Conservation law for multiple four-wave-mixing processes in a nonlinear optical medium

Darlene L. Hart, Arthur Judy, T. A. Brian Kennedy, Rajarshi Roy, and Kroum Stoev
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332
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When two optical waves at frequencies ω_1 and ω_2 interact in a medium with cubic nonlinearity, multiple four-wave-mixing processes can generate pairs of sidebands. A conservation law that connects the amplitudes of the pump waves and sidebands is shown to arise from the Manley-Rowe relations and from the infinite set of conservation relations of the nonlinear Schrödinger equation. The conservation relation is examined theoretically and verified experimentally for a variety of input pump power levels and asymmetries. It can be used as a diagnostic measure to determine if nonlinearities of higher order than cubic are present.

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INTRODUCTION

Intense light waves propagating in nonlinear optical media may generate excitations at new frequencies (sidebands) not contained in the input, and the four-wave-mixing (FWM) interactions that occur between the input waves and those originating in the medium may exhibit complex spatial and temporal characteristics. The exchange of energy among these waves is of great interest for optical communications. The magnitude of these interactions determines the limits of power and frequency spacing of laser sources. From a different perspective, the generation of light at multiple wavelengths with a specified frequency spacing may provide a novel light source with characteristics that are very difficult to obtain otherwise.

Earlier investigations of FWM in a single mode optical fiber [1,2] revealed that both periodic and aperiodic exchange of energy could occur between two input pump waves and the sidebands generated through FWM. For input waves at angular frequencies ω_1 and ω_2, first-order sidebands are generated by four wave mixing at $\omega_3=(2\omega_1-\omega_2)$ and $\omega_4=(2\omega_2-\omega_1)$ for a medium with a third-order nonlinear susceptibility. Second-order sidebands are then generated by FWM at $\omega_5=(2\omega_3-\omega_1)=(3\omega_1-2\omega_2)$ and $\omega_6=(2\omega_4-\omega_2)=(3\omega_2-2\omega_1)$. Higher-order sidebands may evolve subsequently. It was found that a conservation law could be derived for the asymmetries of the power in the pump waves and sidebands from the coupled, nonlinear propagation equations for the complex wave amplitudes. In this paper we will review the derivation of the conservation relation, and show its connection to the Manley-Rowe relations when several FWM processes occur simultaneously. While the above derivations are based on the coupled equations for the amplitudes of a finite set of discrete frequencies, it will be shown that this conservation relation may also be obtained from the nonlinear Schrödinger equation (NLSE) in which a single amplitude is used to describe the entire four-wave-mixing spectrum. We will also comment on how the conservation of asymmetry relates to the hierarchy of conservation relations that exist for the NLSE. Finally, we will present the results of experiments performed to test the validity of the conservation relation for bichromatic pump input to a single mode optical fiber.

THEORETICAL CONSIDERATIONS

For long pulses or continuous wave input and assuming monochromatic waves, the coupled amplitude equations for the pumps and sidebands derived from the wave equation [1] with a cubic nonlinearity $\chi^{(3)}$ are written below,

$$\frac{dU_j}{dz}=i\gamma P_1\left[\left[|U_j|^2+2\sum_{k\neq j}|U_k|^2\right]U_j + {\sum_{kmn}}^{*} d_{kmn}U_kU_mU_n^*\exp(i\Delta\beta_{kmn}z)\right], \quad (1)$$

where $j,k,m,n=1,2,3,4,5,6$ and $k,m\neq n$, and U_j are the complex field amplitudes normalized to the absolute value of the amplitude of the pump frequency component at ω_1 (which has power P_1 at the input end of the fiber). A larger number of sidebands could be included for higher input intensities. Here $\sum_{kmn}^{*}$ denotes the permutations of the indices k, m, and n such that $\omega_k+\omega_m-\omega_n=\omega_j$, and the quantity $\Delta\beta_{kmn}=\beta_k+\beta_m-\beta_n-\beta_j$ is the axial wave-vector mismatch. The quantity d_{kmn} is a degeneracy factor that is unity when $k=m$ and 2 when $k\neq m$. The nonlinearity coefficient γ is given by the relationship

$$\gamma=\frac{\overline{\omega}n_2^I}{cA_{\text{eff}}}, \quad (2)$$

where A_{eff} is the effective core area of the fiber determined by the size of the fundamental mode, n_2^I is the Kerr coefficient for the intensity-dependent refractive index and is directly proportional to $\chi^{(3)}$, c is the speed of light in vacuum, and $\overline{\omega}$ is the average angular frequency of the waves [3]. The linear mismatches $\Delta\beta_{kmn}$ can be simplified using the approximation that the material part of the index difference dominates the mismatch and the waveguiding contribution can be neglected. This approx-

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imation is justified for the frequency separations in these experiments, since the v number characterizing the single transverse mode changes by less than 1% over the entire range of frequencies considered. By using the frequency relationships between the peaks and expanding the propagation constants β_j about one of the pump frequencies, e.g., ω_1, all the mismatches are found to be integer multiples of the quantity $\Delta\kappa=\Omega_{12}^2\beta^{(2)}$ where $\Omega_{12}=|\omega_1-\omega_2|$ is the detuning between the two pump waves and $\beta^{(2)}$ is the second-order dispersion coefficient [3]. These amplitude equations can be solved numerically, and the energy of each frequency component obtained as a function of distance along the fiber.

