Enhanced transient squeezing in a kicked Jaynes-Cummings model

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It is found that in an externally kicked Jaynes-Cummings model the transient squeezing of the field can be sustained for longer times as compared with the usual dynamics.

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From its original formulation the Jaynes-Cummings model [1] remains as a central one in quantum optics, because of its simplicity and powerful predictive nature. The quantum-mechanical description of the electromagnetic field gives rise to one of the most important effects predicted by the model, namely, the collapses and revivals on the difference of atomic level populations [2]. The model is the basis of the micromaster theory [3], where sub-Poissonian electromagnetic fields could be realized. More recently, the possibility of generating pure atomic states has been discussed within this model [4,5]. Also, squeezing of the electromagnetic field is predicted in the transient regime of the atom-field evolution [6]. Collapses and revivals, and the sub-Poissonian photon statistics [7,8] have been experimentally tested in high-Q microwave cavities operating at very low temperatures.

In this work, we study the transient regime of the atom-field interaction in the Jaynes-Cummings model, putting emphasis on the transient squeezing which exhibits the field mode. We consider a theoretical model, which includes a mechanism that sustains the squeezing of the electromagnetic field, in the transient evolution, for longer times than those found in the usual dynamics. The physical is achieved by kicking periodically the atom-field system with an intense electromagnetic field pulse, which modifies the global atom-field state after the kick, to an appropriate condition for the generation of a nonclassical state. Additionally, this model could provide a mechanism to convert a coherent state into an amplitude nonclassical field state.

Let us consider a two-level atom interacting with a quantized electromagnetic field. For simplicity, we assume that the frequency of the field is tuned to the atomic transition. In addition, we consider the system being kicked by an external classical electric field during an interval of time, which is very small when compared with the time between the kicks. The Hamiltonian is given by

$$H=\sum_{i=a,b}\hbar w_i|i\rangle\langle i|+\hbar w a^\dagger a+\hbar_g(a^\dagger|b\rangle\langle a|+|a\rangle\langle b|a)$$
$$+\sum_{n=0}^{\infty}\hbar(\xi_n|b\rangle\langle a|+\text{H.c.})\Theta(t-t_n^-)\Theta(t_n^+-t)\ , \quad (1)$$

where $|i\rangle\langle j|$ represents the atomic operators, a and $a^\dagger$ denote the usual field operators. The amplitude ξ_n corresponds to the Rabi frequency of the classical electrical field, which is coupled to the atom via an electric dipole interaction. The function $\Theta(t-t_n^-)\Theta(t_n^+-t)$ represents the kick of duration $\Delta t_n=t_n^+-t_n^-$, which is applied around the time t_n. The period between kicks is $\tau_n=t_n-t_{n-1}$. We assume that the classical field is perpendicularly polarized with respect to the quantum field in such a way that they are not mutually affected. This theoretical model assumes the existence of a train of intense laser pulses, for which the pulse duration Δt_n is very small as compared with the period τ_n of the nth kick. Within this assumption, it is reasonable to consider $\Theta(t-t_n^-)\Theta(t_n^+-t)=\Delta t_n\delta(t-t_n)$. A similar picture has been previously considered to study the dynamics of a two-level atom kicked by a classical field [9]. We basically consider the same idea, but we are interested in analyzing the evolution between kicks when a two-level atom is coupled to a quantum field. The Hamiltonian between kicks is given by the superposition of a usual Jaynes-Cummings model, $H_{\rm JC}$, plus the kick interaction $V_K(t)$. After the kick, the evolution is only generated by $H_{\rm JC}$. We begin our analysis by considering the unitary evolution of the whole system in the global atom-field Hilbert space. The evolution in the interaction picture from a time t_n^- (smaller than t_n) to a time t_{n+1}^- is given by

$$U(t_{n+1}^-,t_n^-)=P\exp\left[-\frac{i}{\hbar}\int_{t_n^-}^{t_{n+1}^-}[V_{\rm JC}(t')+V_K(t')]dt'\right]\ . \quad (2)$$

