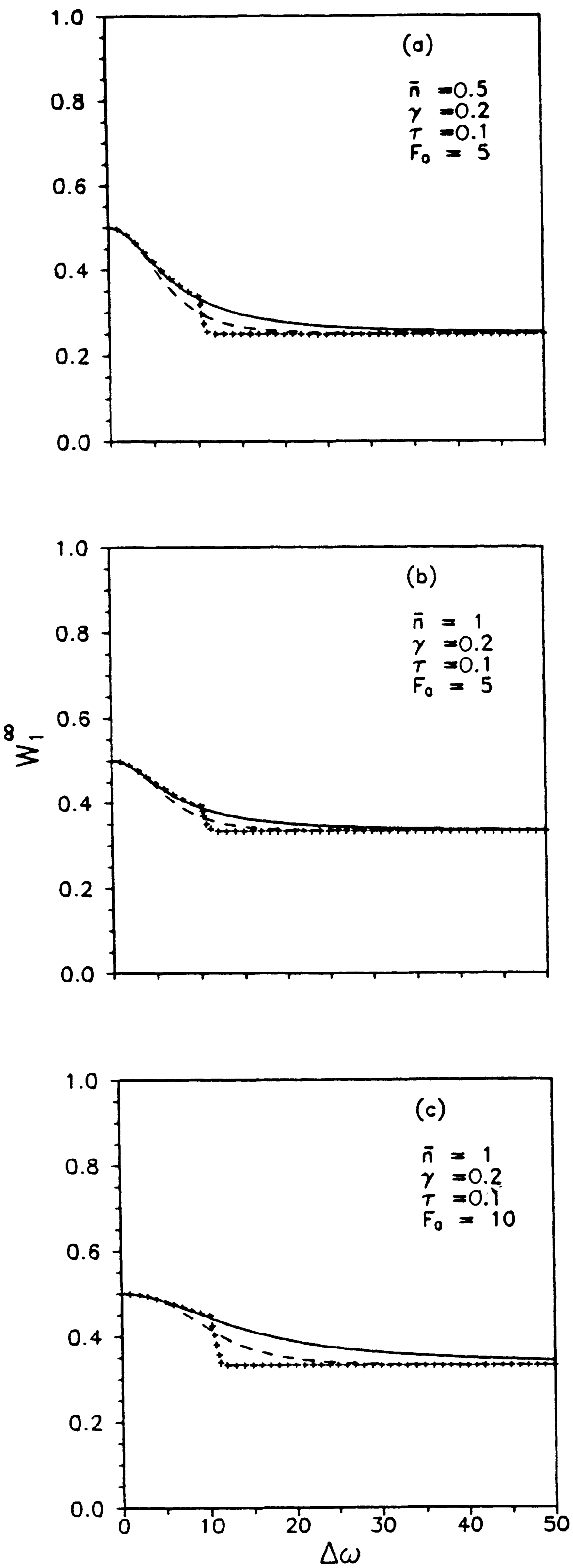

FIG. 1. The occupation probability W_1^∞ as a function of the detuning $\Delta\omega$ for different sets of $\bar{n}$ and F_0 and fixed values of γ and τ. The solid line corresponds to the MA, the dashed line corresponds to the Cauchy distribution, and the crosses correspond to the square distribution.

