Properties of a deformed Jaynes-Cummings model

J. Črnugelj,* M. Martinis,† and V. Mikuta-Martinis
Department of Theoretical Physics, Rudjer Bošković Institute, P.O.B. 1016, 41001 Zagreb, Croatia
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It is shown that various variants of the deformed Jaynes-Cummings model (JCM) correspond to the JCM with an intensity-dependent coupling characterized by two additional phenomenological parameters (p,q). The standard JCM is obtained for $p=q=1$. The quantum collapse and revival effects and the squeezing properties of a particular variant of the (p,q)-deformed Jaynes-Cummings model are studied numerically. The model is based on the q-deformed oscillator algebra $AA^{\dagger}-qA^{\dagger}A=1$ that interpolates between Fermi-Dirac and Bose-Einstein statistics. If the cavity field is prepared initially in a q-deformed coherent state, the quantum collapse and revival effects are observed only for $q\approx 1$. For $q>1$, the atomic inversion $\langle\sigma_3(t)\rangle$ exhibits chaoticlike behavior, which is a feature observed also in other q-deformed JCM's. Strong squeezing is observed only for small positive and small negative q values. If $q\approx 1$, the squeezing is very weak. In the limit $q=1\pm\varepsilon$ with $\varepsilon\ll 1$, the algebra can be interpreted as describing a small violation of Bose-Einstein statistics in the JCM.

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I. INTRODUCTION

The Jaynes-Cummings model (JMC) [1] of a single two-level atom interacting with a single-mode cavity field has been the subject of many recent investigations [2] in laser physics and quantum optics. It has been observed [3] that the long-time behavior of the model is very sensitive to the statistical properties of the radiation field in the initial state, revealing a number of unexpected pure quantum features of the model [2–4]. For example, when the field mode is initially in a coherent state [3] or in a general squeezed state [5,6], the mean atomic excitation energy and the mean photon number exhibit periodic collapses and revivals. The dynamics predicted by the model has been supported in experiments with Rydberg atoms in high-Q microwave cavities [7].

Recently, there have been several generalizations of the JC Hamiltonian in which the interaction between the atom and the radiation field is no longer linear in the field variables [8–11].

It has been shown [12,13] that most of the nonlinear generalizations of the JCM are only particular cases of the JCM in which the creation and annihilation operators of the radiation field are replaced by deformed harmonic-oscillator operators with prescribed commutation relations. The Hamiltonian of this deformed JCM has the following generic form:

$$H=\omega_0\sigma_3+\omega N_A+g(A^{\dagger}\sigma_-+A\sigma_+)\ . \tag{1.1}$$

Here $\sigma_3,\sigma_\pm$ are the standard pseudospin atomic two-level transition operators with $[\sigma_+,\sigma_-]=2\sigma_3$ and $[\sigma_3,\sigma_\pm]=\pm\sigma_\pm$. The operators A, $A^{\dagger}$, and N_A are constructed from the single-mode field operators a, $a^{\dagger}$, and N, with $[a,a^{\dagger}]=1$, $[a,N]=a$, where $N=a^{\dagger}a$ is the number operator. These operators satisfy the deformed-oscillator commutation relations:

$$\begin{aligned}&[A,A^{\dagger}]=\{N_A+1\}-\{N_A\}\ ,\\&[A,N_A]=A\ ,\\&[A^{\dagger},N_A]=-A^{\dagger}\ ,\end{aligned} \tag{1.2}$$

where $\{N_A\}$, is an arbitrary real function of N_A. The knowledge of the function $\{N_A\}$ determines all the properties of the deformed-oscillator algebra (1.2). Note that $A,A^{\dagger}$ become generalized Bose operators [14,15] if $\{N_A+1\}=\{N_A\}+1$.

The intensity-dependent coupling and the multiphoton coupling [8–11] are generically described by the following set of deformed-oscillator operators:

$$\begin{aligned}&A=a^m f(N)\ ,\\&A^{\dagger}=f(N)(a^{\dagger})^m\ ,\\&N_A=\frac{1}{m}N,\quad m=1,2,\ldots,\end{aligned} \tag{1.3}$$

where $f(N)$ is an arbitrary real function of N. It is easy to see that the commutation relations (1.2) are satisfied if

$$\{N_A\}=\frac{N!}{(N-m)!}f^2(N)\ . \tag{1.4}$$

By specifying m and the form of the function $f(N)$ we obtain all the models that have been discussed in the literature so far.