The operator P denotes the Dyson time-ordering operator. The unitary property of the evolution operator allows us to write

$$U(t_{n+1}^-,t_n^-)=U(t_{n+1}^-,t_n^+)U(t_n^+,t_n^-)\ , \quad (3)$$

where t_n^+ is a time greater than the time t_n. For $t>t_n^+$, the atom only interacts with the quantum field. For $t_n^-<t<t_n^+$ both interactions are present. The relation above can be written as

$$U(t_{n+1}^-,t_n^-)=P\exp\left[-\frac{i}{\hbar}\int_{t_n^+}^{t_{n+1}^-}V_{\rm JC}(t')dt'\right]\times P\exp\left[-\frac{i}{\hbar}\int_{t_n^-}^{t_n^+}[V_{\rm JC}(t')+V_K(t')]dt'\right]. \tag{4}$$

We know that the relative importance of $V_{\rm JC}$ and V_K is determined by the strength of their respective coupling constants and the value of the time in which they are considered. Let us consider the situation in which $\Delta t_n=t_n^+-t_n^-$ is very small, in such a way that the contribution of the Jaynes-Cummings part during this time is negligible compared with that of the kick. Under this approximation, from the Hamiltonian in Eq. (1), the evolution operator in Eq. (2), between kicks can be written as

$$U(t_{n+1}^-,t_n^-)=U_{\rm JC}(t_n^++\tau_n,t_n^+)U_K(t_n)\ , \tag{5}$$

where we have replaced $t_{n+1}^-\simeq t_n^++\tau_n$. The evolution between kicks, $U_{\rm JC}(t_n^++\tau_n,t_n^+)$, corresponds to the well known Jaynes-Cummings evolution operator for a two-level atom [10]. The operator $U_K(t_n)$ is given by

$$U_K(t_n)=\exp[-i\xi_n\Delta t_n(|a\rangle\langle b|+{\rm H.c.})]\ . \tag{6}$$

This last operator is the classical version of the Jaynes-Cummings evolution operator. Thus, the temporal evolution between kicks is known. It is convenient to consider the description in terms of the density matrix operator, namely, at time t_{n+1}^- the state of the coupled system is described by

$$\rho_{\rm tot}(t_{n+1}^-)=U_{\rm JC}(t_n^++\tau_n,t_n^+)U_K(t_n)\rho_{\rm tot}(t_n^-)\times U_K^\dagger(t_n)U_{\rm JC}^\dagger(t_n^++\tau_n,t_n^+)\ . \tag{7}$$

In this way we have obtained a quantum map in a discrete time scale, which describes the dynamics of the atom-field interaction in the global Hilbert space of the atom and the field. Because the atom and the field evolves to an entangled state, we explore numerically this recursive map by considering the general density matrix in the global space

$$\rho_{\rm tot}=\begin{bmatrix}\hat\rho_{aa} & \hat\rho_{ab}\\ \hat\rho_{ba} & \hat\rho_{bb}\end{bmatrix}\ . \tag{8}$$

In order to illustrate the physics involved in this process, we consider the kicks with a constant Rabi frequency ξ applied with a constant period $\tau_n=T$. The main question is: What is the role of the kick in the dynamical behavior of the quantum field? To answer this question, we analyze the unkicked dynamics of a field which is initially in a coherent state of average photon number $|\alpha|^2=10$ and the atom initially in its upper level. Figure 1 shows the quadrature fluctuation $(\Delta a_1)^2$ and the upper level population. We observe that the quadrature noise tends to decrease when the atomic upper level population is below $\frac{1}{2}$. Squeezing appears when the upper level population begins to increase from its lower value around $gt=0.43$ to higher values. The noise is reduced up to a maximum level around a time for which the upper level population is dynamically set to $\frac{1}{2}$. After that, the squeezing begins to disappear when the upper level population is greater than $\frac{1}{2}$. For later times, the same picture repeats itself, even when no squeezing is achieved, but the trend to noise reduction appears again when the upper level population is below $\frac{1}{2}$. We ask ourselves the following question: What would happen if we reset the atom-field state every time the atom gets inverted to the lower level? And, how can we do it? In order to see this, let us consider the effects of the kick on the global atom-field state. After the kick the total density matrix is given by