If we let the scaled powers of the waves be $\rho_m=|U_m|^2$, then as was shown in Ref. [2] Eqs. (1) display power conservation, as is expected. It was also shown that another conserved quantity

$$C=[\rho_1(z)-\rho_2(z)]+3[\rho_3(z)-\rho_4(z)]+5[\rho_5(z)-\rho_6(z)] \tag{3}$$

is obtained for the multiple four-wave-mixing processes that occur within the fiber. This relation holds at any distance z of propagation in the nonlinear medium, and connects the asymmetries of the pump waves and sidebands. For a symmetric set of input pump waves, $C=0$, and the sidebands evolve symmetrically as well. When the inputs are asymmetric, there can be complicated changes in the relative power levels of the pump waves and sidebands along the length of the fiber, but the constant $C=[\rho_1(0)-\rho_2(0)]$ is a conserved quantity for the propagation. It was shown in [2] that conservation of power and Eq. (3) are the only two conservation relations that involve linear combinations of the powers at the various frequencies.

We will now examine generalizations and extensions of this conservation law, its physical interpretation, and its experimental validity. Our first step is to show that the conservation relation (3) follows also from the Manley-Rowe relations [4,5]. To this end, we introduce a notation that is conventional in the statement of the Manley-Rowe relations. The input pump frequencies ω_1 and ω_2 are then written as ω_{10} and ω_{01}, respectively, while the notation $\omega_{jk}=j\omega_1+k\omega_2$ is used for the sideband frequencies. This notation has the advantage of explicitly revealing the combination of pump photons necessary to produce a particular sideband. Thus, for $j=2$, $k=-1$, $\omega_{2-1}=(2\omega_1-\omega_2)=\omega_3$; for $j=-1$, $k=2$, $\omega_{-12}=(2\omega_2-\omega_1)=\omega_4$; for $j=3$, $k=-2$, $\omega_{3-2}=(3\omega_1-2\omega_2)=\omega_5$; and for $j=-2$, $k=3$, $\omega_{-23}=(3\omega_2-2\omega_1)=\omega_6$. The notation for the power at the different frequencies is $\rho_{10}=\rho_1$, $\rho_{01}=\rho_2$, $\rho_{2-1}=\rho_3$, $\rho_{-12}=\rho_4$, $\rho_{3-2}=\rho_5$, and $\rho_{-23}=\rho_6$. These relations are tabulated below:

j	k	$\omega_{jk}=\omega_i$	$\rho_{jk}=\rho_i$
1	0	$\omega_{10}=\omega_1$	$\rho_{10}=\rho_1$
1	0	$\omega_{01}=\omega_2$	$\rho_{01}=\rho_2$
2	-1	$\omega_{2-1}=(2\omega_1-\omega_2)=\omega_3$	$\rho_{2-1}=\rho_3$
-1	2	$\omega_{-12}=(2\omega_2-\omega_1)=\omega_4$	$\rho_{-12}=\rho_4$
3	-2	$\omega_{3-2}=(3\omega_1-2\omega_2)=\omega_5$	$\rho_{3-2}=\rho_5$
-2	3	$\omega_{-23}=(3\omega_2-2\omega_1)=\omega_6$	$\rho_{-23}=\rho_6$

Higher-order sidebands are found from other combinations of j and k when $j-k=$(odd integer) and $|j|-|k|=\pm1$ are both satisfied. Then the Manley-Rowe relations, for the coupled amplitude equations given above, are compactly expressed in this notation as [5]

$$\sum_{j,k}\frac{j}{\omega_{jk}}\frac{d\rho_{jk}}{dz}=0 \tag{4}$$

and

$$\sum_{j,k}\frac{k}{\omega_{jk}}\frac{d\rho_{jk}}{dz}=0\ , \tag{5}$$

where the summations are over the values of j and k considered in the table above. For parametric processes such as the multiwave mixing, the relations arise from the permutation symmetry of the nonlinear susceptibility. Beyond the conservation of power, the Manley-Rowe relations can be used to gain more insight into the symmetries of the multiwave mixing processes. Combining the relations above, we find that

$$\frac{d}{dz}\left\{\sum_{j,k}\frac{(j-k)}{\omega_{jk}}\rho_{jk}\right\}=0\ . \tag{6}$$

The conserved quantity for the asymmetry in the waves is then immediately found to be

$$C'=\left[\frac{\rho_{10}}{\omega_{10}}-\frac{\rho_{01}}{\omega_{01}}\right]+3\left[\frac{\rho_{2-1}}{\omega_{2-1}}-\frac{\rho_{-12}}{\omega_{-12}}\right]+5\left[\frac{\rho_{3-2}}{\omega_{3-2}}-\frac{\rho_{-23}}{\omega_{-23}}\right]. \tag{7}$$

For small frequency detunings between the pumps and sidebands, $\omega_{10}\approx\omega_{01}\approx\omega_{2-1}\approx\omega_{-12}\approx\omega_{3-2}\approx\omega_{-23}\approx\bar{\omega}$, we find that Eqs. (6) and (7) are expressions of the same conservation law given in Eq. (3). By summing Eqs. (4) and (5), one arrives at an expression for conservation of power among the pump waves and sidebands. For nonparametric processes, such as stimulated Raman scattering, in which some of the energy will be transferred to the medium, these relations will not be preserved.