Since the JCM is one of the basic models in quantum optics, extensions in different directions are of general interest. It is quite possible that deformations characterized by (1.2) and (1.3) may be applied to some real physical systems with nonlinear effective interactions, which

*Electronic address: crnugelj@thphys.irb.hr
†Electronic address: martinis@thphys.irb.hr

explains the attention given to them in the literature. The aim of the present paper is to study the time evolution of the atomic inversion and the squeezing of the JCM with an intensity-dependent coupling characterized by two additional deformation parameters (p,q) as phenomenological constants to be determined by experiment. We also present the numerical calculation and the results for a particular variant of the (p,q)-deformed JCM, namely, the $(p=1,q)$ case.

II. THE q- AND (p,q)-DEFORMED JCM

The recent development of quantum groups [16] has stimulated considerable interest in applications of q-deformed algebraic structures to various physical problems. Thus Chaichian, Ellinas, and Kulish [17], using a q-deformed oscillator algebra [18], were the first to generalize the JC Hamiltonian with an intensity-dependent coupling by relating it to the quantum $\mathrm{su}_q(1,1)$ algebra. Similarly, Bužek [19], hoping to extract possible information about the physical meaning of the q deformation, studied the atomic inversion of the standard JCM with a q-deformed cavity field initially prepared in a q-deformed coherent state. There also exists a two-parameter deformation of the JCM based on the (p,q)-deformed oscillator and the quantum $\mathrm{u}_{p,q}(1|1)$ superalgebra [20]. The existence of the two-parameter deformations of the JCM implies an infinite number of possible one-parameter deformations with the standard case [19] corresponding to $p=q$.

It is easy to see that the model of Chaichian *et al.* [17], the model of Bužek [19], and the two-parameter model of Chakrabarti and Jagannathan [20] are only particular cases of the deformed JC Hamiltonian (1.1) with an intensity-dependent coupling. In these particular cases the operators A, $A^\dagger$, and N_A are given by (1.3), with $m=1$ and

$$f(N)=\left[\frac{\{N_A\}}{N}\right]^{1/2} . \tag{2.1}$$

The choice

$$\{N_A\}=[N]_{p,q}=\frac{q^N-p^{-N}}{q-p^{-1}} \tag{2.2}$$

gives the model of Chakrabarti and Jagannathan [20] as well as the model of Bužek [19] when $p=q$. The operators A, $A^\dagger$, and N also satisfy

$$\begin{aligned} &AA^\dagger-qA^\dagger A=p^{-N} ,\\ &AA^\dagger-p^{-1}A^\dagger A=q^N , \end{aligned} \tag{2.3}$$

in addition to the commutation relations (1.2).

The choice

$$\{N_A\}=[N]^2_{p,q}=\left[\frac{q^N-p^{-N}}{q-p^{-1}}\right]^2 \tag{2.4}$$

gives the (p,q)-deformed Buck-Sukumar model [8,9]. With this choice, the operators A, $A^\dagger$, and N give the realization of the two-parameter quantum $\mathrm{su}_{p,q}(1,1)$ algebra. By writing $K_-=A, K_+=A^\dagger$, and $K_0=N+\frac{1}{2}$, the commutation relations of the quantum $\mathrm{su}_{p,q}(1,1)$ algebra become

$$\begin{aligned} &[K_-,K_+]_{p,q}=K_-K_+-\frac{q}{p}K_+K_-=[2K_0]_{p,q} ,\\ &[K_0,K_\pm]=\pm K_\pm . \end{aligned} \tag{2.5}$$

In the limit $q=p$, the model reduces to that of Chaichian, Ellinas, and Kulish [17]. Note that $\mathrm{su}_{p,q}(1,1)$ is isomorphic to $\mathrm{su}_Q(1,1)$ if $Q=\sqrt{pq}$. The relations between the generators of $\mathrm{su}_{p,q}(1,1)$ and the generators of $\mathrm{su}_Q(1,1)$ follow from

$$[N]_{p,q}=\left[\frac{q}{p}\right]^{(1/2)(N-1)}[N]_Q , \quad Q=\sqrt{pq} , \tag{2.6}$$

where

$$[N]_Q=\frac{Q^N-Q^{-N}}{Q-Q^{-1}}$$

is given by

$$\begin{aligned} &K_-=\left[\frac{q}{p}\right]^{N/2}(K_-)_Q ,\\ &K_+=\left[\frac{q}{p}\right]^{(N-1)/2}(K_+)_Q ,\\ &K_0=(K_0)_Q . \end{aligned} \tag{2.7}$$

Similar isomorphism between $\mathrm{su}_{p,q}(2)$ and $\mathrm{su}_Q(2)$ has been found in [25].

