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Weak capture of negative muons in hydrogenic media

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The weak capture of negative muons on protons has been calculated in the local $V-A$ theory in the correct spinor representation for μp atoms and $p\mu p$ molecules. The neutrino-mass effects have been explored. The results obtained by this formal theory are compared with earlier calculations, and their importance is discussed in the light of the experiments.

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INTRODUCTION

Molecular and atomic routings for electro-weak interactions of negative muons in matter are well known [1,2]. In particular, the weak capture of negative muons by the nuclei of matter forms a subject of considerable importance for the information it offers on the characteristics of the capturing nucleus as well as the reaction itself [3].

While both positive and negative muons interact electromagnetically with the electrons and nuclei of the matter through which they pass, the distortion is magnified many times in the case of negative muons because of their propensity for capture into Coulomb bound states about the nuclei in the host medium. This increases the overlap of the muon and nuclear wave functions and serves to enhance the weak capture. The influence is most pronounced in the case of heavy nuclei due to the enhanced Coulomb field in this case. Moreover, muonic molecular states in high-Z media are rapidly depleted and transformed into atomic states about the nuclei with the highest Z.

We restrict ourselves to hydrogenic media in this work and investigate the weak capture of the muon in protonic hydrogen only. The relevant reaction is

$$\mu^- + p \rightarrow n + \nu_\mu \ , \tag{1}$$

and can occur within a muonic atom or molecule.

In view of the high-precision experimental results available for muon capture [4], we felt it necessary to reinvestigate the problem theoretically in the framework of the $V-A$ theory. We have thus worked in the correct spinor representation, retaining the current-current form for the baryonic sector, without taking recourse to individual form factors as in the existing descriptions of the problem. We present results for muon capture in μp atoms for both hyperfine states as well as muon capture within the $p\mu p$ system. We compare our results with those obtained by the earlier methods and discuss the relevance in the context of a better understanding of the extraction of the pure capture rate from the sophisticated high-precision experiments.

Before proceeding to the formulation of the problem, we point out an important difference in the role of the Coulomb dressing in the weak capture and decay reactions [2] that are connected by crossing symmetry as reverse reactions. In the case of muon decay from a Coulomb bound state, the role of the Coulomb partner is a passive one for the purpose of the interaction itself, and it participates as a spectator through the final-state kinematics and the initial constraining of the decaying muon due to its binding.

In contrast, for the case of the weak capture of negative muons on the nuclei of matter, the role of the Coulomb partner transforms into a much more dynamic one, as the partner itself is a participant in the weak interaction. In addition, the proximity of the muon to its capturing partner is a crucial parameter, as it controls the availability of the reaction participants for reaction. This scenario is of course also simulated in the weak annihilation of the muonium system into two neutrinos, although the rate in that case is extremely small [5]. It may be noted that the rate for reaction (1) from the free state is greatly reduced due to nonoverlap of muon and proton space functions.

The weak capture of the negative muon on the nuclei of matter has been studied both theoretically and experimentally [6,4]. We mention in passing that, as one transists from hydrogen to the heavier nuclei, the weak capture gradually increases in importance until for $Z \sim 11$ it crosses the muon decay rate, so that for heavier nuclei, capture is the dominant form of muon disappearance. Despite possible molecular routings, muon capture in heavy nuclei occurs almost exclusively from bound atomic states. This is because the muon rapidly cascades to the ground states of the highest Z atom of the molecule into which it was captured. The situation changes for hydrogenic molecules where Z values of the constituent nuclei are degenerate. In hydrogen, therefore, muonic molecular ions are much less transient and the system is able to host the capture reaction on one of its nuclei. The capture rate is $\sim 10^{-3}$ times the muon decay rates in most muonic molecules, although in the fusion-favorable systems like $dt\mu$ and $dd\mu$, nuclear fusion forms the dominant mode of decay of the muonic molecular systems [2]. The muon capture in the μp system either in isolation or within the $p\mu p$ environment forms a topic of primary research interest.

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FORMALISM

The bare weak capture process is described in the standard model by the W exchange Feynman diagram [5]. Dressed in the atomic or molecular states, these devolve to collapsing the W boson propagator to local $(V-A)$ coupling and the relevant matrix element can be written as

$$M=(G/\sqrt{2})\cos\theta[\bar{\psi}_n\gamma_\alpha(1+\alpha\gamma_5)\psi_p\bar{\psi}_{\nu_\mu}\gamma^\alpha(1+\gamma_5)\psi_\mu] , \tag{2}$$

where the ψ represents the particle wave functions, including their spinor part.