The derivation above can be extended to include any number of sidebands that are generated by the nonlinear response of the medium, and to show that seven, nine, etc. times the difference in power of the corresponding higher-order sidebands would be included in the sum to form the conserved quantity. Thus we have found a conservation law for simultaneous, multiple four-wave-mixing processes that occur in a nonlinear medium with an input of discrete, equally spaced frequencies. The frequencies that appear in the medium due to FWM, and the conservation laws themselves, follow from the overall permutation symmetry of the third-order nonlinear susceptibility that was used in the derivation of Eqs. (1) [2,5].

The previous analysis considered a set of coupled ordinary differential equations for the propagation of monochromatic pump waves and sidebands in the nonlinear medium. This system of equations is limited in practice to a finite set of discrete frequency inputs and can be cumbersome when considering many higher-order sidebands. A more general description of wave propagation is achieved using a partial differential equation (PDE) model. A systematic derivation starting from the Maxwell equations for the nonlinear fiber medium (assuming a third-order nonlinearity) results in the nonlinear Schrödinger equation [3], provided the range of frequencies spanned is sufficiently small compared to the optical frequency.

NONLINEAR SCHRÖDINGER EQUATION AND CONSERVATION LAWS

The coupled-mode analysis presented earlier assumed time periodic waves and parabolic group velocity dispersion. The derivation of the NLSE makes similar assumptions about the dispersion and this suggests that the conservation conditions given previously may be found among the infinite set of conservation laws which exist for the NLSE. Here we establish this connection for the NLSE with periodic boundary conditions.

We first show that the coupled-mode equations (1) may be derived from the standard NLSE. The electric field in the fiber is written $\mathcal{E}(t,z)=E(t,z)\exp[-i(\omega_{10}+\omega_{01})t/2]$ +(complex conjugate), where the field envelope $E(t,z)$ satisfies the NLSE

$$i\frac{\partial E}{\partial z}-\frac{1}{2}\beta^{(2)}\frac{\partial^2 E}{\partial t^2}+\gamma|E|^2E=0\ , \tag{8}$$

where t is time measured in a frame moving at the group velocity of the wave and $E(t,z)$ is the electric field in $\mathrm{W}^{1/2}$. Since we are interested in the propagation of a number of discrete equally spaced frequency components, we assume the Fourier series expansion

$$E(t,z)=\sum_{jk}{}^{*}\sqrt{P_1}\,U_{jk}(z)\times\exp[i\beta_{jk}z-i(j-k)(\omega_{10}-\omega_{01})t/2]\ , \tag{9}$$

where $(j-k)$ is an odd integer, and we label the frequency components in the way described in the table above. Substituting (9) into (8) yields an equation identical to (1), when truncated to six modes.

To address conservation laws of the NLSE it is convenient to first scale the equation into the standard Ablowitz-Kaup-Newell-Segur (AKNS) form [6]. Define the scaled variables

$$\zeta=\frac{\beta^{(2)}z}{2t_0^2}\ ,\quad \eta=\frac{t}{t_0}\ ,\qquad q(\zeta,\eta)=\left[\frac{\gamma t_0^2}{\beta^{(2)}}\right]^{1/2}E^*(t,z)\ , \tag{10}$$

where t_0 is a time scale which is as yet undefined, so that Eq. (8) in the normal dispersion regime ($\beta^{(2)}>0$) becomes

$$i\frac{\partial q}{\partial \zeta}+\frac{\partial^2 q}{\partial \eta^2}-2q^*q^2=0\ . \tag{11}$$

The boundary conditions appropriate to the multiwave mixing problem may be written $q(\zeta,\eta)=q(\zeta,\eta+T)$, where the numerical value of the period T may be conveniently set by choice of the time scale t_0 introduced in Eq. (10): $T=4\pi/(\omega_{10}-\omega_{01})t_0$.