III. TIME EVOLUTION

To study the influence of the deformation on the dynamics, it is necessary to find the corresponding time-evolution operator $U(t)=e^{-iHt}$, which has been found in [12] for the Hamiltonian (1.1). For completeness, we briefly repeat the derivation. The Hamiltonian (1.1) is split in two parts H_0 and H_1, in such a way that both H_0 and H_1 commute with each other and are constants of motion:

$$H=H_0+H_1 , \quad [H_0,H_1]=0 ,$$

where

$$\begin{aligned} &H_0=\omega(N_A+\sigma_3) ,\\ &H_1=2\Delta\sigma_3+g(A^\dagger\sigma_-+A\sigma_+) , \end{aligned} \tag{3.1}$$

and $\Delta=\frac{1}{2}(\omega_0-\omega)$ is the detuning parameter. The evolution operator $U(t)$ factorizes as

$$U(t)=U_0(t)U_1(t) ,$$

with

$$\begin{aligned} U_0(t)&=e^{-iH_0t}\\ &=e^{-i\omega tN_A}\begin{bmatrix} e^{-i\omega t/2} & 0\\ 0 & e^{i\omega t/2}\end{bmatrix} \end{aligned} \tag{3.2}$$

and

$$U_1(t)=e^{-iH_1 t}$$
$$=\begin{bmatrix} \cos(f_{N_A+1}t)-i\Delta\dfrac{\sin(f_{N_A+1}t)}{f_{N_A+1}} & -igA\dfrac{\sin(f_{N_A}t)}{f_{N_A}} \\ -ig\dfrac{\sin(f_{N_A}t)}{f_{N_A}}A^{\dagger} & \cos(f_{N_A}t)+i\Delta\dfrac{\sin(f_{N_A}t)}{f_{N_A}} \end{bmatrix}, \tag{3.3}$$

where

$$f_{N_A}=(\Delta^2+g^2\{N_A\})^{1/2} \tag{3.4}$$

is half of the Rabi frequency in the N_A subspace.

The matrix elements of the operators $a, a^2, a^{\dagger}a, \sigma_3, \ldots$ are calculated using the wave function $|\psi(t)\rangle=U(t)|\psi(0)\rangle$ with a given initial state

$$|\psi(0)\rangle=\sum_{n=0}^{\infty}Q_n|n\rangle|\psi_{\text{atom}}(0)\rangle . \tag{3.5}$$

Here $|\psi_{\text{atom}}(0)\rangle$ is assumed to denote the atom in the ground state $|-\rangle$ $(\langle\sigma_-|-\rangle=0)$ and $|n\rangle$ to denote the normalized Fock states of the operators A, $A^{\dagger}$:

$$|n\rangle=\frac{(A^{\dagger})^n}{\sqrt{\{n\}!}}|0\rangle , \quad A|0\rangle=0 , \quad \{N_A\}|n\rangle=\{n\}|n\rangle , \tag{3.6}$$

with $\{n\}!\equiv\{n\}\{n-1\}\cdots\{1\}$ and $\{0\}!=1$. The initial distribution of A quanta is assumed to be the deformed Poisson distribution:

$$|Q_n|^2=[e_A(|\alpha|^2)]^{-1}\frac{|\alpha|^{2n}}{\{n\}!} , \tag{3.7}$$

where $\alpha=|\alpha|e^{i\phi}$ and $e_A(x)$ is the deformed exponential function:

$$e_A(x)=\sum_{n=0}^{\infty}\frac{x^n}{\{n\}!} . \tag{3.8}$$

The mean number of A quanta at $t=0$ is

$$\bar{n}=\sum_{n=0}^{\infty}n|Q_n|^2=|\alpha|^2\frac{e'_A(|\alpha|^2)}{e_A(|\alpha|^2)} . \tag{3.9}$$

The population inversion $\langle\sigma_3(t)\rangle\equiv\langle\psi(t)|\sigma_3|\psi(t)\rangle$ and the mean photon number $\bar{n}(t)\equiv\langle\psi(t)|N|\psi(t)\rangle$ are related as

$$\bar{n}(t)=\bar{n}-\tfrac{1}{2}-\langle\sigma_3(t)\rangle , \tag{3.10}$$

where

$$\langle\sigma_3(t)\rangle=-\frac{1}{2}+\sum_{n=0}^{\infty}|Q_n|^2\left[1-\frac{\Delta^2}{f_n^2}\right]\sin^2(f_n t) . \tag{3.11}$$