G is the weak-interaction constant. $\cos\theta$ is the Cabibbo angle which equals 0.9730 ± 0.0024. The parameter α in the nuclear sector accounts for the nuclear structure effects and permits a compact formulation of the $(V-A)$ baryonic current [7].

Squaring the matrix element in the usual way, one has

$$|M|^2=(G^2/2)\cos^2(\theta)M_N\bar{M}_N M_L\bar{M}_L , \tag{3}$$

where

$$M_N\bar{M}_N=\bar{\psi}_n\gamma_\alpha(1+\alpha\gamma_5)\psi_p\bar{\psi}_p\gamma_\beta(1+\alpha\gamma_5)\psi_n , \tag{4}$$

and

$$M_L\bar{M}_L=\bar{\psi}_{\nu_\mu}\gamma^\alpha(1+\gamma_5)\psi_\mu\bar{\psi}_\mu\gamma_\beta(1+\gamma_5)\psi_{\nu_\mu} .$$

We have used the natural units $\hbar=C=1$, and introducing the usual unpolarized density matrix of the type [7]

$$\sum_s U^k(s)U_i(s)=(\hat{p}+m)_i^k$$

for the spinors $U_i(s)$ corresponding to the spin parts of the Ψ_i', we can write for the spin averaged and summed matrix element

$$\tfrac{1}{2}|\bar{M}|^2=\tfrac{1}{2}\,\mathrm{Tr}M_N\bar{M}_N\mathrm{Tr}M_L\bar{M}_L , \tag{5}$$

where

$$\mathrm{Tr}M_N\bar{M}_N=\mathrm{Tr}[\{(\hat{p}_n+m_n)\gamma_\alpha(1+\alpha\gamma_5)\}\times\{(\hat{p}_p+m_p)\gamma_\beta(1+\alpha\gamma_5)\}] , \tag{6}$$

and

$$\mathrm{Tr}M_L\bar{M}_L=\mathrm{Tr}[\bar{\psi}_{\nu_\mu}\gamma_\alpha(1+\gamma_5)\psi_\mu\bar{\psi}_\mu\gamma_\beta(1+\gamma_5)\psi_{\nu_\mu}] . \tag{7}$$

Simplifying the terms, we have

$$\begin{aligned}\mathrm{Tr}M_N\bar{M}_N=\mathrm{Tr}[&(\hat{p}_n\gamma_\alpha+\alpha\hat{p}_n\gamma_\alpha\gamma_5)(\hat{p}_p\gamma_\beta+\alpha\hat{p}_p\gamma_\beta\gamma_5)+(m_p\hat{p}_n\gamma_\alpha+\alpha m_p\hat{p}_n\gamma_\alpha\gamma_5)(\gamma_\beta+\alpha\gamma_\beta\gamma_5)\\&+m_n\hat{p}_p(\gamma_\alpha\gamma_\beta+\alpha\gamma_\alpha\gamma_\beta\gamma_5+\alpha\gamma_\alpha\gamma_5\gamma_\beta+\alpha\gamma_\alpha\gamma_5\gamma_\beta\gamma_5)+m_nm_p(\gamma_\alpha\gamma_\beta+\alpha\gamma_\alpha\gamma_\beta\gamma_5+\alpha\gamma_\alpha\gamma_5\gamma_\beta+\alpha^2\gamma_\alpha\gamma_5\gamma_\beta\gamma_5)] .\end{aligned} \tag{8}$$

Referring to the rules for the γ matrix outlined by Okun [7] and simplifying further, we get

$$\mathrm{Tr}M_N\bar{M}_N=\mathrm{Tr}[(\alpha^2+1)\hat{p}_n\gamma_\alpha\hat{p}_p\gamma_\beta+2\alpha\gamma_5\hat{p}_n\gamma_\alpha\hat{p}_p\gamma_\beta-(\alpha^2-1)m_nm_p\gamma_\alpha\gamma_\beta] . \tag{9}$$

Proceeding similarly for the leptonic sector, we get from (7)

$$\mathrm{Tr}M_L\bar{M}_L=\mathrm{Tr}[(\hat{p}_\nu+m_\nu)\gamma_\alpha(1+\gamma_5)(\hat{p}_\mu+m_\mu)(\gamma_\beta+\gamma_\beta\gamma_5)] . \tag{10}$$

The neutrino mass terms have been formally retained in the above.