We now discuss a quantum-mechanical interpretation of the NLSE which is useful for motivating the conservation laws. Equation (11) may be regarded as the Heisenberg equation for the second quantized field operators $q(\zeta,\eta)$ and $q^*(\zeta,\eta')$ of a one-dimensional Bose gas with repulsive interactions [7]. The fields satisfy the canonical equal-time commutation relations $[q(\zeta,\eta),q(\zeta,\eta')]=\delta(\eta-\eta')$, where ζ is regarded as time and η as space. Note that the interpretations of ζ and η are reversed for optical fiber propagation as indicated by Eq. (10). Second quantized operators for particle number and momentum are given, respectively, by

$$\begin{aligned}N&=\int q^*(\zeta,\eta)q(\zeta,\eta)d\eta\ ,\\ P&=-i\int q^*(\zeta,\eta)\frac{\partial}{\partial\eta}q(\zeta,\eta)d\eta\\ &=-\frac{i}{2}\int\left\{q^*(\zeta,\eta)\frac{\partial}{\partial\eta}q(\zeta,\eta)-\left[\frac{\partial}{\partial\eta}q^*(\zeta,\eta)\right]q(\zeta,\eta)\right\}d\eta\end{aligned} \tag{12}$$

and the Heisenberg field equation (11) is generated by the Hamiltonian

$$H=\int\left\{\frac{\partial}{\partial\eta}q^*(\zeta,\eta)\frac{\partial}{\partial\eta}q(\zeta,\eta)+q^*(\zeta,\eta)^2q(\zeta,\eta)^2\right\}d\eta\ . \tag{13}$$

For a conservative Hamiltonian system all of these quantities will be conserved.

Now returning to the classical context, the NLSE can be derived from the Hamiltonian (13) using Hamilton's equations with $q(\zeta,\eta)$ and $q^*(\zeta,\eta)$ regarded as classical canonical field variables [8]. The inverse scattering method allows the complete integrability of the NLSE to be demonstrated: the infinite-dimensional Hamiltonian system has an infinite number of conserved quantities. The integrals on the right-hand side of Eqs. (12) and (13), with (q,q^*) now interpreted as classical complex fields, are the first three in this hierarchy. The discussion of the interacting Bose gas serves to motivate the particular conserved integrals in a physically appealing way, although the integral forms hold whenever the NLSE applies, in particular to the present case of propagation in a nonlinear optical fiber.

One further technical point regarding boundary conditions should be made here. In the multiwave mixing problem it is appropriate to apply periodic boundary conditions as described above. The more common situation in physical applications involves field pulses which vanish (solitons in the anomalous dispersion regime), or become constant (dark solitons in the normal dispersion regime) as $\eta\rightarrow\pm\infty$. The distinction in boundary conditions is very important as the inverse scattering technique has to be applied differently to the two situations [9]. Despite this difference, however, the conservation laws are constructed by a somewhat similar asymptotic procedure, the integrals being restricted to one period of η rather than $-\infty<\eta<\infty$ as in the soliton case.

The conserved integrals may be written in the form [9,10]

$$I_n=\int_{\eta_0}^{\eta_0+T} f_n(\eta)d\eta\ , \quad n=1,2,\ldots \tag{14}$$

where η_0 is arbitrary, and

$$\begin{aligned} f_1&=-|q|^2\ ,\\ f_{n+1}&=\sum_{jk}{}^{*} f_j f_k+q\frac{d}{d\eta}\left[\frac{1}{q}f_n\right]\ , \quad n=1,2,\ldots\ . \end{aligned} \tag{15}$$

The asterisk indicates $j+k=n$. The first three conservation laws, corresponding to particle number, momentum, and energy for the Bose gas are, respectively, proportional to

$$\begin{aligned} I_1&=-\int_{\eta_0}^{\eta_0+T}|q|^2 d\eta\ ,\\ I_2&=-\int_{\eta_0}^{\eta_0+T} q\frac{\partial q^*}{\partial\eta}d\eta=\frac{1}{2}\int_{\eta_0}^{\eta_0+T}\left[q^*\frac{\partial q}{\partial\eta}-q\frac{\partial q^*}{\partial\eta}\right]d\eta\ ,\\ I_3&=\int_{\eta_0}^{\eta_0+T}\left[\left|\frac{\partial q}{\partial\eta}\right|^2+|q|^4\right]d\eta\ . \end{aligned} \tag{16}$$

Substituting the Fourier expansion (with $T=2\pi$)

$$q(\zeta,\eta)=\sum_{jk}{}^{*} q_{jk}(\zeta)e^{-i(j-k)\eta} \tag{17}$$

into Eqs. (16) gives

$$\begin{aligned} I_1&=-2\pi\sum_{jk}{}^{*}|q_{jk}(\zeta)|^2\ ,\\ I_2&=-2\pi i\sum_{jk}{}^{*}(j-k)|q_{jk}(\zeta)|^2\ ,\\ \frac{I_3}{2\pi}&=\sum_{jk}{}^{*}[(j-k)^2|q_{jk}(\zeta)|^2+|q_{jk}(\zeta)|^2]+\sum_{\substack{jk,lm\\ np,rs\\ j-k+l-m=n-p+r-s}}{}^{*}\cdots\ , \end{aligned} \tag{18}$$