Relation (3.10) is the consequence of the fact that H_0 is the constant of motion, i.e.,

$$\langle\psi(t)|H_0|\psi(t)\rangle=\langle\psi(0)|H_0|\psi(0)\rangle$$
$$=\omega(\bar{n}-\tfrac{1}{2}) . \tag{3.12}$$

Generally, it is not possible to find a close analytic expression for the infinite sum appearing in $\langle\sigma_3(t)\rangle$, Eq. (3.11). However, the time t_R needed for the most complete revival of the initial value $\langle\sigma_3(0)\rangle$ $(=-\frac{1}{2}$ in our case) can be estimated from

$$t_R=\pi/(f_{\bar{n}+1}-f_{\bar{n}}) , \tag{3.13}$$

where, according to (3.4)

$$f_{\bar{n}}=(\Delta^2+g^2\{\bar{n}\})^{1/2} .$$

IV. SQUEEZING

The squeezing properties of the JCM have been investigated by many authors [5,6,9–11], in particular after it has become possible to produce a squeezed electromagnetic field in the laboratory [21]. States containing a large amount of squeezing are obtained from the JCM with the intensity-dependent coupling and with the multiphoton-transition mechanism. Theoretically, it has been shown that squeezed states can be constructed as generalized coherent states using group-theoretical methods [22]. One of such generalizations is the conjecture [23] that quantum-group coherent states [18] are natural candidates for describing squeezed quantum states of matter.

The squeezing properties of the radiation field are usually studied by introducing two Hermitian time-dependent quadrature operators:

$$\begin{aligned} a_1&=(ae^{i\omega t}+a^{\dagger}e^{-i\omega t})/2 , \\ a_2&=(ae^{i\omega t}-a^{\dagger}e^{-i\omega t})/2i , \end{aligned} \tag{4.1}$$

satisfying $[a_1,a_2]=i/2$. The corresponding uncertainty relation is $(\Delta a_1)(\Delta a_2)\geq\frac{1}{4}$, where variances $(\Delta a_{1,2})$ are defined by $(\Delta a_{1,2})^2=\langle a_{1,2}^2\rangle-\langle a_{1,2}\rangle^2$. A state of the field is considered squeezed if either (Δa_1) or (Δa_2) are smaller than $\frac{1}{2}$.

Let us define the relative variances with respect to $(\Delta a_{1,2})^2_{\text{coh}}=\frac{1}{4}$:

$$S_{1,2}(t)=4(\Delta a_{1,2})^2-1 . \tag{4.2}$$

The squeezing condition then becomes

$$S_i(t)<0 , \quad i=1 \text{ or } 2 . \tag{4.3}$$

In terms of photon operators we have

$$\begin{aligned} S_1(t)=&2\langle a^{\dagger}a\rangle+\langle a^2\rangle e^{2i\omega t}+\langle a^{\dagger 2}\rangle e^{-2i\omega t} \\ &-(\langle a\rangle e^{i\omega t}+\langle a^{\dagger}\rangle e^{-i\omega t})^2 \end{aligned} \tag{4.4}$$

and similarly for $S_2(t)$. Introducing the notation $\langle a\rangle = e^{-i(\omega t-\phi)}A_1(t)$, $\langle a^2\rangle = e^{-2i(\omega t-\phi)}A_2(t)$, and $\langle a^\dagger a\rangle = A_0(t)$, we obtain [24]

$$\begin{aligned} S_1(t) &= 2[A_0(t)-A_2(t)]+4\cos^2\phi[A_2(t)-A_1^2(t)]\ , \\ S_2(t) &= 2[A_0(t)-A_2(t)]+4\sin^2\phi[A_2(t)-A_1^2(t)]\ , \end{aligned} \tag{4.5}$$

where, in the resonant case $\omega_0=\omega$,

$$\begin{aligned} A_0(t) &= \bar{n}(t) \\ &= \bar{n}-\sum_{n=0}^{\infty}|Q_n|^2\sin^2(gt\sqrt{\{n\}})\ , \\ A_1(t) &= |\alpha|\sum_{n=0}^{\infty}\frac{|Q_n|^2}{\sqrt{\{n+1\}}}[\sqrt{n}\,\sin(gt\sqrt{\{n\}})\sin(gt\sqrt{\{n+1\}})+\sqrt{n+1}\cos(gt\sqrt{\{n\}})\cos(gt\sqrt{\{n+1\}})]\ , \\ A_2(t) &= |\alpha|^2\sum_{n=0}^{\infty}\frac{|Q_n|^2}{\sqrt{\{n+1\}\{n+2\}}}[\sqrt{n(n+1)}\sin(gt\sqrt{\{n\}})\sin(gt\sqrt{\{n+2\}}) \\ &\quad +\sqrt{(n+1)(n+2)}\cos(gt\sqrt{\{n\}})\cos(gt\sqrt{\{n+2\}})]\ . \end{aligned} \tag{4.6}$$