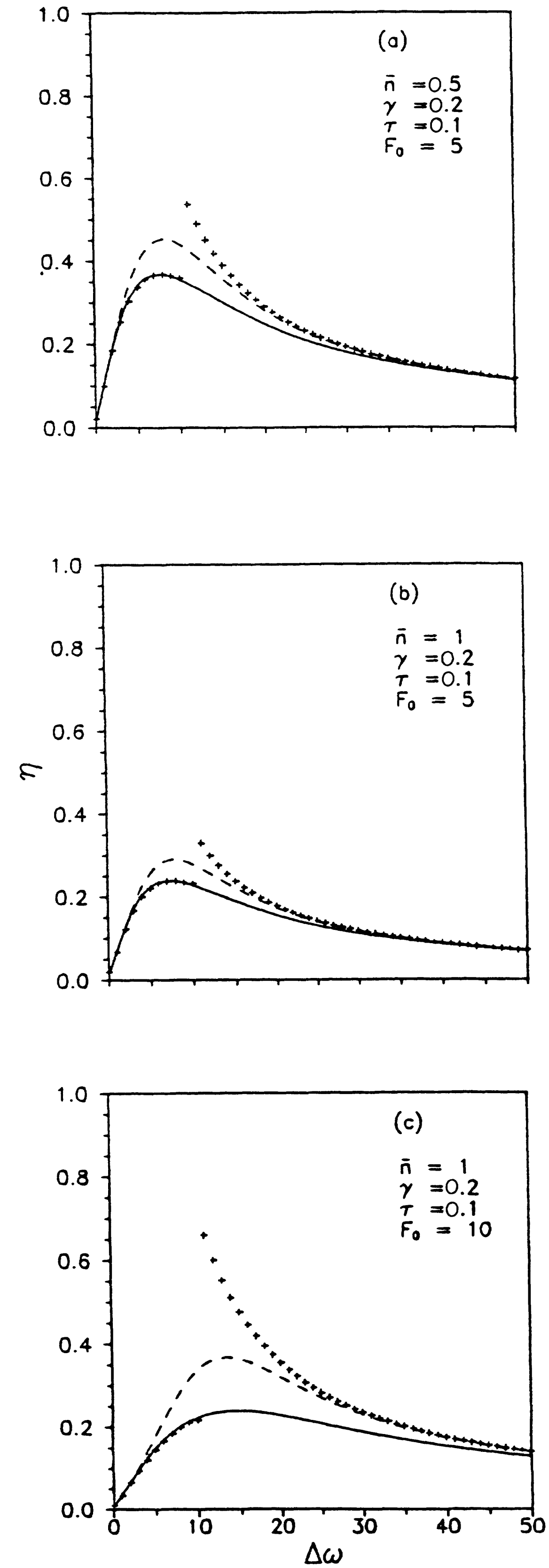

FIG. 2. The coherence parameter η as a function of the detuning $\Delta\omega$. All curves correspond to the same distributions as in Fig. 1.

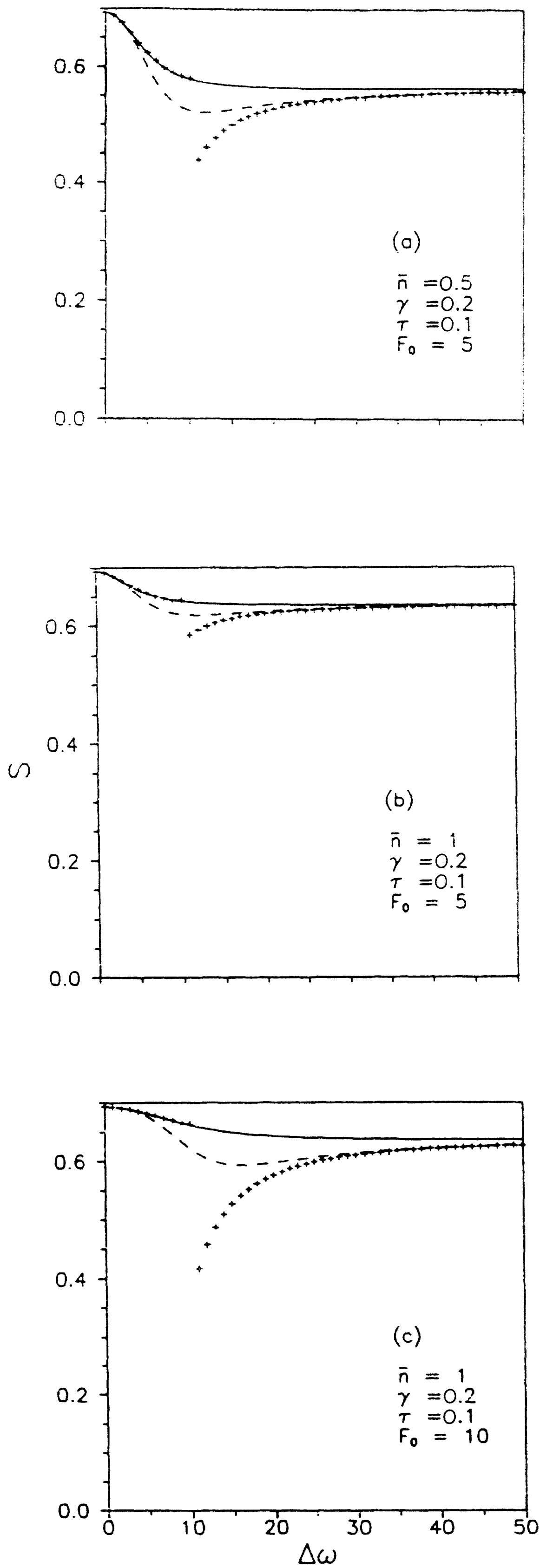

FIG. 3. The Shannon information entropy S as a function of the detuning $\Delta\omega$. All curves correspond to the same distributions as in Fig. 1.

(4) The external force strength F_0 stresses the memory effects much more than in the occupation probability case, Fig. 1. Now, for very strong forces $W_3^\infty \rightarrow 0$ and η vanishes, so coherence is destroyed, except when $\Delta\omega > \tau^{-1}$ for the square distribution.

(5) Again, as the temperature increases, and keeping F_0 constant, the memory effects become less pronounced.