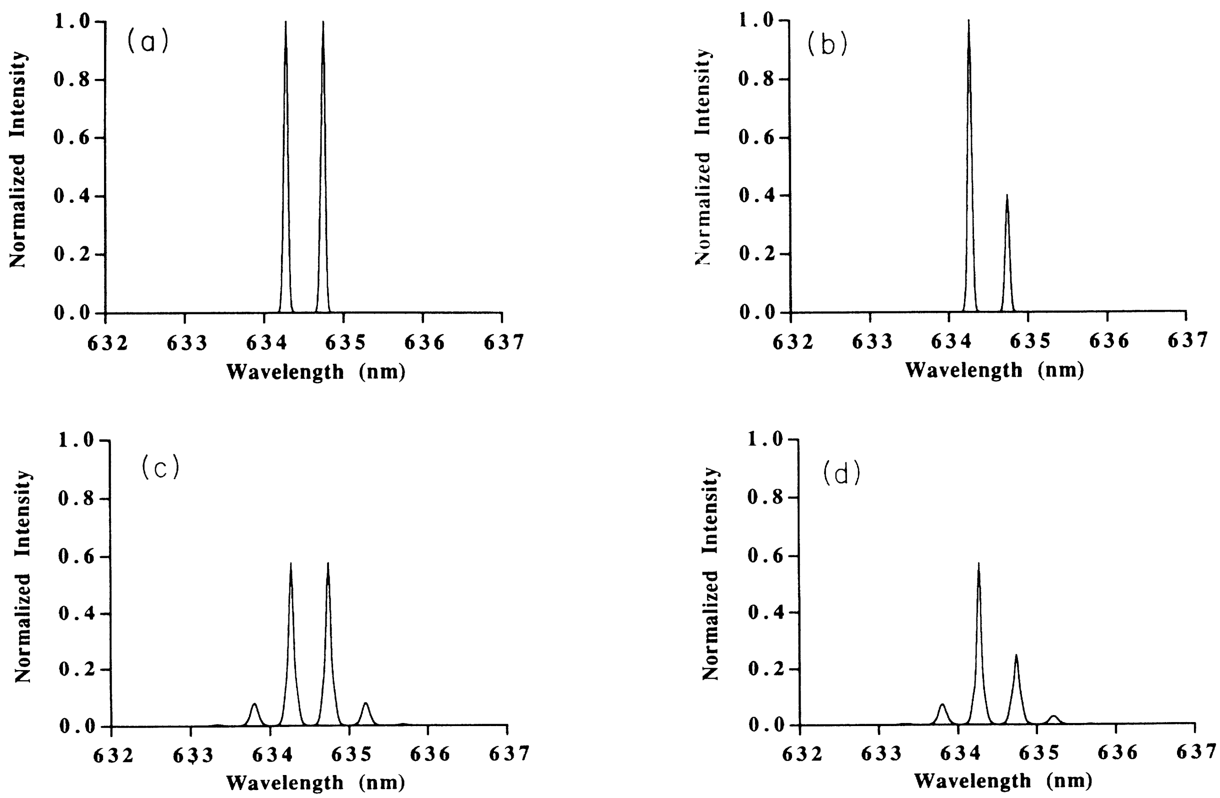

FIG. 1. Input and output spectra for symmetric and asymmetric pulses using the NLSE with a frequency detuning of 342 GHz, $T_0=5$ ps, $\beta^{(2)}=58$ ps^2/km, and $\gamma=1.85\times10^{-2}$ W^{-1}m^{-1}. Bichromatic inputs (a) symmetric $P_1=25$ W and (b) asymmetric [$C(0)=0.6$ in Eq. (3) and $P_1=42$ W]. NLSE output after 1.36 m of propagation, (c) symmetric case and (d) asymmetric case. All spectra are normalized to the input peak power at ω_1.

where $\sum^*$ indicates summation over j and k as governed by the nonlinear processes (see the table above) and $\phi_{jk}(\zeta)$ is the phase of $q_{jk}(\zeta)$. In contrast to the Bose gas I_1 here represents the conservation of power in all Fourier modes, while I_2 relates to the conservation of asymmetry of the Fourier modes established earlier via the Manley-Rowe relations. The latter account for the production of pairs of photons by the simultaneous destruction of other pairs through energy conserving degenerate and nondegenerate four-wave-mixing processes. These appear to be the only conservation laws which depend on the power in the Fourier modes alone. Direct optical detection methods enable these to be measured in a straightforward way. The higher conserved quantities depend on the relative phases of the Fourier amplitudes. For example, the Hamiltonian I_3 depends on combinations of the phases of four modes, and is therefore more difficult to check experimentally.

As alluded to earlier, the conservation laws hold for more general conditions than shown explicitly here, in particular when the input waves are not monochromatic, but pulsed. In Fig. 1 we show input and output spectra from computations based on the NLSE using the split step Fourier (beam propagation) method [3]. The second-order dispersion parameter and nonlinear coefficient were chosen to be $\beta^{(2)}=58$ ps^2/km and $\gamma=1.85\times10^{-2}$ m^{-1}W^{-1}, values corresponding to the fiber used in the experiments described in the next section. Figures 1(a) and 1(b) show the input spectra, for Gaussian pulses of 5 ps width, for symmetric and asymmetric pump inputs, respectively. The asymmetric case corresponds to the weaker pump having 40% of the power in the stronger one. Figures 1(c) and 1(d) show the spectra after propagation through 1.36 m of fiber. Figures 1(a) and 1(c) show that a symmetric input spectrum remains so after propagation. Figures 1(b) and 1(d) illustrate how the initial asymmetry is shared by the sidebands after propagation through the fiber. The conservation law should be regarded as a measure of the accuracy of the numerical integration for discrete frequency waves. For pulsed inputs the Fourier sum, Eq. (17), should be replaced by a Fourier integral to allow for the spectral broadening of the pump and sidebands during propagation. However, if the broadening is small compared with the separation, as in Fig. 1, Eqs. (3) and (18) hold very accurately.