As for the baryonic sector, the leptonic sector finally simplifies to

$$\mathrm{Tr}M_L\bar{M}_L=2\,\mathrm{Tr}[\hat{p}_\nu\gamma_\alpha\hat{p}_\mu\gamma_\beta(1+\gamma_5)] . \tag{11}$$

Combining, we have from Eq. (5)

$$\begin{aligned}\tfrac{1}{2}|\bar{M}|^2=(G^2/2)\cos^2(\theta)\mathrm{Tr}[&(\alpha^2+1)\hat{p}_n\gamma_\alpha\hat{p}_p\gamma_\beta\hat{p}_\nu\gamma_\alpha\hat{p}_\mu\gamma_\beta(1+\gamma_5)+2\alpha\gamma_5\hat{p}_n\gamma_\alpha\hat{p}_p\gamma_\beta\hat{p}_\nu\gamma_\alpha\hat{p}_\mu\gamma_\beta(1+\gamma_5)\\&-m_nm_p(\alpha^2-1)\gamma_\alpha\gamma_\beta\hat{p}_\nu\gamma_\alpha\hat{p}_\mu\gamma_\beta(1+\gamma_5)] .\end{aligned} \tag{12}$$

Using the standard reduction formulas

$$\gamma_\alpha\hat{A}\,\hat{B}\,\hat{C}=-2\hat{C}\,\hat{B}\,\hat{A},\quad \gamma_\alpha\hat{A}\,\hat{B}\gamma_\beta=4(AB) ,\quad \gamma_\alpha\hat{A}\gamma_\beta=-2\hat{A} , \tag{13}$$

we get

$$\tfrac{1}{2}|\bar{M}|^2=(\tfrac{16}{2})G^2\cos^2(\theta)[2(\alpha+1)^2(p_pp_\mu)(p_\nu p_n)+m_nm_p(\alpha^2-1)(p_\nu p_\mu)] . \tag{14}$$

Equation (14) corresponds to the spin averaged case.

For the capture case, it is instructive to study the spin dependence of the process. To investigate the spin dependence of the entrance channel, we introduce a spin projection operator as in Okun [7] and replace $(\hat{p}+m)$ terms in the above calculations by $(\bar{p}+m)\to\frac{1}{2}(p+m)(1-\gamma_5\hat{S})$ for the initial proton and muon. This gives

$$\tfrac{1}{2}|\bar{M}|^2=(\tfrac{16}{2})G^2\cos^2(\theta)\{2(\alpha+1)^2(p_p-m_pS_p)(p_\mu-m'_\mu S_\mu)\times(p_\nu p_n)+(\alpha^2-1)m_nm_p\times[p_\nu(p_\mu-m'_\mu S_\mu)]\} , \tag{15}$$

where $m'_\mu=m_\mu-\epsilon$, ϵ is equal to the magnitude of the binding energy of the $p\mu$ system and $m_R/[2(137.036)^2]$, and $m_R=m'_\mu/[1+(m'_\mu/m_p)]$. This is assumed to be the major correction to the bound muon capture.

The spin averaged matrix elements of Eqs. (14) and (15) must now be simplified and the four-vector products expanded. In particular, to eliminate the angle between the two exit channel particles we use four-momentum conservation,

$$p_\nu+p_n=p_p+p_\mu . \tag{16}$$

As is customary in bound-state problems [8], we assume the three momenta of the bound muon and proton to be zero. Squaring Eq. (16) and continuing to retain m_ν, we have for the product $(p_n p_\nu)$,

$$p_n p_\nu = [(m_p + m'_\mu)^2 - m_n^2 - m_\nu^2]/2 \ . \tag{17}$$

The other four-vector products are evaluated simply and Eq. (14) then reduces to

$$\tfrac{1}{2}|\overline{M}|^2 = (\tfrac{16}{2})G^2\cos^2(\theta)[2(\alpha+1)^2 m_p m'_\mu A + m_n m_p m'_\mu E_\nu(\alpha^2-1)] \ , \tag{18}$$

where

$$A = [(m_p + m'_\mu)^2 - m_n^2 - m_\nu^2]/2 \ .$$

Similarly, the spin-dependent matrix element of Eq. (15) reduces to

$$\tfrac{1}{2}|\overline{M}|^2 = (\tfrac{16}{2})G^2\cos^2(\theta)[2(\alpha+1)^2 m_p m'_\mu(1-\mathbf{s}_p\cdot\mathbf{s}_\mu)A + m_n m_p(\alpha^2-1)(m'_\mu E_\nu - m'_\mu \mathbf{s}_\mu\cdot\mathbf{p}_\nu)] \ . \tag{19}$$