$$\rho_{\rm tot}(t_n^+)=U_K(t_n)\rho_{\rm tot}(t_n^-)U_K^\dagger(t_n)\ , \tag{9}$$

where the operator U_K given in Eq. (6) explicitly reads

$$U_K=\begin{bmatrix}\cos(\xi\Delta t_n) & -i\sin(\xi\Delta t_n)\\ -i\sin(\xi\Delta t_n) & \cos(\xi\Delta t_n)\end{bmatrix}\ . \tag{10}$$

After the kick, the expression for the diagonal $\hat\rho_{aa}$ operator in the expression (8) is given by

$$\hat\rho_{aa}(t_n^+)=\hat\rho_{aa}(t_n^-)\cos^2(\xi\Delta t_n)+\hat\rho_{bb}(t_n^-)\sin^2(\xi\Delta t_n)+i\hat\rho_{ab}(t_n^-)\sin(\xi\Delta t_n)\cos(\xi\Delta t_n)+{\rm H.c.} \tag{11}$$

It is apparent from this equation that choosing a particular value of the adimensional time associated with the kick, it is possible to change the global atom-field state $\rho_{\rm tot}(t_n^-)$ to a new one $\rho_{\rm tot}(t_n^+)$, with modified atomic populations. By taking the trace respect to the field variables, Eq. (11) allows us to observe how the atomic upper level population evolves. However, we have to point out that the dynamics is governed at all times by the Hamiltonian in Eq. (1) and we study only the internal dynamics of the coupled system. From Fig. 1, we observe how the upper level population reaches it first minimum value and the lower one reaches its maximum value, for $gt=0.43$. We can see that when choosing $\xi\Delta t_n=(2n+1)\pi/2$ in Eq. (11), for some integer n, we can transform the global atom-field state $\hat\rho_{\rm tot}(t_n^-)$ to a new state $\hat\rho_{\rm tot}(t_n^+)$ with inverted atomic populations, as is clear from Eq. (11). We sank see that atomic coherence $\hat\rho_{ab}(t_n^-)$ does not contribute because in this case the initial coherence is zero. In

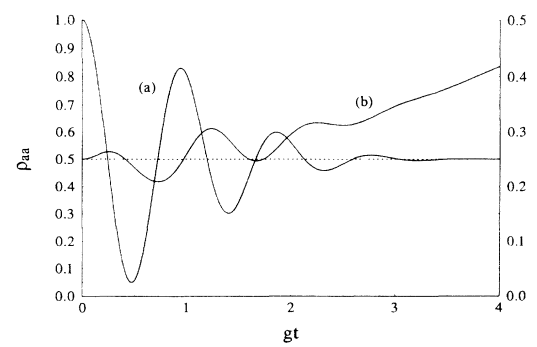

FIG. 1. Usual Jaynes-Cummings dynamics (unkicked case). Evolution of the upper level population (a) and quadrature fluctuations (b) as a function of gt, for an initial coherent state with $|\alpha|^2=10$ and the atom initially in the upper level.

this way, the global atom field given by Eq. (8) is modified in such a way that the global system evolves from a different set of conditions, which changes its later evolution. Figure 2 shows the quadrature noise of $(\Delta a_1)^2$ and the upper level population as a function of gt for $\xi\Delta t=(2n+1)\pi/2$ and the kick period is chosen such that $gT=0.5$. The modification of the transient quadrature noise is evident from Fig. 2. We observe that the population inversion provided by the kick prevents the field noise from increasing. This is because the upper level population never collapses to $\rho_{aa}=\frac{1}{2}$, as in Fig. 1, where the quadrature noise tends to increase far from the standard limit. In Fig. 3, we consider a variable kick period and Rabi frequency, for the same parameter as in Fig. 1, resetting the global field state every time when the upper level population reaches the value $\rho_{aa}=0.4$, with positive slope. The quadrature noise behaves like the regular period case of Fig 2, but we observe a smoother curve. We reset ρ_{aa} at this point because, as shown in Fig. 1, the quadrature noise is near the minimum and the upper level population does not reach $\rho_{aa}=\frac{1}{2}$. If we would choose to reset the atom-field state when the upper level population is near to this value, the kick would have no effects on the transient squeezing, because both the upper and lower level population would be near to $\frac{1}{2}$, allowing the noise to increase.