EXPERIMENTAL TESTS OF THE CONSERVATION RELATION

A laser system consisting of two tunable dye lasers, pumped by a frequency doubled Nd:YAG (where YAG denotes yttrium aluminum garnet) laser, was used to generate pulses that are ~5 ns full width at half maximum (FWHM) in length. The entire experimental setup is shown in Fig. 2. The outputs from the two dye lasers ($\lambda \sim 633$ nm), detuned by 342 GHz, are amplified and then passed through the appropriate delays to ensure overlap of the pulses. The light is then coupled into a single mode polarization maintaining optical fiber (AT&T rectangular fiber, single mode at 633 nm, with a core diameter ~4 μm), after passage through a polarizer and half wave plate. The half wave plate rotates the polarization of the light to coincide with a principal axis of the birefringent fiber. The fiber length chosen for these investigations is 1.36 m, which results in a negligible level of Raman scattering at the power levels used (< 100 W). The dominant optical processes are four wave mixing of the pump waves and sidebands. At the fiber output, a beam splitter is used to monitor the power of the laser beams. A variable neutral density filter was used to attenuate the beams before input to a spectrometer, in which a video camera, charge injection device (CID) array, registered the spectrum. The spectrum was digitized by a frame-grabber board and stored in the memory of a microcomputer. The laser pulses vary in intensity and frequency from shot to shot; the spectrum from several hundred laser pulses may be averaged and stored in the computer. The individual laser pulses have spectral widths of ~3 GHz, and are thus essentially monochromatic when compared with the typical frequency spacings (of several hundred GHz) for the tunable lasers. The resolution of the spectrometer-video-camera system is ~50 GHz.

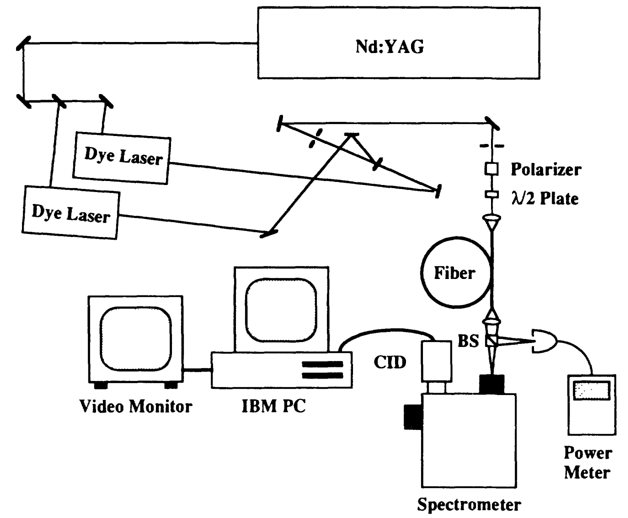

FIG. 2. Experimental apparatus used to study four wave mixing in a birefringent single mode optical fiber. BS denotes a beam splitter. CID denotes charge injection device video camera.

In Fig. 3 we show output spectra measured for symmetric and asymmetric pump inputs, and compare these results with computations based on the NLSE with cw input. The pulses from the lasers are ~5 ns (FWHM) and for the short lengths of fiber considered here this is considered to be an excellent approximation to cw input. Figure 3(a) shows the output spectrum for symmetric pump waves with 25 W peak power in each pump. The power in the two pump waves was equalized by first letting each beam propagate separately through the fiber. Then they were both allowed to simultaneously propagate through the fiber. We find sizable first-order and very small but noticeable second-order sidebands. The output spectrum is very symmetric, and agrees well with the predictions based on the NLSE with cw input, shown