V. THE MODEL

In an attempt to better understand the real nature of the deformation parameter q, we study numerically another q generalization of the JCM. Our model is obtained from (2.2) by setting $p=1$, i.e.,

$$\{N_A\}=[N]_{1,q}=\frac{1-q^N}{1-q}\ . \tag{5.1}$$

The q-boson algebra of this model

$$AA^\dagger-qA^\dagger A=1 \tag{5.2}$$

was studied in different contexts earlier [26–29]. Note that it defines a new form of quantum statistics which interpolates between Bose ($q=1$) and Fermi ($q=-1$) statistics as q goes from 1 to -1 on the real axis. The reason for our specific choice of the q-deformed JCM as given by (5.2) is the possibility of studying the effect of small violation of Bose statistics in the JCM when $q=1\pm\epsilon$ with $\epsilon\ll 1$. In the limit $q=1$, our model reduces to the original JCM.

Now let us first examine the time evolution of the atomic inversion $\langle\sigma_3(t)\rangle$ for a field initially in a q-deformed coherent state (3.7) with $\{n\}=(1-q^n)/(1-q)$. The time behavior of $\langle\sigma_3(t)\rangle$ for different values of the parameter q at the exact resonance ($\Delta=0$) is shown in Fig. 1 for $|\alpha|=0.8$ and $q=-0.2$, 0.2, and 0.9. When $\bar{n}\gg 1$ and $q=1\pm\epsilon$ with $\bar{n}\epsilon<1$, the revival time t_R is easily seen to be given by

$$t_R=t_R^{\mathrm{JCM}}(1\mp\tfrac{3}{4}\bar{n}\epsilon+\cdots)\ , \tag{5.3}$$

where $t_R^{\mathrm{JCM}}=(2\pi/g)\sqrt{\bar{n}}$ is the revival time of the conventional JCM when $q=1$. In Fig. 2 we show that for $|\alpha|=5$ and $q=0.99$, $q=1$, and 1.01 the atomic inversion $\langle\sigma_3(t)\rangle$ exhibits the quantum collapse and revival effects according to relation (5.3), i.e., the revival time may be increased or decreased over the time expected for $q=1$. However, when both $\bar{n}$ and $\bar{n}\ln q$ are $\gg 1$, so that $\{\bar{n}\}\sim q^{\bar{n}}/(q-1)$ the revival time t_R becomes

$$t_R\sim\frac{\pi}{g}q^{-\bar{n}/2}(\sqrt{q}+1)/\sqrt{q-1}\ ,$$

which is very short. The quantum collapse and revival effects are lost, as shown in Fig. 3 for $|\alpha|=5$ and $q=1.5$ and 2.0.

The squeezing properties of the model are shown in Fig. 4 where the long-time behavior of the function $S_{1,2}(t)$ for $\phi=\pi/2$, $|\alpha|=0.8$, and different values of the deformation parameter q are studied. We see that the initial squeezing disappears quickly for $q\approx 1$ and reappears quasiperiodically later with magnitudes between -0.4 and 0. For q small or negative, oscillations are more regular.

In Fig. 5 we show the time behavior of $S_1(t)$ for

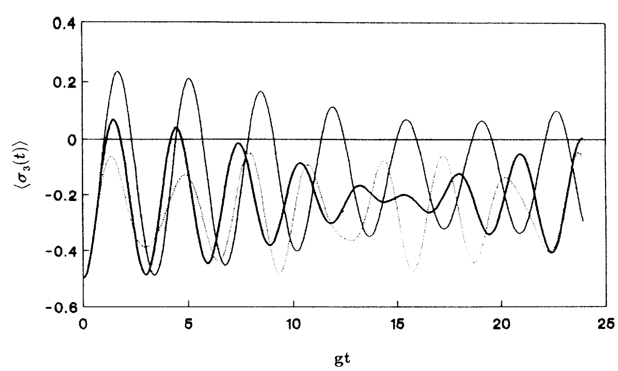

FIG. 1. Time behavior of the atomic inversion $\langle\sigma_3(t)\rangle$ for $|\alpha|=0.8$ and $q=-0.2$ (full line), $q=0.2$ (heavy line), and $q=0.9$ (dotted line).