CAPTURE RATE

The capture rate in $p\mu$ atoms is obtained by integrating the square of the spin-averaged matrix element over the phase space of the final particles,

$$R = (2\pi)^4/[2(4m_p m'_\mu)] \int\int |\overline{M}|^2 N_\mu \delta^4(p_i - p_n - p_\nu) d^3\mathbf{p}_n d^3\mathbf{p}_\nu (\tfrac{1}{4}E_n E_\nu)[1/(2\pi)^6] \ , \tag{20}$$

where N_μ is the density of muon states equal to (μ^3/π) and $\mu = m_R/137.036$.

As in other $V-A$ matrix elements [5], in this case too $|\overline{M}|^2$ is insensitive to neutrino mass, and its effect appears only in the final-state phase space. Integrating over the neutron momentum using the momentum conservation of Eq. (20) reduces to

$$R = 1/[32 m_p m'_\mu (2\pi)^2] \int |\overline{M}|^2 N_\mu d^3\mathbf{p}_\nu \delta^0(E_i - E_n - E_\nu) \times (1/E_n E_\nu) \ , \tag{21}$$

where E_i, E_n, and E_ν refer to energies.

The conservation δ function constraints for μp capture are

$$E_i^2 - m_n^2 + m_\nu^2 = 2E_i E_\nu \ , \tag{22}$$

where

$$E_i = m_p + m'_\mu \ .$$

The angular integration is now trivial. Integrating over E_ν with the energy δ function and writing $f(E_\nu) = E_i - E_n - E_\nu$, we get

$$R = [1/(64 m_p m'_\mu \pi)][(BN_\mu)/E_n E_\nu^0]E_\nu\sqrt{(E_\nu^2 - m_\nu^2)} \times [1/|(d/dE_\nu)f(E_\nu)|_{E_\nu = E_\nu^0}] \ , \tag{23}$$

where B is the spin sector of the matrix element, and differs for spin-averaged and spin-dependent cases.

E_ν^0 satisfies $f(E_\nu) = 0$ to coincide with the energy δ function constraint. Finally, after simplification, we get

$$R = 16G^2\cos^2(\theta) B N_\mu \sqrt{(E_\nu^2 - m_\nu^2)}/(64\pi m_p m'_\mu E_i) \ . \tag{24}$$

SPIN SENSITIVITY OF B

For the spin-averaged case, when we ignore the specific spin structure of B we have

$$B = B_0 = 4(\alpha+1)^2 m_p m'_\mu A + 2 m_n m_p m'_\mu E_\nu(\alpha^2-1) \ . \tag{25}$$

Projecting out the spin states, we have for the singlet case

$$B = B_S = 8(\alpha+1)^2 m_p m'_\mu A + 2(\alpha^2-1) m_n m_p m'_\mu E_\nu \ ,$$

and for the triplet state,

$$B = B_T = 2(\alpha^2-1) m_n m_p m'_\mu E_\nu \ . \tag{26}$$

RESULTS AND DISCUSSION

We computed rates for muon capture from both hyperfine states of the μp atom as well as the spin-averaged rate. These results are presented in Table I, which includes the capture rate for the statistically populated μp system. The results of the earlier theory [6] and experiments [4] are also given. We have taken in the above calculations $m_p = 938.263\,25$ MeV, $m_n = 939.567\,01$ MeV, $m_\mu = 105.658\,75$ MeV, $m_{\nu_\mu} = 0.27$ MeV, $\alpha = 1.253$, and $\cos\theta = 0.9730$. Taking $m_{\nu_\mu} = 0$, cor-

TABLE I. Calculated and experimental results for the rate (in $\sec^{-1}$) of muon nuclear capture by protons.