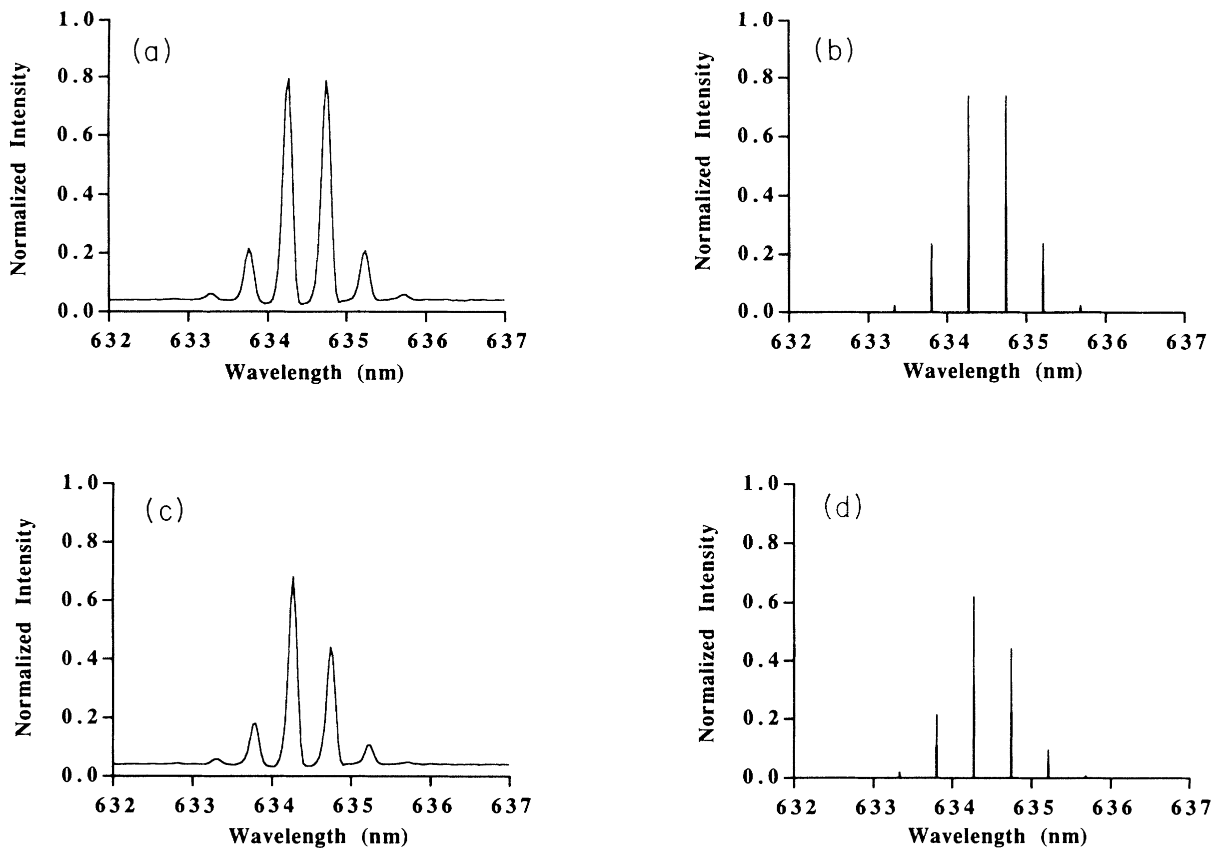

FIG. 3. Comparison of output spectra after a 1.36 m propagation length between the experiment and the NLSE with a cw input. Experimental output spectra with (a) symmetric inputs, $P_1=25$ W, and (c) strong asymmetry in the inputs, $C(0)=0.6$ and $P_1=42$ W. NLSE output with (b) symmetric input and (d) strong input asymmetry, $C(0)=0.6$. The parameter values are 342 GHz frequency detuning, $\beta^{(2)}=58$ ps^2/km and $\gamma=1.85\times10^{-2}$ W^{-1} m^{-1}. All spectra are normalized to the input peak power at ω_1.

in Fig. 3(b). The output spectrum for an input with 40% asymmetry in the pumps is shown in Fig. 3(c). The FWM interactions produce sidebands, and the asymmetry of the pump waves at the output is changed from that at the input. The first-order sidebands are asymmetric in this case, and a very small asymmetric pair of second-order sidebands is seen as well. Figure 3(d) shows the computational results for cw input, and there is quite reasonable agreement with the measured spectra.

In order to examine the validity of the conservation relation, we have adopted the following strategy. It is not possible to observe the evolution of the sidebands within the fiber. Thus we first allowed only one laser beam at a time to propagate to the end of the fiber and measured the average peak power of the pulses. The attenuation of the light in propagating through the short length of fiber ($L=1.36$ m) used was negligible. The value of $C(0)=\{\rho_{10}(0)-\rho_{01}(0)\}$, the initial asymmetry of the pump waves, was determined in this way. Both pump beams are then allowed to simultaneously propagate through the fiber and multiple FWM interactions resulted in the generation of sidebands. The power in each of the spectral components was determined by summing the area under each peak from the averaged spectrum. Four sets of 100 shots each were measured, and the average and standard deviation obtained from this data. The value of

$$C(L)=\{\rho_{10}(L)-\rho_{01}(L)+3[\rho_{2-1}(L)-\rho_{-12}(L)]+5[\rho_{3-2}(L)-\rho_{-23}(L)]\}$$

was thus determined from these measurements.