$|\alpha|=0.8$, $\phi=\pi/2$, and $q=-0.5$. A large magnitude of squeezing is observed in this case. The long-time behavior of $S_{1,2}(t)$ is also characterized by quasiperiodic recoveries of squeezing.

VI. CONCLUSION

In this paper we have studied the time evolution and the squeezing properties of the deformed JCM, which corresponds to the JCM with intensity-dependent coupling controlled by two additional parameters (p,q). Depending on the choice of deformation function $\{N_A\}$ we obtain various q- and (p,q)-deformed JCM's recently studied in the literature.

As an interesting example of a (p,q)-deformed JCM we have studied numerically the case $(p=1,q)$. The q-boson algebra of this model

$$AA^{\dagger}-qA^{\dagger}A=1$$

offers the possibility to investigate small deviations from Bose statistics when $q=1\pm\epsilon$ with $\epsilon \ll 1$. Depending on the value of q we observe different long-time behavior of the atomic inversion $\langle\sigma_3(t)\rangle$ (Figs. 1 and 2). When $q>1$, the quantum collapse and revival effects are lost and $\langle\sigma_3(t)\rangle$ shows chaotic behavior (Fig. 3), which is a feature observed also in other q-deformed JCM's.

The long-time squeezing properties of the model are rather chaotic with magnitudes varying between -0.4 and 0. Note that the maximum possible squeezing is -1. For $q>0$, the squeezing is quasiperiodic and mainly in the variable $S_2(t)$ if $q\approx 1$ and $q>1$, Fig. 4. The variable $S_1(t)$ shows dominant but small squeezing only if q is

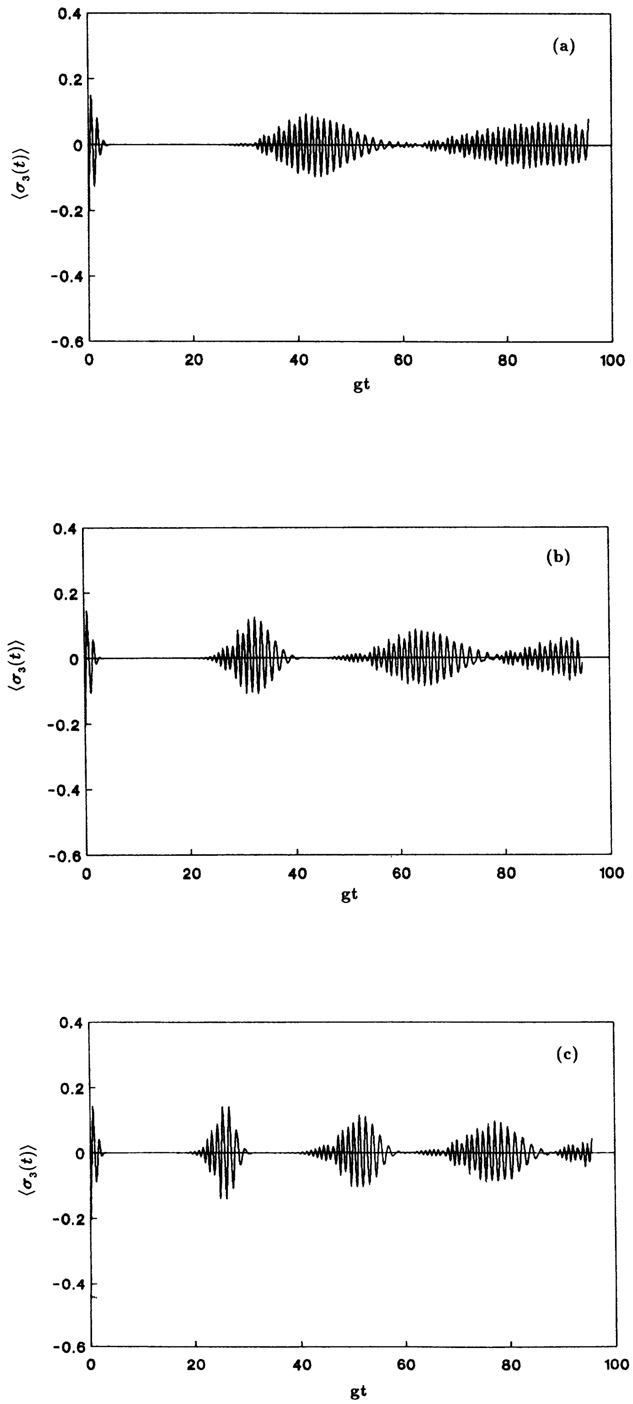

FIG. 2. Long-time behavior of the atomic inversion $\langle\sigma_3(t)\rangle$ for $|\alpha|=5$ and (a) $q=0.99$, (b) $q=1$, and (c) $q=1.01$.