(6) As an important point we note that at resonance, $\Delta\omega = 0$, the coherence is lowest and attains a maximum at a value of $\Delta\omega$ depending, in a form that is not simple, on F_0, γ, τ, and $\bar{n}(\omega_0)$.

Concerning the occupation probability and the coherence parameter, we must note that, excluding the singularity in the square distribution, the shapes in Figs. 1 and 2 do not permit one to distinguish between the Markovian and non-Markovian processes in the stationary regime. However, we will show below that the distinction between the shapes is feasible by using the SIE.

C. Entropy

It becomes crucial to analyze the behavior of the Shannon information entropy S as a function of the several parameters cited since the external force induces asymptotically finite-valued nondiagonal matrix elements for a density matrix, initially diagonal. In Fig. 3 we plot S as function of $\Delta\omega$ for the MA and for the other two non-Markovian expressions. Besides the general features related to the behavior of the curves already mentioned in Secs. IV A and IV B, we additionally observe the following.

(1) Since at $\Delta\omega = 0$ the memory effects vanish and the coherence is the lowest, this implies a high value of S.

(2) As $\Delta\omega \rightarrow \infty$ one gets for all cases $W_3^\infty = 0$, $W_1^\infty = \bar{n}/(2\bar{n}+1)$, and $W_2^\infty = (\bar{n}+1)/(2\bar{n}+1)$, which are the thermal equilibrium values in the absence of the external force, then the entropy becomes lower than its value at resonance.

(3) For intermediate values of $\Delta\omega$ the memory effects are important, leading to lower entropies than at the value $S(\Delta\omega \rightarrow \infty)$. More importantly than the other features we note that, while for the Markovian process the SIE decreases monotonically, for the non-Markovian one the SIE *curve presents a minimum at* $\Delta\omega \sim \tau^{-1}$, which disappears as $\tau \rightarrow 0$.

The fact that the SIE distinguishes between the two processes can be exploited to analyze experimental results, as will be discussed in the next section.

V. SUMMARY AND CONCLUSIONS

In the present work our aim was to analyze the influence of memory effects and strength of an external driving force on a two-level atom. Unlike previous works in this same line, we concentrated our study on essentially two quantities, the coherence parameter and the Shannon information entropy, besides the usual occupation probabilities. We considered the reservoir constituted by an infinite number of HO's and we assumed two types of frequency distributions $g(\omega)$, the Cauchy and the square.

We were not concerned with the transient regime but only with the steady state, since this is easier to work on in experimental setups. As expected, the memory effects owe their existence to the driving force and, moreover, they are enhanced by its strength, these effects being more pronounced in the coherence parameter than in the occupation probabilities. At resonance, $\Delta\omega=0$, these effects are small compared with higher values of $\Delta\omega$, unless for $\Delta\omega\rightarrow\infty$, where the effects vanish. We verified that even when the coherence parameter is initially null, i.e., the system is in a complete mixed state, at stationarity there is a construction of coherence induced by the external force irrespective of the process.

We also note from Eqs. (17)–(19) that for strong driving forces ($|F_0|\gg\gamma,\Delta\omega$), the coherence disappears ($\eta=0$), the occupation probabilities are equal ($W_1^\infty=W_2^\infty=\frac{1}{2}$), and the entropy attains its higher value ($S=\ln 2$). Such a situation occurs either for the MA or the non-Markovian treatments, unless $\Delta\omega>\tau^{-1}$ for the square distribution, meaning the disappearance of memory effects. This behavior was experimentally verified in Ref. [1].

In experimental setups the absorption band-shape function is available and it is related to the imaginary part of the susceptibility χ''; and by using the Kramers-Krönig relations [10] one gets the full complex susceptibility χ. Now, since [8]

$$\chi^{\text{expt}}(\omega)=\frac{1}{3}\frac{|\mu_{12}|^2}{|F_0|^2}W_3^\infty e^{+i\omega_c t}F_0 ,$$

the W_3^∞ is obtained from the experimental results; from Eq. (16b), rewritten as

$$W_3^\infty=e^{-i\omega_c t}iF_0^*\frac{\tilde{W}_1^\infty-\tilde{W}_2^\infty}{i\Delta\omega-\tilde{\tilde{Q}}_{33}(0)} ,$$

together with the condition $W_1(t)+W_2(t)=1$, the quantities W_1^∞ and W_2^∞ are also obtained. Therefore, with W_1^∞, W_2^∞, and W_3^∞ calculated from the experimental data, we can in the steady state draw the coherence parameter, $\eta(\Delta\omega)$, and the SIE, $S(\Delta\omega)$, curves; the presence of a minimum in the $S(\Delta\omega)$ will characterize a non-Markovian process.

ACKNOWLEDGMENT

Two of us (S.S.M and G.A.P.) acknowledge support from CNPq, Brazil.

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