To test the conservation relation, $C(L)$ was plotted versus $C(0)$, as shown in Fig. 4. The data are symbol coded to display the number of sidebands that resulted for a given power in the pump waves. For low power values (≤ 40 W peak power), only the first-order sidebands were generated, while for the highest powers used (~ 100 W peak power), even the third-order sidebands were observed. From Fig. 4 it is seen that the asymmetry of the pump waves was varied from $C=-0.6$ and $C=+0.6$. We find that very good agreement is obtained with the conservation relation over the entire range of power and asymmetry investigated, within the limits of experimental error. The straight line drawn represents perfect agreement with the relation. There is a systematic deviation away from the line at the two extremes of asymmetry values. This small but discernible disagreement is probably due to the fact that there is a threshold of noise which has to be exceeded for the peaks to be visible and accounted for in the computations. While such unaccounted for contributions would cancel each other

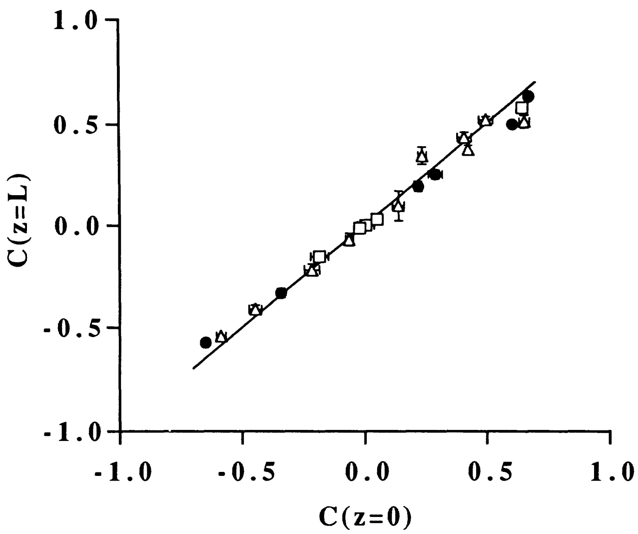

FIG. 4. Measured asymmetry relation, $C(L=1.36$ m) [Eq. (3), vs C at the fiber input, for a variety of input powers. Symbols correspond to the number of sidebands detected at the output; squares—only first-order sidebands, circles—first- and second-order sidebands, and triangles—first-, second-, and third-order sidebands. The solid line represents perfect agreement with the asymmetry relation.

for the symmetric situation, they would affect the measurements most for the highly asymmetric inputs where the higher-order terms have increasing significance in the conservation relation.

DISCUSSION

The exchange of power between multiple waves propagating in an optical fiber can in general be complex. In this paper we have investigated a conservation law which relates the amplitudes of waves generated through multiple four-wave-mixing processes. This relation can be used to check integration of the nonlinear propagation equations; it can also serve as a sensitive diagnostic for higher-order nonlinear effects that may be significant at higher powers or in certain nonlinear media. As long as other nonlinear effects are negligible, this asymmetry relation will still be valid and will provide insight into the relative evolution of the multiple waves.

In this paper we have shown that this conserved quantity [Eq. (3)] can also be found from a simple analysis of the Manley-Rowe relations under the approximation of small frequency detuning. We also obtained this conservation of asymmetry from an analysis of the more general NLSE, assuming periodic boundary conditions. Spectral measurement of conserved quantities of the NLSE has also recently been suggested as a method for extracting information in a wavelength-multiplexed soliton communication system [11]. Numerical integrations of the NLSE have been performed by the split step Fourier method [3] to examine the conservation relation [Eq. (3)] for pulses and continuous waves. Integrations for pulsed two-frequency inputs show that as long as the broadened sidebands do not overlap appreciably, the conservation relation holds very accurately for the power in the sidebands and pump waves. We have tested the conservation relation [Eq. (3)] experimentally, with pump waves that varied from nearly symmetric to very asymmetric power input. This relation was also tested for a variety of inputs powers (≤ 100 W), in which different numbers of sidebands were significant, and was found to be valid within the accuracy of the experiments. For regimes in which Raman scattering becomes significant the conservtion of asymmetry will break down and thus could be used as a sensitive test for the presence of Raman scattering as well as of higher-order nonlinear processes. In conclusion, we find that the conservation law derived here for multiple four-wave-mixing processes is found to be in good agreement with experimental observations for short lengths of optical fiber.

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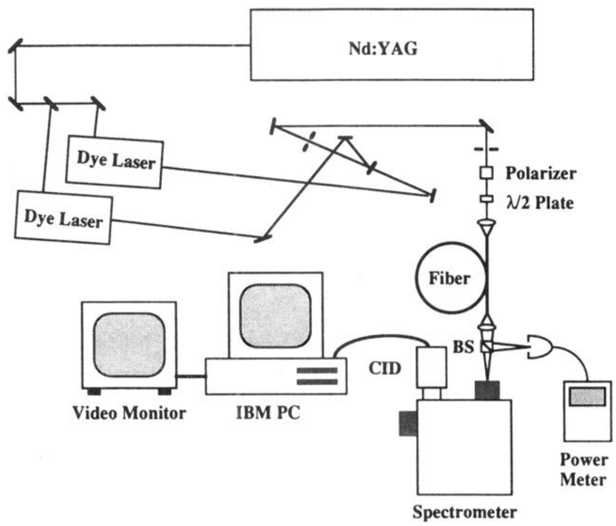

FIG. 2. Experimental apparatus used to study four wave mixing in a birefringent single mode optical fiber. BS denotes a beam splitter. CID denotes charge injection device video camera.