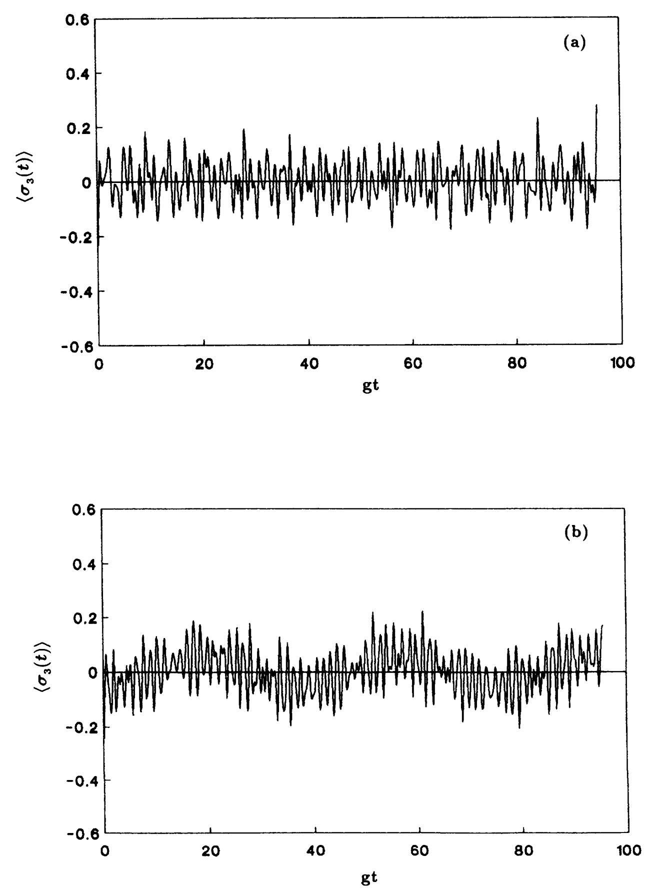

FIG. 3. Long-time behavior of the atomic inversion $\langle\sigma_3(t)\rangle$ for $|\alpha|=5$ and (a) $q=1.5$ and (b) $q=2$.