Now we consider the general case of atoms injected in a coherent superposition. In order to compare our results, we study the situations discussed in Ref. [11]. In this case, because the field acquires, in general, a phase dependence, it is convenient to express the field quadratures in a rotated frame

$$a_1=\tfrac{1}{2}(ae^{-i\theta}+a^{\dagger}e^{i\theta})\ , \tag{12}$$

$$a_2=\frac{1}{2i}(ae^{-i\theta}-a^{\dagger}e^{i\theta})\ , \tag{13}$$

where the phase θ corresponds to the phase of the averaged electric field $\langle a\rangle=Ae^{-i\theta}$. Let us consider atoms initially injected with $\rho_{aa}=\frac{2}{3}$, and the field in an initial coherent state $|\alpha|^2=1$. This set of parameters corresponds to that considered in Ref. [11]. In this case, it is assumed that the atom is initially in a pure state, that is $\rho_{ab}=\sqrt{\rho_{aa}\rho_{bb}}\,e^{i\theta}$ and the phase is chosen as $\theta=\pi/2$. Figure 4 illustrates the upper level population and the quadrature noise fluctuation $(\Delta a_1)^2$ for the unkicked system. Similar considerations to that of the incoherent case apply now. We observe that the quadrature noise tends to decrease when the upper level population is below $\frac{1}{2}$, and reaches its minimum value when ρ_{aa} increase with a positive slope. In order to see the effects of the kick, we have to include the contribution of the off-diagonal $\hat{\rho}_{ab}(t_n^+)$ in Eq. (11), due to the initial phase of the atomic state. In Fig. 5, the upper level population and the quadrature noise $(\Delta a_1)^2$, are shown for the average upper level population reset around the initial value $\rho_{aa}=\frac{2}{3}$ when $\rho_{aa}=0.2$ with a positive slope. This point was chosen because around this value of ρ_{aa} the maximum reduction of $(\Delta a_1)^2$ occurs, as observed in Fig. 4. The squeezing is sustained for longer times as compared with the case with no initial coherence.

Additionally to the enhanced transient squeezing, an amplification of the quantum field is expected because of the energy transference from the classical electric field, which occurs when the atomic populations get inverted due to the kick. This effect is observed in Fig. 6, where the average photon number intensity for the previous

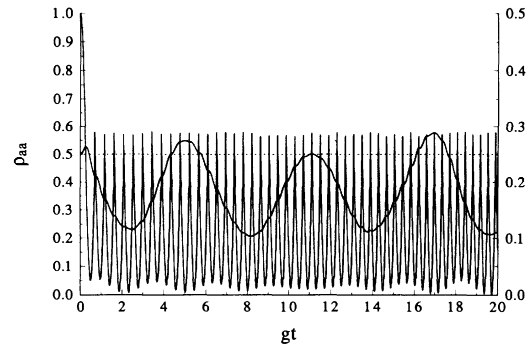

FIG. 3. The upper level population and quadrature noise for variables kick period and Rabi frequency. The frequency initially is in a coherent state with $|\alpha|^2=10$, and the atom in the upper level $\rho_{aa}=1$.

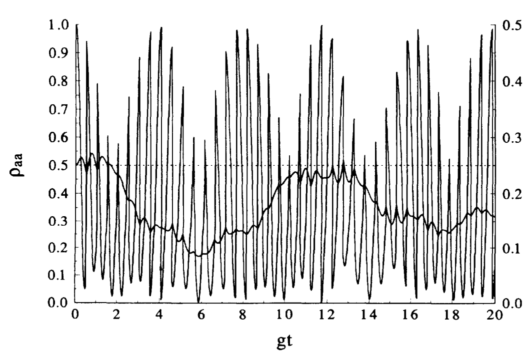

FIG. 2. Kicked Jaynes-Cummings dynamics for the same parameters of Fig. 1. The period and the Rabi frequency are, respectively, $gT=0.5$, and $\xi T=(2n+1)\pi/2$, with n an integer.