(a) Theory

Spin state	Singlet (R_s)	Triplet (R_t)	Spin averaged	Statistical combination $[(\frac{3}{4})R_t + (\frac{1}{4})R_s]$
Present calculation	671.685	16.553	344.119	180.336
Results of Ref. [6]	664±20	11.9±0.7		175±5

(b) Summary of the experimental results

Author	Hydrogen gas target (atm)	Technique	Capture rate ($\sec^{-1}$)
Bystritski *et al.* [4]	41	Counters	686±88
Quaranta *et al.* [4]	8	Counters	651±57

responding to the exact chiral limit does not alter the results within the detectable limits. We have taken the mass [9] of $\nu_\mu = 0.27$ MeV.

We reiterate that our values are obtained by exact evaluation of the $V - A$ matrix element and hence provide an improvement on the existing state of the art, incorporating the low-energy phenomenological limit of the standard model. The baryonic modification of the axial current by the parameter α allows isolation of the vector and axial currents, so that the universality of the former is not disturbed. The leptonic sector can also be evaluated independently and a clean correspondence with the pure leptonic reactions can be obtained for the semileptonic muon capture process. We have also performed the phase-space integrations exactly by using the conservation constraints, and therefore have *ab initio* results for the final rates.

It may be noted that our values are higher than those of Primakoff, which were based on the form factor breakup formalism. These had not been recalculated hitherto in the $(V - A)$ mode despite improved precision in experimental probes.

The sensitivity of the rates for the use of muon mass corrected for the binding energy of the muon, as compared to using the free muon mass, was also studied. We find, using the free muon mass values for singlet and triplet rates, $R_s = 671.770 S^{-1}$ and $R_t = 16.555 S^{-1}$, respectively. While these values differ from those quoted in Table I(a) using the binding-energy corrected mass, the difference is not large enough to explain the increase of our values over those of Primakoff. The difference between our values and those of Primakoff most probably originate in the use of the exact $(V - A)$ formalism used by us, as discussed earlier.

The molecular case can be obtained from the individual atomic cases by modification of the density of states factor. We take these over from the literature [10] as

$$\begin{aligned} R_{\text{ortho}} &= 2\gamma_0[(\tfrac{3}{4})R_S + (\tfrac{1}{4})R_t] , \\ R_{\text{para}} &= 2\gamma_p[(\tfrac{1}{4})R_S + (\tfrac{3}{4})R_t] , \end{aligned} \tag{27}$$

where we have taken [11]

$$2\gamma_0 = 1.009 \pm 0.0001, \quad 2\gamma_p = 1.143 \pm 0.001$$

TABLE II. The rate of muon nuclear capture ($\sec^{-1}$) from ortho and para $p\mu p$ molecules.

	R_{ortho}	R_{para}
Present calculation	512.473	206.124
Results of Ref. [3]	506	200

for orthomolecular and paramolecular ions, respectively. R_s and R_t represent the captures rates in singlet and triplet states for a μp atom.

The factors $2\gamma_0$ and $2\gamma_p$ account for the altered density of states in the molecular environment and the composition of the hyperfine combinations. In the present calculation, we have taken $2\gamma_0 = 1.009$ and $2\gamma_p = 1.143$.

In Table II, we present values for the capture rate from ortho and para $p\mu p$ molecules. It seems pertinent to remark that the recoil effects of the spectator proton for the molecular case are not included in Eq. (27). We have seen in our earlier investigations of analogous Coulomb dressed weak processes that spectator momenta peak near their initial bound-state values. Thus these effects are not expected to modify the values in Table II appreciably. Finally, the contact between theory and experiment merits discussion. The muon decay rate R_0 appears in the expression used for extraction of the capture rate from experimental data [3].

We have reported earlier the modification of R_0 for μ^- decay from the $p\mu p$ molecule to R_0^{mol} where [2] $R_0^{\text{mol}} = 0.999\,917 R_0$. This could affect experiment values for the derived capture rate. In conjunction with our improved estimates for the pure theoretical values (Table I), for the hyperfine structure of the capture rate, the use of the corrected R_0^{mol} instead of R_0 is expected to yield a more precise determination of the molecular capture rate. These values also provoke a reanalysis of the exclusive highly sophisticated data of [12] that provide the ortho-para transition rate in the $p\mu p$ molecule.

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