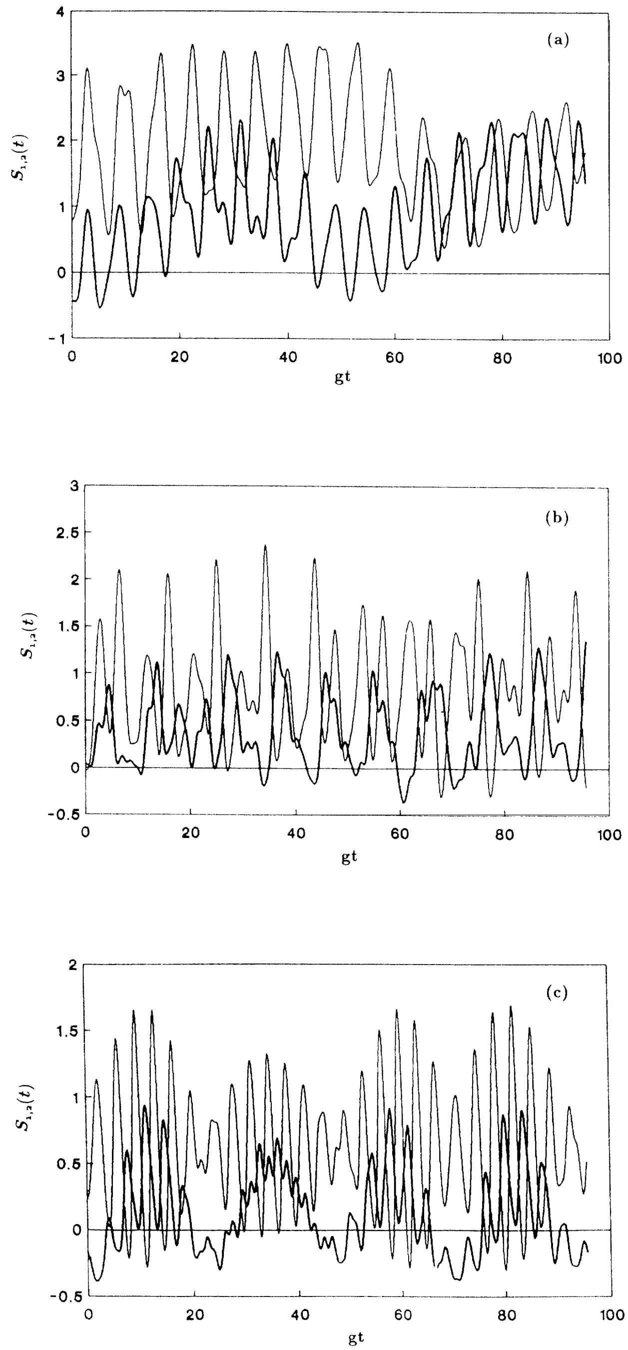

FIG. 4. Time behavior of the $S_1(t)$ (full line) and $S_2(t)$ (heavy line) for $|\alpha|=0.8$ and $\phi=\pi/2$, and $q>0$. The various curves correspond to (a) $q=0.2$, (b) $q=0.9$, and (c) $q=2$.

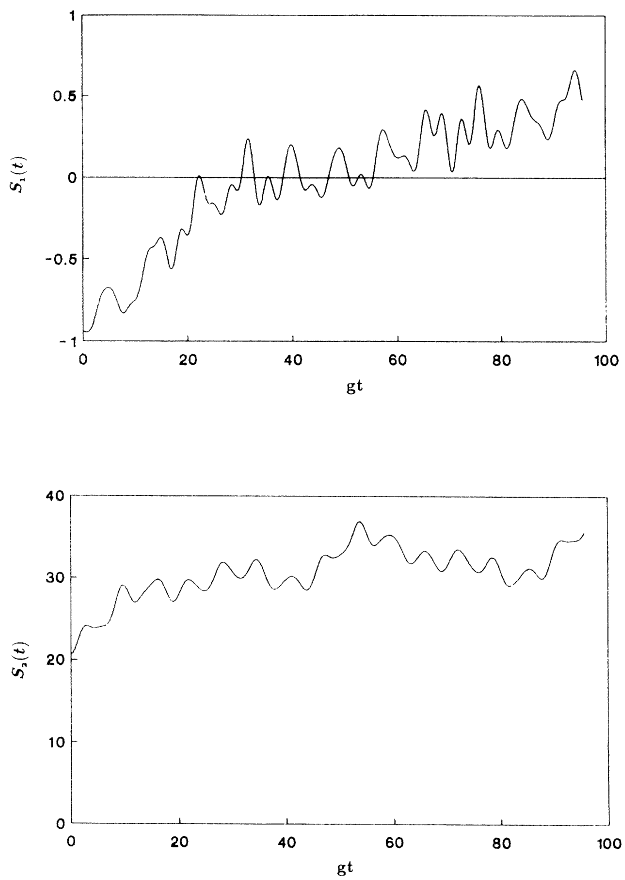

FIG. 5. Time behavior of the $S_1(t)$ and $S_2(t)$ for $|\alpha|=0.8$, $\phi=\pi/2$, and $q=-0.5$.

small, Fig. 4(a). However, for small negative q values, large magnitude of squeezing is observed, but only in the variable $S_1(t)$, Fig. 5.

Since, in our approach, the deformation parameter q may be viewed as a phenomenological constant controlling the strength of the intensity-dependent coupling, it would be interesting to study the properties of the model in the close vicinity of $q\approx 1$. By writing $q=1\pm\epsilon$, with ϵ very small and $\bar{n}\epsilon<1$ it should be possible to study small violation of Bose-Einstein statistics by measuring the revival time t_R and comparing it with $t_R=(2\pi/g)/\sqrt{\bar{n}}$.

We should also mention the problem connected with the convergence of the infinite power series in the definitions of $\langle\sigma_3(t)\rangle$, $A_0(t)$, $A_1(t)$, and $A_2(t)$. It is easy to see that the convergence for real q requires $|\alpha|<|\{n+1\}|$ in the lines $n\rightarrow\infty$. Thus we find that for $|q|<1$ $|\alpha|<1/(1-q)$ and for $q\geq 1$ there is no restriction on $|\alpha|$.

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