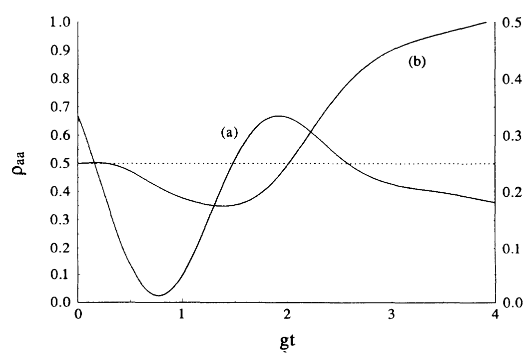

FIG. 4. Usual Jaynes-Cummings dynamics (unkicked case). Evolution of the upper level population (a) and quadrature fluctuations (b) as a function of gt, for an initial coherent state with $|\alpha|^2=1$ and the atom initially in a pure state with $\rho_{aa}=\frac{2}{3}$.

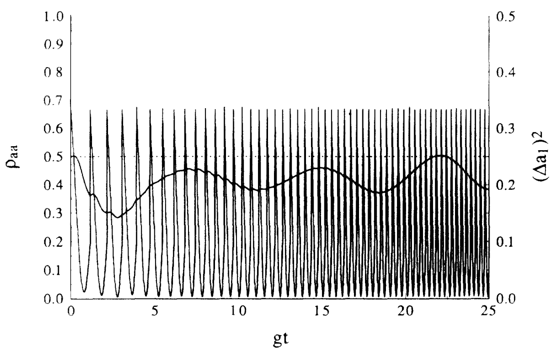

FIG. 5. The upper level population and quadrature noise for variables kick period and Rabi frequency. The frequency initially is in a coherent state with $|\alpha|^2=1$, and the atom in a pure state with $\rho_{aa}=\frac{2}{3}$.

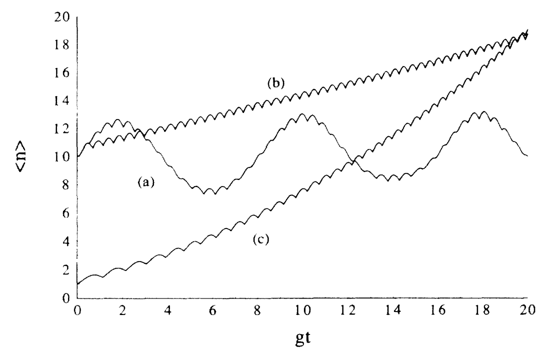

FIG. 6. The average photon number vs gt corresponding to Fig. 3 (a), Fig. 2 (b); and Fig. 5 (c).

cases is shown. We observe that in the case of a regular period of the kick, an oscillator of the average photon number is observed, in agreement with the behavior of the upper level population in Fig. 2. For a variable kick period, there is an amplification of the photon number average because the field always receives energy from the classical field when a population inversion occurs. The amplification could be an important effect when a weak quantum field with classical properties, for example a coherent state, is converted to a nonclassical state with an amplified intensity. This theoretical model result is interesting and opens new questions about the possibility of discussing a more realistic model, and eventually of observing these predictions in a real experiment. These questions are related to recent research in the generation of bright amplitude squeezing [12,13].

In this work, we discussed a theoretical model of an atom coupled to one mode of the electromagnetic field when the system is being externally kicked by a classical electric field. We have found that the transient squeezing, which naturally appears in the usual system, can be sustained for longer times by means of both an appropriate Rabi frequency of the external electric field and period of the kick. This effect is fundamentally due to the reset that the external kick produces on the atom-field state. Finally, the energy transference from the external field could allow the amplification of the quantum field.

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